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intertemporal coordination failure**

2 / 2010

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Beyond DSGE: a Macrodynamic Model with Intertemporal coordination failure

Ronny Mazzocchi

The current consensus in macroeconomics, or New Neoclassical Synthesis (NNS), is based on dynamically stochastic general equilibrium (DSGE) modelling with a RBC core to which nominal rigidities are added by way of imperfect competition. The strategy is to minimize the frictions that are required to reproduce Keynesian results (in terms of persistent real effects of monetary policy) and Wicksellian results (in terms of interaction of interest and prices) in a rigorous framework with intertemporal optimization of consumption, forward-looking behavior and continuously clearing markets. In reality the main contention of Keynes and Wicksell was saving-investment imbalances (i.e. capital market failures and intertemporal disequilibrium in modern parlance) that are notably absent from the NNS. The paper presents a dynamic model with endogenous capital stock whereby it is possible to assess, and hopefully clarify, some basic issues concerning the macroeconomics of saving-investment imbalances and to explore the dynamic properties of the system under different monetary policy rules.

KEYWORDS: macroeconomics, monetary policy, New Neoclassical Synthesis, saving-investment imbalances, intertemporal coordination failure

JEL CLASSIFICATION: E21, E22, E31, E32, E52

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ACKNOWLEDGEMENTS: I wish to thank Roberto Tamborini, Hans-Michael Trautwein, Serena Sordi, Andrea Fracasso, Elisabetta De Antoni and Stefano Zambelli for helpful discussions and comments.

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1 Introduction

After the breakdown of the old Neoclassical Synthesis, macroeconomics was characterized by a persistent disagreement about methodology and substance. The recent debate between the Real Business Cycles theory (RBC) and the New Keynesian Economics (NKE) has led to the development of the so called *New Neoclassical Synthesis* (Goodfriend and King, 1998) which can be regarded as the newly established macroeconomic consensus (Blanchard, 2000). Like the old Neoclassical Synthesis of Hicks, Modigliani, Samuelson and Patinkin, the New Neoclassical Synthesis (NNS) tries to link micro- and macroeconomics, using a general equilibrium framework to model some typically Keynesian features.

The idea of this literature is to furnish a common vision of Neoclassical and Keynesian theories entrusting to them separate roles in the construction of the model: the RBC part of the model explains the evolution of the potential output embodying dynamic stochastic general equilibrium models (DSGE), while the transitory deviations from this trend are explained using the sluggish adjustment of prices and wages which were developed in the 1980s by the NKE. Like the RBC models, the NNS assigns a very important role to the real shocks in the explanation of the short run fluctuations. Differently from the RBC models, however, the NNS does not consider these fluctuations efficient and desirable and does not think that monetary policy is totally ineffective. In fact, because of the delays in the adjustment of prices and wages, the consequences of real shocks are undesirable. An active economic policy can therefore intervene to reduce these distortions.

There are various versions of the NNS. One authoritative contribution is Michael Woodford's *Interest and Prices* (Woodford, 2003), which furnish an excellent representation of the dynamic interaction between interest rates, price level and output¹. As reflected in its title, the book pays a respect to Knut Wicksell's work on monetary policy². The main aspect of the Woodford's contribution, indeed, is rediscovery of the Wicksellian nominal interest rate in relation to the "natural" interest rate prevailing at full-employment general equilibrium as the pivot of rule-based monetary policy.

Many contributions (Boianovsky and Trautwein, 2006; Trautwein and Zouache, 2009; Mazzocchi et al., 2009) have shown that the theoretical structure of the NNS is based on a shaky relationship among the DSGE

¹Many economists have noted the contrast between the title of Patinkin's treatise (Patinkin, 1965), *Money, Interest and Prices*, and Woodford's *Interest and Prices* as making very clear the diminished role of the quantity of money in modern macroeconomic theory.

²In his 1898 treatise (Wicksell, 1898), Knut Wicksell outlined a theory of price-level determination in which a key role was played by the relationship between the money rate of interest and the natural rate of interest. Likewise in Woodford's book the gap between the actual interest rate and the natural rate represents the key channel through which central bank actions affect the economy.

model with rational expectation on one side and a mechanism of price setting which does not depend on excess demand on the other side. At the same time, most of the Wicksellian features are notably absent. In particular, the NNS does not consider frictions in the capital market, which generate the first pillar of the Wicksell's view, i.e. the bank intermediation among savers and investors. Moreover, there is no room for information problems and the intertemporal disequilibrium which could produce the well-known dynamics of money creation, of prices and of nominal income, i.e. the so-called *cumulative process*. These weaknesses do not allow to discuss the effects and the relations between financial market and the real economy, which were the core of Wicksell's and Keynes' work³.

In this paper we try to challenge the NNS view. In section [2] and [3] we clarify some basic theoretical issues underlying the NNS and its inconsistency with a proper "Neo-Wicksellian" model. Section [4] presents a dynamic model whereby it is possible to assess, and hopefully to clarify, some basic issues concerning the macroeconomics of saving-investment imbalances. Section [5] extended the basic model with a more general treatment of inflation expectations. Section [6] discusses some dynamic properties of the model under different interest-rate rules. Section [7] concludes. Proofs and graphs are placed in the Appendix.

2 A NNS in a Neo-Wicksellian Framework

Over the last decade a shift has begun away from a concentration on the Walrasian price-taker models towards a world where firms may be strategic agents. The NNS approach uses the standard tools of New Classical macroeconomics (NCM): consumers, workers and firms are rational, maximizing agents and markets clear. Though the output of NNS models follows Keynesian lines: the aggregate economy has multipliers, economic fluctuations are not Pareto optimal, and finally government interventions can be effective. Imperfect competition⁴ is a key assumption of this approach. It opens new channels of influence of monetary policies but also creates the possibility that an increase in output may be welfare improving (Cooper, 2004). Imperfect competition by itself does not create monetary non-neutrality, but its combination with some other distortions can generate potential real effects (Blanchard, 2000).

³As recognized by Robert Lucas "[...] the problem is that the new theories, the theories embedded in general equilibrium dynamics [...] don't let us think about the US experience in the 1930s or about financial crises and their consequences [...]. We may be disillusioned with the Keynesian apparatus for thinking about these things, but it doesn't mean that this replacement apparatus can do it either" (Lucas, 2004, p.23).

⁴The NNS models use primarily monopolistic competition as a form of imperfect competition (Blanchard and Kiyotaki, 1987). This choice derives mainly from the belief that monopolistic competition is pervasive in a modern economy.

As we have previously discussed in the introduction, there are different versions of NNS models, depending on the purpose (Goodfriend and King, 1998; Clarida et al., 1999; Romer, 2000). We prefer to use the benchmark developed by Michael Woodford in his last book *Interest and Prices* (Woodford, 2003), which represents a good synthesis of New Classical and New Keynesian ideas. This work contains many references to the Wicksell's idea of a pure credit system and his proposal to eliminate inflation by adjusting nominal interest rates to changes in the price level⁵. However, Woodford doubts that the original "*Wicksellian theory can provide a basis for the kind of quantitative analysis in which a modern central bank must engage*" (2003, p.5-6) because it does not conform to modern standards of conceptual rigour, i.e. intertemporal general-equilibrium theory. His book seeks to remedy this shortcoming.

As Woodford (2003, p. 242-243) points out, his basic and best known model (ch.4) "*abstracts from the effects of variations in private spending (including those classified as investment expenditure in the national accounts) upon the economy's productive capacity*", therefore the model should be interpreted "*as if all forms of private expenditure were like nondurable consumer purchases*"⁶. The preference for a model without endogenous capital stock is often justified on the grounds that capital does not exhibit substantial volatility at business cycle frequencies (McCallum and Nelson, 1997). Moreover, sticky price models with endogenous investment imply unrealistically high volatility in the endogenous variables. In other words, changes in nominal interest rates translate one for one into changes in real rates, therefore leading to excessively high volatility of investment. However, neglecting the endogenous determination of investment eliminates one of the main benefits of the DSGE approach begun by Kydland and Prescott (1982), namely that it is inherently intertemporal in nature and incorporates the supply side of the economy. Indeed, as King and Rebelo argue, "*the process of investment and capital accumulation can be very important for how the economy responds to shock*" (King and Rebelo, 2000, p.6). Last but not least, the intertemporal coordination problem between future consumption (saving) and future production (investment), which is the key problem to be solved by the interest rate in general equilibrium theory, vanishes. There remains the sole intratemporal coordination problem between current aggregate demand and supply at each date that is dealt with by the spot price system. For these reasons Woodford (2003, p.352-378; 2004) extends the basic model to include fixed capital and the effects of the related investment dynamic.

Similar to the basic framework, the extended model is based on the assumption of monopolistic competition in the goods market. There is a continuum of differentiated goods i and differentiated labor producing each

⁵For a more complete survey see Mazzocchi et al. (2009) and Mazzocchi (2009).

⁶The same approach is used by Jeanne (1998).

type of good. Individual suppliers have a certain degree of market power and hence can decide how to set their prices. This means that a supplier that fails to immediately adjust the price in response to a demand shock does not suffer an unboundedly relative large change in its sales. Then, it becomes more plausible that prices should not be constantly adjusted. Moreover, in order to have more realistic movements of capital and investment, it might be necessary to posit some form of adjustment costs⁷.

The real sector of the economy has derived analyzing the optimal behavior of the households and of the firms and the respective conditions of equilibrium. Unlike the basic model, the result is not a single equation, but a system of four equations. Starting from a modified production function with an explicit representation of the effects of the variation of the capital-stock and a traditional demand function, we obtain the following *dynamic of capital stock*:

$$\begin{aligned} \hat{\lambda}_t + \epsilon_\psi + (\hat{K}_{t+1} - \hat{K}_t) &= \beta(1 - \delta)E_t \hat{\lambda}_{t+1} + (1 - \beta)(1 - \delta) \\ &[\rho_y E_t \hat{Y}_{t+1} - \rho_k \hat{K}_{t+1} - \omega E_t q_{t+1}] + \beta \epsilon_\psi E_t (\hat{K}_{t+2} - \hat{K}_{t+1}) \end{aligned} \quad (2.1)$$

where

$$\rho_y = \nu \phi_h + \frac{\phi_h}{\phi_h - 1} \omega_p > \rho_k \equiv \rho_y - \nu > 0$$

where $0 < \beta < 1$ is the utility discount factor, $\phi_h > 1$ is the steady-state value of the reciprocal of the elasticity of the production function with respect to the labour input, $\nu > 0$ is the elasticity of the marginal disutility of labour with respect to labour supply and $\omega_p > 0$ is the negative of the elasticity of the marginal product $f'(f^{-1}(\frac{y}{k}))$ with respect to $\frac{y}{k}$ ⁸. $\hat{\lambda}_t$ is the *evolution of the marginal utility of real income* of the representative agent, δ is the rate of depreciation, ϵ_ψ is the degree of adjustment costs and q_t is the exogenous disturbance.

The implied *dynamic of investment spending* is then given by:

$$\hat{I}_t = k[\hat{K}_{t+1} - (1 - \delta)\hat{K}_t] \quad (2.2)$$

where \hat{I}_t is the percentage deviation of investment from the steady-state level and $k = \frac{\bar{K}}{\bar{Y}}$ is the steady state capital-output ratio. The investment dynamic \hat{I}_t is derived as a function of the evolution of the marginal utility

⁷An example of such costs are the costs of installing the new capital and training workers to operate new machines (Eisner and Strotz, 1963; Lucas, 1967). Usually the key assumption of the NNS model with endogenous investment is that firms face costs of adjustment which are a convex function of the rate of change of their capital stock. These assumptions imply that it is costly for a firm to increase or decrease its capital stock, and that the marginal adjustment cost is increasing in the size of the adjustment (Mazzocchi, 2009).

⁸It is useful to note that $\omega = \omega_w + \omega_p$, where ω_w is the elasticity of marginal disutility of work with respect to output (see Woodford (2003), p. 152-153).

of the real income of the representative agent. This is related to aggregate spending through the relation $\lambda_t = u_c(Y_t - C_t - I_t; \epsilon_t)$. The log-lin version becomes:

$$\lambda_t = -\sigma^{-1}(\hat{Y}_t - \hat{I}_t - \hat{g}_t) \quad (2.3)$$

where the composite disturbance \hat{g}_t reflects the effects both of government purchases and of shifts in private impatience to consume. Because of the relation between the marginal utility of the income process and the stochastic discount factor that prices bonds, the nominal interest rate must satisfy $1 + i_t = [\beta E_t \frac{\lambda_{t+1}}{\lambda_t \Pi_{t+1}}]^{-1}$. Therefore the log-lin version of the *nominal interest rate* is:

$$\hat{i}_t = E_t \pi_{t+1} + \hat{\lambda}_t - E_t \hat{\lambda}_{t+1} \quad (2.4)$$

The system of the previous equations (2.1)(2.2)(2.3) and (2.4) then comprise the IS block of the model which is sufficient to determine the paths of the variables $\{\hat{Y}_t, \hat{I}_t, \hat{K}_t, \hat{\lambda}_t\}$ given the initial capital stock and the evolution of the short term interest rate $\hat{i}_t - E_t \pi_{t+1}$.

The second part of the model is composed by an AS-block which investigate the implications of an endogenous capital stock for the price setting decisions of firms. Two things have to be considered: a) the capital stock affects the marginal costs of firms (and therefore output) and b) how the capital stock will evolve over the time that its price remains fixed. The first problem is solved with a simple manipulation of the marginal costs expression:

$$\hat{s}_t = \omega(\hat{Y}_t - \hat{K}_t) + \nu \hat{K}_t - \hat{\lambda}_t - [\nu \bar{h}_t + (1 + \nu)a_t] \quad (2.5)$$

where $\hat{s}_t = \log \frac{s^{(i)}_t}{\bar{s}}$ and ω is the elasticity of the marginal cost with respect to firm's output and \bar{h}_t is an exogenous disturbance to preferences, indicating the percentage increase in labour supply needed to maintain a constant marginal disutility of working. Conversely, the second problem is solved in terms of a New Keynesian Phillips curve in which both the dynamic of the relative capital stock and the optimal firm's price setting are considered:

$$\pi_t = \xi \hat{s}_t + \beta E_t \pi_{t+1} \quad (2.6)$$

where $\xi \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha\phi}$. The underlying assumption is that firms set their prices in a staggered fashion, by way of a modified Calvo lottery (Calvo, 1983). The effects of non-anticipated monetary policies depend mostly on the strategic interactions among the competing firms⁹. In case strategic

⁹In particular, the real effect of a nominal shock will be larger if the enterprises are following a strategic complementary behavior, i.e. they adjust the price less than they would have done if all the enterprises have had the possibility of adjust it. Viceversa, the real effects will be smaller if the enterprises modify the price more than they would have done in presence of a general adjustment, pursuing as strategic substitutability. As a result, the level of prices prior to the shock continues to have an important effect on the general price level even after most prices have been adjusted at least once.

complementarities large enough, the adjustment of general price level can be very slow, even though individual price adjustments are quite frequent. Thus changes in nominal spending will have prolonged effects on real activity¹⁰.

Therefore the AS block is composed by equations (2.5) and (2.6) and it provides the characterization of the inflation dynamic π_t , given the evolution of $\{\hat{Y}_t, \hat{K}_t, \hat{\lambda}_t\}$ from the IS block. It takes into account the effect of changes in the capital stock on the real marginal costs and hence on the short-run trade-off between inflation and output.

Finally, Woodford closes the model by specifying monetary policy in terms of an interest-rate feedback rule of the form:

$$\hat{i}_t = \hat{i}_t^* + \phi_\pi(\pi_t - \bar{\pi}) + \phi_x(x_t - \bar{x}) \quad (2.7)$$

where \hat{i}_t^* is an intercept term that, as will become clear later, should correspond to the nominal value of the natural rate of interest, $r_t^* + \pi^*$. x_t and \bar{x} are the actual and the steady-state output gap respectively¹¹. The weight factors ϕ_π and ϕ_x describe the intensity of the interest-rate reactions to deviations of actual inflation and the output gap from their target values¹². A reaction function of this kind permits the determination of the endogenous variables \hat{i}_t , given \hat{Y}_t and π_t from the IS and AS block.

In the model with endogenous capital stock there are seven equations and, therefore, seven endogenous variables $\{\hat{i}_t, \pi_t, \hat{Y}_t, \hat{L}_t, \hat{K}_t, \hat{\lambda}_t, \hat{s}_t\}$ given the paths of three exogenous processes $\{\bar{i}_t, g_t, q_t\}$, where g_t reflects shocks due to government spending and impatience to consume and q_t represents shocks to the Natural Rate of Interest (NAIRI in modern parlance). Unlike what happens in the basic framework, in the model with endogenous investment the NAIRI depends not only on present and expected future exogenous disturbances, but also on the capital stock, which in turn depends on the past monetary policy when prices were sticky. For this reason we could define the Natural Rate of Output (NAIRO) and the NAIRI as those that would result from price flexibility now and in the future, given all the exogenous and predetermined state variables at the present time, *including* the capital stock. Since the equilibrium depends only on K_t and on current and

¹⁰This hypothesis help us to explain the predictions of the so called "*structural VAR*" literature on the effects of monetary policy shocks on nominal and real output. In particular, it shows a delayed (significant effect is observed with a roughly 2-quarter delay) and persistent effects of monetary policy (peak effect reached at about 6-quarter horizon and persisting) (Christiano et al., 2001).

¹¹Note that $x_t = \hat{Y}_t$ (see later). The steady-state output gap \bar{x} corresponds to the steady-state inflation rate $\bar{\pi}$.

¹²Our specification of equation (2.7) differs from Woodford's in assuming identical time horizons for all variables, whereas Woodford uses quarterly values for interest and inflation, and annual values for output (which consequently have to be divided by four).

expected disturbances, we will have:

$$\begin{aligned}\hat{Y}_t^N &= \hat{Y}_t^{Ncc} + \eta \hat{K}_t \\ \hat{r}_t^N &= \hat{r}_t^{Ncc} + \eta \hat{K}_t\end{aligned}$$

where Ncc represents the *constant-capital natural rate*, i.e. the level of output and interest we *would* have if prices were flexible and the capital stock did not differ from its steady-state level¹³. In other words the Ncc refers only to exogenous processes, which are a function of real disturbances¹⁴. Now the gaps are defined as follows:

$$\tilde{Y}_t = \hat{Y}_t - \hat{Y}_t^N = \hat{Y}_t - \hat{Y}_t^{Ncc} - \eta \hat{K}_t$$

The same thing happens for the remaining variables.

3 NNS and older mainstreams: a comparison

As we discussed at the beginning of the paper, the strategy of the NNS to minimize the frictions that are required to reproduce Keynesian results (in terms of persistent real effects of monetary policy) and Wicksellian results (in terms of interaction of interest and prices) comes at the price of some ad-hocery and other shortcomings that have been criticized by way of many papers (Boianovsky and Trautwein, 2006; Trautwein, 2006; Trautwein and Zouache, 2009; Laidler, 2004; Tamborini, 2006; Tamborini, 2007). Some of the ad-hocery might be refined and made redundant in future version of the NNS, but some of it are indispensable for intertemporal equilibrium modelling of the current kind (Canzoneri et al., 2004; Blanchard and Galí, 2005).

No doubt, there are many points of apparent coincidence between the NNS and the ideas of Wicksell and Keynes. Probably for many readers the idea that Keynes and Wicksell can coexist in a common framework can be confusing or rather troublesome¹⁵. However, in light of the contribution made by the NNS, this interminable theoretical dispute disappears by means of a simple and very popular mechanism: *sticky prices*. In Woodford's treatment, sticky prices are necessary and sufficient to translate Wicksell's interest-rate theory of the price level into a theory of output and prices fluctuations with apparent Keynesian features. Yet the NNS theoretical framework differs from both ancestors substantially, so much that one wonders

¹³On the various definitions of NAIRI and NAIRO used by Woodford (Woodford, 2003), see Boianovsky and Trautwein (2006).

¹⁴Same way for \hat{I}_t^N and $\hat{\lambda}_t^N$. For the capital stock \hat{K}_t^N we will have: $\hat{K}_t^N = \hat{K}_t^{Ncc} + \eta \hat{K}_t$.

¹⁵Certainly there were common elements among the two that became apparent in Keynes' *Treatise on Money* (1930), explicitly built around Wicksell's idea of the natural interest rate as the gravity center of price fluctuations. However, after all, Wicksell remained a champion of neoclassical theory, whereas Keynes eventually paved the way to radically alternative paths (Leijonhufvud, 1981).

whether the points of coincidence may survive to closer inspection (Mazzocchi et al., 2009).

Let us consider Wicksell first. In the first place, we have seen that the NNS concentrates the non-Walrasian features of the economy in the goods (and/or labour) market whereas the capital market remains perfectly Walrasian granting continuous intertemporal equilibrium. As a consequence, *"it is only with sticky prices that one is able to introduce the crucial Wicksellian distinction between the actual and the natural rate of interest, as the discrepancy between the two arises only as a consequence of a failure of prices to adjust sufficiently rapidly"* (Woodford, 2003, p.232). On the contrary, Wicksell cast his theory in a competitive, flex-price, pure credit economy with no use of cash, that is one with Walrasian goods markets but with a capital market which is not Walrasian (Leijonhufvud, 1981). In the Wicksell's framework the problem is not the price of goods but the price of loans:

- A first departure from the assumption of perfect capital markets is the existence of intermediaries (the banking system) between savers (households) and investors (companies). If the banking system plays an active role, the idea that the equilibrium on the capital market - defined by the forces of productivity and thrift - is found at the full-employment level is no longer valid. Differences often arise between the natural rate and the market rate of interest and this happens because intermediaries operate with incomplete and limited information.
- The interference of the banking system with the natural rate can occur because Wicksell's cashless economy is not a moneyless economy (Laidler, 2004, p.3). The key problem to be explained remains how a single agent can have his/her virtual account increased. Apart from selling goods and services, the only other way for an agent to increase his/her nominal purchasing power is to borrow. Consequently, the appropriate concept of money demand is the one expressed by borrowers, whereas the appropriate concept of money supply is the one expressed by lenders. Borrowers are investing firms and lenders are saving households, intermediated by banks. As long as non-bank agents borrow and lend one with the other, the total amount of nominal purchasing power in the economy is redistributed but cannot increase. By contrast, a bank is in a position to grant additional nominal purchasing power to any of its depositors' accounts with no one else in the economy undergoing an equivalent reduction. Likewise, a bank can increase its own nominal purchasing (lending) power by borrowing from the central bank. Thus the problem is that the banking system as a whole might both expand the total nominal purchasing power in the economy and allocate it at terms that differ from those dictated by full-employment saving-investment equilibrium. Over-investment or

over-saving allowed for by imperfect bank intermediation are the keys to price changes in the Wicksellian economy.

- In the NNS, whenever the market real interest rate deviates from the natural rate, households reallocate resources towards present/future consumption along a new intertemporal equilibrium path with an equivalent impact on aggregate demand. This is a consistent transmission mechanism as long as there are no capital goods (Woodford, 2003, sec. 4.1), but it becomes not acceptable in a model with endogenous variation of the stock of capital. In Wicksell's (and Keynes') theory, where there are capital goods to be purchased by means of money and there is a market for loanable funds made by independent borrowers and lenders, the consequence of the market real interest rate on loans being higher (lower) than the natural rate is that households wish to save more (less) whereas firms wish to invest less (more): neither side of the market can achieve intertemporal equilibrium of plans. This problem is not contemplated neither in the framework we presented in section [2] nor in the other models that endogenize investment into the NNS (Casares and McCallum, 2000). Yet ruling saving-investment imbalances out of the theory constitutes a major theoretical weakness (see also Leijonhufvud, 1981 and Van der Ploeg, 2005).
- In a Wicksellian framework the connection between money creation and nominal income - the well-known *cumulative process* - is necessarily examined as out-of-equilibrium dynamics from one level of money and nominal income to another (Wicksell, 1898, p.75; Leijonhufvud, 1981, p.132). Inflation and deflation are a *disequilibrium phenomenon*, the symptom that excess investment or saving are being accommodated at the "wrong" market rate and the economy driven out of the intertemporal equilibrium path. This interpretation of changes in the price level is in sharp contrast with the one put forward in the NNS model, where they are consistent with all markets being cleared and households and firms being in intertemporal equilibrium continuously. On the policy front, whereas the distortionary effects of sticky prices are the *raison d'être* of monetary policy in the NNS, Wicksell argued that interest rates should be brought under policy control not because prices do not move enough, but because unfettered interest rates may force prices to move out of equilibrium. On the other hand, changes in the price level are a means to re-equilibrate the economy only if they induce the nominal interest rate to close the gap with the natural rate (Wicksell, 1898, p. 80).

If sticky prices are not present and are not necessary in Wicksell's interest-rate theory of the price level, it might still be argued that they are necessary to extend that theory to changes in real economic activity. This step takes

us to Keynes. The equation Keynes = Wicksell + sticky prices that seems to emerge from the NNS (at least in Woodford's interpretation) is contentious¹⁶. There is ample textual evidence (notably Keynes, 1937a, 1937b, 1937c) that in the search of a consistent explanation of fluctuations in *real* income, Keynes divorced from Wicksell not on the grounds of imperfect goods market. He realized that a different theory of the interest rate was needed. The idea of the monetary nature of the interest rate related to liquidity preference was conceived as the wedge to be driven in the self-equilibrating mechanism of saving and investment without postulating the role of the banking system¹⁷. Also, Keynes (like some later Swedish followers of Wicksell) was convinced that saving-investment imbalances by themselves would require adjustments in savers' real incomes irrespective of the flexibility of wages and prices. Since much confusion ensued subsequently, it should be stressed that Keynes' point holds true irrespective of which wedge causes savings to diverge from investments.

For reasons that we cannot consider here, Keynesian macroeconomics took the easier, perhaps realistic, shortcut of sticky prices at the cost of obscuring one of the most important keys to understanding business cycles, that is their dimension of *intertemporal coordination failures*. Other scholars of Keynesian inspiration, however, maintained the focus on the role of saving-investment imbalances and the underlying capital-market imperfections (see Minsky, 1975; Leijonhufvud, 1981; Greenwald and Stiglitz, 1987, 1993; Hahn and Solow, 1995; Van der Ploeg, 2005). Despite the methodological differences, common to these views is the idea that the older macroeconomics of saving-investment imbalances does offers guidance for consistent foundations of the interest-rate theory and practice of monetary policy precisely because it focuses on the interest rate as "the wrong price" in the system and lead us to investigate how the monetary authority can manage to "get it right".

4 The Model

4.1 Basic Setup

In order to assess and hopefully clarify some basic issues concerning the macroeconomic of saving-investment imbalances, we introduce a simple competitive, flex-price model economy focused on its intertemporal out of equilibrium dynamics (Mazzocchi et al., 2009). The economy will have the following characteristics: a) Households own the inputs and assets of the

¹⁶Long standing Keynes' hexegetics highlights the paradigmatic divorce that occurred in his thought between the early Wicksellian inspiration of the *Treatise on Money* and the *General Theory* (see Chick, 1983; Rogers, 1989)

¹⁷Paving the way of the liquidity preference hypothesis, Keynes ultimately negated the possibility of stability (De Antoni, 2009, p.11).

economy, including ownership rights in firms, and choose the fractions of their income to consume and save; b) firms hire inputs and use them to produce goods that they sell to households or other firms. Moreover firms have access to a technology that allows them to transform input into output; c) markets exist on which firms sell commodities to households or other firms and on which households sell the inputs to firms. The quantities demanded and supplied determine the relative prices of the inputs and the produced goods.

For sake of concreteness and comparison with the standard NNS model, we have posited specific functional forms for the production function and the utility function. The model presented here is a simplified and modified version of Casares and McCallum (2000) and Woodford (2003, ch.5). The main simplifications consist of homogenous output and price-taking for all agents. Unlike Woodford's NNS model, this framework includes *explicit* interaction of saving and investment in the capital market.

We assume that the supply side of the economy is characterized by the following technology:

$$Y_t = K_t^a L_t^{1-a} \quad (4.1)$$

Where Y_t is the flow of output, K_t is the available capital stock and L_t is the labour input. The chosen production function¹⁸ satisfies the traditional neoclassical properties. For sake of simplicity we assume that capital depreciates at a constant rate $\delta = 1$. At each point in time all the capital stock wears out and, hence, can no longer be used for production. Moreover, we assume a capital accumulation technology such that the share of output transformed into capital at time t takes one period to become operative (Christiano and Todd, 1996; Kydland and Prescott, 1982)¹⁹. These two conditions permit us to write the transition law of capital stock as:

$$K_{t+1} = I_t + (1 - \delta)K_t \quad (4.2)$$

where I_t is net investment and $I'_t = I_t + K_t$ is the gross investment (inclusive capital replacement). Firms are price takers and seek to maximize their expected profit stream, given (4.1) and the gross income distribution

¹⁸Wicksell (1898, 1901, p.128) was one of the pioneers of the concept. However, the concept was not employed in connection with macroeconomic analysis (Humphrey, 1997; Velupillai, 1973).

¹⁹The accumulation function may take different forms, with (slight) differences in result. Here we have followed the "time-to-build" (Kydland and Prescott, 1982) or "time-to-plan" hypothesis (Christiano and Todd, 1996): the underlying idea reflects the fact that firms often decide to carry out new projects and there is a lag between the decision and its implementation. As argued by Blanchard and Fischer (1989), many capital expenditure decisions take long periods to achieve fruition justifying the assumption of a planning before actual investment expenditure is implemented.

constraint, namely:

$$\max_{L_{t+1}; K_{t+1}} E_t \left(\sum_{s=0}^{\infty} Y_{t+s} - w_{t+s} L_{t+s} - R_{t+s} K_{t+s} \right) \quad (4.3)$$

subject to:

$$Y_t = K_t^a L_t^{1-a}$$

The first order conditions for the firms are the following:

$$\tilde{K}_{t+1} = \left(\frac{a}{R_{t+1}} \right)^{\frac{1}{1-a}} \quad (4.4)$$

$$L_{t+1} = \left(\frac{a}{w_{t+1}} \right)^{\frac{1}{a}} \quad (4.5)$$

where w_t is the real wage rate, and R_t is the real gross return to be paid on the capital stock operative at time t and purchased at time $t - 1$.

The households hold claims to the capital stock of the economy, supply their whole labour force L_t inelastically, and choose a consumption plan in order to maximize their utility under the following budget constraint:

$$C_t + S'_t = Y_t \quad (4.6)$$

where S'_t is gross saving. All the previous variables are in real terms, while factors and output are traded at nominal prices denominated in a single unit of account. As far as capital is concerned, firms can finance gross investment out of households' gross saving by issuing one-period bonds bearing a nominal interest rate i_t . By analogy with physical capital, bonds are indexed by their maturity, i.e. $t + 1$ denotes bonds issued at time t with maturity $t + 1$. Note, therefore, that the market real interest rate relevant to the saving/investment decisions in period t is given by $R_{t+1} = E_t \frac{1+i_t}{1+\pi_{t+1}}$, where π_{t+1} is the rate of inflation of the output price P_t (whereas the actual real interest rate that households earn in each t is given by $\frac{1+i_{t-1}}{1+\pi_t}$). In consideration of the assumptions concerning the capital accumulation technology, and of the definition of Y_t , the households' budget constraint can also be written as:

$$B_{t+1} = H_t + R_t B_t - C_t \quad (4.7)$$

where $H_t = w_t L_t$ is labour income and B_t is the outstanding real stock of bonds. Note, therefore, that $B_{t+1} > B_t$ if $H_t + R_t B_t - C_t \equiv S'_t > B_t$, i.e. if gross saving is equal or greater than the existing capital stock of bonds. By analogy with physical capital, bonds are indexed by their maturity, i.e. B_t are bonds purchased in $t - 1$ with maturity in t , etc. Therefore the

households' intertemporal maximization problem is²⁰:

$$\max_{C_t} U(C_t) + E_t \left[\sum_{s=1}^{\infty} \Theta^{-(t+s)} U(C_{t+s}) \right] \quad (4.8)$$

subject to the iterated budget constraint (4.7):

$$C_t + \sum_{s=1}^{\infty} E_t \frac{C_{t+s}}{\prod_{s=1}^{\infty} R_{t+s}} + \frac{B_{t+s+1}}{\prod_{s=1}^{\infty} R_{t+s}} = H_t + \sum_{s=1}^{\infty} E_t \frac{H_{t+s}}{\prod_{s=1}^{\infty} R_{t+s}} + R_t B_t$$

where the transversality condition imposes:

$$\lim_{s \rightarrow \infty} \frac{B_{t+s+1}}{\prod_{s=1}^{\infty} R_{t+s}} = 0$$

The households' first order condition, as of time t , is thus characterized by:

$$U'(C_t) = E_t \left[\frac{U'(C_{t+1})}{\Theta} R_{t+1} \right]$$

Preferences of the households are described by the following logarithmic utility function²¹:

$$U(C_t) = \ln C_t$$

therefore the household's optimal consumption is:

$$C_t = E_t \left[\frac{C_{t+1}}{R_{t+1}} \Theta \right] \quad (4.9)$$

In order to isolate the macroeconomic effects of the interest-rate gaps, the analysis should start at a point in which the economy is in intertemporal general equilibrium²². Therefore we can consider the (stationary) steady-state solution where the employment of labour is normalized to 1, such that $L^* = 1$ and $L^* \leq L$, $F'_L(1) = w$, and $R_{t+1} = \Theta = R^*$, so that for all t , $C_{t+1} = C_t = C^*$. For the given production function, the constant real interest rate also yields a constant capital stock K^* and hence constant output and factor incomes, that is $F'_K = R^*$, $B_t = K_{t+1} = K^*$, $Y_t = Y^* = H^* + R^* K^*$. As long as optimal saving is equal to optimal investment, the

²⁰where $U'(C_t) > 0$, $U''(C_t) < 0$, $\Theta \equiv (1 + \theta) > 1$

²¹This functional form has the advantages that the optimal consumption does not depend on the time horizon of the model.

²²Note that the system has three key elements: a) the rate of time preferences of households, Θ ; b) the market rate of interest, R_{t+1} ; c) the marginal product of capital, R_{t+1}^{PM} . In order to have steady state we need that $\Theta = R_{t+1} = R_{t+1}^{PM} = R^*$. Intuitively a variation of R_{t+1} will influence (with some delay due to time lag or adjustment costs of capital) R_{t+1}^{PM} , so we can assume that on average $R_{t+1} = R_{t+1}^{PM}$. But as long as $R_{t+1} \neq \Theta$ a gap remains. This point will be clearer with the continuation of the paper.

real value of bonds with maturity at any time t coincide with the operating capital stock, $B_t = K^*$. Consequently, the resource constraint is satisfied for:

$$C^* \left(1 + \sum_{i=1}^{\infty} \frac{1}{R^{*i}} \right) = H^* \left(1 + \sum_{i=1}^{\infty} \frac{1}{R^{*i}} \right) + R^* K^*$$

Since $\lim_{i \rightarrow \infty} \sum_{i=1}^{\infty} \frac{1}{R^{*i}} = \frac{1}{r^*}$, we could obtain the optimal consumption, the optimal capital stock and the equilibrium output as follows:

$$C^* + K^* = Y^* \quad (4.10)$$

$$K^* = \left(\frac{a}{R^*} \right)^{\frac{1}{1-a}} \quad (4.11)$$

$$Y^* = K^{*a} \quad (4.12)$$

The first-order condition of the cost minimum yields the equilibrium relative price of factors:

$$\frac{w^*}{R^*} = \frac{1-a}{a} K^* \quad (4.13)$$

The real interest rate R^* associated with the intertemporal general equilibrium is the so called *natural rate of interest*. Note, also, that $S'_t = I_t + K_t = I'_t = K^*$, that is, in steady-state net saving and investment are nil in all t , and the economy only replaces the optimal stock of capital K^* . Finally, it should be $(1 + i_t) = R^*(1 + E_t \pi_{t+1})$ for all t .

In this framework the single policy maker is the central bank which represents the banking system as a whole. It operates by setting an official inflation target π^* and exerting control on the nominal interest rate i_t by trading bonds. At the beginning of each period t the central bank announces the inflation target π^* and pegs the nominal interest rate i_t . Households and firms make up their expectations according to the announced inflation $\pi_{t+1}^e = \pi^*$ ²³. Then transactions take place and output and inflation for the period are realized.

4.2 Three-Gaps analysis

Let us suppose that, during any period t , the economy may be hit by a shock to the capital market. The shock may be *real* (a change in the determinants of thrift or productivity) and imply a new natural interest rate, or it may be *nominal* (a disturbance to the nominal interest rate) and generate a

²³This is a difference with respect to the traditional NNS model, in which we observed an "un-anchored" one-period expected inflation $E_t \pi_{t+1}$. It can be shown that this difference does not entail major theoretical implications. This assumption (Tamborini, 2006) is both convenient and consistent with the usual treatment of policy games where the central bank moves first, and the problem is the conditions such that the target is also the rational expectation of the inflation rate, regardless of transitory inflation rates (Evans and Honkapohja, 2001).

divergence between the market real rate R_{t+1} and the existing natural rate R^* . As will be seen, the only key variable in the problem at hand is the gap between the market and the natural rate, while which of the two has been shocked is immaterial here. Before proceeding, it should be born in mind that the Wicksellian problem is neither one of trades at the "right" rate, nor is it one of quantity rationing at a fixed rate, but it is one of market-clearing trades at the "wrong" rate allowed for by bank intermediation. In other words, the following proposition hold:

Proposition 1 *Given $R_{t+1} \neq R^*$ in any period t , although the bonds market clears, the ensuing levels of saving and investment are not consistent with the output market clearing, both at t and in the subsequent periods, at the intertemporal general equilibrium values of output and at the general price level that would obtain at the natural rate of interest R^* .*

In general, as long as $R_{t+1} \neq R^*$, ceteris paribus an excess investment/saving in the capital market arises. The "wrong" interest rate fixed by the central bank allows excess investment/saving to be financed at the current nominal interest rate i_t (that is, it buys/sells bonds with regard to excess investment/saving) and allows households and firms to carry on their saving and investment plans, respectively. However, the inconsistency between the underlying households' consumption plans and firms' capital-stock choices will spill over across markets and time. In other words, we are in *intratemporal* equilibrium but in *intertemporal* disequilibrium.

Let me recall briefly the disequilibrium relations in the goods market associated with $R_{t+1} \neq R^*$ in the bond market:

Bond Market (at time t)	$R_{t+1} > R^*$	$R_{t+1} < R^*$
Output Market (at time t)	exc. supply	exc. demand
Output Market (at time $t + 1$)	exc. demand	exc. supply

If we abide by the principle that *market always clear*, we should understand how these *intratemporal* and *intertemporal* inconsistencies among notional plans can be brought into equilibrium. In the Appendix [A.1] we proof the following proposition:

Proposition 2 *Given $R_{t+1} \neq R^*$ in any period t , there exists one single sequence of output and price realizations in t and onwards that clears the output market.*

Of course, such a sequence cannot be the same that would obtain with $R_{t+1} = R^*$. Therefore, we could explain it in terms of gaps²⁴ with respect to

²⁴As a matter of simplicity we have hitherto assumed that the capital stock is always adjusted to its optimum level. In the subsequent sections of the paper this assumption is removed in favor of a more general one, in line with the NNS literature. In any case, this simplification does not affect the main conclusions presented here (see Appendix).

the intertemporal general equilibrium output Y^* and the associated expected inflation $\pi_{t+1}^e = \pi^*$:

$$\hat{Y}_{t+1} = \left(\frac{R_{t+1}}{R^*} \right)^{-\frac{a}{1-a}}, \hat{Y}_t = \left(\frac{R_{t+1}}{R^*} \right)^{-\frac{1}{1-a}} \quad (4.14)$$

$$\hat{\Pi}_{t+1} = \left(\frac{\hat{Y}_{t+1}}{\tilde{\hat{K}}_{t+1}^{\frac{a}{1-a}}} \right)^{\frac{a}{1-a}}, \hat{\Pi}_t = \left(\frac{\hat{Y}_t}{\tilde{\hat{K}}_t^{\frac{a}{1-a}}} \right)^{\frac{a}{1-a}} \quad (4.15)$$

where $\hat{Y}_t \equiv \frac{Y_t}{Y^*}$, $\hat{\Pi}_t \equiv \frac{1+\pi_t}{1+\pi^e}$ and $\tilde{\hat{K}}_{t+1} = \left(\frac{R^*}{R_{t+1}} \right)^{\frac{1}{1-a}}$.

To understand the reasons underlying these results, consider again the case in which $R_{t+1} \neq R^*$. As we have seen, households in period t compute a real value of wealth (bonds) that is larger/smaller than the real value of capital that bonds are supposed to represent. In other words, the interest-rate gap generates wrong accounting of *real resources* in the economy. The intuition is that the economy needs a *real correction* of resources: to this effect, output (real incomes accruing to households) should be lower/greater along the whole consumption path of households. Parallely, in order for profit-maximizing firms to reduce/increase output with respect to potential, the inflation rate, too, should be (unexpectedly) lower/greater than the normal rate embedded in nominal contracts. Note that unexpected inflation is an integral part of the process, in the sense that, as long as there exist interest-rate gaps, and hence output gaps, the economy must be off whatever price level path was expected by agents. As a matter of logic of the rational-expectations hypothesis, inflation expectations can, at best, be elaborated by agents consistently with their notional plans (i.e. the inflation rate that would result if the plans based on saving and investment were in fact the intertemporal general equilibrium plans) but these turn out to be unfeasible in the output market. Hence also the related expectation of the inflation rate will be falsified.

In summary, the model yields the following conclusions:

- interest-rate gaps in any period t give rise to an intertemporal sequence of output and inflation gaps: this is a major difference with the NNS model where interest-rate gaps are associated with contemporaneous output gaps only (and therefore future output gaps only depend on future interest-rate gaps);
- interest-rate gaps produce nominal as well as real effects (gaps) even in a competitive, flex-price economy: this is essentially a Keynesian result, which again marks a major difference with the NNS model (where real effects are only ascribed to sticky prices) but also with

Wicksell himself (who did not consider - though did not deny either - real effects)²⁵;

- (excess) inflation or deflation are disequilibrium phenomena in three distinct, but interconnected, meanings: a) excess investment or saving are being accommodated at the "wrong" real interest rate, b) the goods market clears at the "wrong" levels of output and inflation, c) the expected inflation rate is "wrong" with respect to the actual inflation rate.

4.3 A log-linear version

To set the stage for further analyses and facilitate comparison with the NNS framework, we present now a log-linear version of the previous model. A more precise proof is provided in sections [A.1] and [A.3] in Appendix.

4.3.1 The AS curve

Let us begin with the AS function (4.15) which has a straightforward translation into logs:

$$\hat{\pi}_{t+1} = \frac{a}{1-a}\hat{y}_{t+1} + \frac{a^2}{1-a}\hat{k}_{t+1} \quad (4.16)$$

As Ellison and Scott (2000) pointed out, models with endogenous capital stock imply unrealistically high volatility of the investment²⁶. In sticky prices model of NNS framework, this depends on the fact that changes in the nominal interest rate translate *one-to-one* into changes in the real rates leading to the excessively high volatility of all the endogenous variables. This problem also arises in our model: it is sufficient to look at equation (4.4) to note that a change in market interest rate is fully reflected on investment and thus on the capital stock of the next period, causing large and immediate adjustments. The usual shortcut is to assume investment adjustment costs which are able to "buffer" the capital adjustment (Casares and McCallum, 2000; Woodford, 2003)²⁷. We follow the same path.

²⁵Wicksell made incidental mention of the real side of the "cumulative process" (Wicksell, 1898a, p.77). It was Keynes, with his principle of effective demand, who understood that as long as the market real interest rate is "wrong" (e.g. too high) output should take care of adjusting excess saving no matter how deep deflation may be (Keynes, 1936, ch.19). Later, Lindahl (1939), drawing on Wicksell's theory, included unemployment in his analysis, foreshadowing the modern distinction between cyclical and structural unemployment (Boianovsky and Trautwein, 2006).

²⁶This does not represent a problem in standard RBC models because technology shocks have a large impact on the real interest rate, consequently the response of investment mimics its empirical counterpart well.

²⁷Of course, this raises an additional problem in that the adjustment costs are difficult to quantify.

We already know from the first order condition that the optimal capital stock $\tilde{K}_{t+1} = \left(\frac{a}{R_{t+1}}\right)^{\frac{1}{1-a}}$ and that the steady-state capital stock is $K^* = \left(\frac{a}{R^*}\right)^{\frac{1}{1-a}}$. Let us suppose now that there exists some costs given by the following expression:

$$Z = \mu_1(K_{t+1} - \tilde{K}_{t+1})^2 + \mu_2(K_{t+1} - K_t)^2 \quad (4.17)$$

where $\mu_1 + \mu_2 = 1$. We could interpret μ_1 as unitary *disequilibrium costs*, i.e. the cost to have a capital stock different from the optimal one (given the rate of interest). Vice-versa, we could interpret μ_2 as the unitary *change costs* of the capital stock from one period to another. Given \tilde{K}_{t+1} e K_t , the firm seeks to minimize Z . Therefore:

$$\min_{K_{t+1}} Z = \frac{\partial Z}{\partial K_{t+1}} = 2\mu_1(K_{t+1} - \tilde{K}_{t+1}) + 2\mu_2(K_{t+1} - K_t) = 0 \quad (4.18)$$

and thus:

$$K_{t+1} = \psi \tilde{K}_{t+1} + (1 - \psi)K_t \quad (4.19)$$

where $\psi = \frac{\mu_1}{\mu_1 + \mu_2}$.

If the disequilibrium costs are equal to zero, $\mu_1 = 0$ (and thus $\psi = 0$), we have that $K_{t+1} = K_t$, i.e. the capital stock does not change over time²⁸. Vice-versa, if the change costs are equal to zero, $\mu_2 = 0$ (and thus $\psi = 1$), we have that $K_{t+1} = \tilde{K}_{t+1}$, i.e. the *actual* capital stock is always equal to the optimal one, given the interest rate.

We can approximate the deviation of actual capital at time t , \hat{k}_{t+1} , as follows:

$$\hat{k}_{t+1} = \psi \tilde{\hat{k}}_{t+1} + \psi(1 - \psi)\tilde{\hat{k}}_t \quad (4.20)$$

Substituting in the expression (4.16) we have:

$$\hat{\pi}_{t+1} = \kappa \hat{y}_{t+1} + \theta \hat{i}_t + \sigma \hat{i}_{t-1} \quad (4.21)$$

where $\kappa = \frac{a}{1-a}$ measures the link between output and inflation gap while $\theta = \frac{a^2}{(1-a)^2}\psi$ and $\sigma = \frac{a^2}{(1-a)^2}\psi(1-\psi)$ represent the elasticity of the capital stock to changes in the present and past interest rate respectively. We must point out important differences compared to the model without endogenous variation of the capital stock (Mazzocchi et al., 2009). In that basic framework the AS describes the price/output dynamics *off the AS curve* taking into account only the deviation of current inflation from the expected inflation rate that is necessary for competitive firms to supply one unit of profit-maximizing output above/below potential. In other words, if $\hat{\pi}_{t+1} \neq \hat{\pi}_{t+1}^e$, then we will move *along* a traditional upward sloping AS curve²⁹. Conversely, in the

²⁸This is like taking a model without endogenous determination of the stock of (fixed) capital (see Mazzocchi et al., 2009).

²⁹Of course, if $\hat{\pi}_{t+1} = \hat{\pi}_{t+1}^e$, then the AS curve will be vertical.

present model the dynamic of the inflation gap will also depend from the adjustment dynamics of the capital stock caused by an interest rate gap. We know that $\tilde{K}_{t+1} = \frac{R_{t+1}}{R^*}$. If $R_{t+1} = R^*$ then $K_{t+1} = K^*$, i.e. we will have zero net investment (only capital stock replacement) and the capital stock will be consistent with the full employment hypothesis³⁰. On the contrary, if $R_{t+1} \neq R^*$, then $K_{t+1} \neq K^*$, i.e. we will have positive (negative) net investment and the capital stock will be above (below) the full employment level. This means that an interest rate gap affects not only the AD, but also modify the *position* of the AS curve. This phenomenon is often referred to as saying that aggregate supply "moves together" with the aggregate demand (Greenwald and Stiglitz, 1987). The final equilibrium depends of the relative movement of the two curves. As we shall see, this result will significantly change the policy conclusions that can be drawn from the model.

4.3.2 The IS curve

Let us begin with equation (4.14). The main implication for an interest rate gap is a sequence of intertemporal output gaps, each depending on the current interest rate gap. In the Appendix [A.1] we show that, since \hat{Y}_t and \hat{Y}_{t+1} share two common factors, $\frac{R^*}{R_{t+1}}$ and $\frac{R^*}{R_t}$, it is in general possible to express them in a single reduced form displaying autocorrelation. The log-linear version of the IS curve will be:

$$\hat{y}_{t+1} = \rho \hat{y}_t - \alpha \hat{i}_t - \beta \hat{i}_{t-1} \quad (4.22)$$

where ρ can be interpreted as a spurious correlation between \hat{y}_{t+1} and \hat{y}_t and

$$\alpha = \frac{a[\rho(\psi - 1) - \psi] + \rho}{a - 1}$$

$$\beta = \frac{a\psi(1 - \psi)(\rho - 1)}{a - 1}$$

There is a clear analogy with the IS in the NNS model. Yet there are also substantial differences. First, equation (4.22) describes output dynamics *off the IS schedule*. As a consequence of the intertemporal "feed-forward effect" of interest-rate gaps, which is not in the NNS model, these generate time series of output gaps that, ex-post, display two main features: 1) dependence on the lagged values of interest-rate gaps, 2) some degree of (spurious) serial correlation or "inertia" measured by the parameter ρ ³¹.

³⁰Of course this conclusion holds only in a stationary steady-state analysis. If we introduce a positive rate of growth of population (and possibly a technological progress) and if $R_{t+1} = R^*$, then we will observe a positive net investment which allow us to keep constant the per-capita capital stock.

³¹Notably, a dynamic structure like (4.22) is consistent with recurrent empirical estimates of IS equations, which almost invariably find both 1) and 2), that are instead not

4.3.3 A model check

Equations (4.22) and (4.21) form a first-order difference system in the two gaps $[\hat{y}_{t+1}, \hat{\pi}_{t+1}]$ with exogenous nominal interest rates. This formulation is sufficient for a preliminary check of its dynamic properties³². We have thus a non-homogeneous system

$$\begin{bmatrix} \hat{y}_{t+1} \\ \hat{\pi}_{t+1} \end{bmatrix} = \begin{bmatrix} \rho & 0 \\ \kappa\rho & 0 \end{bmatrix} \begin{bmatrix} \hat{y}_t \\ \hat{\pi}_t \end{bmatrix} + \begin{bmatrix} -\alpha \\ \theta - \kappa\alpha \end{bmatrix} \hat{i}_t + \begin{bmatrix} -\beta \\ \sigma - \kappa\beta \end{bmatrix} \hat{i}_{t-1}$$

which, for any initial value $\hat{i}_t = \hat{i}_{t-1} = i_0 \neq 0$, possesses the following steady-state solutions:

$$\hat{y} = -\left(\frac{\alpha + \beta}{1 - \rho}\right) \hat{i}_0 \quad (4.23)$$

$$\hat{\pi} = \left[\theta + \sigma - \frac{\kappa}{1 - \rho}(\alpha + \beta)\right] \hat{i}_0 \quad (4.24)$$

That is to say:

Proposition 3 *A permanent interest-rate gap determines permanent output and inflation gaps. Conversely, the output and inflation gaps are nil only if the interest-rate gap is also nil (Leijonhufvud, 1981, p.136).*

Proposition 4 *If $\rho \in [0, 1]$ output and inflation converge to, and remain locked in, the the steady-state values, with both output and inflation being inefficiently high or low, and being inconsistent with their intertemporal general equilibrium expected values.*

This proposition captures the essence of a cumulative processes as disequilibrium phenomena. If $i_0 \neq 0$ and if the initial inflation rate is nil so that $\pi^e = 0$, then the price level is set on the path given by (4.24) where it grows/declines indefinitely at a constant rate³³. The model solution highlights that a cumulative process on the real side of the economy unfolds

easily accommodated in the framework of the NNS . We shall see that this specification entails considerable differences also in the dynamic properties of the economy.

³²It is convenient to recall the variable $i \equiv r^* + \pi^e$, that is, the nominal value of the natural interest rate. It corresponds to the so-called NAIRI, *non-accelerating-inflation rate of interest*, that provides the benchmark for the nominal interest rate i_t . Correspondingly, let $\hat{i}_t = i_t - i$ be the nominal interest-rate gap.

³³Cumulative processes of the price level in the Wicksellian literature are often associated with accelerating inflation rates. This possibility, however, is closely related by Wicksell to the mechanism of expectations formation: *as long as the change in prices (...) is believed to be temporary, it will in fact remain permanent; as soon as it is considered to be permanent, it will become progressive, and when it is eventually seen progressive it will turn into an avalanche*" (Wicksell, 1922, XII, n.1). The assumption that inflation expectations are held constant at $\pi^e = 0$ corresponds to the first case mentioned by Wicksell. However, as long as the interest-rate gap is not closed, changes in the price level persist. This fact raises the problem of how expectations are possibly revised, and how the revision mechanism impinges upon the dynamic process. This problem will be reconsidered in section [5].

too³⁴. As Keynes argued quite clearly, deflation *per se* cannot be the solution to the problem originating from a saving-investment imbalance as long as the interest-rate gap is not closed³⁵.

We can now say a few words on the sign of the gaps. For the structural values of the parameters, the coefficients are both positive (see proof in Appendix [A.4]). This means that for a $\hat{i}_0 < 0$, we will have a positive output gap ($\hat{y} > 0$) and a negative inflation gap ($\hat{\pi} < 0$). This last result seems to run contrary to the standard macroeconomics models used nowadays³⁶. It does not depend neither on the structural construction of the model nor on its microfoundation and is due to the choice of the production function and its underline hypothesis. In particular, it is due to excessive elasticity of the capital stock to the interest rate, measured by the parameter θ (and also σ), which amplifies thus the adjustment of the aggregate supply. Intuitively, if the central bank implements a tighten monetary policy $\hat{i}_0 > 0$, we have a big decrease of the the capital stock (with some delay due to time lag and/or adjustment costs) and the AS curve will move much more than the AD. Hence, the dynamics will conclude with a positive and slight inflation gap [Figure 2]. Conversely, if the reactivity of the capital stock to the interest rate is lower (as the empirical evidence seems to suggest), the AS will move slight and the system will end with a negative inflation gap [Figure 1]. In both cases the co-movements of the two curves lead to a pronounced change in the output gap without causing appreciable inflation gaps. In Section [6] we will see how this conclusion significantly alter certain policy provisions provided by the NNS.

5 Inflation expectations

In the basic version of the model we have assumed exogenous expectations reflecting the "normal" growth rate (whether zero or positive) of the price level. The problem with this assumption is that, in the event of a persistent interest-rate gap, the future path of the price level will no longer be the same as in the past. In modern parlance, in the course of the cumulative process, expectations of return to normality are systematically falsified.

³⁴This aspect was underscored by Wicksell whereas it was brought into full light by Keynes with his theory of effective demand. As seen above, the model shows that Keynes was right arguing that saving-investment imbalances are real misallocations that require real adjustments in resources, irrespective of the degree of flexibility of nominal prices. Moreover, contrary to interpretations of the Old and New Synthesis, Keynes did not ignore that (unexpected) deflation (equation (4.24)) was the counterpart to the real adjustment process of supply and demand (Keynes, 1936, ch.19)

³⁵Notably, this was the same conclusion, as far as the price level was concerned, reached by Wicksell in his critique of the limitations of the classical quantity theory of money, (Wicksell, 1898a, p.80)).

³⁶In any case we should note that a negative inflation gap is present also in Casares and McCallum (2000) and Ellison and Scott (2000).

Whereas modern macroeconomists tend to rule this problem out of analysis by focusing exclusively on states of the economy where expectations are statistically correct (the so-called *short-run rational expectations*), older mainstreams (see Lindahl, 1939; Lundberg, 1930; Myrdal, 1927) introduced the hypothesis of learning in the cumulative process that shifts expectations from static to adaptive to forward-looking and eventually rational in the sense of self-fulfilling. Therefore, it may be interesting to re-examine the model in the case where a share ξ of agents engage in the short-run anticipation of the inflation process, $E_t\pi_{t+1}$, whereas only $1 - \xi$ sticks to the expectation of "normal" inflation π^* . To this effect, in equations (4.21) and (4.22) we replace $\pi_{t+1}^e = \pi^*$ with $\pi_{t+1}^e = \xi\pi_{t+1} + (1 - \xi)\pi^*$, and redefine $\hat{\pi}_{t+1} = \pi_{t+1} - \pi^*$. As a result (see the proof in Appendix [A.5]):

$$\hat{y}_{t+1} = \rho'\hat{y}_t - \alpha'\hat{i}_t - \beta'\hat{i}_{t-1} \quad (5.1)$$

$$\hat{\pi}_{t+1} = \kappa'\hat{y}_{t+1} + \theta'\hat{i}_t + \sigma'\hat{i}_{t-1} \quad (5.2)$$

where

$$\alpha' = \alpha \frac{1 - \xi}{1 - \xi(1 + \alpha\kappa - \theta)}, \quad \rho' = \rho \frac{1 - \xi(1 - \theta)}{1 - \xi(1 + \alpha\kappa - \theta)}$$

$$\beta' = \beta \frac{[1 - \xi(1 - \theta)] - \alpha\xi\sigma}{1 - \xi(1 + \alpha\kappa - \theta)}, \quad \kappa' = \frac{\kappa}{1 - \xi(1 - \theta)}$$

$$\theta' = \frac{\theta}{1 - \xi(1 - \theta)}, \quad \sigma' = \frac{\sigma}{1 - \xi(1 - \theta)}$$

The steady-state solution for $[\hat{y}_{t+1}, \hat{\pi}_{t+1}]$ can simply be restated as follows

$$\hat{y} = - \left(\frac{\alpha' + \beta'}{1 - \rho'} \right) \hat{i}_0 \quad (5.3)$$

$$\hat{\pi} = \left[\theta' + \sigma' - \frac{\kappa'}{1 - \rho'} (\alpha' + \beta') \right] \hat{i}_0 \quad (5.4)$$

These new solutions are ambiguous as to their sign, magnitude and stability, not only because of what we said in the previous section, but also for the role played by the parameter ξ . In general (proofs in Appendix [A.5]),

- for \hat{y} to maintain the normal negative relationship with \hat{i}_0 , ξ should be bounded, that is, too large a share of forward-looking expectations would invert the relationship between interest-rate gap and output gap (e.g. a positive interest-rate gap would raise output permanently).

$$\xi < \frac{(1 - a)[a(\rho(\psi^2 - 2\psi + 1) - \psi(\psi - 2)) - \rho]}{a^2[\rho(\psi - 1) + \psi(\psi - 2)] + a[\rho(\psi^2 - 2\psi + 2) - \psi(\psi - 2)] - \rho}$$

On the contrary, the coefficient of $\hat{\pi}$ is always positive, whatever the value of ξ .

- for structural values of the parameter the coefficient of \hat{y} decreases with ξ in absolute value only if certain conditions are met³⁷, whereas with empirical values the behavior is the opposite. On the contrary, the coefficient of $\hat{\pi}$ increases with ξ in absolute value (i.e., forward-looking expectations are *deviation-amplifying* in steady state) only if $a + \rho < 1$.
- if ξ satisfies the sign condition, the system also converges to $[\hat{y}, \hat{\pi}]$; if ξ exceeds the sign condition, the system may take different trajectories some of which may be explosive.
- the limit solution for $\xi \rightarrow 1$, is $[\hat{y}, \hat{\pi}] = [0, \hat{i}_0]$. There are no dynamics, the system "jumps" to an inflation gap equal to the interest-rate gap and forward-looking expectations are (self-)fulfilled (McCallum, 1986).

6 Dynamic Properties of the model under different interest-rate rules

The central part of section [4.3.3] elicits a conception of monetary policy as a visible hand possibly keeping the interest rate on the right track. The search of reliable indicators to guide the conduct of policy continues to be an ongoing process. In what follows we evaluate the model with different monetary policy rules. The analysis also includes the inflation expectations, and therefore all the parameters shall be construed as presented in section [5]. However, for reasons of convenience we have omitted the "prime". Some charts placed at the end of Appendix will (hopefully) make clearer the dynamics of the system.

6.1 Optimizing Taylor rule

As Boianovsky and Trautwein (2006, 2006a) pointed out, Wicksell and his followers were aware of (and worried about) the indeterminacy of the inflation rate over pure credit cycles, the crux being the price level taken as "normal" by banks and their borrowers. Theoretical as well as empirical research suggested to search for an interest rate rule that supports a determinate rational-expectation equilibrium (Blanchard and Kahn, 1980). Technically speaking this consists of endogenizing the nominal interest rate, and hence the interest-rate gap. This operation transforms the system from non-homogenous to homogenous in that all three gaps now appear as endogenous variables with no exogenous variables.

³⁷ $\rho < \frac{a\psi}{a(\psi-1)+1}$. This condition is more stringent than the previous one (see Appendix [A.5]).

In the NNS increasing emphasis has been placed on the design of optimal monetary policy rules with reference to the welfare benchmark of the economy. In general, it is shown that Taylor-type interest-rate reaction functions (Taylor, 1993) can be derived from the optimization principle (Woodford, 2003). Following Clarida et al. (1999), let us consider the following problem:

$$\min_{\hat{y}_{t+1}, \hat{\pi}_{t+1}} L_{t+1} = - \sum_{s=1}^{\infty} \frac{1}{2} (\eta_y \hat{y}_{t+s}^2 + \eta_{\pi} \hat{\pi}_{t+s}^2) \quad (6.1)$$

subject to:

$$\hat{\pi}_{t+1} = \kappa \hat{y}_{t+1} + \theta \hat{i}_t + \sigma \hat{i}_{t-1}$$

The central bank aims at minimizing the absolute value of the output and inflation gaps along the dynamic path of the system, where η_{π} and η_y measures the weight assigned to each variable³⁸. By applying the same procedure as Clarida et al. (1999) we obtain an "optimizing" Taylor rule:

$$i_{t+1} = i^* + \gamma_y \hat{y}_{t+1} + \gamma_{\pi} (\hat{\pi}_{t+2}^e | i_t) \quad (6.2)$$

where

$$\gamma_y = \frac{\sigma \rho}{\alpha \sigma - \beta \theta} \text{ and } \gamma_{\pi} = \frac{\eta_{\pi} \kappa (\sigma - \beta \kappa) - \beta \eta_y}{\eta_y (\sigma \alpha - \beta \theta)}$$

This formulation presents some important features. First, there is an explicit target for the interest rate, namely the NAIRI i^* . Second, the informational inflation rate used to assess the cyclical position of the economy is $\hat{\pi}_{t+2}^e | i_t$, i.e. the forecast of the inflation rate in absence of policy interventions (Woodford, 2003, ch.8; Svensson, 1997). Third, the coefficients γ_y and γ_{π} are not arbitrary, but are determined by the central bank's loss function and by the structural parameters of the economy. Forth, the system supports a rational-expectations equilibrium in the target inflation rate set by the central bank: this solves the problem of coordinating inflation expectations in the economy.

Let us check the dynamic properties of the economy under (6.2). By moving to the l.h.s. the NAIRI and using the structural model to solve $\hat{\pi}_{t+2}^e | i_t$, we get the following expression:

$$\hat{i}_{t+1} = \gamma_y \hat{y}_{t+1} + \gamma_{\pi} \hat{\pi}_{t+1} \quad (6.3)$$

³⁸Using this formulation, it is easier to see if the policy regime is a *pure inflation targeting* ($\eta_{\pi} = 1$ and $\eta_y = 0$), or a *flexible inflation targeting* ($\eta_{\pi} > 1$ and $\eta_y > 0$). Furthermore, if $\eta_{\pi} > \eta_y$, the central bank gives greater weight to inflation than output and is therefore called "conservative".

By adding (6.3) to (4.22) and (4.21), we obtain the homogenous system we were looking for (see Appendix [B.1]):

$$\begin{bmatrix} \hat{y}_{t+1} \\ \hat{\pi}_{t+1} \\ \hat{i}_{t+1} \\ \hat{z}_{t+1} \end{bmatrix} = \mathbf{A} \cdot \begin{bmatrix} \hat{y}_t \\ \hat{\pi}_t \\ \hat{i}_t \\ \hat{z}_t \end{bmatrix}$$

where

$$\mathbf{A} = \begin{bmatrix} \rho & 0 & -\alpha & -\beta \\ \kappa\rho & 0 & \theta - \kappa\alpha & \sigma - \kappa\beta \\ \rho(\gamma_\pi\kappa + \gamma_y) & 0 & \gamma_\pi(\theta - \kappa\alpha) - \gamma_y\alpha & \gamma_\pi(\sigma - \kappa\beta) - \gamma_y\beta \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Clearly, this system admits a zero-gap steady-state solution. Yet, an endogenous interest-rate equation is a necessary, but not sufficient, condition for convergence to the zero-gap state of the economy.

Proposition 5 *Given the structural parameters ρ , a and ψ , for the system to converge to, and to be stable around, the zero-gaps steady state, the parameters η_π and η_y should satisfy the following conditions:*

$$\Upsilon_1 < \bar{\eta} < \Upsilon_2 \quad (6.4)$$

where $\bar{\eta} = \frac{\eta_\pi}{\eta_y}$ and:

$$\begin{aligned} \Upsilon_1 &= -\frac{\alpha + \beta}{\kappa[\kappa(\alpha + \beta) + (\theta + \sigma)(\rho - 1)]} \\ \Upsilon_2 &= \frac{\alpha(\beta\kappa + \sigma) - \beta[\beta\kappa + 2\theta(\rho + 1) - \sigma(2\rho + 1)]}{\kappa(\sigma - \beta\kappa)[\kappa(\alpha - \beta) - (\theta - \sigma)(\rho + 1)]} \end{aligned}$$

The first important implication concerns one of the key elements of modern monetary theory, namely the so-called *Taylor Principle*. The main contributions of the literature on this subject say that η_π should be greater than 1. The underlying idea is that when there is a positive inflation gap, the central bank should proceed to a more than proportional increase in the real interest rate. However Woodford (2003, p.253-54) has shown that the Taylor principle should apply not only to the inflation coefficient, but to the whole reaction of the interest rate to the inflation gap. As we have seen, also in our model the output and inflation gap are mutually connected through the AS curve, then the reaction depends on both the inflation coefficient and the output coefficient.

The second thing we notice is that, unlike what happens in the NNS, we have convergence towards the steady-state equilibrium if and only if the policy parameters (namely η_y and η_π) are bounded. This is a general feature of the stability conditions for a model with intertemporal coordination

failure: we can call it *boundedness principle* of the interest-rate rules. The difference between the NNS models, which have only a lower bound, and our model, which instead presents also an upper bound, lies in microfoundations underlying the IS equation. While in our framework an interest rate gap influences both the present *and future* output and inflation gaps, in those of the NNS it has only temporary effect, limited to the period when the shock occurs.

The third consideration that we can make concerns the choice of parameters η_π and η_y . As Clarida et al. (1999, p.1668-69) pointed out, these parameters have no clear foundations and interpretations. Usually, they are meant to capture the relative importance of price and output stability respectively (Uhlig, 2001). In this view it seems that they can be a matter of taste. We find this explanation grossly inaccurate. First, the choice of parameters must take into account the relationship between inflation and output. As we saw in Section [4.3.3], this relationship is far from being unique and depends on the reactions of each variables to interest rates gaps. If the reaction of the stock of capital is slight (namely θ and σ are small), the inflation and output gaps are positively correlated. In this case stabilizing inflation also stabilizes output, and vice versa: once the targets of inflation and output have been chosen consistently with the steady-state of the system, the loss function parameters co-determine the dynamic paths of both inflation and output gaps (Tamborini, 2010). Conversely, if the responsiveness of capital is high (namely θ and σ are large), the inflation and output gap are negatively correlated and the central bank faces conflicting objectives. In this case the choice of the policy parameters reflects the relative importance of one objective over the other. However, as we already have said, the co-movements of aggregate supply and demand curves determine small (and ambiguous) changes in the inflation gap and contemporary large variation of the output gap (see also Casares and McCallum, 2000). Thus, inflation appears to be not the best indicator on which to base a monetary policy. There is the possibility that the correction implemented by the central bank is insufficient or that the convergence dynamics of the real variables to the steady state is too slow. In some cases we can also observe a divergent dynamics of the system.

The choice on η_π and η_y should also take into account the second dynamic property of the system, namely the type of convergence. It is curious that this issue is virtually ignored by the NNS: in fact the type of convergence towards the steady-state is not indifferent. Of course, this is not the right place to discuss widely this issue. However, it is easy to see how a oscillatory dynamic, although convergent, can produce a destabilization of the expectations of agents, with deleterious effects on the whole economy. By contrast, a monotonic convergence, though not optimal from the standpoint of minimizing the welfare loss, is more desirable. At first glance the study of eigenvalues of matrix \mathbf{A} suggests that there is always oscillatory con-

vergence³⁹. However, a more careful analysis reveals that the under certain circumstances, the convergence can be monotonic. Nevertheless, the identification of these combinations of parameters requires on one hand the presence of certain structural parameters and on the other hand a choosing-process of the parameters that should not be based solely on welfare considerations but should also consider the formation of expectations.

Finally some brief comments on inflation expectations. *Ceteris paribus*, more weight of short-term rational expectation will increase the lower bound and decrease the upper bound of $\bar{\eta}$. For sufficiently high value of ξ the upper bound becomes less than 1: this means that η_y must be greater than η_π .

Summing up, our exploration of the optimal interest-rate rule leads to quite problematic conclusions. The choice of the policy parameters must take account of structural constraints, of the values of the parameters associated with individual variables, of the type of desired convergence and of the expectations of agents. As we have seen, that approach, despite the existence of a central bank with a detailed knowledge of the structural model of the economy, could lead to choices which could be suboptimal or even disruptive for the system.

6.2 Adaptive Taylor Rule

In the previous subsection we saw that the central bank sets its policy interest rate in response to deviations of inflation from an inflation target, the actual output from the natural rate of output (or NAIRO) and an estimate of the long-run natural rate of interest (or NAIRI). Despite the success of many central banks at achieving low and stable inflation over the past decades, a reliable tool-kit of indicators of inflationary pressures and other underlying economic imbalances had remained elusive. Although the obvious potential role the natural rates could play in the conduct of monetary policy, the fact that both cannot be observed draws into question its practical usefulness. Their estimations are not straightforward and are associated to a very high degree of uncertainty (see also Hauptmeier et al., 2009; Clark and Kozicki, 2005; Laubach and Williams, 2003).

³⁹The general rule states that a sufficient condition for monotonic convergence is that all the eigenvalues are real and positive. In our model this does not ever happen, since one of the three eigenvalues is always negative (regardless of the combination of the values of the parameters). This result is due to the way in which the model is constructed. In the periods immediately following the arising of an interest-rate gap, the output gap continues to widen, despite the existence of a monetary policy rule that seeks to bring the system towards the steady-state. After a certain number of times this trend reverses and the output gap begins to shrink. This explains why, computing the eigenvalues, we always observe an oscillatory convergence. Since each eigenvalue is related to the dynamics of one of the variables, it is clear that in order to observe an oscillatory dynamic which leaves aside the above considerations, it is necessary to have at least two eigenvalues that do not meet the classic conditions for the convergence (i.e. are negative or complex).

Scepticism about the use of the natural rates for monetary policy was largely prevailing in the past. Wicksell himself thought that the natural rate is inherently unobservable and would be difficult to measure in practice (Wicksell, 1898a). Keynes was even more radical, casting doubts on the existence itself of a single general equilibrium real rate of interest (Keynes, 1937a). Friedman still made the point when he linked the natural rate of unemployment to the natural rate of interest in his Presidential Address (Friedman, 1968, p.8), but he also warned that attempts at conducting monetary policy with reference to natural rates might be fallacious. Doubts concerning the practical use of natural rates for monetary policy are now mounting again (Garnier and Wilhelmsen, 2005; Gnan and Ritzberger-Gruenwald, 2005; Amato, 2005).

Even if we neglect these criticism on the opportunity to use the NAIRI and the NAIRO, there is the undue neglect of central banks' problems with information about both variables. A growing literature shows that wrong informations might seriously destabilize the system (see Orphanides and Williams, 2002, 2002a, 2006; Primiceri, 2006). Indeed the common view of these models is that poor stabilization performance may be due not to the lack of the "right" rule but to the lack of the "right" information about that rule. Moreover, the risk of this information deficiency is not only the worsening of the stabilization performance, but the driving of the economy on an altogether non-convergent path. In fact, as seen in the previous section the Taylor rule works as it transform a non-homogenous system into a homogeneous one. In other words, it works if the interest-rate target is always equal to the "true" NAIRI and the output gap target is always equal to the NAIRO. Any discrepancy between the two implies a non-zero-gaps steady state for the whole system.

Let us assume that the central bank has no exact information about the NAIRI and the NAIRO and let \hat{i}^* and \hat{y}^* be replaced by \hat{i} and \hat{y} respectively. The system that is obtained in this case is:

$$\begin{bmatrix} \hat{y}_{t+1} \\ \hat{\pi}_{t+1} \\ \hat{i}_{t+1} \\ \hat{z}_{t+1} \end{bmatrix} = \mathbf{A} \cdot \begin{bmatrix} \hat{y}_t \\ \hat{\pi}_t \\ \hat{i}_t \\ \hat{z}_t \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} (\hat{i} - \hat{i}^*) + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} (\hat{y} - \hat{y}^*)$$

If the central bank has misinformations about the NAIRI and / or NAIRO (i.e. $\hat{i} \neq \hat{i}^*$ and/or $\hat{y} \neq \hat{y}^*$), at each point in time it may be the case that $\hat{i}_t \neq 0$ and therefore the system go back to the case with an exogenously pegged nominal interest rate that is inconsistent with the intertemporal equilibrium (see section [4.3.3]). The most important conclusion we can draw is that an optimizing Taylor rule is not robust in face of a small informational errors of the central bank. The idea that the central bank may eventually understand that the target set is wrong is not so ob-

vious⁴⁰. The persistence of the error also makes rather impossible to switch off the rule or to take any correction.

The informational requirements of the optimal interest-rate rule and the related problems suggests to look for more robust rules that do not make use of "natural" variables. As Orphanides and Williams (2002) show, these rules may not match theoretical criteria of optimality, but allow for reliable stabilization policy.

To address this issue, we may conveniently begin with a simple representation of an adaptive Taylor rule as the following:

$$i_{t+1} = \gamma_i i_t + \gamma_\pi (\pi_{t+1} - \pi^*) + \gamma_y (y_{t+1} - y_t) \quad (6.5)$$

Let us express the rule in terms of gaps:

$$\hat{i}_{t+1} = \gamma_i \hat{i}_t + \gamma_\pi \hat{\pi}_{t+1} + \gamma_y (\hat{y}_{t+1} - \hat{y}_t) \quad (6.6)$$

Now equations (5.1), (5.2) and (6.6) form the homogeneous system as follows (see Appendix):

$$\begin{bmatrix} \hat{y}_{t+1} \\ \hat{\pi}_{t+1} \\ \hat{i}_{t+1} \\ \hat{z}_{t+1} \end{bmatrix} = \mathbf{B} \cdot \begin{bmatrix} \hat{y}_t \\ \hat{\pi}_t \\ \hat{i}_t \\ \hat{z}_t \end{bmatrix}$$

where

$$\mathbf{B} = \begin{bmatrix} \rho & 0 & -\alpha & -\beta \\ \kappa\rho & 0 & \theta - \kappa\alpha & \sigma - \kappa\beta \\ \gamma_\pi\kappa\rho + \gamma_y(\rho - 1) & 0 & \gamma_i + \gamma_\pi\theta - \alpha(\gamma_y + \gamma_\pi\kappa) & \gamma_\pi\sigma - \beta(\gamma_y + \gamma_\pi\kappa) \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

As in the previous case, this system admits of a zero-gap steady-state solution. Yet, the dynamic properties of the system depend on the interplay between the parameters γ_π , γ_y , γ_i and ξ . We will show in the Appendix [B.2] that the following proposition holds:

Proposition 6 *With an adaptive interest-rate rule like (6.6), for the system to converge to the zero-gaps steady state, the policy coefficients should satisfy the following conditions:*

$$\gamma_i < 1 \quad (6.7)$$

⁴⁰The estimates and the simulations presented by Orphanides and Williams (2002) do not lend empirical support to the convergence prediction. Instead the simulation presented by Primiceri (2006) suggests that, in the long run, has eventually been successful. However, in that paper the long run covers around fifteen years.

$$\gamma_i + \gamma_\pi \frac{\kappa(\alpha + \beta) + (\rho - 1)(\theta + \sigma)}{\rho - 1} < 1 \quad (6.8)$$

$$\frac{\zeta_1 - \sqrt{\zeta_2}}{2\beta(\alpha + \beta)} < \gamma_y < \frac{\sqrt{\zeta_2} - \zeta_1}{2\beta(\alpha + \beta)} \quad (6.9)$$

where both ζ_1 and ζ_2 are function of $\alpha, \beta, \kappa, \rho, \sigma, \theta, \gamma_\pi, \gamma_i$, namely⁴¹:

$$\begin{aligned} \zeta_1 &= \alpha(\beta\gamma_\pi\kappa - \gamma_\pi\rho\sigma - 1) - \beta(\gamma_\pi(\theta + 2\rho\sigma) + \rho + \gamma_i - 1) \\ \zeta_2 &= \alpha^2 [\beta^2\gamma_\pi^2\kappa^2 + 2\beta\gamma_\pi\kappa(\gamma_\pi\rho\sigma - 1) + (\gamma_\pi\rho\sigma + 1)^2] - \\ &\quad - 2\alpha\beta[\beta\gamma_\pi\kappa(\gamma_\pi\theta + \rho + \gamma_i + 1) + \gamma_\pi^2\theta\sigma\rho + \gamma_\pi(\theta(2\rho - 1) + \\ &\quad \sigma(\rho^2 + \rho(\gamma_i - 1) - 2)) + \rho(2\gamma_i - 1) - \gamma_i - 1] - \\ &\quad - \beta^2[4\beta\gamma_\pi\kappa - \gamma_\pi^2\theta^2 + 2\gamma_\pi(\theta(\rho - \gamma_i + 1) + 2\sigma(\rho - 1)) \\ &\quad - \rho^2 + 2\rho(\gamma_i + 1) - \rho^2 + 2\gamma_i - 5] \end{aligned}$$

At first sight these results looks similar to the contribution of the new consensus. Nevertheless, the underlying theoretical discontinuity among the NNS and our macrodynamic framework is substantial.

First, also in this case we find an upper bound condition for all the three policy parameters rather than a lower bound. As we noticed above, this is due to the microfoundations underlying the IS equation. A second observation concerns the effects of inflation expectations on the various policy parameters. In particular, an increase of ξ , i.e. an higher share of forward-looking agents, determines a decrease of the upper bound of γ_π and has no substantial effects on γ_y . That is to say, more weight of short-term inflation expectations in the economy calls for a more aggressive monetary policy but, at the same time, the leeways of the central bank are reduced. These effects are related to the deviation-amplifying role of the expectations. Third, looking at condition (6.8) it becomes clear that there is an inverse relationship⁴² between the inflation coefficient γ_π and the interest rate coefficient γ_i . Fourth, an increase in γ_π and/or in γ_i increases the upper bound of γ_y . On the contrary, the upper bound of γ_π and γ_i is not affected by γ_y . The table below shows the upper bound for γ_y in relation to various values of γ_π and γ_i , for the structural values of the model⁴³, assuming $\xi = 0$:

⁴¹The lower bound of condition (6.9) becomes binding only for implausible values of the parameters, in particular if γ_π becomes very high. Otherwise it is sufficient that $\gamma_y > 0$.

⁴²The inverse relationship holds if $(\theta + \sigma) > \frac{\kappa(\alpha + \beta)}{1 - \rho}$.

⁴³We should remember that all the coefficient of our model (except for the policy coefficient γ_i, γ_π and γ_y) depends only on three "originating" parameters: ρ , i.e. the degree of spurious correlations among the output gaps, a , namely the share-distribution coefficient of the production function, and the cost of adjustment ψ . Since their values change slowly over time and since they are not under complete control of the single agents, we chose to study the behavior of the system assuming (realistic) fixed values of these parameters, namely $\rho = 0.3$, $a = 0.4$ and $\psi = 0.8$. This implies that other parameters of the model will take the following values: $\alpha = 0.073$, $\beta = 0.074$, $\kappa = 0.666$, $\theta = 0.355$, $\sigma = 0.071$.

$\xi = 0$	$\gamma_i = 0$	$\gamma_i = 0.7$	$\gamma_i = 0.99$
$\gamma_\pi = 0$	$\gamma_y < 10.61$	$\gamma_y < 12.51$	$\gamma_y < 13.47$
$\gamma_\pi = 0.7$	$\gamma_y < 11.29$	$\gamma_y < 13.37$	eq.(6.8) violated
$\gamma_\pi = 1$	$\gamma_y < 11.60$	$\gamma_y < 13.75$	eq.(6.8) violated
$\gamma_\pi = 1.5$	$\gamma_y < 12.14$	eq.(6.8) violated	eq.(6.8) violated

It is possible to notice the inverse relationship between γ_i and γ_π as well as the relationships between γ_y and the other two parameters. If we increase the fraction of agents who form forward-looking expectations, we could observe a loss of degrees of freedom for the central bank, as shown in the table below:

$\xi = 0.5$	$\gamma_i = 0$	$\gamma_i = 0.7$	$\gamma_i = 0.99$
$\gamma_\pi = 0$	$\gamma_y < 10.66$	$\gamma_y < 12.48$	$\gamma_y < 13.44$
$\gamma_\pi = 0.7$	$\gamma_y < 11.33$	eq.(6.8) violated	eq.(6.8) violated
$\gamma_\pi = 1$	$\gamma_y < 11.64$	eq.(6.8) violated	eq.(6.8) violated
$\gamma_\pi = 1.5$	$\gamma_y < 12.19$	eq.(6.8) violated	eq.(6.8) violated

The main important message of these tables is that the choice of the policy parameters by the central bank can not just be a matter of taste, and a wrong combination of these can lead to the destabilization of the whole system.

The dynamics depends also on the values of the parameters of the model. As we have previously pointed out, a key role is played by the parameters which measure the responsiveness of the capital stock to the rate of interest (θ and σ). If the reactivity is low, the output-gap and inflation gap will have the same sign. Conversely, if the elasticity is high, it will emerge a conflict of objectives for the central bank. In both cases it becomes dangerous to rely on both variables to implement a consistent monetary policy. On one side, if the output gap and the inflation gap were positively correlated, a good rule need not (and should not) react to both gaps: stabilizing output also stabilizes inflation and viceversa [Figures 3-5-7-9]. On the other side, if we had an output/inflation trade-off, trying to simultaneously correct both the gaps can prolong the adjustment dynamic towards the equilibrium [Figures 4-8], or even steer the system towards different divergent trajectories [Figures 8-10]. Therefore, we have to choose one of the two gaps on which to base the rule. Unlike traditional NNS models (as well as neo-wicksellian models without capital stock (Mazzocchi et al., 2009)), the inflation gap is not a reliable indicator to manage economic policy. The adjustment of the structure of production determined by the interest-rate gap generates a little variation in prices. Recent episodes of over-investment, such as the U.S. "New Economy" bubble in the late 1990s and the housing and mortgages boom in the last years seem to confirm that the *missing inflation* is a critical element in the picture, which has probably played a role in driving monetary policy onto a wrong track (Borio and Lowe, 2002). In this situation even

an adaptive feedback rule based only on inflation gap may break down: the central bank discovers whether his market rate is too low or too high by the price level starting to rise or fall, and he can then adjust his rate accordingly. The problem is that this crucial feedback loop can be short circuited by the arising of a saving-investment imbalances. The trouble with a rule based on inflation targeting is that a constant inflation rate gives no information about whether monetary policy is right or not. And a wrong monetary policy allows the financial imbalances to grow without end.

All these elements suggest that it could be better for the policy-maker to focus only on the evolution of the output along the cumulative process:

$$i_{t+1} = \gamma_i i_t + \gamma_y (y_{t+1} - y_t) \quad (6.10)$$

Subtracting the (constant) NAIRI from both sides of the equation (6.10), we have:

$$\hat{i}_{t+1} = \gamma_i \hat{i}_t + \gamma_y (\hat{y}_{t+1} - \hat{y}_t) \quad (6.11)$$

The equilibrium is determinate if and only if the following holds⁴⁴:

Proposition 7 *With an adaptive interest-rate rule like (6.11), for the system to converge to the zero-gaps steady state, the policy coefficients should satisfy the following conditions:*

$$\gamma_i < 1 \quad (6.12)$$

$$\gamma_y < \frac{\sqrt{\zeta_3} + \alpha + \beta(1 + \rho + \gamma_i)}{2\beta(\alpha + \beta)} \quad (6.13)$$

where:

$$\zeta_3 = \alpha^2 - 2\alpha\beta[\rho(2\gamma_i - 1) - \gamma_i - 1] + \beta^2[\rho^2 - 2\rho(1 + \gamma_i) + \gamma_i^2 - 2\gamma_i + 5]$$

Of course, even in this case are valid all the considerations we made before for the general case. In particular, we have an upper-bound for both parameters and we note that an increase of γ_i determines an increase of γ_y . Moreover the choice of the policy parameters of policy need careful scrutiny: once we fixed γ_i , too high values of γ_y lead to an oscillatory convergence toward equilibrium. The type of convergence also depends on the inflation expectation of the agents and the underlying learning process [Figures 11-12-13-14].

⁴⁴A general proof is given in the Appendix [B.3]. Nevertheless, it is sufficient to take the condition (6.7), (6.8) and (6.9) and put $\gamma_\pi = 0$.

7 Conclusions

Despite to the NNS, we have seen that in the model we presented the cycles are driven *saving-investment imbalances* which generate an intertemporal spillover effect that transmits the effects of present interest-rate gaps to present *and future* output and inflation. Nominal price (or wage) stickiness is not the exclusive problem, price (or wage) flexibility is not the exclusive solution. The focus is mainly on the fact that the NAIRI is volatile and that it is not easily transmitted to the capital market. Since the NAIRI consists of the marginal efficiency of capital and core inflation, these requirements should apply to both components or at least one. In developed countries with relatively stable and predictable inflation, the candidate to trouble-making remains the *marginal efficiency of capital*, and in this respect the inflexibility of the nominal market rate of interest determined by the asymmetric information, the heterogeneity of firms, and other New Keynesian explanations may have a role to play (Mazzocchi, 2009).

As long as the system has a "nominal anchor" (for example, a given core inflation rate in which agents have reason to believe), and the market interest rate is driven to close the gaps with the natural rate of interest (with a monetary feedback rule), the system will converge to the steady-state equilibrium. Nonetheless, this class of cycles remains relevant to the extent that interest rate gaps are likely, substantial and persistent. Even when long-run dynamic is converging toward the equilibrium, frequency, amplitude and persistence of these cycles may make them problematic enough in the short and medium run.

Looking at monetary policy, the main conclusion to be drawn so far is that the critical elements that eventually determine whether a rule is good or bad are not the parameters but the crucial piece of information about the NAIRI: none of the traditional rules produces good results if the central bank is misinformed about this variable. If informational problems with a volatile marginal efficiency of capital are the crux, then interest-rate mechanisms relying upon timely and precise knowledge of the NAIRI are inapplicable (Orphanides and Williams, 2002; 2002a).

Finally, three considerations can be made. First, an "adaptive" rule, using step-by-step adjustment of the interest rate with respect to the different observable conditions in the economy is preferable in that it produces adjustment paths which are generally slower, but safer. Second, saving-investment imbalances could build up also in a low inflation environment. The main reason may be that as long as firms over-invests, the stock of physical capital and thus the productive capacity increase. As a result output grows, excess demand is offset over time and inflation is damped. This type of prediction is like the one made by Casares and McCallum (2000), where the output gap is very sensitive to interest rate, whereas the opposite can be said of inflation. As Leijonhufvud (2007) recently argued, inflation targeting not

only will not protect by itself against financial instability, but it might mislead into pursuing a policy that is actively damaging to financial stability. Recent episodes in the US seem to confirm this view. Third, an adaptive interest-rate rule specified solely in terms of output is safer and performs better than the other rules considered in the paper.

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A Appendix

A.1 Interest rate gaps and output gaps

To begin with, let us examine the notional plan of households. Given $\Theta = R^*$, $B_t = K^*$ and $R_{t+1} \neq R^*$, their optimal consumption path would be:

$$C_t = E_t \left[\frac{C_{t+1}}{R_{t+1}} R^* \right] \quad (\text{A.1})$$

Therefore, from the main text we know that:

$$S'_t = H_t + R_t K_t - C_t \quad (\text{A.2})$$

Ceteris paribus, with respect to the steady state, $R_{t+1} \neq R^*$ shifts notional consumption to the present ($R_{t+1} < R^*$) or to the future ($R_{t+1} > R^*$). As

a result, notional saving is decreased or increased, respectively. Now let us see the notional investment of firms, that is:

$$I'_t = \tilde{K}_{t+1} = \left(\frac{a}{R_{t+1}} \right)^{\frac{1}{1-a}} \quad (\text{A.3})$$

Hence notional investment is increased (if $R_{t+1} < R^*$) or decreased (if $R_{t+1} > R^*$). Consequently, there is a unique relationship between interest-rate gaps and saving-investment gaps, namely if $R_{t+1} > R^*$ then $S'_t > I'_t$ and if $R_{t+1} < R^*$ then $S'_t < I'_t$. These inconsistent notional plans are transformed into mutually consistent actual plans thanks to a re-combination in a vector of present and future output and prices. Following the same procedure as in the NNS models, we plug each period budget constraint (A.2) into households Euler equation (A.29):

$$H_t + R_t K_t - B_{t+1} = E_t \left[\frac{H_{t+1} + R_{t+1} B_{t+1} - B_{t+2}}{R_{t+1}} R^* \right] \quad (\text{A.4})$$

As long as $R_{t+1} \neq R^*$ the actual consumption path consistent with $B_{t+s} = \tilde{K}_{t+s}$ should satisfy:

$$Y_t = Y_{t+1} \frac{R^*}{R_{t+1}} + \tilde{K}_{t+1} - \tilde{K}_{t+2} \frac{R^*}{R_{t+1}}$$

where $Y_t = H_t + R_t K_t$ and $Y_{t+1} = H_{t+1} + R_{t+1} K_{t+1}$. This reformulation of households' consumption path leads to the following propositions:

1. *Given the capital stock chosen by firms for $R_{t+1} \neq R^*$, there exists a unique intertemporal vector of output realizations associated with consistent ex-post output market clearing.*
2. *These output realizations correspond to non-zero gaps with respect to the level of "potential output" given by the capital stock that would obtain with the natural rate of interest R^* .*

The proof goes as follows. We know that:

$$Y_{t+1} = K_{t+1}^a \quad (\text{A.5})$$

By applying the Uhlig's procedure (Uhlig, 1999) we have that $Y_t = Y^*(1 + \hat{y}_t)$, $Y_{t+1} = Y^*(1 + \hat{y}_{t+1})$, $K_{t+1}^a = K^{*a}(1 + \hat{a}\hat{k}_{t+1})$, $\tilde{K}_{t+1} = K^*(1 + \hat{k}_{t+1})$ and $\tilde{K}_{t+2} = K^*(1 + \hat{k}_{t+2})$. Moreover we know that $R_{t+1} = R^*e^{\hat{r}}$, thus $\frac{R^*}{R_{t+1}} = e^{-\hat{r}} = -\hat{r}$. Substituting in the expression above we have:

$$Y^*(1 + \hat{y}_t) = K^{*a}(1 + \hat{a}\hat{k}_{t+1})(1 - \hat{r}_{t+1}) + K^*(1 + \hat{k}_{t+1}) - K^*(1 + \hat{k}_{t+2})(1 - \hat{r}_{t+1})$$

$$Y^*(1 + \hat{y}_{t+1}) = K^{*a}(1 + \hat{a}\hat{k}_{t+1})$$

Dividing for Y^* :

$$(1 + \hat{y}_t) = (1 + a\hat{k}_{t+1})(1 - \hat{r}_{t+1}) + \phi(1 + \tilde{\hat{k}}_{t+1}) - \phi(1 + \tilde{\hat{k}}_{t+2})(1 - \hat{r}_{t+1})$$

$$(1 + \hat{y}_{t+1}) = (1 + a\hat{k}_{t+1})$$

where $\phi = \frac{K^*}{Y^*}$. From (4.20) we know that:

$$\hat{k}_{t+1} = \psi\tilde{\hat{k}}_{t+1} + \psi(1 - \psi)\tilde{\hat{k}}_t$$

Iterating one period we have:

$$\hat{k}_{t+2} = \psi\tilde{\hat{k}}_{t+2} + \psi(1 - \psi)\tilde{\hat{k}}_{t+1}$$

For \hat{r}_{t+1} constant, $\tilde{\hat{k}}_{t+2} = \tilde{\hat{k}}_{t+1}$. Therefore:

$$\hat{k}_{t+2} = \psi(2 - \psi)\tilde{\hat{k}}_{t+1}$$

And substituting in the above equations we have:

$$(1 + \hat{y}_t) = (1 + a(\psi\tilde{\hat{k}}_{t+1} + \psi(1 - \psi)\tilde{\hat{k}}_t))(1 - \hat{r}_{t+1}) + \phi(1 + (\psi\tilde{\hat{k}}_{t+1} + \psi(1 - \psi)\tilde{\hat{k}}_t)) - \phi(1 + \tilde{\hat{k}}_{t+1}\psi(2 - \psi))(1 - \hat{r}_{t+1})$$

and

$$(1 + \hat{y}_{t+1}) = (1 + a(\psi\tilde{\hat{k}}_{t+1} + \psi(1 - \psi)\tilde{\hat{k}}_t))$$

Substituting for:

$$\tilde{\hat{k}}_{t+1} = -\frac{1}{1-a}\hat{r}_{t+1} \tag{A.6}$$

$$\tilde{\hat{k}}_t = -\frac{1}{1-a}\hat{r}_t \tag{A.7}$$

We have:

$$\begin{aligned} \hat{y}_t = & \frac{\hat{r}_{t+1}^2\phi\psi(2-\psi)}{a-1} + \frac{a\hat{r}_{t+1}^2\psi}{1-a} + \frac{\hat{r}_{t+1}\hat{r}_t\phi(\psi-1)^2}{a-1} + \frac{a\hat{r}_{t+1}\hat{r}_t\psi(\psi-1)}{a-1} + \\ & + \frac{\hat{r}_{t+1}\phi(a+\psi^2-\psi-1)}{a-1} + \frac{\hat{r}_{t+1}[a(\psi-1)+1]}{a-1} + \\ & + \frac{\hat{r}_t\phi\psi(1-\psi)}{a-1} + \frac{a\hat{r}_t\psi(1-\psi)}{a-1} \end{aligned}$$

$$\hat{y}_{t+1} = \frac{a\psi}{a-1}\hat{r}_{t+1} + \frac{a\psi(1-\psi)}{a-1}\hat{r}_t$$

The quadratic terms \hat{r}_{t+1}^2 , the cross terms $\hat{r}_{t+1} \cdot \hat{r}_t$ and the product of the two decimal numbers $\hat{r}_{t+1} \cdot \phi$ and $\hat{r}_t \cdot \phi$ are small log-deviations, and therefore can be neglected. We get:

$$\hat{y}_t \approx \frac{[a(\psi - 1) + 1]}{a - 1} \hat{r}_{t+1} + \frac{a\psi(1 - \psi)}{a - 1} \hat{r}_t \quad (\text{A.8})$$

$$\hat{y}_{t+1} = \frac{a\psi}{a - 1} \hat{r}_{t+1} + \frac{a\psi(1 - \psi)}{a - 1} \hat{r}_t \quad (\text{A.9})$$

Consider again the (A.8) and (A.9). Indicating with $z_1 = \frac{a\psi}{a-1}$, $z_2 = \frac{a\psi(1-\psi)}{a-1}$, $z_3 = \frac{a(\psi-1)+1}{a-1}$ e $z_4 = \frac{a\psi(1-\psi)}{a-1}$, we get:

$$\begin{aligned} \hat{y}_{t+1} &= z_1 \hat{r}_{t+1} + z_2 \hat{r}_t \\ \hat{y}_t &= z_3 \hat{r}_{t+1} + z_4 \hat{r}_t \end{aligned}$$

We can write:

$$\hat{y}_{t+1} = \rho \hat{y}_t - \alpha \hat{r}_{t+1} - \beta \hat{r}_t \quad (\text{A.10})$$

for an appropriate linear combination of parameters α and β and where ρ can be interpreted as a spurious correlation between y_{t+1} and y_t ⁴⁵. We have that:

$$\begin{aligned} \alpha &= \frac{a[\rho(\psi - 1) - \psi] + \rho}{a - 1} \\ \beta &= \frac{a\psi(1 - \psi)(\rho - 1)}{a - 1} \end{aligned}$$

We know that $\hat{r}_{t+1} = r_{t+1} - r^*$. Thus, if $r_{t+1} = i_t - \pi^*$, then we could write $\hat{i}_t = i_t - \pi^* - r^*$. That is:

$$\hat{y}_{t+1} = \rho \hat{y}_t - \alpha \hat{i}_t - \beta \hat{i}_{t-1}$$

which is equation (4.22) in the main text.

A.2 Inflation gaps

As to price determination in relation to output gaps, let me assume that all nominal prices and wages are fully indexed to the inflation rate π . Given the general-equilibrium real wage rate w^* and capital stock K^* , potential output at any time t can also be expressed as:

$$Y^* = K^{*a} \left(\frac{1 - a}{w^*} \right)^{\frac{1-a}{a}} \quad (\text{A.11})$$

Let the nominal wage rate for t be given by indexing w^* with the expected inflation rate $\pi_{t+1}^e = \pi^*$, i.e. $W_{t+1} = w^* P_t (1 + \pi_{t+1}^e)$. Therefore, firms can

⁴⁵We have that $\rho \hat{y}_t - \alpha \hat{r}_{t+1} - \beta \hat{r}_t = z_1 \hat{r}_{t+1} + z_2 \hat{r}_t$, thus $\alpha = z_3 \rho - z_1$ e $\beta = z_4 \rho - z_2$.

still adjust output for $t+1$ by choosing the labour input upon observing the current real wage rate $w_{t+1} = \frac{W_{t+1}}{P_{t+1}}$, where $P_{t+1} = P_t(1 + \pi_{t+1})$. As a result,

$$w_{t+1} \equiv \frac{W_{t+1}}{P_{t+1}} = \frac{w^* P_t (1 + \pi_{t+1}^e)}{P_t (1 + \pi_{t+1})} = \frac{w^* (1 + \pi_{t+1}^e)}{1 + \pi_{t+1}}$$

Moreover we know that $Y_{t+1} = K^{*a} \left(\frac{1-a}{w_{t+1}} \right)^{\frac{1-a}{a}}$, therefore we obtain:

$$Y_{t+1} = K^{*a} \left(\frac{1-a}{w^*} \frac{1 + \pi_{t+1}}{1 + \pi^*} \right)^{\frac{1-a}{a}} \quad (\text{A.12})$$

Ceteris paribus, profit-maximizing firms are ready to expand (contract) output as long as π_t , being greater (smaller) than π^* , increases (reduces) the current nominal value of the marginal product of labour with respect to W_t . Conversely, we can derive the Marshallian supply curve of firms, that is, the inflation gap $\hat{\Pi}_t \equiv \frac{1+\pi_t}{1+\pi^*}$ which supports a given output gap. Let me divide (A.12) for Y^* :

$$\frac{Y_{t+1}}{Y^*} = \frac{K_{t+1}^a}{Y^*} \left(\frac{1-a}{w} \frac{1 + \pi_{t+1}}{1 + \pi^*} \right)^{\frac{1-a}{a}}$$

Putting $\hat{Y}_{t+1} = \frac{Y_{t+1}}{Y^*}$ we have:

$$\hat{Y}_{t+1} = \hat{K}_{t+1}^a \frac{1}{\left(\frac{1-a}{w} \right)^{\frac{1-a}{a}}} \left(\frac{1-a}{w} \hat{\Pi}_{t+1} \right)^{\frac{1-a}{a}}$$

and therefore:

$$\hat{Y}_{t+1} = \hat{K}_{t+1}^a \left(\hat{\Pi}_{t+1} \right)^{\frac{1-a}{a}}$$

and thus:

$$\hat{\Pi}_{t+1} = \left(\frac{\hat{Y}_{t+1}}{\hat{K}_{t+1}^a} \right)^{\frac{a}{1-a}}$$

which is equation (4.15) in the main text. Log-linearizing the expression:

$$\hat{\pi}_{t+1} = \frac{a}{1-a} \hat{y}_{t+1} - \frac{a^2}{1-a} \hat{k}_{t+1}$$

which is equation (4.16) in the main text.

A.3 Interest rate gaps and inflation gaps

Let us start with equation (4.19) in the main text. Let us indicate with $\hat{k}_{t+1} = \log \frac{K_{t+1}}{K^*}$, $K_{t+1} = K^*(1 + \hat{k}_{t+1})$, $\tilde{K}_{t+1} = K^*(1 + \tilde{k}_{t+1})$ and $K_t = K^*(1 + \hat{k}_t)$. Log-linearizing around the steady-state (Uhlig, 1999) we get:

$$K^*(1 + \hat{k}_{t+1}) = \psi K^*(1 + \tilde{k}_{t+1}) + (1 - \psi)K^*(1 + \hat{k}_t) \quad (\text{A.13})$$

dividing both sides for K^* we get:

$$\hat{k}_{t+1} = \psi \tilde{k}_{t+1} + (1 - \psi)\hat{k}_t \quad (\text{A.14})$$

iterating the terms over time we have:

$$\hat{k}_{t+1} = \psi \tilde{k}_{t+1} + \psi(1 - \psi)\tilde{k}_t - \sum_{j=2}^{\infty} (-1)^j \psi(1 - \psi)^j \tilde{k}_{t-j+1} \quad (\text{A.15})$$

For $0 \leq \psi \leq 1$ we have that:

$$\lim_{j \rightarrow \infty} \sum_j (-1)^j \psi(1 - \psi)^j \tilde{k}_{t-j+1} = 0$$

Therefore we get:

$$\hat{k}_{t+1} = \psi \tilde{k}_{t+1} + \psi(1 - \psi)\tilde{k}_t$$

which is equation (4.20) in the main text.

Let us divide \tilde{K}_{t+1} for K^* :

$$\tilde{K}_{t+1} = \left(\frac{R^*}{R_{t+1}} \right)^{\frac{1}{1-a}} \quad (\text{A.16})$$

and applying the logarithm we obtain:

$$\tilde{k}_{t+1} = \frac{1}{1-a}(r^* - r_{t+1}) \quad (\text{A.17})$$

Similarly for \tilde{K}_t we have:

$$\tilde{K}_t = \left(\frac{R^*}{R_t} \right)^{\frac{1}{1-a}} \quad (\text{A.18})$$

$$\tilde{k}_t = \frac{1}{1-a}(r^* - r_t) \quad (\text{A.19})$$

replacing (A.17) and (A.19) in expression (4.20) we have:

$$\hat{k}_{t+1} = \frac{1}{1-a}\psi(r^* - r_{t+1}) + \frac{1}{1-a}\psi(1 - \psi)(r^* - r_t) \quad (\text{A.20})$$

and thus:

$$\hat{k}_{t+1} = \frac{1}{1-a} [\psi(r^* - r_{t+1}) + \psi(1-\psi)(r^* - r_t)] \quad (\text{A.21})$$

Substituting this expression in the supply curve (4.16), we have:

$$\hat{\pi}_{t+1} = \frac{a}{1-a} \hat{y}_{t+1} - \frac{a^2}{(1-a)^2} [\psi(r^* - r_{t+1}) + \psi(1-\psi)(r^* - r_t)] \quad (\text{A.22})$$

knowing that $\kappa = \frac{a}{1-a}$, $r_{t+1} = i_t - \pi^*$ and $\hat{i}_t = i_t - \pi^* - r^*$, we have:

$$\hat{\pi}_{t+1} = \kappa \hat{y}_{t+1} + \frac{a^2}{(1-a)^2} [\psi \hat{i}_t + \psi(1-\psi) \hat{i}_{t-1}] \quad (\text{A.23})$$

and putting $\theta = \frac{a^2}{(1-a)^2} \psi$ and $\sigma = \frac{a^2}{(1-a)^2} \psi(1-\psi)$ we have:

$$\hat{\pi}_{t+1} = \kappa \hat{y}_{t+1} + \theta \hat{i}_t + \sigma \hat{i}_{t-1}$$

which is equation (4.21) in the main text.

A.4 A model check

The equation (4.23) can be rewritten as:

$$\hat{y} = - \frac{a[\rho(\psi-1)^2 - \psi(\psi-2)] - \rho}{(a-1)(1-\rho)} \hat{i}_0$$

where the coefficient $\frac{a[\rho(\psi-1)^2 - \psi(\psi-2)] - \rho}{(a-1)(1-\rho)}$ is positive only if:

$$\rho < \frac{a\psi(\psi-2)}{a(\psi-1)^2 - 1} \quad (\text{A.24})$$

For the structural values of the parameters, it is easy to show that this condition is always satisfied. The equation (4.24) can be decomposed as follows:

$$\hat{\pi} = \left(\theta - \frac{\kappa\alpha}{1-\rho} \right) \hat{i}_0 + \left(\sigma - \frac{\kappa\beta}{1-\rho} \right) \hat{i}_0 \quad (\text{A.25})$$

It can be seen that $\left(\theta - \frac{\kappa\alpha}{1-\rho} \right) = \frac{a\rho}{(a-1)(\rho-1)} > 0$ for $0 < a < 1$ and $0 < \rho < 1$, while $\left(\sigma - \frac{\kappa\beta}{1-\rho} \right)$ is always equal to zero. Then the interest-rate gap and the inflation-gap will always have the same sign. Anyway, empirically it is still possible that the sign of the coefficient is negative.

A.5 Inflation expectations

We can write equations (4.22) and (4.21) as follows:

$$\hat{y}_{t+1} = \rho \hat{y}_t - \alpha(i_t - \pi_{t+1}^e - r^*) - \beta \hat{i}_{t-1} \quad (\text{A.26})$$

$$\pi_{t+1} - \pi_{t+1}^e = \kappa \hat{y}_{t+1} + \theta(i_t - \pi_{t+1}^e - r^*) + \sigma \hat{i}_{t-1} \quad (\text{A.27})$$

Since $E_t \pi_{t+1} - \pi_{t+1} = 0$ (*short run rational expectation hypothesis*), we substitute $\pi_{t+1}^e = \xi \pi_{t+1} + (1 - \xi) \pi^*$ in the (A.27) and we get:

$$\pi_{t+1} - \xi \pi_{t+1} - (1 - \xi) \pi^* = \kappa \hat{y}_{t+1} + \theta \hat{i}_t - \theta \xi (\pi_{t+1} - \pi^*) + \sigma \hat{i}_{t-1}$$

e then, by putting again $\hat{\pi}_{t+1} = \pi_{t+1} - \pi^*$, we have:

$$\hat{\pi}_{t+1} = \kappa' \hat{y}_{t+1} + \theta' \hat{i}_t + \sigma' \hat{i}_{t-1} \quad (\text{A.28})$$

where

$$\begin{aligned} \kappa' &= \frac{\kappa}{1 - \xi(1 - \theta)} \\ \theta' &= \frac{\theta}{1 - \xi(1 - \theta)} \\ \sigma' &= \frac{\sigma}{1 - \xi(1 - \theta)} \end{aligned}$$

Let us reconsider equation (A.26). By similar substitutions we have:

$$\hat{y}_{t+1} = \rho \hat{y}_t - \alpha \hat{i}_t + \alpha \xi \pi_{t+1} + \alpha(1 - \xi) \pi^* + \alpha r^* - \beta \hat{i}_{t-1}$$

thus:

$$\hat{y}_{t+1} = \rho \hat{y}_t - \alpha \hat{i}_t + \alpha \xi (\pi_{t+1} - \pi^*) - \beta \hat{i}_{t-1}$$

substituting we get:

$$\hat{y}_{t+1} = \rho \hat{y}_t - \alpha \hat{i}_t + \alpha \xi (\kappa' \hat{y}_{t+1} + \theta' \hat{i}_t + \sigma' \hat{i}_{t-1}) - \beta \hat{i}_{t-1}$$

and inserting the values of κ' , θ' e σ' we get:

$$\hat{y}_{t+1} = \rho' \hat{y}_t - \alpha' \hat{i}_t - \beta' \hat{i}_{t-1} \quad (\text{A.29})$$

where:

$$\begin{aligned} \rho' &= \rho \frac{1 - \xi(1 - \theta)}{1 - \xi(1 + \alpha\kappa - \theta)} \\ \alpha' &= \alpha \frac{1 - \xi}{1 - \xi(1 + \alpha\kappa - \theta)} \\ \beta' &= \frac{\beta[1 - \xi(1 - \theta)] - \alpha\xi\sigma}{1 - \xi(1 + \alpha\kappa - \theta)} \end{aligned}$$

The model will then consist of the equations (5.1) and (5.2) in the main text.

Let us start from equation (5.3). We know that:

$$\frac{\alpha' + \beta'}{1 - \rho'} = \frac{(\xi - 1)[a(\rho(\psi - 1) - \psi) + \rho]}{a(\xi + \rho - 1) + \xi(\rho - 1) - \rho + 1} + \frac{a\psi(\psi - 1)}{a - 1}$$

this quantity is greater than zero only if⁴⁶:

$$\xi < \frac{(1 - a)[a(\rho(\psi^2 - 2\psi + 1) - \psi(\psi - 2)) - \rho]}{a^2[\rho(\psi - 1) + \psi(\psi - 2)] + a[\rho(\psi^2 - 2\psi + 2) - \psi(\psi - 2)] - \rho}$$

Let us compute the effect of ξ on the output-gap:

$$\frac{d}{d\xi} \frac{\alpha' + \beta'}{1 - \rho'} = \frac{a\rho[a(\rho(\psi - 1) - \psi) + \rho]}{[a(\xi + \rho - 1) + (\xi - 1)(\rho - 1)]^2}$$

which is negative when:

$$\rho < \frac{a\psi}{a(\psi - 1) + 1} \quad (\text{A.30})$$

So if it (A.30) holds, the steady-state output-gap decreases as ξ increases.

Note: condition (A.30) is more stringent than (A.24). We distinguish three situations:

1. if $\rho < \frac{a\psi}{a(\psi - 1) + 1}$ and if $\hat{i} > 0$, then $\hat{y} < 0$ and an increase of ξ determines a decrease (in absolute value) of \hat{y} , i.e. $\frac{d\hat{y}}{d\xi} < 0$.
2. if $\frac{a\psi}{a(\psi - 1) + 1} < \rho < \frac{a\psi(\psi - 2)}{a(\psi - 1)^2 - 1}$ and if $\hat{i} > 0$ then $\hat{y} < 0$ and an increase of ξ determines an increase (in absolute value) of \hat{y} , i.e. $\frac{d\hat{y}}{d\xi} > 0$.
3. if $\rho > \frac{a\psi(\psi - 2)}{a(\psi - 1)^2 - 1}$ and if $\hat{i} > 0$ then $\hat{y} > 0$ and an increase of ξ determines an increase (in absolute value) of \hat{y} , i.e. $\frac{d\hat{y}}{d\xi} > 0$.

Let us consider now equation (5.4). The steady-state is:

$$\hat{\pi} = \frac{a\rho}{a(\xi + \rho - 1) + (\xi - 1)(\rho - 1)} \hat{i}_0$$

It is simply to check that this coefficient, for the structural values of the parameters, is always greater than zero. Therefore, if $\hat{i} > 0$ we have $\hat{\pi} > 0$. Moreover we have that:

$$\frac{d\hat{\pi}}{d\xi} = -\frac{a\rho(a + \rho - 1)}{[a(\xi + \rho - 1) + (\xi - 1)(\rho - 1)]^2} > 0 \text{ if } a + \rho < 1$$

⁴⁶This condition could be obtained also with respect to ρ :

$$\rho < \frac{a\psi(2 - \psi)(a - 1)(\xi - 1)}{a^2(\xi + \psi - 1)(\psi - 1) + a(\xi - 1)(\psi^2 - 2\psi + 2) - \xi + 1}$$

Note that if $\xi = 0$ we get back equation (A.24).

hence, if $a + \rho < 1$ we have that an increase of ξ determines an increase (in absolute value) of $\hat{\pi}$. In other words, forward-looking expectations are *deviation-amplifying* in steady state.

B Endogenizing the interest-rate gap

B.1 Optimizing Taylor rule

Let us consider the structural log-linearized model:

$$\begin{cases} \hat{y}_{t+1} = \rho \hat{y}_t - \alpha \hat{i}_t - \beta \hat{i}_{t-1} \\ \hat{\pi}_{t+1} = \kappa \hat{y}_{t+1} + \theta \hat{i}_t + \sigma \hat{i}_{t-1} \\ \hat{i}_{t+1} = \gamma_\pi \hat{\pi}_{t+1} + \gamma_y \hat{y}_{t+1} \end{cases}$$

Making the appropriate substitutions we have:

$$\begin{aligned} \hat{y}_{t+1} &= \rho \hat{y}_t - \alpha \hat{i}_t - \beta \hat{i}_{t-1} \\ \hat{\pi}_{t+1} &= \kappa \rho \hat{y}_t + (\theta - \kappa \alpha) \hat{i}_t + (\sigma - \kappa \beta) \hat{i}_{t-1} \\ \hat{i}_{t+1} &= (\gamma_\pi \kappa \rho + \gamma_y \rho) \hat{y}_t + [\gamma_\pi (\theta - \kappa \alpha) - \gamma_y \alpha] \hat{i}_t + [\gamma_\pi (\sigma - \kappa \beta) - \gamma_y \beta] \hat{i}_{t-1} \end{aligned}$$

To write the system in the standard form (ie, $X_{t+1} = f(X_t)$) we insert auxiliary variable Z defined as follows:

$$\begin{aligned} \hat{y}_{t+1} &= \rho \hat{y}_t - \alpha \hat{i}_t - \beta \hat{z}_t \\ \hat{\pi}_{t+1} &= \kappa \rho \hat{y}_t + (\theta - \kappa \alpha) \hat{i}_t + (\sigma - \kappa \beta) \hat{z}_t \\ \hat{i}_{t+1} &= (\gamma_\pi \kappa \rho + \gamma_y \rho) \hat{y}_t + [\gamma_\pi (\theta - \kappa \alpha) - \gamma_y \alpha] \hat{i}_t + [\gamma_\pi (\sigma - \kappa \beta) - \gamma_y \beta] \hat{z}_t \\ \hat{z}_{t+1} &= \hat{i}_t \end{aligned}$$

It is easy to note that the second equation can be expressed in terms of the other three and therefore can be omitted. By writing the system in matrix form we have:

$$\begin{bmatrix} \hat{y}_{t+1} \\ \hat{i}_{t+1} \\ \hat{z}_{t+1} \end{bmatrix} = \begin{bmatrix} \rho & -\alpha & -\beta \\ (\gamma_\pi \kappa + \gamma_y) \rho & \gamma_\pi (\theta - \kappa \alpha) - \gamma_y \alpha & \gamma_\pi (\sigma - \kappa \beta) - \gamma_y \beta \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \hat{y}_t \\ \hat{i}_t \\ \hat{z}_t \end{bmatrix}$$

By applying the classical criteria of resolution will be:

$$|\mathbf{A} - \lambda \mathbf{I}| = \begin{vmatrix} \rho - \lambda & -\alpha & -\beta \\ \gamma_\pi \kappa \rho + \gamma_y \rho & [\gamma_\pi (\theta - \kappa \alpha) - \gamma_y \alpha] - \lambda & [\gamma_\pi (\sigma - \kappa \beta) - \gamma_y \beta] \\ 0 & 1 & -\lambda \end{vmatrix}$$

From which we obtain the characteristic polynomial:

$$\begin{aligned} \lambda^3 - \{\rho + [\gamma_\pi (\theta - \kappa \alpha) - \gamma_y \alpha]\} \lambda^2 - \{[\gamma_\pi (\sigma - \kappa \beta) - \gamma_y \beta] - [\gamma_\pi (\theta - \kappa \alpha) - \gamma_y \alpha] \rho\} \lambda \\ + \alpha (\gamma_\pi \kappa \rho + \gamma_y \rho) + \rho [\gamma_\pi (\sigma - \kappa \beta) - \gamma_y \beta] + \beta (\gamma_\pi \kappa \rho + \gamma_y \rho) = 0 \end{aligned}$$

Denote by:

$$\begin{aligned}
a_0 &= 1 \\
a_1 &= -\rho - [\gamma_\pi (\theta - \kappa\alpha) - \gamma_y\alpha] \\
a_2 &= -[\gamma_\pi (\sigma - \kappa\beta) - \gamma_y\beta] + [\gamma_\pi (\theta - \kappa\alpha) - \gamma_y\alpha] \rho \\
a_3 &= \alpha (\gamma_\pi \kappa \rho + \gamma_y \rho) + \rho [\gamma_\pi (\sigma - \kappa\beta) - \gamma_y\beta] + \beta (\gamma_\pi \kappa \rho + \gamma_y \rho)
\end{aligned}$$

The stability conditions will be the following (Sydsaeter et al., 2005; Gandolfo, 1971; Shone, 2002)⁴⁷:

$$\begin{aligned}
1 + a_1 + a_2 + a_3 &> 0 \\
3 - a_1 - a_2 + 3a_3 &> 0 \\
1 - a_1 + a_2 - a_3 &> 0 \\
-a_3^2 + a_1 a_3 - a_2 + 1 &> 0
\end{aligned}$$

It is easy to check that the first and the third conditions are binding. Since we know that $\gamma_y = \frac{\sigma\rho}{\alpha\sigma - \beta\theta}$ and $\gamma_\pi = \frac{\eta_\pi \kappa (\sigma - \beta\kappa) - \beta\eta_y}{\eta_y (\sigma\alpha - \beta\theta)}$, we will get the following condition:

$$\frac{\tau_3}{\tau_1} < \frac{\eta_\pi}{\eta_y} < \frac{\tau_4}{\tau_2}$$

where:

$$\begin{aligned}
\tau_1 &= \frac{\kappa [\kappa(\alpha + \beta) + (\theta + \sigma)(\rho - 1)] (\beta\kappa - \sigma)}{\alpha\sigma - \beta\theta} \\
\tau_2 &= \frac{\kappa [\kappa(\alpha - \beta) - (\theta - \sigma)(\rho + 1)] (\beta\kappa - \sigma)}{\alpha\sigma - \beta\theta} \\
\tau_3 &= \frac{(\alpha + \beta)(\sigma - \beta\kappa)}{\alpha\sigma - \beta\theta} \\
\tau_4 &= \frac{\beta(\sigma - \theta) [\beta\kappa - \sigma(2\rho + 1)]}{\sigma(\alpha\sigma - \beta\theta)} - \frac{\beta\kappa}{\sigma} - 1
\end{aligned}$$

Therefore, putting $\bar{\eta} = \frac{\eta_\pi}{\eta_y}$, $\Upsilon_1 = \frac{\tau_3}{\tau_1}$ and $\Upsilon_2 = \frac{\tau_4}{\tau_2}$ we should have the following condition:

$$\Upsilon_1 < \bar{\eta} < \Upsilon_2 \quad (\text{B.1})$$

where:

$$\begin{aligned}
\Upsilon_1 &= -\frac{\alpha + \beta}{\kappa [\kappa(\alpha + \beta) + (\theta + \sigma)(\rho - 1)]} \\
\Upsilon_2 &= \frac{\alpha(\beta\kappa + \sigma) - \beta [\beta\kappa + 2\theta(\rho + 1) - \sigma(2\rho + 1)]}{\kappa(\sigma - \beta\kappa) [\kappa(\alpha - \beta) - (\theta - \sigma)(\rho + 1)]}
\end{aligned}$$

⁴⁷Another simple way to compute the stability conditions is to study directly the eigenvalues of the coefficient matrix, i.e. the (three) roots of the characteristic polynomial. The steady-state solution is stable (i.e. the variables converge towards the equilibrium from whatever initial condition) if all the (real) eigenvalues are smaller than 1 in absolute value (complex eigenvalues have to be included in the unit circle).

B.2 Adaptive Taylor rule

Let us consider the structural log-linearized model:

$$\begin{cases} \hat{y}_{t+1} = \rho \hat{y}_t - \alpha \hat{i}_t - \beta \hat{i}_{t-1} \\ \hat{\pi}_{t+1} = \kappa \hat{y}_{t+1} + \theta \hat{i}_t + \sigma \hat{i}_{t-1} \\ \hat{i}_{t+1} = \gamma_i \hat{i}_t + \gamma_\pi \hat{\pi}_{t+1} + \gamma_y (\hat{y}_{t+1} - \hat{y}_t) \end{cases}$$

Making the appropriate substitutions we have:

$$\begin{aligned} \hat{\pi}_{t+1} &= \kappa \rho \hat{y}_t + (\theta - \kappa \alpha) \hat{i}_t + (\sigma - \kappa \beta) \hat{i}_{t-1} \\ \hat{i}_{t+1} &= \gamma_i \hat{i}_t + \gamma_\pi \hat{\pi}_{t+1} + \gamma_y (\rho \hat{y}_t - \alpha \hat{i}_t - \beta \hat{i}_{t-1} - \hat{y}_t) \end{aligned}$$

Thus the final system will be given by:

$$\begin{aligned} \hat{y}_{t+1} &= \rho \hat{y}_t - \alpha \hat{i}_t - \beta \hat{i}_{t-1} \\ \hat{i}_{t+1} &= [\gamma_\pi \kappa \rho + \gamma_y (\rho - 1)] \hat{y}_t + [\gamma_i + \gamma_\pi \theta - \alpha (\gamma_y + \gamma_\pi \kappa)] \hat{i}_t + [\gamma_\pi \sigma - \beta (\gamma_y + \gamma_\pi \kappa)] \hat{i}_{t-1} \end{aligned}$$

To write the system in the standard form (ie, $X_{t+1} = f(X_t)$) we insert auxiliary variable Z defined as follows:

$$\begin{aligned} \hat{y}_{t+1} &= \rho \hat{y}_t - \alpha \hat{i}_t - \beta \hat{z}_t \\ \hat{i}_{t+1} &= [\gamma_\pi \kappa \rho + \gamma_y (\rho - 1)] \hat{y}_t + [\gamma_i + \gamma_\pi \theta - \alpha (\gamma_y + \gamma_\pi \kappa)] \hat{i}_t + [\gamma_\pi \sigma - \beta (\gamma_y + \gamma_\pi \kappa)] \hat{z}_t \\ \hat{z}_{t+1} &= \hat{i}_t \end{aligned}$$

By writing the system in matrix form we have:

$$\begin{bmatrix} \hat{y}_{t+1} \\ \hat{i}_{t+1} \\ \hat{z}_{t+1} \end{bmatrix} = \begin{bmatrix} \rho & -\alpha & -\beta \\ \gamma_\pi \kappa \rho + \gamma_y (\rho - 1) & \gamma_i + \gamma_\pi \theta - \alpha (\gamma_y + \gamma_\pi \kappa) & \gamma_\pi \sigma - \beta (\gamma_y + \gamma_\pi \kappa) \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \hat{y}_t \\ \hat{i}_t \\ \hat{z}_t \end{bmatrix}$$

By applying the classical criteria of resolution will be:

$$|\mathbf{B} - \lambda \mathbf{I}| = \begin{vmatrix} \rho - \lambda & -\alpha & -\beta \\ \gamma_\pi \kappa \rho + \gamma_y (\rho - 1) & \gamma_i + \gamma_\pi \theta - \alpha (\gamma_y + \gamma_\pi \kappa) - \lambda & \gamma_\pi \sigma - \beta (\gamma_y + \gamma_\pi \kappa) \\ 0 & 1 & -\lambda \end{vmatrix}$$

From which we obtain the characteristic polynomial:

$$\begin{aligned} &\lambda^3 + [\alpha (\gamma_\pi \kappa + \gamma_y) - \gamma_\pi \theta - \rho - \gamma_i] \lambda^2 + \\ &[\rho \gamma_i + \beta (\gamma_\pi \kappa + \gamma_y) - \alpha \gamma_y - \gamma_\pi (\sigma - \theta \rho)] \lambda + \gamma_\pi \rho \sigma - \beta \gamma_y = 0 \end{aligned}$$

Following the procedure described in section [B.1] we have:

$$\gamma_i < 1 \tag{B.2}$$

$$\gamma_i + \gamma_\pi \frac{\kappa(\alpha + \beta) + (\rho - 1)(\theta + \sigma)}{\rho - 1} < 1 \quad (\text{B.3})$$

$$\frac{\zeta_1 - \sqrt{\zeta_2}}{2\beta(\alpha + \beta)} < \gamma_y < \frac{\sqrt{\zeta_2} - \zeta_1}{2\beta(\alpha + \beta)} \quad (\text{B.4})$$

where:

$$\zeta_1 = \alpha(\beta\gamma_\pi\kappa - \gamma_\pi\rho\sigma - 1) - \beta(\gamma_\pi(\theta + 2\rho\sigma) + \rho + \gamma_i - 1)$$

$$\begin{aligned} \zeta_2 = & \alpha^2 [\beta^2\gamma_\pi^2\kappa^2 + 2\beta\gamma_\pi\kappa(\gamma_\pi\rho\sigma - 1) + (\gamma_\pi\rho\sigma + 1)^2] - \\ & - 2\alpha\beta[\beta\gamma_\pi\kappa(\gamma_\pi\theta + \rho + \gamma_i + 1) + \gamma_\pi^2\theta\sigma\rho + \gamma_\pi(\theta(2\rho - 1) + \\ & \sigma(\rho^2 + \rho(\gamma_i - 1) - 2)) + \rho(2\gamma_i - 1) - \gamma_i - 1] - \\ & - \beta^2[4\beta\gamma_\pi\kappa - \gamma_\pi^2\theta^2 + 2\gamma_\pi(\theta(\rho - \gamma_i + 1) + 2\sigma(\rho - 1)) \\ & - \rho^2 + 2\rho(\gamma_i + 1) - \rho^2 + 2\gamma_i - 5] \end{aligned}$$

Some considerations:

- an increase in γ_i *decreases* the upper bound of γ_π . In fact:

$$\frac{\partial\gamma_\pi}{\partial\gamma_i} = -\frac{\rho - 1}{\kappa(\alpha + \beta) + (\rho - 1)(\theta + \sigma)} < 0$$

Since the coefficient of γ_π is always positive, then if $\gamma_i = 1$ we have that $\gamma_\pi < 0$ (we go back to the previous case).

- an increase in ξ *decreases* the upper bound of γ_π . In fact:

$$\frac{\partial\gamma_\pi}{\partial\gamma_i} = \frac{(1 - \xi(1 - \theta)) [(1 - \xi(1 - \theta))\rho + \xi(1 + \alpha\kappa - \theta) - 1]}{\zeta_7 + (\alpha + \beta)\kappa - \sigma - \theta} < 0$$

where

$$\begin{aligned} \zeta_7 = & [(\theta^2 - \theta - (1 - \theta)\sigma)\xi + \sigma + \theta] \rho \\ & + [\theta + (\beta\sigma - \alpha(1 - \theta - \sigma(1 - \beta)) - \beta)\kappa - \theta^2 - \sigma(1 - \theta)] \xi \end{aligned}$$

This effect is related to the deviation-amplifying role of the expectations.

- an increase in γ_i *increases* the upper bound of γ_y . In fact:

$$\frac{\partial\gamma_y}{\partial\gamma_i} = \frac{-2\alpha\beta(\gamma_\pi\beta\kappa + \gamma_\pi\rho\sigma + 2\rho - 1) - \beta^2(2\rho - 2\gamma_\pi\theta + 2)}{2\sqrt{\zeta_8} + \beta} > 0$$

where

$$\begin{aligned} \zeta_8 = & \alpha(\beta^2\gamma_\pi^2\kappa^2 + 2\beta\gamma_\pi\kappa(\gamma_\pi\rho\sigma - 1) + (\gamma_\pi\rho\sigma + 1)^2)^2 - \\ & - 2\alpha\beta(\beta\gamma_\pi\kappa(\gamma_\pi\theta + \rho + \gamma_i + 1) + \gamma_\pi^2\theta\sigma\rho + \gamma_\pi(\theta(2\rho - 1) + \\ & + \sigma(\rho^2 + \rho(\gamma_i - 1) - 2)) + \rho(2\gamma_i - 1) - \gamma_i - 1) - \\ & - \beta^2(4\beta\gamma_\pi\kappa - \gamma_\pi^2\theta^2 + 2\gamma_\pi(\theta(\rho - \gamma_i + 1) + 2\sigma(\rho - 1)) - \\ & - 2\rho^2 + 2\rho(\gamma_i + 1) + 2\gamma_i - 5) \end{aligned}$$

- an increase in ξ *decreases* the upper bound of γ_y ⁴⁸.
- ceteris paribus, an increase in γ_π increases the upper bound of γ_y . On the contrary, the upper bound of γ_π is not affected by γ_y ⁴⁹.

B.3 Adaptive rule with only output gap and smoothed interest rate

Let us consider the following case:

$$\begin{cases} \hat{y}_{t+1} = \rho\hat{y}_t - \alpha\hat{i}_t - \beta\hat{i}_{t-1} \\ \hat{\pi}_{t+1} = \kappa\hat{y}_{t+1} + \theta\hat{i}_t + \sigma\hat{i}_{t-1} \\ \hat{i}_{t+1} = \gamma_i\hat{i}_t + \gamma_y(\hat{y}_{t+1} - \hat{y}_t) \end{cases}$$

The final system will be given by:

$$\begin{aligned} \hat{y}_{t+1} &= \rho\hat{y}_t - \alpha\hat{i}_t - \beta\hat{i}_{t-1} \\ \hat{i}_{t+1} &= \gamma_y(\rho - 1)\hat{y}_t + (\gamma_i - \gamma_y\alpha)\hat{i}_t - \gamma_y\beta\hat{i}_{t-1} \end{aligned}$$

To write the system in the standard form (ie, $X_{t+1} = f(X_t)$) we insert auxiliary variable Z defined as follows:

$$\begin{aligned} \hat{y}_{t+1} &= \rho\hat{y}_t - \alpha\hat{i}_t - \beta\hat{z}_t \\ \hat{i}_{t+1} &= \gamma_y(\rho - 1)\hat{y}_t + (\gamma_i - \gamma_y\alpha)\hat{i}_t - \gamma_y\beta\hat{z}_t \\ \hat{z}_{t+1} &= \hat{i}_t \end{aligned}$$

By writing the system in matrix form we have:

$$\begin{bmatrix} \hat{y}_{t+1} \\ \hat{i}_{t+1} \\ \hat{z}_{t+1} \end{bmatrix} = \begin{bmatrix} \rho & -\alpha & -\beta \\ \gamma_y(\rho - 1) & \gamma_i - \gamma_y\alpha & -\gamma_y\beta \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \hat{y}_t \\ \hat{i}_t \\ \hat{z}_t \end{bmatrix}$$

By applying the classical criteria of resolution will be:

$$|\mathbf{C} - \lambda\mathbf{I}| = \begin{vmatrix} \rho - \lambda & -\alpha & -\beta \\ \gamma_y(\rho - 1) & \gamma_i - \gamma_y\alpha - \lambda & -\gamma_y\beta \\ 0 & 1 & -\lambda \end{vmatrix}$$

From which we obtain the characteristic polynomial:

$$\lambda^3 + (\alpha\gamma_y - \rho - \gamma_i)\lambda^2 + (\beta\gamma_y + \rho\gamma_i - \alpha\gamma_y)\lambda - \beta\gamma_y = 0$$

Following the procedure described in section [B.1] we have:

$$\begin{aligned} \gamma_i &< 1 \\ \gamma_y &< \frac{\sqrt{\zeta_3} + \alpha + \beta(1 + \rho + \gamma_i)}{2\beta(\alpha + \beta)} \end{aligned}$$

⁴⁸This proof is available upon request.

⁴⁹See previous footnote.

where:

$$\zeta_3 = \alpha^2 - 2\alpha\beta[\rho(2\gamma_i - 1) - \gamma_i - 1] + \beta^2[\rho^2 - 2\rho(1 + \gamma_i) + \gamma_i^2 - 2\gamma_i + 5]$$

If we put $\gamma_i = 0.7$, we have that $\gamma_y < 12.51$. The two extreme cases are the following: $[\gamma_i = 0.99; \gamma_y < 13.47]$ and $[\gamma_i = 0; \gamma_y < 10.61]$.

C Simulations and charts

In this section we present some simulations of the model under different monetary policy rules. We should remember that all the coefficient of our model (except for the policy coefficient γ_i , γ_π and γ_y) depends only on three "originating" parameters: ρ , i.e. the degree of spurious correlations among the output gaps, a , namely the share-distribution coefficient of the production function, and the cost of adjustment ψ . Since their values change slowly over time and since they are not under complete control of the single agents, we chose to study the eigenvalues assuming (realistic) fixed values of these parameters, namely $\rho = 0.3$, $a = 0.4$ and $\psi = 0.8$. This implies that other parameters of the model will take the following values: $\alpha = 0.073$, $\beta = 0.074$, $\kappa = 0.666$, $\theta = 0.355$, $\sigma = 0.071$. We also plot the same graphs assuming empirical values of the parameters, namely $\alpha = 0.15$, $\beta = 0.07$, $\kappa = 0.66$, $\theta = 0.10$, $\sigma = 0.05$ (see Laubach and Williams, 2003; Garnier and Wilhelmsen, 2005; Tamborini, 2010). Each simulation assumes an initial interest rate gap equal to 10 basis points, i.e. $\hat{i}_t = 100$. Obviously this implies a corresponding variation of the output gap and inflation gap as described by equations (A.8) and (A.9)⁵⁰. Series 1,2,3 represent the output gap, the inflation gap and the interest-rate gap respectively.

⁵⁰For example, if $\xi = 0$, the vector of initial condition will be $[\hat{y}_t, \hat{\pi}_t, \hat{i}_t] = [-154.4; -102.93; 100.00]$.

C.1 Permanent interest-rate gap

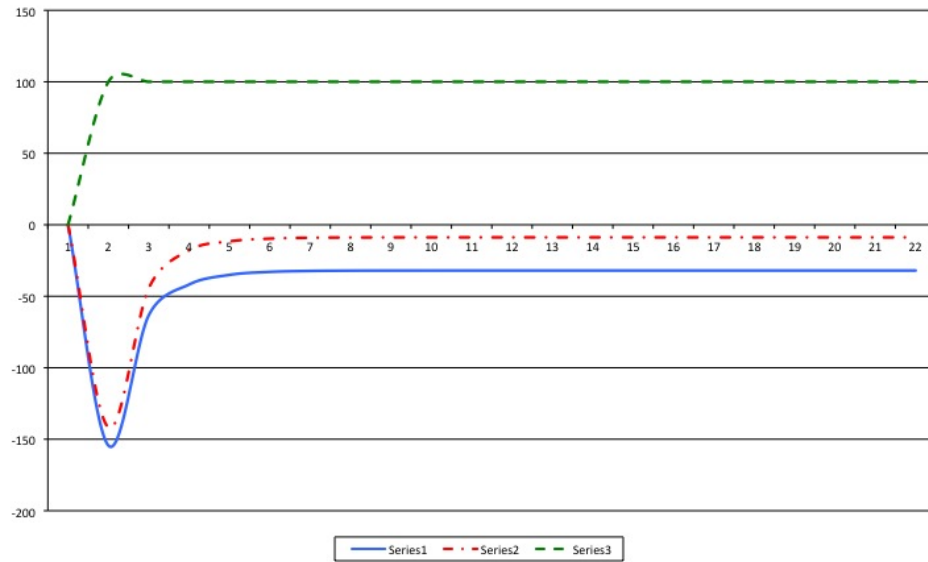


Figure 1 - Empirical parameters. Permanent interest rate gap of 100 basis points.
 $\xi = 0.3$.

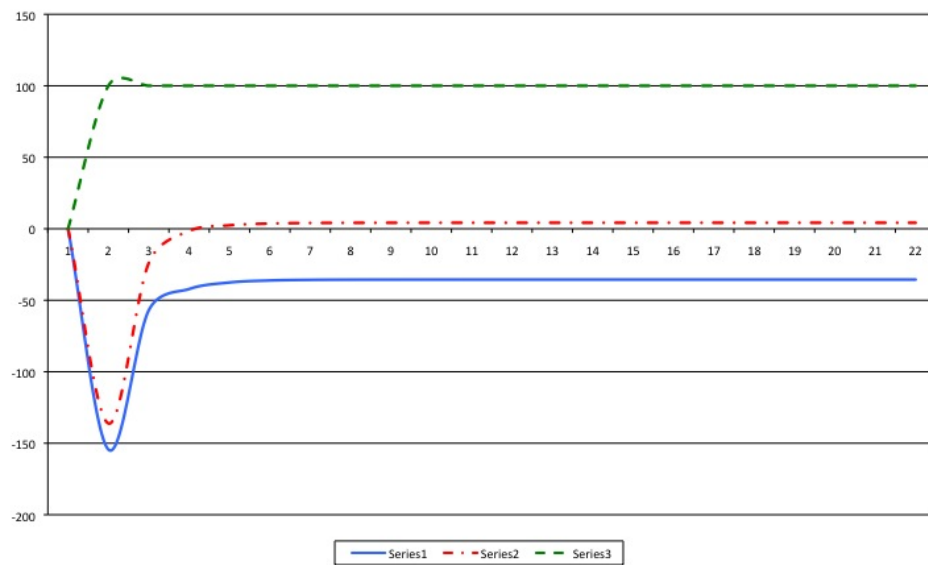


Figure 2 - Structural parameters. Permanent interest rate gap of 100 basis points.
 $\xi = 0.3$.

C.2 Adaptive rule with three policy parameters

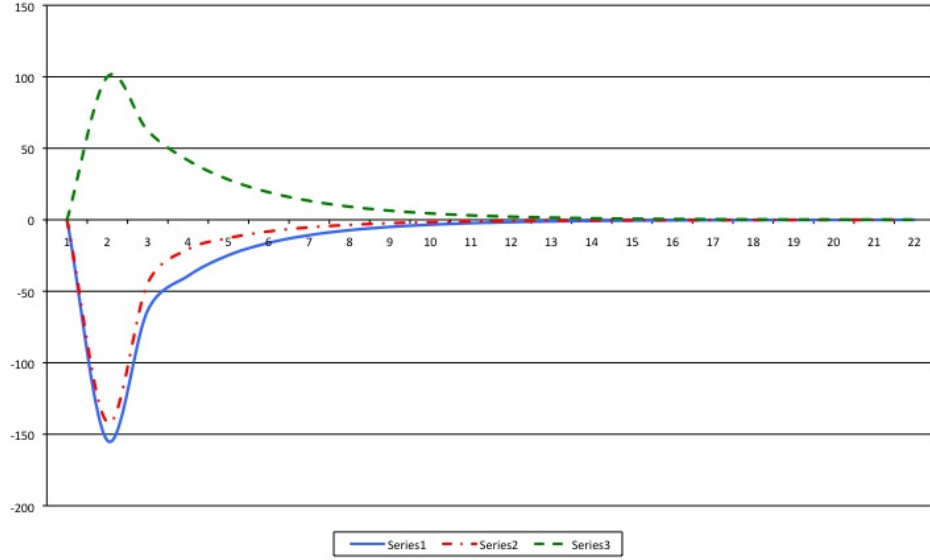


Figure 3 - Empirical parameters. $\xi = 0.3$, $\gamma_i = 0.8$, $\gamma_\pi = 0.4$, $\gamma_y = 0$.

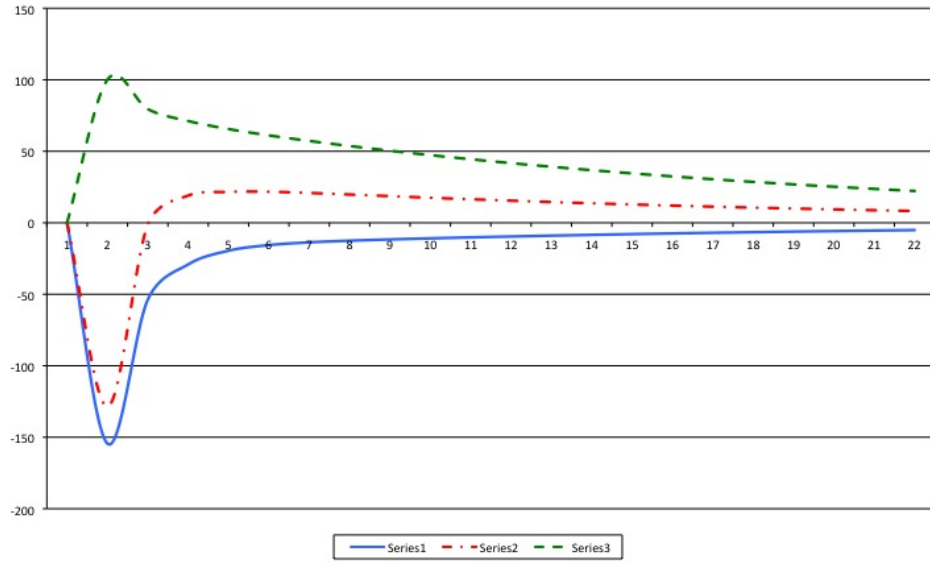


Figure 4 - Structural parameters. $\xi = 0.3$, $\gamma_i = 0.8$, $\gamma_\pi = 0.4$, $\gamma_y = 0$.

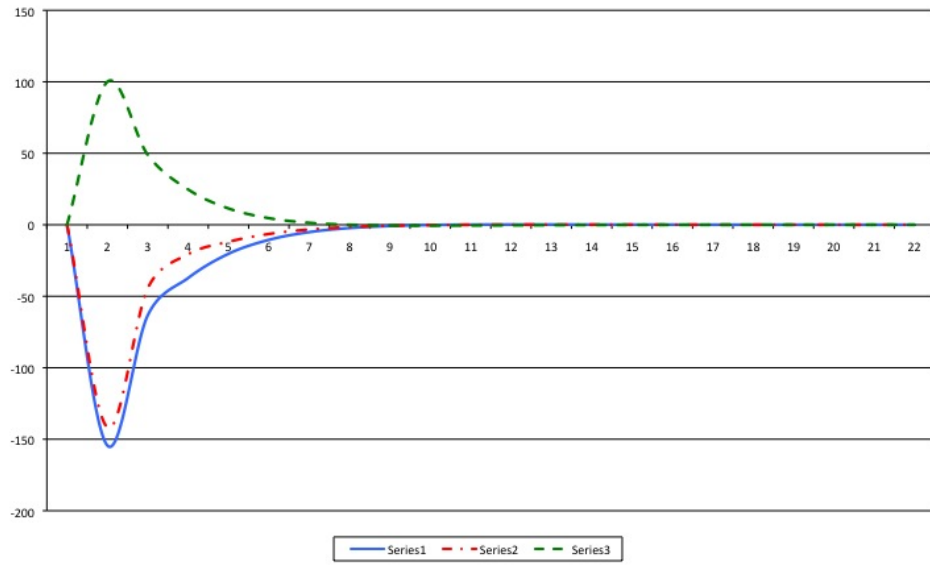


Figure 5 - Empirical parameters. $\xi = 0.3$, $\gamma_i = 0.8$, $\gamma_\pi = 0.7$, $\gamma_y = 0$.

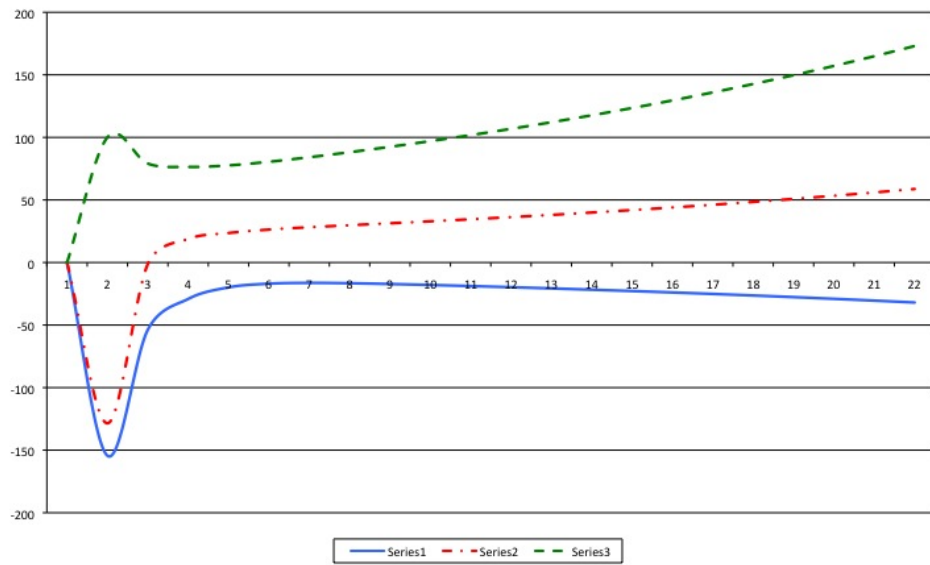


Figure 6 - Structural parameters. $\xi = 0.3$, $\gamma_i = 0.8$, $\gamma_\pi = 0.7$, $\gamma_y = 0$.

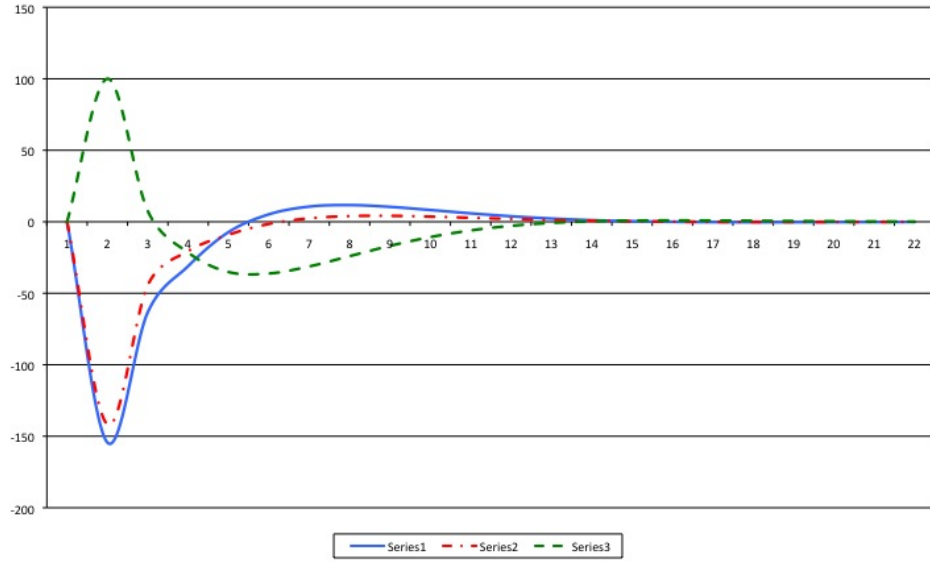


Figure 7 - Empirical parameters. $\xi = 0.3$, $\gamma_i = 0.8$, $\gamma_\pi = 0.4$, $\gamma_y = 0.6$.

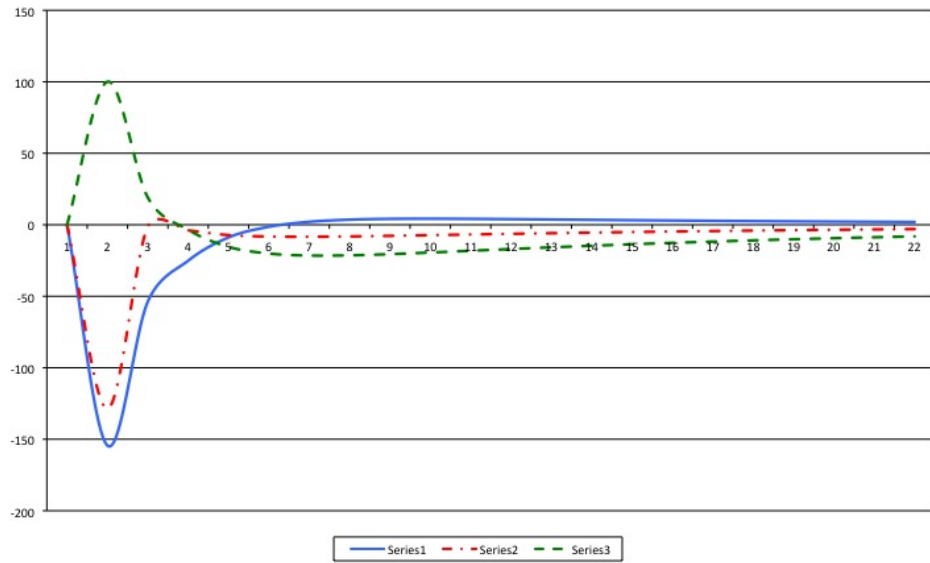


Figure 8 - Structural parameters. $\xi = 0.3$, $\gamma_i = 0.8$, $\gamma_\pi = 0.4$, $\gamma_y = 0.6$.

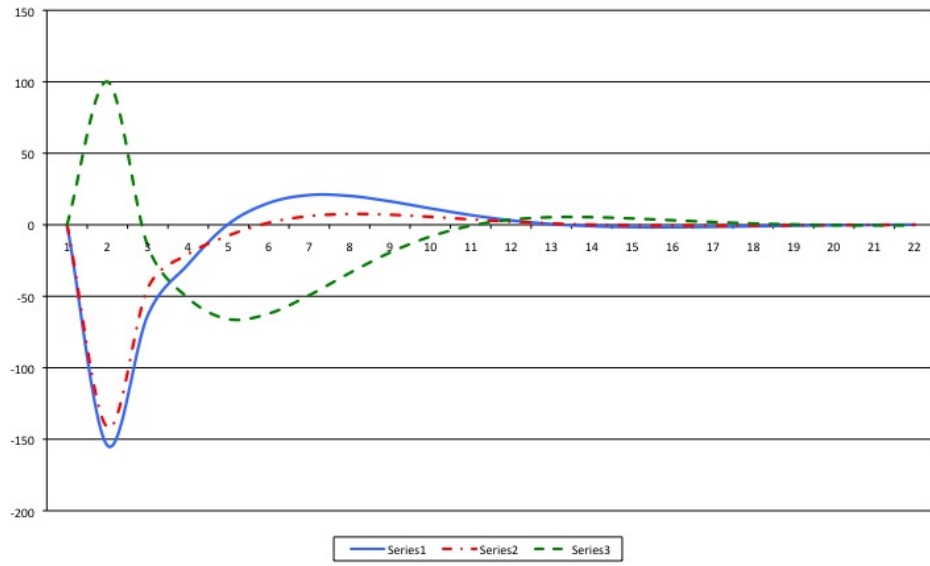


Figure 9 - Empirical parameters. $\xi = 0.3$, $\gamma_i = 0.8$, $\gamma_\pi = 0.7$, $\gamma_y = 0.7$.

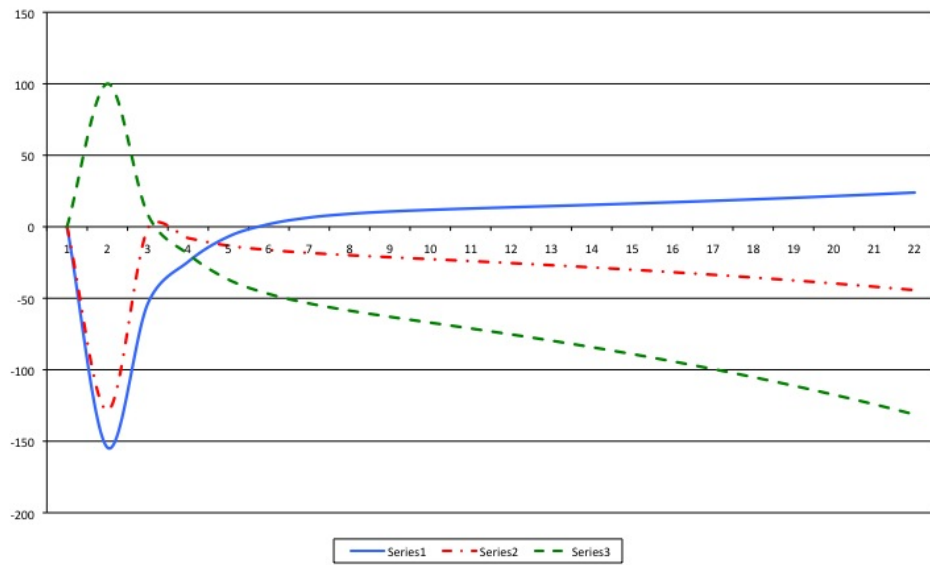


Figure 10 - Structural parameters. $\xi = 0.3$, $\gamma_i = 0.8$, $\gamma_\pi = 0.7$, $\gamma_y = 0.7$.

C.3 Adaptive rule without inflation parameter

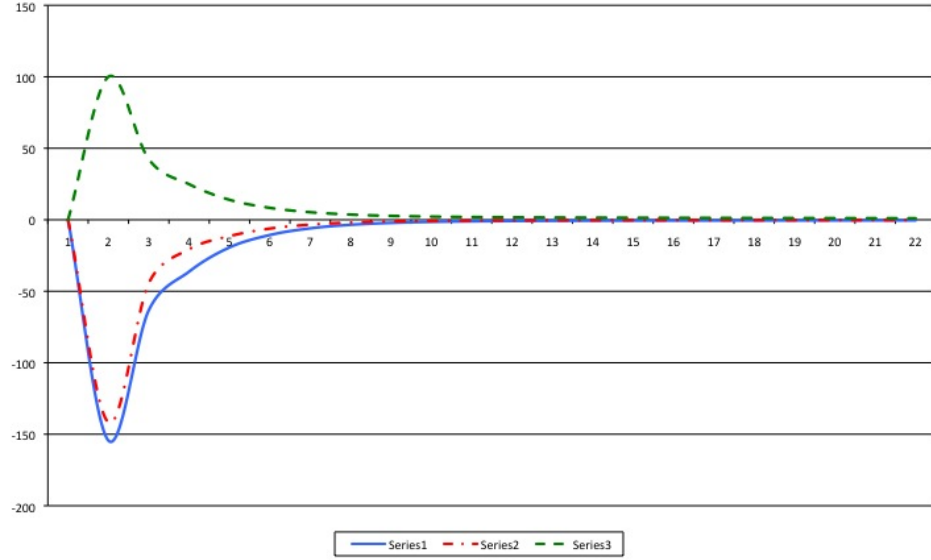


Figure 11 - Empirical parameters. $\xi = 0.3$, $\gamma_i = 0.8$, $\gamma_\pi = 0$, $\gamma_y = 0.7$.

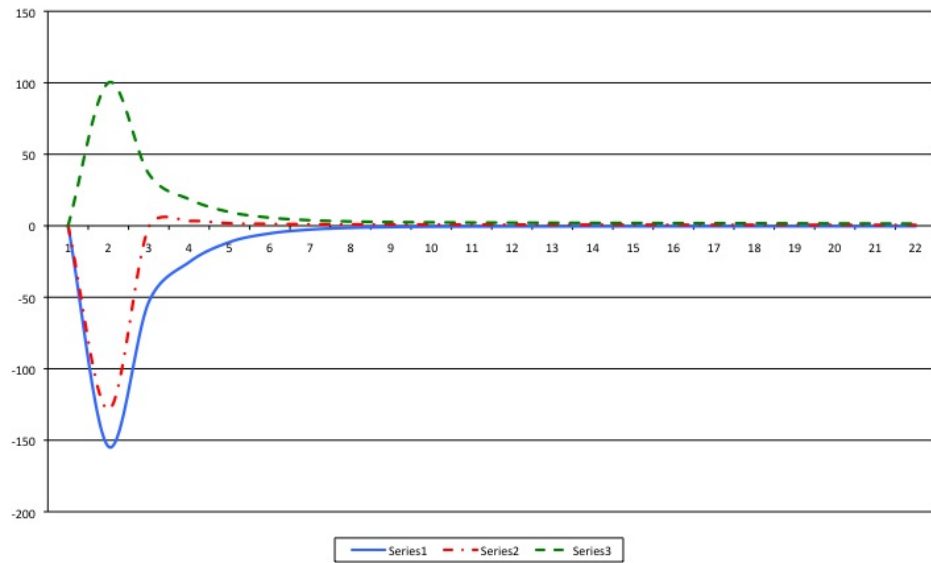


Figure 12 - Structural parameters. $\xi = 0.3$, $\gamma_i = 0.8$, $\gamma_\pi = 0$, $\gamma_y = 0.7$.

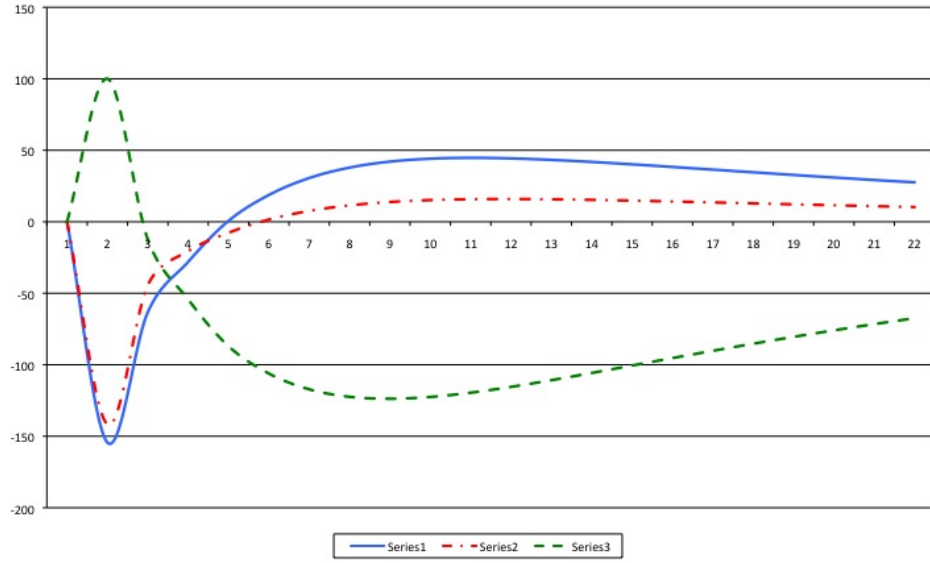


Figure 13 - Empirical parameters. $\xi = 0.3$, $\gamma_i = 0.8$, $\gamma_\pi = 0$, $\gamma_y = 1.2$.

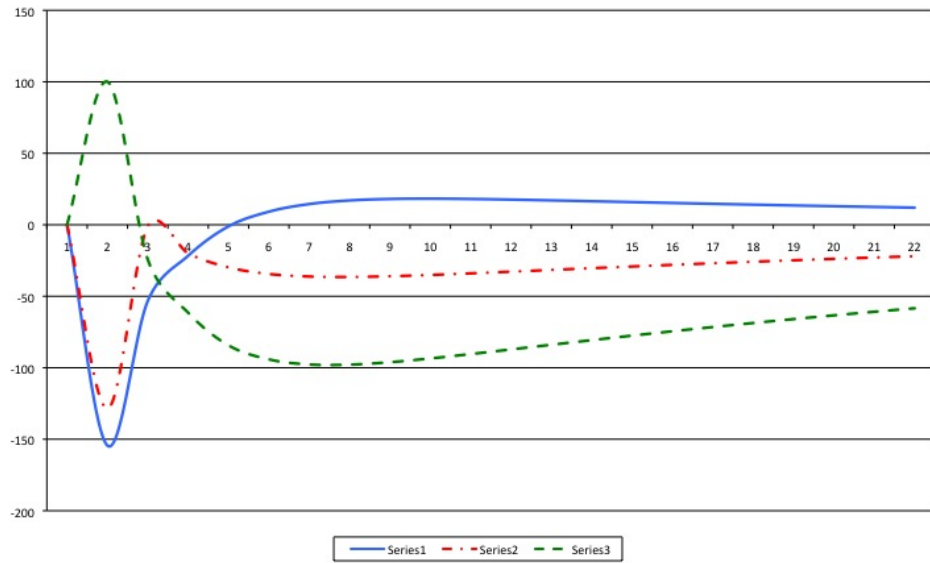


Figure 14 - Structural parameters. $\xi = 0.3$, $\gamma_i = 0.8$, $\gamma_\pi = 0$, $\gamma_y = 1.2$.



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UNIVERSITÀ DI SIENA
PIAZZA S. FRANCESCO 7 I- 53100 SIENA
<http://www.depfid.unisi.it/WorkingPapers/>
ISSN 1972 - 361X

