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Abstract

In recent years internet auctions have attracted much attention. This paper discusses a possible explanation for empirical evidence, notably in fixed-end auctions such as eBay, showing a low number of offers early in the auction, with most of the bids concentrated towards the end of the auction. This "last minute bidding" (sniping) phenomenon was first investigated by Roth and Ockenfels (2002). Unlike standard auctions where each offer is processed by the auctioneer, due to system traffic and connection time in eBay close-end auctions very late bids may get lost and left unprocessed by the auction site. Based on this observation, in a simple two-player game theoretic model with private values, we analyze how the possibility that a bid might not be accepted by the system could affect bidders' strategic decision to offer late. We find that late bidding, with no offer early in the auction, could be a Nash Equilibrium if players' values for the object on sale are not too different, with the difference depending upon the relevant probabilities for last minute bids to be successfully placed.

KEYWORDS: eBay auctions, last minute bidding, congestion

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1. Introduction

Internet auctions have recently attracted much attention (Varian, 2000; Bajari-Hortacsu, 2003, 2004a-2004b; Wintr 2005), both because of their vast commercial success but also because of some interesting empirical evidence concerning bidders' behaviour, which was considered as puzzling. This paper focuses on one of their main intriguing features given by the so called "last minute bidding" phenomenon, also named *sniping*, first identified by Roth-Ockenfels (2002) in auctions ran by the eBay and Amazon sites (Schindler 2003; Barbaro-Bracht 2006). The two auction sites adopt a "second price" design, which can either be seen as an extension to a dynamic context of the Vickrey (1961) second-price sealed bid auction, or alternatively as a generalization of the standard oral English ascending auction.

Though they share a number of similarities, a main difference between the two auctions sites is their end-rule, which can account for rather different bidding behaviour by participants (Ariely-Loewenstein-Prelec, 2005). In particular, the phenomenon of late bidding appears to be much more visible in eBay auctions, characterized by a fixed-end length, and this is why in the paper we concentrate on them. Unlike Amazon, where an auction can be extended beyond the initially predefined length until no offer is received within the last ten minutes from the last valid bid, in e-Bay the auction length can not be changed, independently of when bidders make their offers.

Single-object eBay auctions work as follows. Participants submit their offers, which the system receives and conceals to the other participants; the system interprets the offers as bidders' maximum willingness to pay for the object. What instead becomes commonly known, publicly visible on the site screen, is the current price, namely the price at which the object would be sold should the auction end at that time. The current price is computed by adding to the second highest offer, received by the system, a (small) monetary amount fixed by eBay before the auction starts. This is the price at which the bidder who submitted the highest offer wins and pays the object. For this reason, this is a second-price type of auction; moreover, the auction is ascending because bidders, while competing, can not submitted offers lower than the ones that they placed earlier.

Given that the winner will pay less than what she offers, and since offers are kept secret by the system, the eBay site suggests participants to place their maximum willingness to pay early in the auction, which would then be handled by the system in the way we described. In doing so the site seems to translate to its generalized version of the Vickrey design a main feature of the standard sealed-bid format, namely that bidding the own object evaluation is a weakly dominant strategy. Clearly, if followed, the indication would induce participants to bid early in the auction and prevent multiple offers. Empirical evidence however shows the contrary. Indeed, during the auction they may offer more than once, though early offers are not competitive, while most of the (winning) bids concentrate around the very end of the competition. The *intensity* of late bidding has however different degrees, depending upon (i) the end-rule of the auction and (ii) the type of good being auctioned. In particular, as we remarked earlier, it is more likely to occur with fixed-end but also common value auctions, for example such as those in which antiques are sold.

Among the explanations offered for late bidding, recently summarised and extensively discussed by Bajari-Hortacsu (2004a), Roth-Ockenfels (2002) identify the possibility to prevent incremental bidding, or naive bidding (when the eBay auction is confused with a first price English Auction). Bajari-Hortascu (2003) emphasised the presence of late bidding as a way to prevent other bidders to learn from one's offer, which instead might occur when bids are placed early in the competition. Indeed bidding at the last minute would reduce, if not eliminate, the possibility of being outbid. This argument would be particularly cogent in common value auctions, where participants are uncertain about the object value, and learning from others is important. Indeed, it is in a common value framework that they characterize late bidding as a symmetric Nash Equilibrium of the eBay auction game. Some other contributions have instead emphasized the sequential nature of eBay auctions (Kolodziejczyk, 2003; Wang 203), within which late bidding could be rationalized.

More recently Ockenfels-Roth (2006) proposed a game theoretic characterization of the eBay auctions in which they show that no weakly dominant strategy exist, as well as formalize late bidding in terms of Nash Equilibrium with both common and private values. Their results hinge on the probability p, for a last minute bid to be successfully placed. In their model however, as long as p is less than one its value does not play any specific role.

Within a simple single object private value, two players and two periods, model this paper instead focuses on the possible strategic content of internet traffic, *system congestion*, as formalized by the value of the probability for a successful bid. If subjects are aware that because of traffic the system may not accept some late offers, then their bidding decisions may be influenced. Hence, quite plausibly, the choice between bidding early or late may stem from the resolution to the following trade-off. Bidding early one's reservation value would imply placing the offer for sure, optimizing the probability to win. However, if all players bid their object value soon in the auction the winner will pay the highest possible price and expected profits will be at their lowest level. In submitting her best offer late instead there is a risk that it would not be placed successfully, since internet congestion could prevent the offer from being processed. However, should the bid get through, it's likely that in case of victory the object price will be lower than with early bidding only, just because some of the other offers might have not been processed. Hence, intuitively, late bidding only may be a choice when the risk of not placing the bid is low relatively to the own object value. If an object is particularly valuable to a bidder, then she may not want to run the risk of losing the auction, and decide to bid (also) early. Therefore, what could drive the decision solving the trade-off between early offers and late bids is the way the object value and the probability of a successful last minute bid are related. Though this intuition seems to be shared no attempt has been made to explore the conditions, on the object value and the success probability, characterizing last minute bidding only as an optimal decision.

This is the goal of the paper where the main finding is that late bidding, with no offers in the early part of auction, could be a Nash Equilibrium of the game when players' values for the object are not too different, and the degree of difference is determined by the relevant probabilities for successful last minute bids. Moreover, such equilibrium is Pareto superior to the one suggested by the eBay site. Therefore, the paper could be seen as a contribution to the understanding of last minute bidding as an equilibrium phenomenon, as well as why it prevails on other possible bidding behaviour.

2. The Model

We consider an independent, private value, auction with a single object on sale and two players (*1* and *2*), whose object (reservation) values are given by $v_1>v_2>0$. To simplify the analysis, time t will be discrete and such that $t=0, 1$. We interpret $t=0$ as the "early part", and $t=1$ as the final part ("last minute"), of the auction. The main difference between the two periods is that at $t=0$ all bids are accepted with certainty by the system while, instead, at $t=1$ this may not be the case.

We begin with the following definition of a pure bidding strategy. This is the notion of strategy that we shall work with in the paper where, with no meaningful loss of generality we shan't discuss mixed strategies.

Definition 1 *(Bidding Strategy) A (pure) bidding strategy, for player* $i=1,2$ *in the auction game is a pair of non negative numbers* $b_i=(b_i(0); b_i(1))$ *specifying, at each* t*, the bid submitted to the system. No bidding at* t*, with* $t=0,1$*, is indicated by* $b_i(t)=0$*. If* $b_i(1)>0$ *then* $b_i(0)<b_i(1)$*.*

Namely, either a player does not bid at $t=1$ or otherwise her bid must be higher than $b_i(0)$. We assume that the auction system will automatically reject any strategy which does not satisfy such property. Finally, the above notion of strategy does not depend upon the opponent's bidding behaviour. In other words, we consider players planning their course of actions independently of what the opponent does at $t=0$. However, by definition in equilibrium players will react optimally to the opponent's strategy. Here too, with such simplification will shall not lose much generality. We now develop and formalise the idea of *system congestion* due to internet traffic.

Assumption 1 *Let* $b_{-i}=(b_{-i}(0), b_{-i}(1))$ *stands for a strategy played by player i's opponent. Then*

(a) The probability that $b_i(t)$*, with* $i=1,2$*, will be accepted by the system is*

i) 1 *if* $t=0$

ii) α *if* $t=1$ *and* $b_{-i}(1)=0$

ii) λ *if* $t=1$ *and* $b_{-i}(1)>0$ *with* $\lambda\leq\alpha$*.*

(b) The probability that both $b_i(1)>0$*, namely the pair* $(b_1(1)>0; b_2(1)>0)$*, will be accepted by the system is* δ*, with* $\lambda\geq\delta$*.*

(c) The probability that no $b_i(1)>0$ will be accepted by the system is $\mu = 1- 2\lambda +\delta$.

Point (i) of the previous assumption establishes that at *t=0* all bids are accepted by the system; namely, there is no meaningful system traffic in the early part of the auction. In other words, we imagine *t=0* to be a "long enough period" during which, even if network traffic was intense and bids processing delayed, there would always be enough time for the players to resubmit successfully.

At *t=1* we consider two possible cases. Conditional only on player *i* bidding, there is a probability that her offer would not get through just because of internet traffic; such probability is given by α. However, if both players make an offer there are four possibilities: no bid is accepted by the system, only player *1's* bid goes through, or otherwise only player *2's* bid or both. The probability that player *i*'s offer will be accepted is λ. We assume $\lambda\leq\alpha$ to formalize the idea that at *t=1* there may be an increase in network traffic, due to the opponent bidding. If $\alpha=1$ and $1>\lambda>0$, then the only reason for traffic would be the opponent's bid, whereas $\alpha=\lambda$ means that the opponent offer would impose no negative externality on player *i*'s probability for a successful late bid. This last case is the one Roth-Ockenfels (2006) consider. Finally, δ is the probability that both last minute offers would go through, which implies that the probability that at least one bid will be accepted is $2\lambda -\delta$. Hence, $\mu = 1 - 2\lambda +\delta$ is the probability that no late bid would be accepted by the system.

We conclude this part noting that (conditional) independence would imply $\delta=\lambda^2$, and so that $\mu=1-2\lambda+\lambda^2=0$ is uniquely solved by $\lambda=1$; hence, with independence, for all $\lambda<1$ there will be strictly positive probability that no offer will be accepted. In the more general case of dependence, namely $\delta\neq\lambda^2$, $\mu=0$ when $\lambda=(1+\delta)/2$; consequently $\lambda-\delta=(1-\delta)/2$ that can take as maximum value *1/2*.

We now introduce the following notions.

Definition 2 *(Highest and second highest bid) By b(I) and b(II) we indicate, respectively, the highest and second highest bid accepted by the system.*

Notice that the above definition allows the possibility for both the highest and second highest accepted bids to have been placed by the same player. For example, if

$b_1=(b_1(0)=10;\ b_1(1)=20)$ are the only bids accepted by the system then $b(I)=b_1(1)=20$ and $b(II)=b_1(0)=10$, are both proposed by player *1*.

Definition 3 *(Offer) For each* $i=1,2$, *we define* $O(b_i)= Max\ \{b_i(0);\ b_i(1)\ |\ \text{accepted by the system}\}$ *to be player i's offer associated to strategy* b_i.

Again, suppose $b_1 = (b_1(0)=10;\ b_1(1)=20)$ and imagine that only $b_1(0)=10$ is accepted by the system. Then, $O(b_1)=b_1(0)=10$, despite $b_1(1)=20$ being higher than $b_1(0)$.

The payoff function of player *1* (the one of player *2* is defined analogously) is then given by

$$\Pi_1(b_1;b_2)= \begin{cases} v_1 - b(II) - \varepsilon & \text{if } O(b_1)\geq O(b_2) \\ 0 & \textit{otherwise} \end{cases}$$

The payoff function is defined in terms of the two players' bidding strategies; its components are however specified by the players' offers. The quantity $\varepsilon\geq 0$ is a (small) monetary amount fixed by the internet auction site which, for simplicity, we assume it as constant; more in general, however, it could depend upon the amount offered. To avoid trivial conclusions, throughout the paper we'll also assume $v_2>\varepsilon$.

We can now give the definition of Pure Strategy Nash Equilibrium (PSNE)

Definition 3 *(PSNE) A PSNE in the auction game is a pair of strategies* $(b_1^*;\ b_2^*)$ *such that*

$$E\Pi_i(b_i^*;b_{-i}^*)\ \geq\ E\Pi_i(b_i;b_{-i}^*)$$

for $i=1,2$ and $b_i\in R^2$.

Though we only consider pure strategies, the above definition is given in terms of expected payoffs, since when players bid at $t=1$ their offers become random variables.

The first result that we present concerns the bidding strategy suggested by eBay

$$b_{i(E)} = (b_i(0)=v_i\ ,\ b_i(1)=0)$$

that consists in placing one's reservation price early in the auction. Assuming throughout a zero reservation price, the finding may provide a first explanation on why bidders do not follow the recommendation of choosing strategy $b_{i(E)}$. Though the two models are somewhat different, as remarked in the introduction an analogous result is also obtained by Ockenfels-Roth (2006).

Proposition 1 *For no* $\varepsilon>0$ *strategy* $b_{i(E)}$ *is weakly dominant.*

Proof Take player i, with $i=1,2$, and consider strategy $b_{-i}{}^{*}= (b_{-i}(0)>0;\ b_i(1) =0)$ chosen by the opponent, such that $b_{-i}(0)< v_i < b_{-i}(0)+ \varepsilon$. Then it is easy to see that

$$E\Pi_i(b_i{}^{**};b_{-i}{}^{*})=0> (v_i - b_{-i}(0)-\varepsilon) = E\Pi_i(b_{i(E)};b_{-i}{}^{*})$$

where $b_i{}^{**}= (b_i(0)< b_{-i}(0);\ b_i(1) =0)$, which implies that $b_{i(E)}$ is not weakly dominant for player i.

In the above proposition *system congestion* plays no role, since when players bid early in the auction $b_{i(E)}$ fails being weakly dominant simply because of the presence of $\varepsilon>0$.

But there could be other reasons for $b_{i(E)}$ not being weakly dominant. On this point the following result makes it explicit that internet traffic can have a role when players consider late bidding.

Proposition 2 *Suppose* $(v_1 - v_2 - \varepsilon)>0$*; then, for no* λ *such that* $1-\alpha\, v_2/(v_1- \varepsilon)+\delta v_2/(v_1 - \varepsilon)<\lambda\leq\alpha$*, with* $\alpha>[(v_1 - \varepsilon) + \delta v_2]/ (v_1 - v_2- \varepsilon)$*, strategy* $b_{i(E)}$ *is weakly dominant.*

Proof Consider strategy $b_i= (b_i(0)=0;\ b_i(1) =v_i)$ for player i, with $i=1,2$. We shall prove that the statement holds for both players.

Starting with player 1 we have

$$E\Pi_1(b_1;b_2)= (\lambda -\delta)(v_1 - \varepsilon) + \delta(v_1 - v_2- \varepsilon)$$

while

$$E\Pi_1(b_{1(E)};b_2)= (1-\alpha)(v_1 - \varepsilon) + \alpha (v_1 - v_2- \varepsilon)$$

and $E\Pi_1(b_1;b_2)> E\Pi_1(b_{1(E)};b_2)$ if and only if

$$\lambda > 1 - \alpha v_2/(v_1- \varepsilon) + \delta v_2/(v_1 - \varepsilon) = f(\alpha) \qquad (1)$$

As a negatively sloped linear function of α it's immediate to see that

$$f(0)\geq 1 \text{ and } f(1)= (v_1- v_2- \varepsilon)/(v_1 - \varepsilon) + \delta v_2/(v_1 - \varepsilon)\geq \delta .$$

Finally, since $f(\delta)=1$ and $\lambda\geq\delta$ from the definition of α the interval $[\lambda,1]$ determines the domain of $f(\alpha)$. Notice also that the condition could be satisfied only when $\alpha^*=(v_1 - \varepsilon + \delta v_2/(v_1 + v_2 - \varepsilon)\leq \alpha$, with α^* solving the equation $f(\alpha^*)=\alpha^*$.

Take now player 2; then

$$E\Pi_2(b_1;b_2) = (\lambda - \delta)(v_2 - \varepsilon); E\Pi_2(b_1;b_{2(E)})=(1-\alpha)(v_2 - \varepsilon)$$

so that $E\Pi_2(b_1;b_2)> E\Pi_2(b_1;b_{2(E)})$ if and only if

$$\lambda > 1 - \alpha + \delta = g(\alpha) \qquad (2)$$

As a negatively sloped linear function of α, it's easy to see that $f(\alpha)\geq g(\alpha)$, for all α in the relevant domains, and the result is proved.

□

The above result formulates sufficient conditions on λ for $b_{i(E)}$ failing to be weakly dominant: the following corollary specifies necessary conditions.

Corollary 1 *The condition $1-\alpha v_2/(v_1 - \varepsilon)+\delta v_2/(v_1 - \varepsilon)<\lambda\leq\alpha$ can be satisfied by some λ only if $\lambda\geq(v_1 - v_2- \varepsilon)/(v_1 - \varepsilon) + \delta v_2/(v_1 - \varepsilon)=f(1)$.*

Proposition 2 and its corollary formalize the intuition that for high enough α and λ the strategy suggested by eBay can not be weakly dominant. If the opponent contemplates late bidding only, then for high enough values of the relevant success probabilities is not optimal to follow the eBay suggestion. This suggests that network traffic can have a strategic role when auction participants decide their bidding strategies.

Notice that the condition $1-\alpha\, v_2/(v_1 - \varepsilon)+\delta v_2/(v_1 - \varepsilon)<\lambda$ can be conveniently rewritten as

$$v_1 <[(\alpha-\delta)/(1-\lambda)]v_2+\varepsilon \qquad (3)$$

showing late bidding to be profitable when the two object evaluations are neither too different, nor not too close since $(v_1 - v_2 - \varepsilon)>0$, with the difference determined by the relevant probabilities. If v_1 is much higher than v_2, namely the object is very valuable to player 1, then she would not want to take the risk of losing the auction by bidding at the last minute.

Furthermore, observe that for λ close to 1 inequality (3) is always satisfied while if $\alpha=1$ and $\lambda=\delta<1$ then (3) does not hold true since $(v_1- v_2- \varepsilon)>0$. Finally, when $\alpha=1$ and in case no "complete congestion" would occur, namely $\mu = 0$ and $(1+\delta)/2=\lambda$, then (3) takes the particularly simple expression $v_1 \leq 2v_2+\varepsilon$.

The conditions established by Proposition 2 go beyond simply preventing weak dominance of $b_{i(E)}$; indeed, in the most interesting case of $\mu=0$ they suffice for last minute bidding only to be a Pure Strategy Nash Equilibrium.

Proposition 3 *Suppose* $(v_1 - v_2 - \varepsilon)>0$, $\mu=0$ *and* $v_1< [(\alpha-\delta)/(1-\lambda)]v_2+\varepsilon$; *then the strategy pair* $b_i^*= (b_i(0)=0;\ b_i(1)=v_i)$, *with* $i=1,2$, *is a PSNE of the auction game.*

Proof First notice that since $\mu=0$ it is $(\alpha-\delta)/(1-\lambda)=(1+\alpha-2\lambda)/(1-\lambda)$, which is increasing in α and obtains its maximum value of 2 at $\alpha=1$. Hence, any v_1 satisfying $v_1< [(1+\alpha-2\lambda)/(1-\lambda)]v_2+\varepsilon$ will also satisfy inequality $v_1<2v_2+\varepsilon$.

Start with player 1 and suppose player 2 chooses $b_2^* = (b_2(0)=0;\ b_2(1) =v_2)$. Then, if $b_1 = (b_1(0);\ b_1(1))$ is the generic strategy adopted by player 1, there are two possibilities:

(i) Player 1 bids at the last minute, namely $b_1(1)>0$

In this case, her expected payoff is given by

$$E\Pi_1(b_1;b_2^*) = \begin{cases} (\lambda-\delta)(v_1 - b_1(0) - \varepsilon) & \text{if } (b_1(0) < v_2;\ 0 < b_1(1) < v_2) \\ (\lambda-\delta)(v_1-b_1(0)-\varepsilon) + (v_1 - v_2 - \varepsilon)\delta & \text{if } (b_1(0) < v_2;\ b_1(1) \geq v_2) \\ \lambda(v_1 - b_1(0) - \varepsilon)+(\lambda-\delta)(v_1 - v_2 - \varepsilon) & \text{if } (b_1(0) \geq v_2;\ b_1(1) \geq v_2) \end{cases}$$

It is easy to see that the highest value $(\lambda-\delta)(v_1-b_1(0)-\varepsilon) + (v_1 - v_2 - \varepsilon)\delta$ can take is $(\lambda-\delta)(v_1-\varepsilon) + (v_1 - v_2 - \varepsilon)\delta$, obtained when strategy $b_1 = (b_1(0)=0;\ b_1(1) \geq v_2)$ is chosen. Moreover, $\lambda(v_1 - b_1(0) - \varepsilon)+(\lambda-\delta)(v_1 - v_2 - \varepsilon)$ can take its maximum value $(2\lambda-\delta)(v_1 - v_2 - \varepsilon)$ by choosing strategy $b_1 = (b_1(0)= v_2;\ b_1(1) \geq v_2)$. Therefore it is $(\lambda-\delta)(v_1-\varepsilon) + (v_1 - v_2 - \varepsilon)\delta > (2\lambda-\delta)(v_1 - v_2 - \varepsilon)$ if and only if $v_1<2v_2+\varepsilon$ which, by assumption, is satisfied. Hence $E\Pi_1(b_1;b_2^*)= (\lambda-\delta)(v_1 - \varepsilon) + (v_1 - v_2 - \varepsilon)\delta$ is the maximum value that player 1 expected profit can take when adopting strategy $b_1 =(b_1(0)=0;\ b_1(1) \geq v_2)$.

(ii) Player 1 does not bid at the last minute, namely $b_1(1)=0$

In this case her expected payoff is given by

$$E\Pi_1(b_1;b_2^*) = \begin{cases} (1-\alpha)(v_1 - \varepsilon) & \text{if } (0 < b_1(0) < v_2;\ b_1(1)=0) \\ \alpha(v_1 - v_2 - \varepsilon)+(1-\alpha)(v_1 - \varepsilon) & \text{if } (b_1(0) \geq v_2;\ b_1(1)=0) \end{cases}$$

which attains its maximum value $E\Pi_1(b_1;b_2^*)= \alpha(v_1 - v_2 - \varepsilon)+(1-\alpha)(v_1 - \varepsilon)$, when player 1 chooses $b_1=(b_1(0) \geq v_2;\ b_1(1)=0)$. Hence, it is under the same conditions $1-\alpha v_2/(v_1 - \varepsilon)+\delta v_2/(v_1 - \varepsilon)<\lambda\leq\alpha$ characterizing Proposition 2, which by assumption hold, that $b_1^* = (b_1(0)=0;\ b_1(1) =v_1)$ is best reply for player 1 against player 2 choosing $b_2^* = (b_2 0)=0;\ b_2(1) = v_2)$.

Now consider player 2 and suppose player *1* adopts $b_1^* = (b_1(0)=0;\ b_1(1) =v_1)$. Then, if $b_2 = (b_2(0);\ b_2(1))$ is the generic strategy chosen by player *2*, she also has two possibilities:

(i) Player 2 bids at the last minute, namely $b_2(1)>0$

In this case, her expected payoff is given by

$$E\Pi_2(b_1^*;b_2) = \begin{cases} (\lambda-\delta)(v_2 - b_2(0) - \varepsilon) & \text{if } (b_2(0) \leq v_1;\ 0 < b_2(1) \leq v_1) \\ (\lambda-\delta)(v_2 - b_2(0) - \varepsilon) + (v_2 - v_1 - \varepsilon)\delta & \text{if } (b_2(0) \leq v_1;\ b_2(1) > v_1) \\ \lambda(v_2 - b_2(0) - \varepsilon) + (\lambda-\delta)(v_{2-} v_1 - \varepsilon) & \text{if } (b_2(0) > v_1;\ b_2(1) > v_1) \end{cases}$$

which can take its maximum value $E\Pi_2(b_1;b_2^*)= (\lambda-\delta)(v_2 - \varepsilon)$ at $b_2^* = (b_20)=0;\ b_2(1) \leq v_1)$

(ii) Player 2 does not bid at the last minute, namely $b_2(1)=0$

In this case her expected payoff is given by

$$E\Pi_2(b_1^*;b_2) = \begin{cases} (1-\alpha)(v_2 - \varepsilon) & \text{if } (0 < b_2(0) \leq v_1;\ b_2(1)=0) \\ \alpha(v_2 - v_1 - \varepsilon) + (1-\alpha)(v_2 - \varepsilon) & \text{if } (b_2(0) > v_1;\ b_2(1)=0) \end{cases}$$

taking as its highest value $E\Pi_2(b_1^*;b_2)=(1-\alpha)(v_2-\varepsilon)$ when player 2 chooses strategy $b_2=(0<b_2(0) \leq v_1;\ b_2(1)=0)$. As a consequence, also in this case it is under the same conditions $\lambda > 1 - \alpha + \delta$ of Proposition 2 that $b_2^* = (b_2(0)=0;\ b_2(1) = v_2)$ is best reply for player 2 against player 1 choosing $b_1^* = (b_1(0)=0;\ b_1(1) = v_1)$, and the result is proved.

In full analogy with the sealed bid version of the second-price Vickrey auction, equilibrium is not unique and, moreover, inefficient equilibria where the object goes to player 2 exist. The following proposition, of which we omit the simple proof, formalizes this.

Proposition 4 *In the auction game there are multiple equilibria. Among them the pair* $(b_{1(E)},b_{2(E)})$*, as well as inefficient equilibria such as the pair* $b_1=(\ b_1(0)=<v_2-\varepsilon,\ b_1(1)=0)$ *and* $b_2=(\ b_2(0)> v_1 - \varepsilon,\ b_2(1)=0)$

Though last minute bidding only could be characterized as a Nash Equilibrium, given the multiplicity of possible equilibria why should bidders choose it, as according to empirical evidence, rather then the eBay recommendation? The following corollary might provide an answer to this last question.

Corollary 2 *Let* $(v_1 - v_2 - \varepsilon)>0$, $\mu=0$ *and* $(v_1-v_2-\varepsilon)/(v_1-\varepsilon)+\delta v_2/(v_1-\varepsilon)<\lambda\leq\alpha$; *then the last minute bidding equilibrium* (b_1^*,b_2^*) *is strictly Pareto superior to the eBay equilibrium* $(b_{1(E)},b_{2(E)})$.

Since the condition $(v_1-v_2-\varepsilon)/(v_1-\varepsilon)+\delta v_2/(v_1-\varepsilon)<\lambda$ corresponds to the one in Corollary 1, Corollary 2 basically says that (whenever existence is guaranteed) the last minute bidding equilibrium is always preferred to the eBay equilibrium by both players. This could provide an explanation on why players may prefer late, rather than early, bidding of reservation values.

3. Conclusions

In the paper we discussed conditions for last minute bidding, with no offer in the early part of the auction, to be characterized as a Nash Equilibrium of an eBay auction type of game. In particular we focused our attention on the strategic role played by the probability of successfully placing a last minute bid. We found that late bidding could be profitable when players' object evaluations are not too different, with the difference being determined by the relevant success probabilities. More specifically, in equilibrium the higher the success probability the less risky is late bidding the more far apart the two players' object evaluation could be. We believe our focus on the critical values for the relevant success probabilities could provide an additional contribution to the understanding of why, in close-end internet auctions, bidders do not tend to bid early in the auction but rather towards its end.

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