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6 / 2010

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DEPFID Working Papers - 6 / 2010 - Abstract

Weight of argument and economic decisions

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This paper aims to clarify the controversial concept of "weight of argument" as introduced by Keynes in the *Treatise of Probabilities* and explicitly resumed in crucial passages of the *General Theory* (GT), in order to assess its influence on the theoretical framework and methodological approach of the GT. To this end the paper carries on a preliminary examination whether, and for what reason, we should expect that the weight of argument has a significant impact on economic decisions. A few recent development in epistemology and decision theory under uncertainty have reopened the issue, providing at the same time new analytical instruments capable to translate Keynes's intuitions in rigorous and operational instruments. We suggest an interpretation of the concept of weight of argument that we believe consistent with the spirit of Keynes's approach and that vindicates its substantial correctness and analytical potential in the light of the recent advances in decision theory under hard uncertainty. This interpretation confirms the practical relevance of the concept getting over the doubts expressed by Keynes himself, as well as its crucial role as a foundation of the theoretical and policy message of Keynes. It is thus possible and opportune to resume the research programme, suggested by Keynes with some timidity, meant to analyze the role of the weight of argument in economic decisions.

KEYWORDS: weight of argument, decision theory under hard uncertainty

JEL CLASSIFICATION: B2, B4, D03

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1. Introduction

The *Treatise on Probability* (Keynes, 1921; henceforth TP) is a crucial reference to understand in depth the *General Theory* (henceforth GT) published a few years later (Keynes, 1936). This nexus has been neglected or downplayed for many decades. Only recently its importance has been fully recognized (among the early contributions we mention Carabelli, 1988, and O'Donnell, 1989). However, its importance and scope remain quite controversial.

In this paper I intend to clarify the controversial concept of "weight of argument" as introduced in the TP and explicitly resumed in crucial passages of the GT, in order to assess its influence on the theoretical framework and methodological approach of the GT. To this end I will carry on a preliminary examination whether, and for what reason, we should expect that the weight of argument has a significant impact on economic decisions. As is well known, Keynes himself admitted to be puzzled over the importance to be attributed to this 'somewhat novel' concept (TP, p.77); in particular, he admitted on a few occasions to be uncertain about its 'practical significance' (TP, pp.83, 345, 348), also because 'it is difficult to think of any clear example of this' (TP, p.83) that exemplifies and corroborates its importance. As a matter of fact the concept of weight of argument was not really new when Keynes wrote the TP. Keynes himself refers back to writers on probability who at the end of the 19th century explicitly, although briefly, raised the question: in particular Meinong and Nitsche '(TP, p.84-85). From the theoretical point of view, the contribution of Keynes is not much more than a reappraisal of the concept within the framework of his own theory of probability. What is really new is his tentative application of the concept to the explanation of economic decisions in the GT. In our opinion Keynes's innovative method suggested in the GT cannot be properly understood without understanding the crucial role played in it by the weight of argument.

As is well known, in the GT Keynes explicitly refers to the weight of argument in crucial passages of his reasoning showing its significant impact on economic decisions. This provides a crucial example of its practical role that was missing in the TP and shows why the weight of argument should play a crucial role in a satisfactory account of how a monetary economy works. Unfortunately the passages of the GT referring to

the weight of argument have been long neglected by followers and interpreters. This depended on many factors. One is the difficulty of providing an operational definition of the weight of argument integrating it within analytic or econometric models. A second factor has been the emergence since the early 1930s of decision theories under uncertainty characterized by rigour and operational power (Ramsey, 1931; De Finetti, 1937; Morgenstern-von Neumann, 1944; Savage, 1954); these theories have deeply influenced the foundations of economics, including mainstream Keynesian macroeconomics after the death of Keynes, in such a way to exclude any possible role for the weight of argument. A few recent development in epistemology and decision theory under uncertainty have reopened the issue, providing at the same time new analytical instruments capable to translate Keynes's intuitions in rigorous and operational instruments.

2. Definitions of 'weight of argument'

Given two sets of propositions, the set *h* of premises and the set *x* of conclusions, an argument *x\h* is according to Keynes a logical relation the knowledge of which permits one to infer *x* from *h* with a certain degree of rational belief *p* that defines the probability of *x* given *h*. The epistemic and pragmatic relevance of an argument depends on his view not only from its probability but also from its 'weight' (TP, pp. 72-85 and 345-349; GT, pp. 148 and 240). The concept of 'weight of argument' (also called by Keynes 'weight of evidence') has been interpreted in different ways by the readers of the TP and the GT. We find in the TP different definitions that, at least at first sight, do not seem altogether congruent:

i) According to a first definition often repeated in chapter 6 of the TP headed 'The weight of argument', 'one argument has more *weight* than another if it is based upon a greater amount of relevant evidence' (TP, p.84).
ii) According to an alternative definition that may be found in the same chapter, the weight of argument 'turns upon a balance … between the *absolute* amounts of relevant knowledge and of relevant ignorance respectively' (TP, p.77).

iii) Finally, in chapter 26 the weight of argument is defined as ‘the degree of completeness of the information upon which a probability is based’ (TP, p.345).

Each of these three definitions aims to measure the degree of knowledge relevant for the probability; however, the first measure is presented as absolute, the second measure is relative to relevant knowledge, the third is relative to the complete relevant knowledge. In my opinion, contrary to the first-sight appearance the three definitions, correctly understood, are fairly consistent and may be represented by the same analytic measure.

Most interpreters picked up the ‘absolute’ definition identifying the weight of argument simply with the amount of relevant knowledge *K*. The choice of this definition may depend on the fact that it appears at the very beginning of the chapter 6 on the weight of argument and on the frequency of its explicit and implicit references in the TP. Therefore most interpreters believe that a satisfactory measure of the weight of argument may be given simply by:

$$(1) \qquad V(x\backslash h) = K.$$

In my opinion, however, this measure is inconsistent with Keynes’s crucial assertion that additional evidence may increase the relevant knowledge without increasing the weight of argument: ‘the new datum strengthens or weakens the argument, although there is no basis for an estimate how much stronger or weaker the new argument is than the old.’ (TP, p.34). This reflection clarifies the *rationale* of the second definition. Unfortunately, this important clarification may be found not in chapter 6 on the weight of argument, but in chapter 3 on the fundamental ideas of the TP, before having explicitly introduced the concept of weight of argument, what may explain why the assertion of Keynes has been often neglected. We notice that, according to Keynes, the new evidence may reduce the weight of argument as it may alert the awareness of the agent that the gap between her relevant knowledge and complete relevant knowledge is bigger than she believed before.

Consistently with the preceding considerations and the second definition of weight of argument, Runde (1990) suggests the following measure:

(2) $V(x\backslash h) = K \backslash I$

This simple ratio between relevant knowledge and relevant ignorance takes account of the exigency, emphasized by Keynes in his second definition, of taking in due account the relevant ignorance. This measure implies, differently then in the first one, that the weight of argument increases only if the relevant knowledge increases more (decreases less) than the relevant ignorance. Also this measure, however, is not fully satisfactory because it is meaningless when the relevant ignorance is zero (complete relevant knowledge), and takes values tending towards infinite, absurd and hardly operational, for values of the relevant ignorance tending to zero.

We may overcome these shortcomings by introducing the following measure that descends from the third definition:

(3) $V(x\backslash h) = K \backslash (K + I)$

In this case the weight of argument increases only if the relevant ignorance diminishes. This measure has the advantage of being clearly defined also in the extreme case of complete relevant knowledge ($V(x\backslash h) = 1$). In addition, its range of values from zero to one is consistent with the usual measures of uncertainty, such as probability, and conforms to the range of values that also Keynes seems to have in mind (see TP, p.348).[1] Therefore I conclude that the third definition, as expressed in the relation (3), is the most general and satisfactory of the three definitions of weight of argument as it takes account explicitly of the relation between relevant knowledge and both relevant ignorance and complete knowledge. Therefore, in what follows, I will define and measure the weight of argument as in relation (3).

[1] By introducing the usual criterion of normalization of probability measures $K + I = 1$ (criterion that has been utilized by Keynes himself in the TP, for example in p.348), this measure subsumes also the first definition.

3. Weight of argument and modalities of uncertainty

In the TP Keynes often illustrates general concepts by examining specific instances considered as particularly important or emblematic. This exposition method has the advantage of favouring a constant intuitive control of the meaning of arguments, but it may jeopardize the rigorous definition of concepts. In the preceding section we have seen a significant example of this kind of difficulty. In order to clarify the distinction between probability and weight of argument Keynes insists on the particular case of a new evidence that may increase or decrease the probability of an event while increasing at the same time the weight of argument. This happens, we may add, only when the new piece of evidence reduces the relevant ignorance of the DM. This emblematic example illustrates in an intuitive way the semantic difference between probability and weight of argument but it may mislead the reader if he is induced to believe that an increase of relevant evidence necessarily implies an increase in the weight of argument. This is not necessarily true because an increase in relevant knowledge may increase the awareness that relevant ignorance is deeper than he believed before. As Socrates, Plato and many other eminent philosophers often maintained, "wisest is he who knows he knows not".

In order to go deeper into the problem, we have to clarify the concept of uncertainty. We may start from a generic definition that I believe to be coherent with Kenes's epistemic approach: uncertainty is in its essence "rational awareness of ignorance". We have to distinguish between ignorance relative to a conclusion x of an argument A (given the premises h) expressed by the probability, and the ignorance relative to the argument as expressed by the weight of argument $V(x\backslash h)$. The weight of argument is expressed through a proposition having as object the argument A. This establishes a hierarchical relation between probability and weight of argument that may be expressed in the following way: the probability of a proposition expressing the conclusion of an argument given the premises is a first-order measure of uncertainty while the weight of argument is a second-order measure having as object the reliability of the first-order measure.

The concept of weight of argument, as defined and measured above, permits a clear definition of different modalities of (first order) uncertainty, that may be ordered on the basis of an homogeneous criterion. The range of values of $V(x\backslash h)$, as defined in relation

(3), goes from 0 to 1 and allows a distinction between three modalities of (first-order) uncertainty that play a different role in Keynes's analysis in the TP and then in the GT. Uncertainty may be defined as "radical" when the weight of argument is nil. In other words, the decision maker (DM) is aware that he does not know anything relevant about the occurrence of a certain event. For example: "… the prospect of a European war is uncertain, or the price of copper and the rate of interest twenty years hence…about these matters there is no scientific basis on which to form any calculable probability whatever. We simply do not know" (Keynes, XIV, pp.113-114). On the contrary, the uncertainty may be defined as "soft" (or weak) when the weight of argument is one. In other words, in this case the DM is uncertain only in the weak sense that he does not know which from a set of possible events will occur but believes that he knows their "true" probability distribution. This is the typical case in a game of chance as the emblematic case of roulette games well illustrates. If the roulette is fair, the DM knows exactly the complete lists of possible events and knows the "objective" or "true" probability of each of the possible events. Traditionally only these two extreme cases have been considered. The weight of argument clarifies that between the two extremes, the white of soft uncertainty and the black of complete relevant ignorance, there is a wide grey zone characterized by the awareness on the part of the DM that his relevant knowledge is not complete but not nil. It is thus rational to exploit all the relevant knowledge. The weight of argument allows a measure of the degree of incompleteness of relevant knowledge and provides a guide for its rational exploitation.

(Fig.1 about here)

This threefold classification of uncertainty that we may represent in graphical terms as in fig.1 emerges naturally –we believe- from the interpretation here suggested of weight of argument, but it is not universally accepted. Many interpreters of Keynes believe that what Keynes had in mind was a simple dichotomy between weak uncertainty (that may be expressed by probability) and radical uncertainty (or 'uncertainty' in its strict sense): see e.g. Davidson, 1988 e 1991. There is no doubt that a dichotomy of this kind often

appears in Keynes's economic arguments but, in our opinion, its role is that of emphasising the hierarchy between first-order and second-order uncertainty. In any case, we want to show that the interpretation that focuses on a simple dichotomy hurts against a series of textual and contextual difficulties.

The first observation refers to radical uncertainty and takes into account a qualification by Keynes. In this case the knowledge relevant for probability is altogether absent and this prevents the use of probability. Keynes maintains that this is already true for a value of the weight of argument inferior to ε that defines the minimum degree of relevant knowledge that makes probability meaningful. As Keynes emphasizes (TP, p.78):

> 'A proposition cannot be the subject of an argument, unless we at least attach some *meaning* to it, and this meaning, even if it only relates to the form of the proposition, may be relevant in some arguments relating to it. But there may be no other relevant evidence… in this case the weight of the argument is at its lowest'

According to the dichotomic view of uncertainty modalities, when the weight of argument is maximum (one in our interpretation) the probability is either one or zero. This assertion seems at first sight inescapable as the completeness of relevant knowledge seems to imply the convergence of probability towards one of its extreme values (zero or one; see on this point in particular O'Donnell, 1989). This thesis, however, is misleading. The knowledge that intervenes in the definition of weight of argument is, according to Keynes, the relevant knowledge that may be acquired by an epistemic subject characterized by bounded rationality. In general, thus, as the weight of argument increases, the probability converges towards a more reliable value that may be whatever value between zero and one (extremes included). This conclusion should be obvious as soon as we refer to games of chance. If the DM knows that a dice is fair, the probability of any of the numbers written on its faces is assumed to be equal to 1\6 and this assertion has the maximum weight (equal to one). Even if we believe that the outcome of the dice toss ultimately depends on deterministic factors and we share the opinion of Laplace that a demon knowing all the relevant initial conditions would be able to forecast exactly the outcome, this is patently beyond human reach. It would be meaningless in a case like this to maintain that an argument based on the probability 1\6

for each number on the dice has a weight inferior to the maximum one. We have thus to conclude that the probability converges toward its extreme values (zero or one) only in a deterministic argument. In addition, we have to emphasize that the weight of argument has a significant role in decision theory only when uncertainty is hard. As a matter of fact, if uncertainty is radical, probabilities are groundless, while when uncertainty is weak probabilities are seen as fully reliable. Only when uncertainty is hard, a change in the weight of argument may affect economic decisions (see sections 4 and 5). In the GT Keynes refers to the weight of argument within a conceptual framework based on the distinction between probability (when the weight of argument is maximum) and genuine "uncertainty" in the other cases. He wants to stress how demanding is the hypothesis of soft uncertainty underlying classical economics and how fragile is the approach based on such an extreme assumption: a small deviation from the assumption that the agents have complete relevant knowledge is enough to produce deep modifications in financial and real choices and in the theories that account for them. The prevailing interpretation identifies with radical uncertainty what Keynes calls simply uncertainty in contraposition to probability. This seems justified by a few passages where Keynes refers uncertainty to complete relevant ignorance (as the famous passage, too well-known to be reported here, qualifying his reply in 1937 to the early critiques to the GT (CW XIV, pp.113-114)). However, this interpretation does not work in different crucial passages of the GT where Keynes focuses on the effects of changes in one or more crucial variables on the degree of uncertainty perceived by the economic agents. The interpretation of these variations as a jump between extreme values of the weight of argument would be misleading. The weight of argument can play an active role in the causal analysis only in the hypothesis of hard uncertainty when a change in the weight may bring about a different behaviour.

To avoid confusion we believe that the distinction between probability and uncertainty should be interpreted not as a dichotomy between two extreme modalities of first-order uncertainty, but as a distinction between two levels of a hierarchy: *probability* is a first-order uncertainty measure while 'uncertainty' in its strict sense refers to second order uncertainty as measured by the weight of argument. We may understand in this way why Keynes relates the weight of argument to uncertainty in the

strict sense and why the classical economists by neglecting altogether this dimension of analysis limit themselves to consider probability in the hypothesis of soft uncertainty.

4. Practical relevance of the weight of argument: preliminary analysis

Keynes does not make clear why one should, *ceteris paribus*, prefer to give adequate foundations to a decision on the basis of an argument having a higher weight. As he stresses, the Bernoulli's advice that we 'must take into account all the information we have, amounts to an injunction that we should be guided by the probability of that argument, amongst those of which we know the premisses, of which the evidential weight is the greatest' (TP, p.83 and 345-6). He then recalls the decision rule suggested by Locke in the following maxim 'he that judges without informing himself to the utmost that he is capable, cannot acquit himself of judging amiss' (quoted in TP, p.84, n.2; see also p. 6). This second rule links in a crucial way weight of argument and learning: 'when our knowledge is slight but capable of increase, the course of action which will, relative to such knowledge, probably produce the greatest amount of good, will often consist in the acquisition of more knowledge' (TP, p.83). If we take Locke's prescription too literally, we could undermine the practical relevance of the weight of argument by inferring that a rational agent should take a decision only when the relevant knowledge is complete. However Keynes rightly observes that as soon as we take account of the practical constraints to decisions (time horizon of the decision and costs of acquiring new information), it is rational to reduce the relevant ignorance and thus to reinforce the weight of argument only up to a threshold that in general is short of its maximum value (*ibidem*). Therefore the weight of argument cannot be ignored as the pursuit of a satisficing value is part of the decision strategy of a rational decision maker (TP, p.342).

The reason for taking into account the weight of argument in decision making has not made explicit neither by Keynes, nor Bernoulli nor Locke in the passages cited. We could speculate that the higher is the weight of argument the lower is the probability of deviating from the target. However, Keynes maintains that, in principle, the weight of argument is independent of the expected error or 'probable error': 'there is…no reason

whatever for supposing that the probable error must necessarily diminish, as the weight of argument is increased' (TP, p.82). This observation is a source of perplexity for Keynes himself; we believe that this dilemma can be solved by recalling the distinction, routinely made in statistical inference and econometric estimation between stochastic and systematic error. We suggest that the weight of argument is altogether independent of the stochastic error but is strictly correlated with the expected systematic error. A higher weight of argument reduces the expected systematic error and it is exactly this property that confers practical relevance to the weight of argument. The stochastic errors are by definition inevitable as they depend on a host of (by definition) unknowable exogenous factors; however, the stochastic errors have a practical relevance for decision making because their properties determine the nature and size of the risk associated to a decision. If we admit only the presence of stochastic errors, the weight of argument is maximum because the relevant knowledge cannot be further increased. On the contrary, a weight of argument inferior to one implies the awareness of possible systematic errors that are the more significant the deeper is the relevant ignorance. Systematic errors diminish with learning to the extent that the latter diminishes the relevant ignorance.

Although the confidence in the conclusions of a non-demonstrative argument relies upon the expected value of both the stochastic and systematic errors, these two determinants should be kept separated as they depend on completely different factors: the risk and the weight of argument. The practical relevance of risk depends on the attitude towards risk, while the practical relevance of the weight of argument depends on the attitude towards (second order) uncertainty. The weight of argument, however, is important also for a second fundamental reason that breaks the symmetry with the analysis of risk as it may be interpreted as an index of potential learning. The crucial nexus between the weight of argument and learning is already altogether evident in the TP, but it is only in the GT that Keynes formulates the fundamental principle that gives the weight of argument a high degree of practical relevance, in particular in his analysis of liquidity preference: the lower the weight of argument, and thus the higher the potential learning, the higher the degree of intertemporal flexibility sought by a rational agent (Basili and Vercelli, 1998).

5. The weight of argument in the light of theory of decision under uncertainty

To assess in depth the practical relevance of the Keynesian concept of weight of argument it is sensible to take into account the remarkable advances of decision theory under uncertainty (henceforth DTU) since the publication of TP (see Camerer and Weber, 1992).

The DTU reaches a stage of maturity with von Neumann e Morgenstern (1944) who succeed to provide sound foundations by axiomatizing it from the point of view of objective probabilities. Its empirical scope is however limited to what we have called soft uncertainty. Probabilities are considered by the DM as "known" that is as fully reliable. In this case the weight of argument does not have any practical role since by assumption is always equal to 1.

A few years later Savage (1954), building on ideas put forward by Ramsay (1931) and De Finetti (1937), suggests a different axiomatized DTU that pretends to be applicable to whatever situation characterized by uncertainty, In this subjectivist theory, often called Bayesian, probabilities are conceived as epistemic weights that assure the coherence of the decisions of a rational agent. De Finetti e Savage believe that the distinction of different modalities of uncertainty, and thus also of concepts such as the weight of argument that presupposes it, is inconsistent with rationality. The main argument has been put forward by de Finetti who developed in the form of a theorem intuitions put forward by Ramsay. He showed that if the beliefs of the DM are not represented in the form of a unique distribution of additive probabilities, as in Bayesian theory, he is vulnerable to accept a *Dutch book*, that is a system of bets whose acceptation is irrational as it does not involve a possible positive payoff. The assumption that the beliefs are represented by a unique distribution of additive probabilities implies that the DM has complete relevant knowledge so that his uncertainty is soft. Therefore in this view only weak uncertainty is consistent with rationality and this excludes any normative role for the weight of argument. Savage reinforced this conclusion by observing that the introduction of a second-order measure of uncertainty would trigger an infinite regress that in his opinion would be unacceptable from the logical point of view (Savage, 1954).

The state-of-the-art textbook exposition of decision theory under uncertainty makes a basic distinction between “known” and “unknown” probabilities to articulate a simplistic division of labour between objectivist and subjectivist theories of decision (Vercelli, 1999). According to this approach when the probabilities are “known” (as in the case of a “roulette game”) it is prescribed the use of the objectivist theory introduced by von Neumann e Morgenstern, while when the probabilities are “unknown” (as in the case of a “horse race”) it is prescribed the use of the subjectivist theory introduced by Savage. This widespread eclectic view seems to introduce a distinction between two different modalities of uncertainty providing an opportunity for the use of the weight of argument seen as degree of knowledge of probabilities. However a deeper analysis shows that the distinction between known and unknown probabilities is confined to their source (stable frequencies in the objective approach and coherent subjective assessment in the Bayesian theory) and does not affect the modality of uncertainty that in both cases is represented through a unique distribution of additive probabilities (ibidem). Also the axioms of the two theories are expressed in a different language but are substantially equivalent. In particular, in both cases the axioms exclude that a rational agent makes systematic mistakes. It is assumed that the probability distribution is not modified by the choices of the agents and that its structural characteristics are perfectly known by them: the DM knows the complete list of possible world states, of the available options and the consequences of each option or *act* in each possible state of the world. These assumptions presuppose that the world be closed and stationary and that the agent has fully adapted to such a ‘world’ (Vercelli, 2002 and 2005). In both cases the weight of argument is maximum and its explicit introduction would be irrelevant. This common approach of mainstream DTU explains why, up the middle 1980s, most economists and decision theorists expressed sheer hostility against the concept of weight of argument or any other concept presupposing different modalities of uncertainty. However, starting from the second half of the ’1980s, a series of innovative contributions to DTU has progressively spread a more favourable climate to the understanding of the Keynesian insights on the weight of argument (Kelsey and Quiggin, 1992). First, the strength of the obstruction arguments by de Finetti and Savage proved to be weaker than they were believed to be before. The Dutch book argument by Ramsay and de Finetti is based upon implicit assumptions that

are quite implausible in situations in which the weight of argument has a role, i.e. when the uncertainty is hard reflecting an open and non-stationary world. This is true in particular for the assumption that the DM is expected to bet pro or con a certain event; this does not take account of the fact that the refusal to bet could be altogether rational when the weight of argument is far from its extreme values. In addition the argument by Savage about the infinite regress is not convincing since the introduction of a second order measure of uncertainty implies only the possibility of a higher-order measure, not its necessity: the issue whether is useful to introduce a measure of uncertainty of higher order is a pragmatic question not a logical one.

It is now spreading the opinion that do not exist binding in principle objections that preclude the analysis of different modalities of uncertainty and thus bar the use of the Keynesian weight of argument. This shift of attitude is cause and effect of the emergence of new DTUs, not less rigorous than the classical ones mentioned before that presuppose, or at least are consistent with, hard uncertainty and a weight of argument different from its extreme values. Some of these DTUs assume that the beliefs of DMs are to be expressed through a plurality of probability distributions none of which is considered fully reliable. This amounts to evaluate the probability of the occurrence of an event or state of the world through an interval of probability. Other DTUs assume that the beliefs of DMs may be expressed through a unique distribution of non-additive probabilities. This expresses the awareness of the DM that his relevant knowledge is incomplete; in particular in the subadditive case it reveals that the list of possible states or events is not exhaustive (see Vercelli, 1999). The latter assumption may be assumed to be a crucial reference to assess the theoretical and empirical scope of the Keynesian theory of the weight of argument. It is possible to demonstrate that the measure of uncertainty aversion advanced by Dow e Werlang (1992) within this theory

$$c(P,A) = 1 - P(A) - P(A^c), \qquad (4)$$

where A is an event and A^c is its complement, is strictly related to the weight of argument as here defined. In fact we may interpret the relation (4) as a measure of the relevant ignorance; this is true in general of measures of subadditivity of the probability

distribution. In this case, by utilizing the normalization mentioned in section 2, we obtain that the weight of argument is the complement to unity of the measure of uncertainty aversion suggested by Dow e Werlang:

(5) $$c(P,A) = 1 - P(A) - P(A^c) = I/(K + I) = 1 - V(x/h).$$

We can thus conclude that the recent advances of DTU are rediscovering in the context of a different language and formalization the importance of the ideas underlying the Keynesian concept of weight of argument.

6. The weight of argument in the GT

As is well known, the weight of argument is explicitly mentioned by Keynes in the GT in two occasions (the footnotes at pag. 148 and 240). The importance of these references is confirmed also by his correspondence (see, e.g. the well-known letter to Townshend del 1938, CW XXIX, p.293). Although the explicit hints to the weight of argument are scarce, we contend that its role is important as it intervenes in a crucial way, although sometimes only implicitly, in the reasoning. Its role looms wherever Keynes refers to uncertainty in a meaning different from that of classical economics and classical decision theory. Actually, as we have seen in section 3, uncertainty, as distinguished from probability, implies a weight of argument lower than one and vice versa. Therefore the weight of argument is potentially relevant in all the passages in the GT where uncertainty, confidence and expectations play a crucial role. This is true in particular for the passages where expectations affect economic behaviour. In particular sudden and discontinuous shifts of the weight of argument induced by variations in the macroeconomic and policy context of agents decisions determine significant shifts also in the two crucial functions that determine aggregate income: liquidity preference and marginal efficiency of capital.

Before discussing the role of the weight of argument in the GT we have to get rid of a preliminary objection. According to some interpreters,[2] the references in the GT to the TP should not be taken too seriously since Keynes had changed his ideas on probability under the influence of Ramsey (Keynes, 1931). Extrapolating from a famous passage contained in Keynes's review (Keynes, 1931) of Ramsey's posthumous book (Ramsay 1931) many interpreters claimed that Keynes accepted Ramsey's criticisms to the TP and adhered from then on to his subjective approach to probability theory. If true, this assertion would destroy the continuity in Keynes's thought not only in the field of probability philosophy but also between TP and GT making meaningless his subsequent references to the weight of argument. O'Donnell rightly maintained that this crucial interpretive issue should be discussed in the light of all the relevant writings of Keynes before and after the alleged conversion around 1931 (O'Donnell, 1990). After an accurate analysis of this kind he advances seven arguments in favour of the continuity thesis. Further arguments have been put forward by other interpreters (in particular Carabelli, 1988). On this occasion, I do not want to assess these arguments that on the whole seem to me compelling, but only to complement them with some further considerations from the specific point of view of the weight of argument.

I observe first that both Keynes (1921) and Ramsey (1931) believe that the theory of probability is a normative discipline since its rules of inference are based on well-precise rationality requirements. However, according to Ramsey, probability theory has to be seen as a branch of formal logic while, according to Keynes, it has to be treated as an extension of logic to non-demonstrative inference. Before reading Ramsey's own version, Keynes had rejected subjective probability theory because of its psychological and arbitrary nature: "…in the sense important to logic, probability is not subjective. It is not, that is to say, subject to human caprice. A proposition is not probable because we think it so. When once the facts are given which determine our knowledge, what is probable or improbable in these circumstances has been fixed objectively, and is independent of our opinion. The theory of probability is logical, therefore, because it is concerned with the degree of belief which is rational to entertain in given conditions…" (TP, p.4). However in his review of Ramsey's posthumous book, he readily admits that

[2] A list of interpreters emphasizing discontinuity in Keynes's views on probability may be found in O'Donnell (1990, p. 56), while a list of interpreters claiming continuity may be found in Bateman (1990,

"Ramsey… succeeds in showing that the calculus of probabilities simply amounts to a set of rules for ensuring that the system of degrees of belief which we hold shall be a *consistent* system. Thus the calculus of probabilities belongs to formal logic" (X, pp.338-9). According to Keynes, Ramsey's demonstration that the subjective theory may be conceived as a normative theory, qualifies such a theory as an acceptable theory, but only in the case of demonstrative arguments where the inference may be conceived as logical implication and the weight of argument is one. This, however, is an extreme case that applies only when probabilities are numerical. On this point Keynes did not change his mind as explicitly confirmed in his correspondence with Townshend in 1938: "a main point to which I would call your attention is that, on my theory of probability, the probabilities themselves are non-numerical" (*XXIX,* p.289). This argument is sufficient to exclude a conversion of Keynes to subjective probability theory with the only possible exception of the extreme case of numerical probabilities coupled with a weight of argument equal to one. In any case, whenever the relevant knowledge is incomplete and the weight of argument is less than one, the probability inference, to be distinguished from the classical probability calculus to which Ramsey referred, follow different rules that Keynes discussed in the TP and tries to apply to economic decisions in the GT. This is the case in particular of induction, statistical inference (see Carabelli, 1989, chaps.4-7) and causal inference (see Vercelli, 1991 and 2001).

The second issue discussed by Keynes in his alleged retreat is the nature of initial, or a priori, probabilities that provide the basis of the inference. Here Keynes declares himself to yield to Ramsay, agreeing with him that "the basis of our degrees of belief – or the *a priori* probabilities, as they used to be called – is part of our human outfit, perhaps given us merely by natural selection" (Keynes, 1938, *ibidem*). This is not far from his previous point of view as expressed in the TP (see Carabelli, 1989). It is however inconsistent with the assertion, often iterated in the TP, that probability statements are 'objective' in the sense of logic (see quotation above). This assertion was a source of countless misunderstandings with his readers since, as it became evident with Ramsey's criticisms, many interpreters mainly focused on this specific point. With this assertion Keynes wanted to emphasize the irreconcilable distance between his

p.73).

theory and the pre-Ramsay subjective theory by emphasizing that the degree of probability is not to be taken as psychological or arbitrary belief but as the one "which it is rational for *us* to entertain" (TP, p.35). The word "objective" does not aim to have deontological overtones but only to emphasize its non-arbitrary relation with rationality and the word logic does not refer to formal logic, or to the logic of implication, characterizing demonstrative arguments but the extension of logic to non-demostrative arguments. The acceptation of Ramsey's assertion that initial probabilities are inter-subjective rather than objective does not change Keynes's view of probabilistic inference. The crucial difference with Ramsey, before and after 1931, lies in a radically different view of the relationship between probability theory and rationality: "in attempting to distinguish 'rational' degrees of belief from beliefs in general he was not yet, I think, quite successful" (Keynes, 1931, *ibidem*). In fact in Ramsay the rationality requirements of probabilistic inference are too strong for a general theory of probability as they are the same of formal logic and demonstrative argument and apply only to a very limited subset of probabilities when they can be expressed as numerical and the weight of argument is one. On the contrary the initial probabilities may be explained in terms of logic of discovery that in Ramsay has no clear-cut rationality requirements. In Keynes, on the contrary, the probabilistic inference continues to be conceived as "relative … to the principles of *human* reason…does not presume perfect logical insight, and it is …relative to human powers" (TP, p.35). Ramsay's approach, as pursued in particular in his sketch of natural logic, induces Keynes to broaden the scope of non-demonstrative inference whose validity is now seen as relatives not only to the premises and background knowledge but also to the pragmatic and semantic context. This may explain the growing attention for social psychology in the GT, particularly in the passages where he gives a crucial role to uncertainty but this does not change the essential outlines of his theory of probability inference.

The new point of view adopted by Keynes blurs the clear demarcation put forward in the TP between rational and non-rational choice based on 'objective' criteria. This stimulates Keynes to investigate in more depth the grey zone between rational choice in the TP sense and non-rational choices[3] based, as in the real world, on the interaction

[3] This does not mean that these choices are necessarily irrational in the light of more comprehensive concepts of rationality (see Vercelli 2005)

between subjective beliefs and intersubjective beliefs (conventions). As we have seen in section 4, the practical role of the weight of argument refers exactly to this borderline zone of bounded rationality so that its revival in the GT is altogether appropriate and must be taken very seriously (Vercelli, 2005).

In the GT the weight of argument plays a crucial role in the central argument of Keynes. In its absence it would be very difficult to demonstrate the inability of the market to self-regulate itself. In particular Keynes does not deny that the excess supply of labour may bring about a reduction in money wages; this would reduce the money supply in real terms and this should reduce the rate of interest increasing the investment and so reabsorbing the involuntary unemployment. The reason why this virtuous interaction between real and monetary markets is unreliable depends on the increase of perceived uncertainty triggered by deflation leading to a reduction of the weight of argument that shifts the liquidity preference schedule upwards and the marginal efficiency of capital downwards offsetting the potentially positive effects of deflation.

If we assume soft uncertainty as classical economics and DTU do, an increase in the perceived risk of a recession associated to wage deflation does not necessarily shift the two curves in a perverse direction since, at least in principle, the additional risk may be insured through hedging techniques or issuing *Arrow securities* (see Arrow, 1964). On the contrary in the case of hard uncertainty the effects of a change in the weight of argument cannot be insured so that uncertainty (or second-order risk) aversion shifts the curves in the wrong direction jeopardizing the process of adjustment of the market. From the analytic point of view a way out has been concocted by assuming that the liquidity preference curve becomes horizontal a level of the rate of interest higher than that would assure full employment equilibrium (Modigliani, 1944). This way to give foundations to Keynesian analysis and policy pursued by the Neoclassical synthesis should be discarded being ad hoc from the point of view of theory and empirical evidence. On the contrary the approach based on the weight of argument provides proper foundations to the central message of Keynes that the market may be unable to self-regulate itself so that full employment equilibrium can be restored and maintained only through judicious policies of intervention of the state in the economy.

7. Concluding remarks

The concept of weight of argument has been neglected for a long time even by interpreters sympathetic with the basic messages of the GT who believed that this concept was unimportant, outdated or even wrong. In this essay we suggested an interpretation of the concept of weight of argument that we believe consistent with the spirit of Keynes's approach and that permitted us to vindicate its substantial correctness and analytical potential in the light of the recent advances of DTUs. This interpretation confirmed the practical relevance of the concept getting over the doubts expressed by Keynes himself, as well as its crucial role as a foundation of the crucial theoretical and policy message of Keynes. It is thus possible and opportune to resume the research programme, suggested by Keynes with some timidity, meant to analyze the role of the weight of argument in economic decisions.

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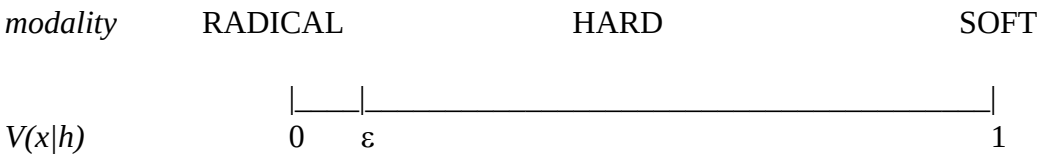

Figure 1: Weight of argument and uncertainty

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