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We examine the role of expectations in a model aimed to explain financial fluctuations. The model restates the core of Minsky's financial instability hypothesis, focusing on the role of expectations. The hypotheses concerning the process of formation and revision of expectations are discussed in light of Keynes's epistemological view of the behaviour of boundedly rational agents under conditions of strong uncertainty. These hypotheses are formalized by drawing on recent advances in complex dynamics, decision theory and behavioural economics. We show that widespread use of extrapolative expectations by economic agents produces a high degree of financial instability that may lead to a serious financial crisis, and that the use by economic agents of a mix of extrapolative and regressive expectations reduces the dynamical instability of the model but may give rise to complex dynamics.

KEYWORDS: financial instability, heterogeneous expectations, extrapolative expectations, regressive expectations, complex dynamics

JEL CLASSIFICATION: G01, C61, D84

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1. Introduction

The deep global recession triggered by the subprime mortgage crisis has revived a strong interest in the explanation and policy implications of financial crises. The latter typically emerge just before or during the slumps that terminate business cycles. However, only some of them have a significant impact on the economic activity and the wealth of economic agents. It is therefore important to understand why financial crises are recurrent and why only some of them generate deep and long recessions. With this in mind, this paper restates the financial instability hypothesis (FIH) originally developed by Minsky (see e.g., Minsky, 1975, 1982, 2008/1986) in order to make it a more manageable instrument of analysis. The focus is limited to what we believe to be the core of this conceptual framework, i.e. the interaction between the financial conditions of the economic units and their fluctuations. Therefore in this paper the interactions between the financial and the real sides of the economy remain implicit. The analysis, however, is not restricted to the financial sector but applies to all the economic units (firms, banks, and households), although the focus is restricted to the dynamics of their financial conditions.

The proximate starting point of the analysis is the model recently suggested by one of the authors to update and extend the core of the FIH (Vercelli, 2009a,b; see also Vercelli, 2000, Sordi and Vercelli, 2006 and Dieci, Sordi and Vercelli, 2006) from which we draw the underlying vision of the interaction between liquidity and solvency conditions, as well as a classification of the financial structures of the units that we believe to be more general than that of Minsky and more suitable to formal analysis. The model is here modified in order to analyse in some depth the role of expectations in financial fluctuations. In particular, the functional specification of the feedback between the liquidity and solvency conditions is here derived from precise and reasonable assumptions on expectations formation. To the best of our knowledge a thorough analysis of the role of expectations within the FIH is missing. Minsky emphasised the importance of expectations in a sophisticated financial economy characterised by strong uncertainty, often stressing their crucial role within his seminal version of the FIH, but he did not clarify satisfactorily the precise process of expectations formation. On this specific issue, as in many other crucial steps of the argument, he referred back to Keynes's contributions as providing the ultimate foundations of his own analysis (see e.g., Minsky, 1975). However, Keynes himself did not go beyond a stimulating but open-ended analysis of expectations dynamics. He suggested far-reaching epistemological foundations to the analysis of economic agents' behaviour in conditions of strong uncertainty and these included inspiring reflections on the crucial role of expectations formation (see e.g., O'Donnel, 1989; Carabelli, 1988). However he never tried to formulate, in precise analytical terms, his insights regarding expectations dynamics. In the model suggested in this paper we aim to do so within the core of the FIH. Although our formalisation borrows from recent advances in complex dynamics, decision theory and behavioural economics, we believe that it is substantially consistent with the basic epistemological tenets of Minsky and Keynes.

The paper is structured as follows. Section 2 provides the epistemological foundations of the model and provides the rationale for the underlying hypothesis of expectations formation and revision. Section 3 introduces the state variables that we use in the model to measure the liquidity and solvency (or net worth) conditions of economic units. These variables provide a bivariate measure of the economic units' financial conditions that is related to that suggested by Minsky but aims to generalise it while making it fitter for a formal analysis. Section 4 presents the model that is based on the dynamic interaction between the liquidity and solvency conditions of the economic units. The specification of the interaction is derived from the hypothesis of expectations dynamics justified in Section 2 and formalised in this section. Section 5 studies the dynamic behaviour of the model. We show that the adoption of extrapolative expectations by economic agents brings about a high degree of financial instability that may lead to a serious financial crisis and that the use by economic agents of a mix of extrapolative and regressive expectations, as justified in Section 2, reduces the dynamical instability of the model but may give rise to complex dynamics. Section 6 concludes.

2. Behavioural rules, rationality and heterogeneous expectations in financial fluctuations

The FIH was not conceived by Minsky as a full-fledged macroeconomic theory but only as an updating and extension of Keynes's *General Theory* (1936) focusing on the crucial role of the financial structure of economic units in macroeconomic dynamics. Such an extension is based on a reading of the *General Theory* that rejects the interpretations suggested by both the classical economists and conventional Keynesianism (also called the 'neoclassical synthesis'). In Minsky's interpretation, the insights of Keynes concerning economic behaviour under conditions of strong uncertainty play a crucial role. As he often repeated, 'Keynes without uncertainty is something like *Hamlet* without the Prince' (Minsky, 1975, p. 57). In particular he emphasised that the crucial role played by expectations in such a theory depends strictly on Keynes's philosophy of uncertainty as stated in *A Treatise on Probability* (1921) and developed throughout his intellectual career.

Unfortunately, Keynes never provided a comprehensive and systematic account of his views on the role of expectations in economic behaviour and in his writings the most inspiring hints are sparsely distributed. We discuss here only a few of the crucial assumptions underlying his point of view – assumptions that provide the foundations for the treatment of expectations dynamics in our model (see Carabelli, 1988, pp. 224-225). As is well known, Keynes distinguishes between short-run expectations, relative to the proceeds obtainable by the employment of the existing stock of capital (Keynes, 1936, pp. 46 and 148), and long-run expectations that are concerned with investment and the value of capital assets for all their residual life (Keynes, 1936, pp. 47, 147-148 and 246). We are here concerned exclusively with long-term expectations since the financial conditions of economic units crucially depend on them rather than on short-term expectations. We warn the reader that the Keynesian terminology 'long-term expectations' can be misleading as it could suggest that they extend to a fairly distant future. However, Keynes himself clarifies that, under conditions of strong uncertainty, as assumed in the *General Theory* and in this essay, boundedly rational economic agents when making decisions take into account a very limited sequence of periods. One crucial reason is that 'owing to the operation of compound interest combined with the likelihood of obsolescence with the passage of time, there are many individual investments of which the prospective yield is legitimately dominated by the returns of comparatively near future' (Keynes, 1936, p. 163). This depends on the fact that (*ibid*., pp.149-150)

> 'our basis of knowledge for estimating the yield ten years hence of a railway, a copper mine, a textile factory, the goodwill of a patent medicine, an Atlantic liner, a building in the city of London amounts to little and sometimes to nothing; or even five years hence. In fact, those who seriously attempt to make any such estimate are often so much in minority that their behaviour does not govern the market'

According to Keynes, the basic principle of long-term expectations formation is that the agents project into the future the current trends unless they have good reasons to anticipate a change (Keynes, 1936, p. 152; CW XIV, pp. 114 and 124). The agents are generally aware that 'the future never resembles the past' (Keynes, CW XIV, p. 124) and that the present enters into their expectations in a disproportionate manner (Keynes, 1921, p. 275). They are therefore aware that, by extrapolating expectations from recent experience, they are liable to systematic mistakes that will emerge ex post; however their attitude is not necessarily irrational since they do not know in which direction the future will change.

We may clarify this point further by recalling a metaphor used by Descartes in his *Discourse on the Method* (1637, pp. 24-25) to express an illuminating general maxim:

> 'a traveller (…) upon finding himself lost in a forest, should not wander about turning this way and that (…) but should keep walking as straight as he can in one direction, never changing it for slight reasons; for in this way (…) he will at least end up in a place where he is likely to be better off than in the middle of the forest'

This prescription clearly assumes strong uncertainty (on the characteristics of the forest) and suggests that under these circumstances it is rational to stick to a rigid rule of conduct.

A reasonable extension of this metaphor to financial fluctuations suggests a specification of expectations for the object of our analysis. Let us assume that an explorer has to cross a forest of unknown size and dimension to reach a desired destination. It is rational for him to proceed in a straight direction to minimize the risk of getting lost inside the forest. He will then expect that the distance from his base camp will progressively increase according to a principle of extrapolative expectations. However, he has to take account of the fact that his reserves of food are limited so that he will pursue this strategy only up to a well-defined threshold, when about half of his food has been consumed. At that point a rational explorer will go back following the same path in reverse because he does not know how far he is from the border of the forest and whether he will be able to survive by continuing in the same direction. After that threshold, he has to abandon pursuit of the original goal since he has to focus on mere survival. He will then anticipate a progressive reduction of the distance from the base camp according to a principle of regressive expectations.

This metaphor captures something crucial that typically happens in financial behaviour. During the upward phase of the cycle, the economic units try hard to increase their returns, being confident that their financial structure is within the desired solvency safety margin. This leads the units to increase their financial exposure, which we assume to be positively correlated with expected returns, until they have to recognise, typically after a shock, that they have breached their safety margin. After this threshold is reached, their focus switches from the attempt to withstand competition and increase returns to the struggle to secure their own financial survival going back to a safe financial structure by deleveraging. This new attitude translates into a shift from extrapolative expectations pointing towards a progressive increase of financial exposure to regressive expectations pointing towards predicting a progressive reduction of financial exposure. The model presented in this paper is built on this basic intuition about expectations dynamics. The aim is to show that this assumption on expectations formation clarifies the nature of the interaction between the two fundamental financial conditions that characterise the financial structure of economic units and may help to explain why this interaction produces fluctuations of the two variables of varying period and amplitude.

The positive correlation between degree of uncertainty and rigidity in the behavioural rule has been asserted by many decision theorists. According to Heiner, for example, a boundedly rational agent sticks to behavioural rules that are the more rigid the higher the degree of uncertainty: ‘greater uncertainty will cause rule-governed behaviour to exhibit increasingly predictable regularities, so that uncertainty becomes the basic source of predictable behaviour.’ (Heiner, 1983, p. 570). In the case of the FIH, as here revisited, the repertoire of reliable rules of expectations formation is restricted to the choice between extrapolative and regressive expectations. Extrapolative expectations are chosen so long as the focus is on improving returns, while regressive expectations are chosen beyond the safety threshold when the survival motive prevails.

Experimental economics has gathered a very large number of observations based on laboratory experiments, confirming that economic agents generally behave as boundedly rational agents in an environment characterised by strong uncertainty. They base their decisions and expectations on simple heuristics (see, e.g., Kahneman, Slovic and Tversky, 1986). In particular, survey evidence suggests that agents typically resort to a combination of extrapolative and regressive expectations (Ito, 1990; Takagi, 1991). More specifically, these results are confirmed by experimental evidence in asset pricing (Smith, 1991; Sonnemans, Hommes, Tuinstra and van de Velden, 2004).

The hypothesis of expectations formation here analysed also has an evident kinship with the hypothesis of heterogeneous expectations that has been recently applied in macroeconomic dynamics (for a recent survey, see Lines and Westerhoff, 2010). This literature has in common the adoption of a weighted average of two different hypotheses of expectation formation, such as chartists’ and fundamentalists’ expectations (see, e.g., Day and Huang, 1990; Levin, 1997; Brock and Hommes, 1998; Westerhoff and Reitz, 2003 and Chiarella, Dieci and He, 2007, 2009), extrapolative and rational expectations (e.g., Lines and Westerhoff, 2010), extrapolative and regressive expectations (e.g., Westerhoff, 2006a,b,c; 2008). The macroeconomic circumstances change the weight of the hypotheses leading to the prevalence of one or the other mechanism of expectations formation and this feeds back to macroeconomic dynamics. Heterogeneity thus has two dimensions: the variations in the macroeconomic environment and the differences between agents. In our model, for the sake of simplicity, we explore only the consequences of the first source of heterogeneity by assuming that the weight of each hypothesis can only be zero or one. However, this conceptual framework lends itself to a generalisation to the second source of heterogeneity.

Summing up, the process of expectations formation plays a crucial role in Minsky's FIH and in its Keynesian theoretical background but this is a field where the criticism of 'implicit theorizing' as levelled against both Minsky's contributions (Tobin, 1989; Toporowski, 2005) and Keynes's contributions (Leontief, 1937) have some ground. In this paper we aim to make the theory of expectations formation in the FIH more explicit, consistently with its Keynesian epistemological background and in the light of the recent advances in complex dynamics, decision theory and behavioural economics.

3. A bivariate measure of the economic units' financial conditions

The decisions of economic units (banks, firms and households) are heavily affected by their current and expected financial conditions. Thus, in order to study their influence on the economy, we need a measure of the financial conditions of economic units. It appears that the two crucial financial dimensions of units are their liquidity and solvency features, as measured by specific indexes.

As regards units' *liquidity* at time t (f_{it}), we measure it in each period t as the surplus of financial inflows y_{it} over financial outflows e_{it}, all flows being measured in the same time unit, let us say the financial year. This gives:

$$f_{it} = y_{it} - e_{it} \qquad i = 1, 2, \dots, n$$

The *solvency* of an economic unit, on the other hand, may be measured by its net worth (f_{it}^*), defined as the capitalisation of its expected surplus and deficits, namely:

$$f_{it}^* = \sum_{s=0}^{T_i} \frac{\mathrm{E}_{t-1}\left[f_{it+s}\right]}{\left(1+r\right)^s} \qquad i = 1, 2, \dots, n \tag{1}$$

where $\mathrm{E}_{t-1}[\cdot]$ denotes the conditional expectation operator based upon information available at the end of period $t-1$, $T_i \geq 1$ denotes the time horizon of unit i (expressed in terms of financial years), while the nominal interest rate r is used as the discount factor. Given what we have stressed above about the basic principles of long-term expectations formation, we can expect that T_i is rather short (in most cases less or equal to four).

Given this definition of f_{it}^*, the *condition of financial sustainability* for each economic unit i is $f_{it}^* \geq 0$. We can understand this condition in intuitive terms by observing that when $f_{it}^* < 0$, i.e., when the 'net worth' of the financial unit i is negative, the unit is virtually insolvent unless it undertakes a prompt and radical financial restructuring or is bailed out by other units or by the state. It may seem that equation (1) implies rationality requirements inconsistent with the assumption of strong uncertainty underlying the model (see *retro* Section 2). However, this formulation is consistent with a basic insight by Keynes: 'We assume that the existing state of opinion (…) is based on a correct summing up of future prospects' (CW XIV, p. 114). This procedure is correct not in the sense of an actual correspondence with truth – something that is much beyond the capability of a boundedly rational agent obliged to act in a strongly uncertain environment – but only in the *as-if* sense of a useful convention (see Carabelli, 1988, p. 225).

For the sake of simplicity we call f_{it} the *liquidity index* of the unit, and f_{it}^* its *solvency index*. We may apply these two variables describing the crucial financial conditions of the economic units to classify their financial structure by adapting to this model the approach suggested in Vercelli (2009a) (see Fig. 1).

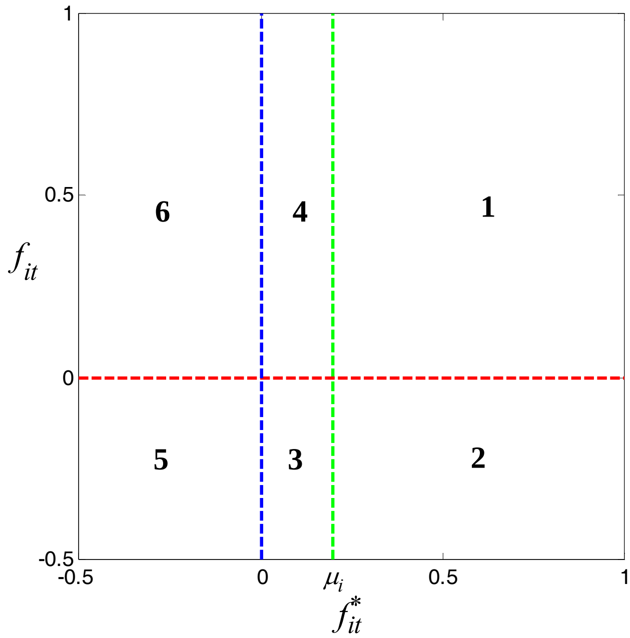

Fig. 1. *A classification of economic units on the basis of their solvency and liquidity conditions*

We adopt this classification rather than the well-known classification of economic units suggested by Minsky (e.g., 2008/1986) because we believe it to be more suitable for a formal analysis (see Vercelli, 2009a). It is possible, however, to establish a link between our representation and Minsky's classification. The vertical dotted line drawn at $f_{it}^* = 0$ represents the *solvency barrier* in the sense that the units to its left have a negative net worth and are thus virtually insolvent. The horizontal dotted line at $f_{it} = 0$ represents the *liquidity line* in the sense that the units beneath it are characterised by a financial deficit. If we consider the space to the right of the solvency barrier, we can easily verify that the units with a solvency/liquidity combination above the red dotted horizontal line (representing the liquidity barrier) may be defined as hedge units in the language of Minsky, while the units underneath may be defined as speculative or Ponzi units. Usually Minsky does not explicitly consider in his classification the units with a negative solvency condition that are virtually insolvent. We believe on the contrary that the units with a solvency/liquidity combination to the left of the solvency line should also be carefully considered. The behaviour of such financially *distressed* economic units, as we shall call them, is crucial to the description, explanation and forecasting of financial crises and to the choice of the best policy measures to keep them under control.

In order to use this Cartesian space for the study of financial fluctuations we need a further essential ingredient. We assume that each economic unit, in order to deal with the risk of bankruptcy, chooses a margin of safety, i.e. a minimum value of its net worth sufficiently higher than zero, beneath which it does not want to go. Let us call this safety margin μ_i. So we have to draw a further vertical line to the right of the solvency barrier (the vertical dotted line drawn at $f_{it}^* = \mu_i$) which we call the *safety line*; this additional threshold allows a refinement of the classification *through the specification of six types of financial structures* (see Fig. 1). Units in region 1 may be called *hyper-hedge* as they do not have problems either from the liquidity point of view or from the solvency point of view ($f_{it} > 0$, $f_{it}^* > \mu_i$). Units in region 2 are *speculative* as they have liquidity problems but do not perceive solvency problems ($f_{it} < 0$, $f_{it}^* > \mu_i$). Units in region 3 are *hyper-speculative* as they have both liquidity problems and solvency problems ($f_{it} < 0, 0 < f_{it}^* < \mu_i$). Units in region 4 are *hedge* units as they do not have liquidity problems but perceive that they may easily incur solvency problems in the future as their safety margin is too small ($f_{it} > 0$, $0 < f_{it}^* < \mu_i$). Finally, we have to consider the units in financial distress, which are virtually bankrupted. We can distinguish between units in region 5, which we define as *highly distressed* financial units as they are both illiquid and virtually insolvent ($f_{it} < 0$, $f_{it}^* < 0$), and units in region 6, which we define as *distressed* units. The latter are virtually insolvent units that have managed in the current period to obtain financial inflows higher than their financial outflows, thus raising hopes of survival ($f_{it} > 0$, $f_{it}^{*} < 0$). This six-fold classification of the financial conditions of economic units keeps a bridge with Minsky's ternary classification but aims to extend its analytic potential and empirical scope.

4. Financial instability and extrapolative expectations

The cyclical dynamics of the financial conditions of economic units in the $\left(f_{it}^{*}, f_{it}\right)$-plane can be described in quantitative terms once the following assumptions are introduced. First, we assume that each unit prefers higher financial returns and that the latter are positively correlated with the liquidity indexes within the desired margin of safety. Beyond such a threshold the units try hard to recover a safe financial posture, even at the cost of reducing returns. Second, we assume that units conform to a common 'herd behaviour' due to the pressure of market and mass psychology so that we may focus on a *representative* unit without forcing too much the realism of the overall picture. Under these assumptions it turns out that there is a tendency to a clockwise dynamics of the financial conditions of the representative economic unit in the space of financial conditions. Let us assume that the representative economic unit initially has safe liquidity and solvency conditions such as to be in region 1 of Fig. 1. This unit may thus improve its financial returns by increasing the outflows more than the inflows without getting into liquidity troubles; in addition, since in this region it has an excess of financial inflows, the unit reduces further its perceived risk of insolvency. This leads the unit into region 2. Beyond the liquidity line the unit has an incentive to increase further the excess of outflows in order to increase its returns until it reaches its margin of safety μ_i. As soon as the unit enters region 3, it then tries to reduce the excessive risk of insolvency by reducing the excess of outflows but it only succeeds in reducing the deficit so that its perception of the insolvency risk continues to increase. Only when the unit breaches the upper liquidity line and enters region 4, does it succeed in recovering a surplus of inflows progressively reducing the risk of insolvency. The unit eventually succeeds in entering region 1 again where it is safe from both liquidity and perceived insolvency problems.

Most units often follow this sequence of financial conditions describing a financial cycle. However, it may happen that the actual margin of safety becomes too small as the unit approaches the solvency line. In this situation the economic unit may be pushed, by a shock, beyond the solvency barrier becoming virtually insolvent (region 5). After the solvency barrier is crossed, the behaviour of the unit has to change radically to avoid bankruptcy. This goal may be reached either through a financial restructuring that abates current and prospective outflows much more than inflows or through a bail-out by the state or another firm. If the unit is sufficiently artful and lucky, it may shift to region 6 and immediately after, to region 4, starting a new financial cycle. In any case it undergoes a sudden and huge reduction of its outflows that in turn reduces the inflows of other units, which are pushed towards the solvency barrier and sometimes beyond it.

Our purpose is now to derive a dynamical system in the two variables f_{it} and f_{it}^{*} capable of generating persistent dynamics of the type we have verbally described.

The crucial behavioural assumptions can be expressed by using the following equation:

$$f_{it+1} = f_{it} - \alpha_i \left(f_{it}^{*} - \mu_i \right) \tag{2}$$

According to (2), when a unit at time t expects the value of its solvency index to be less than the safety margin μ_i, it reacts in the next period by reducing its outflows relative to its inflows in order to recover a safer financial stance. When, on the contrary, the solvency index is expected to be greater than μ_i the unit increases the outflows relative to inflows in order to increase its returns, which are assumed to be positively correlated with f_{it}.

On the other hand, the definition itself of the solvency index f_{it}^{*} in (1) shows that it depends on the capitalisation of the expected values of f_{it} from the current period to the final period within the unit's time horizon. Thus, once we have specified a mechanism of expectations formation and revision, this relation will give us a second feedback relation between the liquidity and solvency indexes of the representative unit. However, this is not an easy task because, by definition, the solvency index is an intertemporal variable. Thus, in order to obtain from it a reasonably simple, workable feedback between the two financial indexes, we need to introduce some simplifying assumptions. In particular, a dynamic recurrent equation for the solvency index can be justified on the basis of the following line of reasoning.[1]

[1] This follows the analysis presented in Dieci, Sordi and Vercelli (2006, pp. 606-607). In order to apply it to the present case, some adjustments are required for the following reasons. First, in line with Keynes's vision, which we discussed in Section 2, we are in this paper considering the case of an extrapolative, instead of an adaptive, expectations-formation mechanism, the latter being more suitable to model *short-run* expectations (see Keynes, CW XIV, p. 182). Second, the variables we are using to describe the financial conditions of economic units are defined in the present paper in a different way. Finally, all the analysis carried out in this paper is for the case in which the representative economic unit uses a finite-time (rather than infinite-time) horizon in order to determine its solvency condition. As a result of this assumption, the analysis gains in plausibility although at the expense of some algebraic elegance. As we shall see at the end of the following section, the parameter that measures the length of the time-horizon of

Let us assume that the economy in the initial period $t-1$ is in a 'tranquil' situation (in the sense of Minsky), characterised by a robust financial system where 'rapid disruptive changes are not taking place' (Minsky, 2008/1986, p. 197). It is reasonable to assume that in a period of this type the unit holds constant expectations about its future liquidity condition, so that:

$$\mathrm{E}_{t-1}\left[f_{it+s}\right]=\bar{f}_i \qquad s=0,1,\ldots,T_i \tag{3}$$

where $\bar{f}_i$ is a level of the liquidity index the unit considers 'normal'.

Inserting (3) into (1), we obtain:

$$f_{it}^{*}=\bar{f}_i\sum_{s=0}^{T_i}\frac{1}{(1+r)^s} \tag{4}$$

Now, denoting by $a=a(r,T_i)$ the ratio $r(1+r)^{T_i}/[(1+r)^{T_i+1}-1]\leq 1$, such that (see Fig. 2):

$$\frac{\partial a}{\partial r}=-\frac{(1+r)^{T_i-1}\left[1+r+T_i r-(1+r)^{T_i+1}\right]}{\left[(1+r)^{T_i+1}-1\right]^2}>0$$

$$\frac{\partial a}{\partial T_i}=-\frac{r(1+r)^{T_i}\ln(1+r)}{\left[(1+r)^{T_i+1}-1\right]^2}<0$$

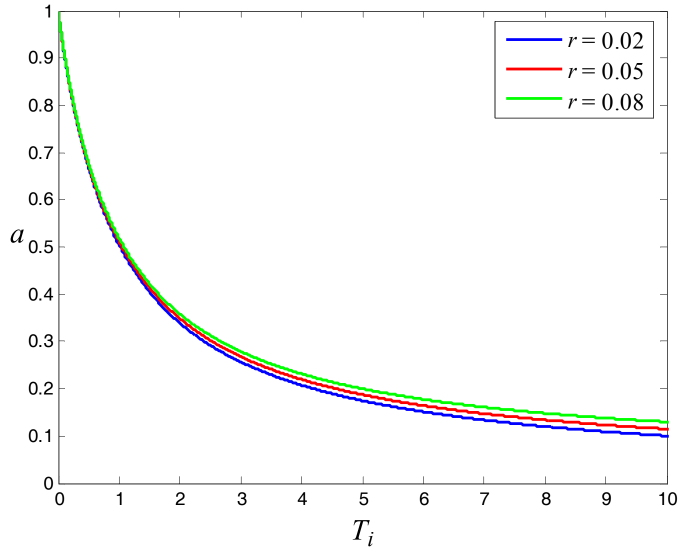

Fig. 2. *The curve $a=a(r,T_i)$ in the (T_i,a)-plane for different values of the interest rate*

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the unit plays a crucial role in the qualitative analysis of the dynamics generated by the model.

(4) simply becomes:

$$f_{it}^{*} = \frac{\overline{f}_i}{a} = \overline{f}_{it}^{*} \tag{5}$$

Thus, in a period of tranquillity the solvency index of the unit is constant and turns out to be parameterized by both the interest rate and the time-horizon of the unit: when the latter becomes longer (shorter), the solvency index increases (decreases), whereas when the former increases (decreases), it decreases (increases).

Minsky's FIH, however, suggested that *financial stability may be destabilizing* (e.g., Minsky, 1985, p. 12). During a period of tranquillity, the expectations of the unit are systematically validated by the market and, as a consequence, there is a progressive reduction of the perception of risk associated with forecasting mistakes. Therefore, the representative unit eventually convinces itself that there is an unexploited margin of safety that may justify an increase in its financial exposure to boost its returns on equity. This brings about a progressive deviation of expectations from the initial value that triggers a fluctuation of its financial conditions. Therefore, as Keynes (1936, p. 49) observes, 'a mere change in expectations is capable of producing an oscillation of shape as a cyclical movement, in the course of working itself out'.

The simplest way to take account of this – and of the point we have made in Section 2 (see p. 4) – is to assume that if at time t the representative financial unit observes a liquidity index smaller than what it considered to be 'normal' at the end of the previous, tranquil, period then it starts to extrapolate the trend and expects its liquidity index further to decrease below that level in the next period, i.e.:

$$\mathrm{E}_t\left[f_{it+1}\right] = f_{it} + \rho\left(f_{it} - \overline{f}_i\right) = \left(1+\rho\right)\left(f_{it} - \overline{f}_i\right) + \overline{f}_i$$

where $\rho > 0$. Moreover, by a repeated substitution process, we can compute all expected liquidity indexes within the time-horizon of the unit, which are given by:

$$\mathrm{E}_t\left[f_{it+s}\right] = \left(1+\rho\right)^s\left(f_{it} - \overline{f}_i\right) + \overline{f}_i \qquad s = 0,1,\ldots,T_i. \tag{6}$$

Using (6), from the definition of the solving index we obtain the following expression for the solvency index at time $t+1$ (the first 'period of euphoria'), in terms of the liquidity index in the previous – 'tranquillity' – period:

$$f_{it+1}^{*} = \sum_{s=0}^{T_i} \frac{\mathrm{E}_t\left[f_{it+s+1}\right]}{\left(1+r\right)^s} = \sum_{s=1}^{T_i+1} \frac{\mathrm{E}_t\left[f_{it+s}\right]}{\left(1+r\right)^{s-1}} = \sum_{s=1}^{T_i+1} \frac{\left(1+\rho\right)^s\left(f_{it} - \overline{f}_i\right) + \overline{f}_i}{\left(1+r\right)^{s-1}}$$

$$= (1+\rho)\left(f_{it} - \overline{f}_i\right)\sum_{s=1}^{T_i+1}\left(\frac{1+\rho}{1+r}\right)^{s-1} + \overline{f}_i\sum_{s=0}^{T_i}\left(\frac{1}{1+r}\right)^{s}$$
$$= \beta\left(f_{it} - \overline{f}_i\right) + \frac{\overline{f}_i}{a}$$

which, given (5), can be rewritten as:

$$f^*_{it+1} = \overline{f}^*_{it} + \beta\left(f_{it} - a\overline{f}^*_{it}\right) \qquad (7)$$

where the parameter β,

$$\beta = \beta\left(\rho, r, T_i\right) = \frac{(1+\rho)\left[(1+r)^{T_i+1} - (1+\rho)^{T+1}\right]}{(1+r)^{T_i}(r-\rho)} > 0\,,$$

is such that (see Fig. 3):[2]

$$\frac{\partial\beta}{\partial\rho} = \frac{\left[(1+r)^{T_i+1} - (1+\rho)^{T_i+1}\right](r+1) - (1+T_i)(r-\rho)(1+\rho)^{T_i+1}}{(1+r)^{T_i}(r-\rho)^2} > 0$$

$$\frac{\partial\beta}{\partial T_i} = -(r+1)^{1-T_i}(\rho+1)^{T_i+1}\frac{\ln(\rho+1) - \ln(r+1)}{r-\rho} > 0$$

$$\frac{\partial\beta}{\partial r} = -\frac{(\rho+1)\left[(1+\rho)(r+1)^{T_i} - (1+\rho+T_i r - T_i\rho)(\rho+1)^{T_i}\right]}{(r-\rho)^2(r+1)^{T_i}} < 0$$

Equation (7) is consistent with Keynes's opinion that realized results play a crucial role in macroeconomic dynamics 'in so far as they cause a modification in subsequent expectations' (1936, p. 47; see also CW XIV, p. 179). To make it more operative, we rewrite it as a recurrent equation in f_i^*:

$$f^*_{it+1} = f^*_{it} + \beta\left(f_{it} - af^*_{it}\right) = \beta f_{it} + (1-a\beta)f^*_{it} \qquad (8)$$

and use it, in what follows, to study the dynamics of the model. Although it is strictly valid for the case we have considered, namely the passage from a tranquillity to an euphoria period, we use it as a recurrent equation in the conviction that, together with

[2] The parameter $\beta = \beta(\rho, r, T_i)$ turns out to be very sensitive to the variation in the length of the time-horizon, but much less so to the level of the interest rate. For this reason, in this section we will limit ourselves to illustrating only the case for a given value of r which we have chosen to be 0.05.

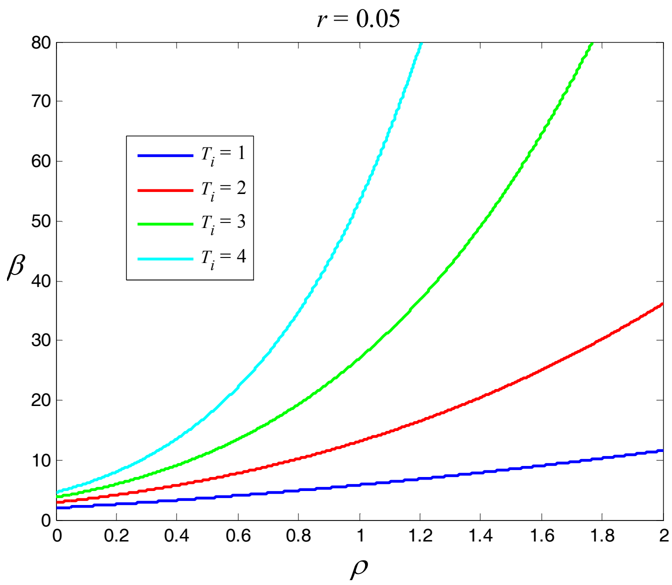

Fig. 3. *The curve β = β(ρ,r,T_i) in the (ρ,β)-plane for r = 0.05 and different values of T_i*

equation (2), it gives a reasonable, first approximation to the dynamical system of the model. This specification simply implies that the solvency index at time $t+1$ derives from a revision of its previous value in the light of the realized value of the liquidity index.

Thus, the dynamics of the financial conditions of the economic unit depends on a simple linear feedback between its realized liquidity and its expected solvency as described by equations (2) and (8). The dynamical system of the model in matrix notation becomes:

$$\begin{bmatrix} f^*_{it+1} \\ f_{it+1} \end{bmatrix} = \mathbf{A} \begin{bmatrix} f^*_{it} \\ f_{it} \end{bmatrix} + \begin{bmatrix} 0 \\ \alpha_i \mu \end{bmatrix} \tag{9}$$

where the coefficient matrix **A**:

$$\mathbf{A} = \begin{bmatrix} 1 - a\beta & \beta \\ -\alpha_i & 1 \end{bmatrix}$$

is such that $\mathrm{tr}(\mathbf{A}) = 2 - a\beta$ and $\det(\mathbf{A}) = 1 - a\beta + \alpha_i \beta$.

Simple passages show that (9) has a unique steady state defined by $P \equiv (f_i^{*e}, f_i^e) \equiv (\mu_i, a\mu_i)$. To study its asymptotic stability and ascertain the type of dynamics we consider the characteristic equation of the system, given by:

$$\lambda^2 - (2 - a\beta)\lambda + (1 - a\beta + \alpha_i\beta) = 0 \tag{10}$$

The dynamics of f_i and f_i^* is cyclical when the discriminant Δ of (10) is negative, i.e. when:

$$\Delta = (2 - a\beta)^2 - 4(1 - a\beta + \alpha_i\beta) = \beta\left(a^2\beta - 4\alpha_i\right) < 0$$

or:

$$\alpha_i > \frac{a^2\beta}{4} \tag{11}$$

Moreover, to determine the asymptotic stability of the solution, we consider the necessary and sufficient conditions for (10) to have only roots of absolute value less than one (see, for example, Gandolfo, 2009, pp. 60-62). In terms of the trace and determinant of the matrix **A**, these conditions can be expressed as 1 + tr(**A**) + det(**A**) > 0, 1 + tr(**A**) + det(**A**) > 0 and 1 − det(**A**) > 0, which in the present case reduce to:

$$\alpha_i > \frac{2a\beta - 4}{\beta} \tag{12}$$

$$\alpha_i\beta > 0 \tag{13}$$

$$\alpha_i < a \tag{14}$$

respectively. While condition (13) is always satisfied, so that we can disregard it in what follows, the other two are satisfied or not depending on the combination of the parameters ρ and α_i. The regions where the conditions (11), (12) and (14) are satisfied or not are plotted in different colours in Fig. 4 for $r = 0.05$ and two different values of T_i (= 1, 4).[3]

All combinations of the two parameters such that at least one of the two conditions (12) and (14) does not hold and the cycle condition (11) is not satisfied are coloured in grey, whereas they are coloured in yellow if at least one of the stability conditions does not hold and the cycle condition is satisfied. Accordingly, all parameter combinations in the grey region produce monotonic divergent dynamics of the two variables whereas the ones in the yellow region produce fluctuations of increasing amplitude.

[3] A number of other figures we have generated using different values of r suggest that both the stability of the motion and the type of dynamics are not significantly influenced by the value of the interest rate. Thus, again, we limit ourselves to considering the case in which $r = 0.05$.

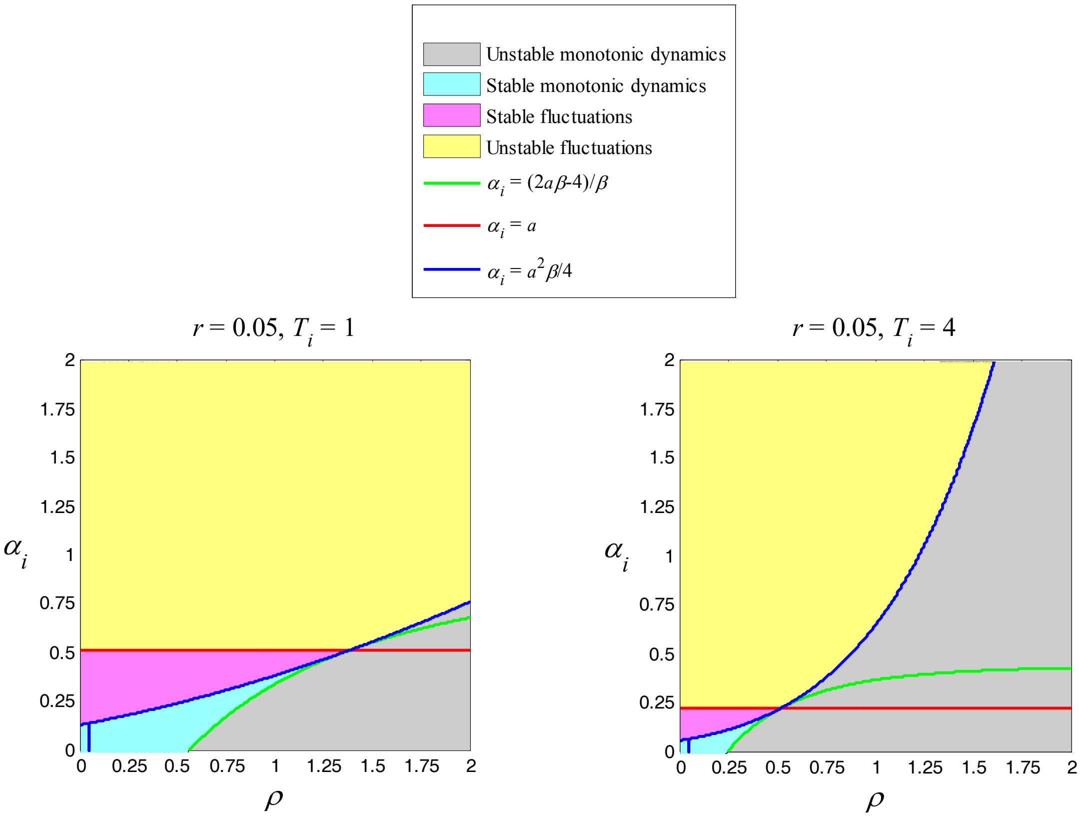

Fig. 4. *The full spectrum of typologies of dynamics with $r = 0.05$ and $T_i = 1, 4$*

In turn, the region in the bottom left-hand corner of the figure where both stability conditions hold is made up of two sub-regions, one, light-blue-coloured, where the cycle condition is not satisfied, the other, magenta-coloured, where the cycle condition holds. Parameter combinations in the first sub-region produce monotonic convergent dynamics, whereas combinations in the second sub-region produce fluctuations of decreasing amplitude. It clearly appears that the typical dynamic behaviour of the solvency and liquidity conditions of the representative unit consists in fluctuations of increasing amplitude (yellow region) or of explosive monotonic dynamics (grey region), both implying the virtual bankruptcy of the unit. Moreover, it is evident that the region of stability decreases in size as the time-horizon of the unit becomes longer. An example of the resulting dynamics is given in Fig. 5 where we have fixed r and T_i at 0.05 and 1 respectively and we have chosen a value of ρ (ρ = 0.3) such that $\alpha_i = a \approx 0.5122$ and condition (12) is redundant. In this special case the two eigenvalues of the matrix **A** are complex conjugate with modulus equal to one. Thus, depending on whether the frequency of the oscillation is a rational or an irrational

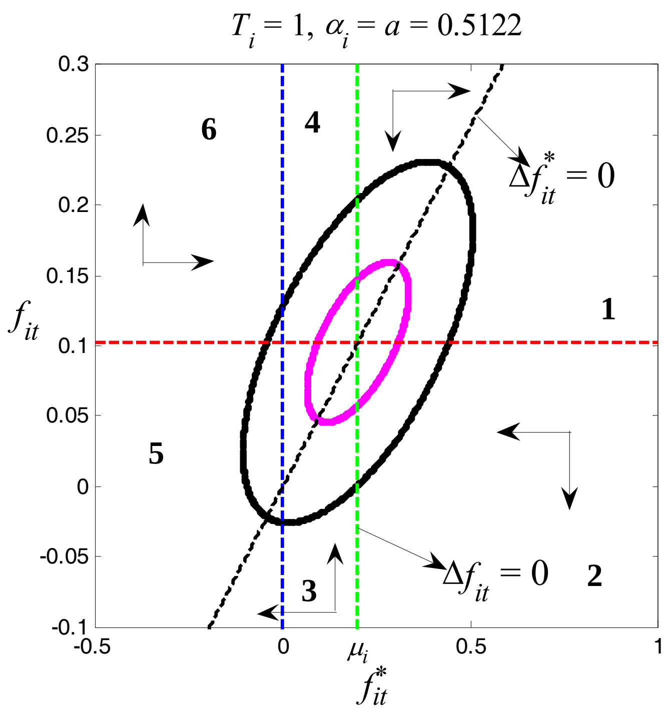

Fig. 5. *Quasi-periodic fluctuations for $\alpha_i = a$ and $T_i = 1$*

number, the dynamics is periodic or quasi-periodic. An example, for two given initial conditions, is shown in the figure. The resulting clockwise cyclical dynamics in f_{it} and f^*_{it} is of the type we have described at the beginning of this section.[4] It turns out that for some initial conditions, the financial unit never becomes virtually insolvent over the cycle (see the magenta trajectory); whereas for others (see the black trajectory) the financial unit is virtually insolvent in some phases of the cycle. However, it is then enough that the value of α_i, becomes even slightly greater to have the system generate explosive fluctuations.

Thus, the fact that the economic unit – having started from a 'period of tranquillity' to form extrapolative expectations – keeps to the same mechanism of expectations formation (regardless of the state of its financial conditions) makes the model strongly unstable, and the more so, the longer the time horizon of the unit. The analysis of expectations dynamics under strong uncertainty developed in Section 2, however, suggests a more realistic picture, which involves taking account of the fact that the

[4] It should be stressed, however, that there is a minor difference between Fig. 1, which we have used to classify financial units on the basis of their solvency and liquidity conditions, and the figures we are now discussing. Indeed, in the latter, the dotted red line – as a result of the specification of the dynamical system – is drawn at $f_{it} = a\mu_i$ rather than at $f_{it} = 0$. It should be remembered however that a is very small and quickly approaches zero as the time horizon of the unit increases.

unit's expectations regarding its future solvency and liquidity conditions endogenously evolve over the financial cycle.

5. Heterogeneous expectations and persistent dynamics

The simplest way to model the hypothesis of heterogeneous expectations suggested in Section 2 is to assume that the economic unit forms extrapolative expectations until it reaches its safety margin, beyond which it starts to form regressive expectations – reflecting the effort to reduce its liquidity index towards its normal value. This assumption seems to be consistent with what Keynes had in mind in his 'Notes on the trade cycle' (chap. 22 of the *General Theory*), since 'the illusions of the boom cause particular types of capital-assets to be produced in such excessive abundance that some part of the output is on any criterion, a waste of resources (…) when the disillusion comes, this expectation is replaced by a contrary 'error of pessimism'' (1936, pp. 321-22). Here Keynes has in mind mainly the investment in real capital as explained by the marginal efficiency of capital; however, his view adapts well to the more comprehensive view of the investment in financial assets that also encompasses the special case of real assets seen as relatively illiquid financial assets (as in chap. 17 of the *General Theory*). Notice that, from the purely financial point of view adopted in this paper, the solvency index f_{it}^* may be interpreted as a generalization of the concept of marginal efficiency of capital. Its oscillations are the main cause of the financial cycles, as here modelled, in analogy with Keynes's conviction that the fluctuations of the marginal efficiency of capital are the main determinant of the trade cycle.

Thus, depending on the phase of the cycle, the unit either extrapolates the trend, in the same way as we have assumed in the previous section,

$$\mathrm{E}_t^e\left[f_{it+1}\right] = f_{it} + \rho^e\left(f_{it} - \overline{f}_i\right) \qquad \rho^e > 0$$

or uses a regressive expectation formation rule which we formalise as:

$$\mathrm{E}_t^r\left[f_{it+1}\right] = f_{it} - \rho^r\left(f_{it} - \overline{f}_i\right) \qquad 0 < \rho^r < 1$$

A possible way to incorporate this idea into the model is to assume that the unit forecasts its future liquidity index by using a mix of the two basic mechanisms, with weights that vary over the financial cycle as a function of the difference between the f_i and its 'normal' value as defined above with reference to a 'tranquillity period'. Thus, we write:

$$\mathrm{E}_t\left[f_{it+1}\right] = w_{it}\mathrm{E}_t^e\left[f_{it+1}\right] + \left(1-w_{it}\right)\mathrm{E}_t^r\left[f_{it+1}\right] = \left(1+\rho_{it}^{er}\right)\left(f_{it}-\overline{f}_i\right)+\overline{f}_i$$

where $\rho_{it}^{er} = w_{it}\rho^e + \left(1-w_{it}\right)\left(-\rho^r\right)$ is nothing other than a weighted average of the two individual coefficients ρ^e and $-\rho^r$. With this specification, the behaviour we described in Section 2 by referring to Descartes' metaphor is obtained in the simple case in which the weight w_{it} can only take the two extreme values 0 or 1 according to the following mechanism:

$$w_{it} = \begin{cases} 0, \text{ if } \left|f_{it} - a\mu_i\right| \geq b_i \\ 1, \text{ if } \left|f_{it} - a\mu_i\right| < b_i \end{cases} \tag{15}$$

where b_i is a small positive magnitude.

When the simple 'switch' mechanism (15) is assumed to hold, the extrapolative mechanism is used by the unit when its liquidity index is not too far from its equilibrium level, whereas the regressive mechanism is used in the opposite case:

$$\rho_{it}^{er} = \begin{cases} -\rho^r < 0, \text{ if } \left|f_{it} - a\mu_i\right| \geq b_i \\ \rho^e > 0, \text{ if } \left|f_{it} - a\mu_i\right| < b_i \end{cases}$$

Using (15), the same procedure we have used to derive equation (8) now yields:

$$f_{it+1}^* = \beta_{it}^{er} f_{it} + \left(1 - a\beta_{it}^{er}\right) f_{it}^* \tag{16}$$

where the parameter β_{it}^{er},

$$\begin{aligned} \beta_{it}^{er} &= \beta^{er}\left(\rho_{it}^{er}, r, T_i\right) = \frac{\left(1+\rho_{it}^{er}\right)\left[\left(1+r\right)^{T_i+1} - \left(1+\rho_{it}^{er}\right)^{T_i+1}\right]}{\left(1+r\right)^{T_i}\left(r-\rho_{it}^{er}\right)} \\ &= \begin{cases} \dfrac{\left(1-\rho^r\right)\left[\left(1+r\right)^{T_i+1} - \left(1-\rho^r\right)^{T_i+1}\right]}{\left(1+r\right)^{T_i}\left(r+\rho^r\right)} = \beta_{it}^r, \text{ if } \left|f_{it} - a\mu_i\right| \geq b_i \\ \dfrac{\left(1+\rho^e\right)\left[\left(1+r\right)^{T_i+1} - \left(1+\rho^e\right)^{T_i+1}\right]}{\left(1+r\right)^{T_i}\left(r-\rho^e\right)} = \beta_{it}^e, \text{ if } \left|f_{it} - a\mu_i\right| < b_i \end{cases} \end{aligned}$$

is such that:

$$\frac{\partial\beta_{it}^{er}}{\partial\rho^{e}}=\frac{\partial\beta_{it}^{e}}{\partial\rho^{e}}=\frac{\left[\left(1+r\right)^{T_i+1}-\left(1+\rho^{e}\right)^{T_i+1}\right]\left(r+1\right)-\left(1+T_i\right)\left(r-\rho^{e}\right)\left(1+\rho^{e}\right)^{T_i+1}}{\left(1+r\right)^{T_i}\left(r-\rho^{e}\right)^{2}}>0$$

$$\frac{\partial\beta_{it}^{er}}{\partial\rho^{r}}=\frac{\partial\beta_{it}^{r}}{\partial\rho^{r}}=\frac{\left[\left(1+r\right)^{T_i+1}-\left(1-\rho^{r}\right)^{T_i+1}\right]\left(r+1\right)-\left(1+T_i\right)\left(r+\rho^{r}\right)\left(1-\rho^{r}\right)^{T_i+1}}{\left(1+r\right)^{T_i}\left(r+\rho^{r}\right)^{2}}<0$$

$$\frac{\partial\beta_{it}^{er}}{\partial T_i}=-\left(r+1\right)^{1-T_i}\left(\rho_{it}^{er}+1\right)^{T_i+1}\frac{\ln\left(\rho_{it}^{er}+1\right)-\ln\left(r+1\right)}{r-\rho_{it}^{er}}>0$$

$$\frac{\partial\beta_{it}^{er}}{\partial r}=-\frac{\left(\rho_{it}^{er}+1\right)\left[\left(1+\rho_{it}^{er}\right)\left(r+1\right)^{T_i}-\left(1+\rho_{it}^{er}+T_i r-T_i\rho_{it}^{er}\right)\left(\rho_{it}^{er}+1\right)^{T_i}\right]}{\left(r-\rho_{it}^{er}\right)^{2}\left(r+1\right)^{T_i}}<0$$

Thus, the existence of two regimes in the expectations-formation mechanism of the economic unit makes the dynamical system of this version of the model – formed by equations (2) and (16) – piecewise-linear and as such able to generate a wide range of dynamics, including persistent fluctuations and complex dynamics. A rigorous investigation of its mathematical properties, although worth pursuing, would take us too far from the purposes of the present paper and for this reason we prefer to postpone it to future research. In what follows we will limit ourselves to illustrating, by means of numerical simulations, some possible cases of dynamical behaviour that can be generated by this version of the model.

Some examples are given in Figs. 7, 8 and 9, where, for illustrative purposes only, we have taken $\rho^{e}=0.2$, $\rho^{r}=0.8$, $b=0.02$, $\alpha=0.25$, $r=0.07$, $\mu=0.1$ and several values of T_i.

As shown in Figs. 7 and 8, in all cases we have considered the model is able to generate a persistent financial cycle of the type described at the beginning of Section 4, one that neither dies away nor explodes. Along these cycles, the fluctuations of the liquidity and solvency indexes turn out to be the more irregular and of a larger amplitude, the longer the time-horizon used by the unit to calculate its solvency condition. This is confirmed by Fig. 9 where, with T_i = 5, the fluctuations of the two variables are even more irregular and more spread in the phase plane than in the previous cases. For this case, Fig. 10 clearly shows the property of sensitive dependence to initial conditions, which we take as an indicator that in this case the dynamics are chaotic.

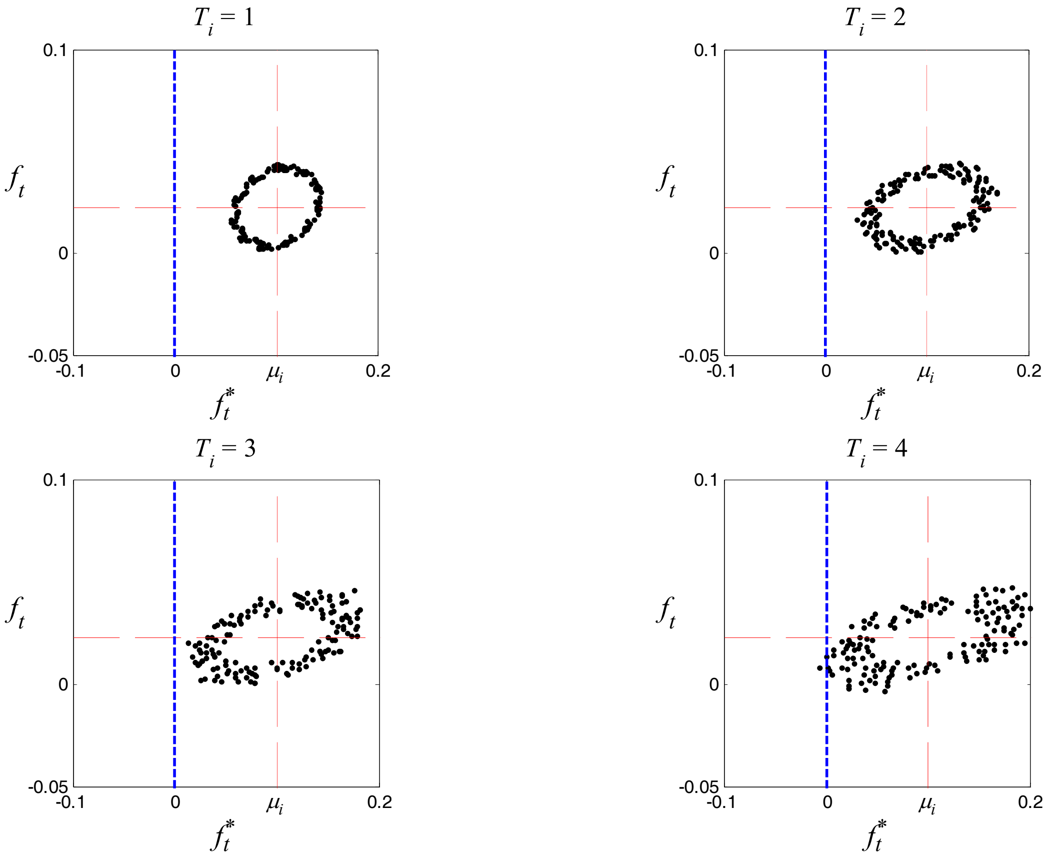

Fig. 7. *Some examples of persistent dynamics generated by the model in the -phase plane for* ρ^e *= 0.2,* ρ^r *= 0.8, b = 0.02,* α *= 0.25, r = 0.07,* μ *= 0.1 and different values of* $T_i \leq 4$

To conclude, the use by the representative economic unit of a mix of extrapolative and regressive expectations reduces the dynamical instability of the model but at the same time causes the resulting dynamics to become more complex. The fact that this is the more so, the longer is the time-horizon of the unit is not surprising since, under the hypotheses of this model, a longer time horizon is not in general a sign of far-sightedness but rather a mere amplifier of the effects of current trends, the more so the longer the time horizon. In this context, far-sightedness would be rather reflected by a higher margin of safety, as a greater awareness of the likelihood of systematic mistakes in long-term expectations should be reflected by a larger and more intertemporally consistent margin of safety.

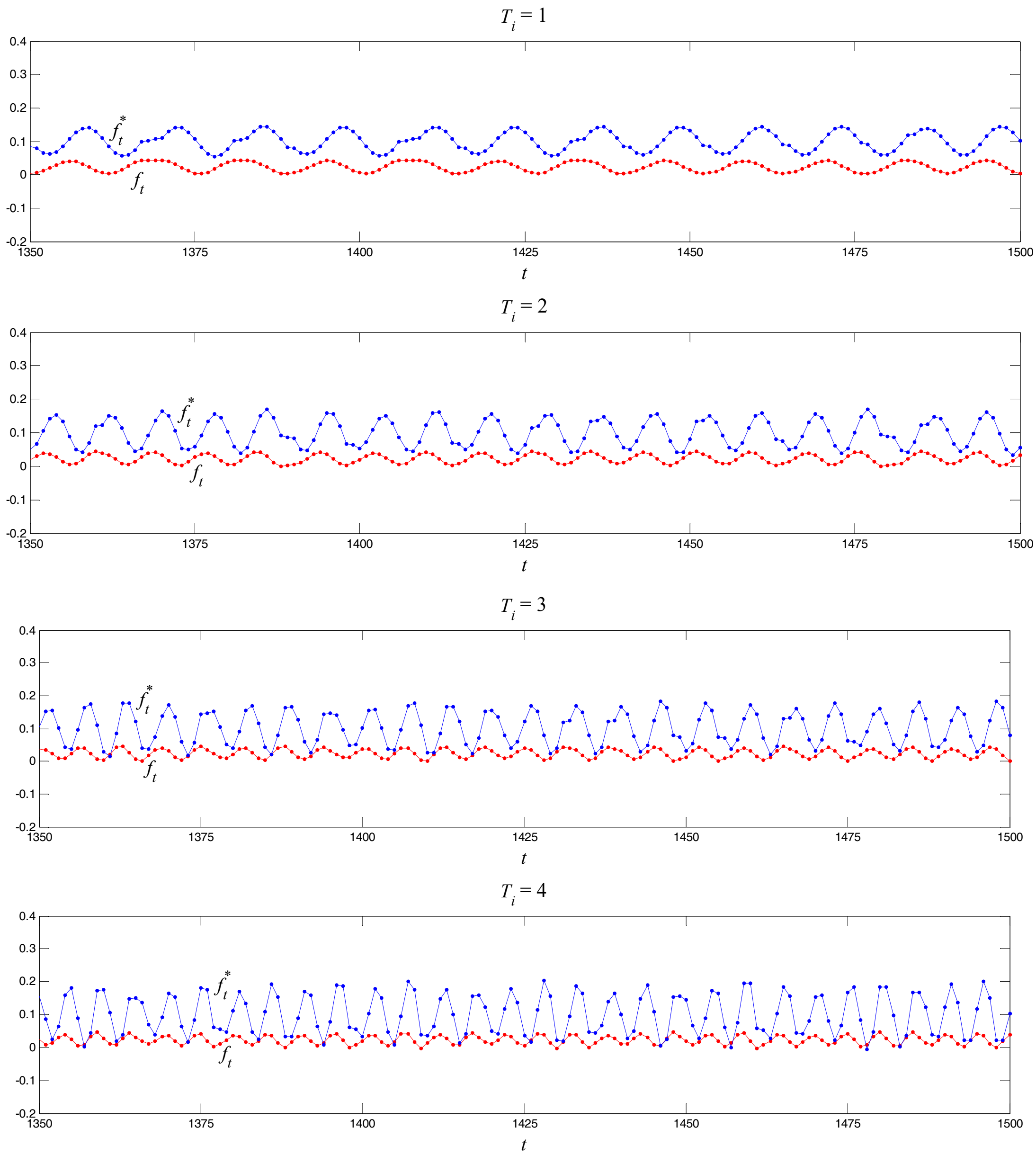

Fig. 8. *The succession of liquidity and solvency indexes over time for the four cases of Fig. 7*

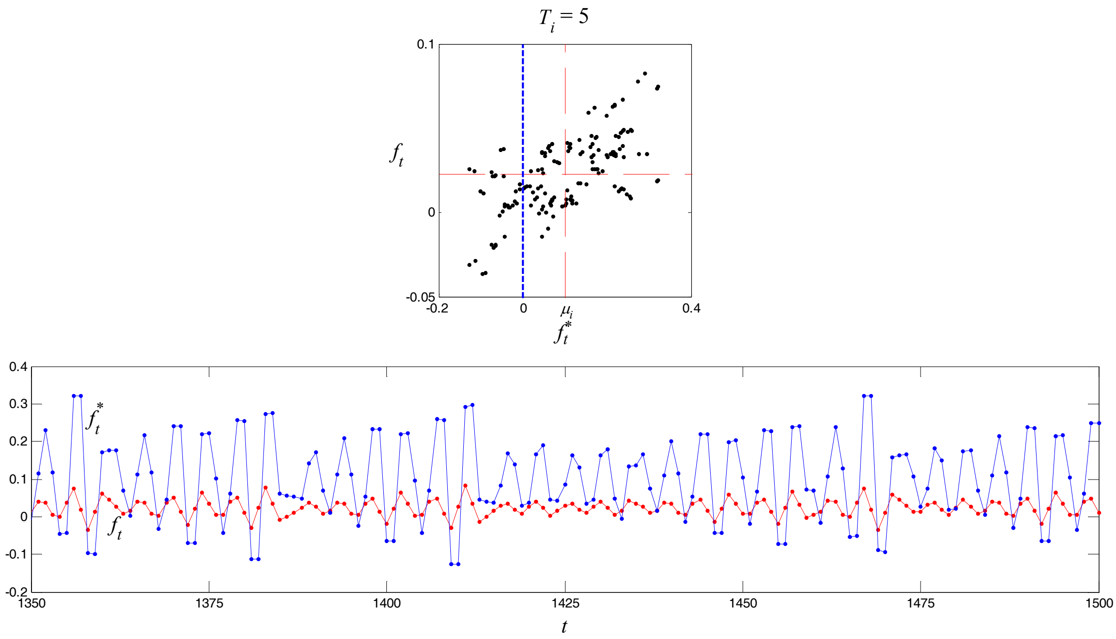

Fig. 9. *The same as in Figs. 7 and 9 for T_i = 5*

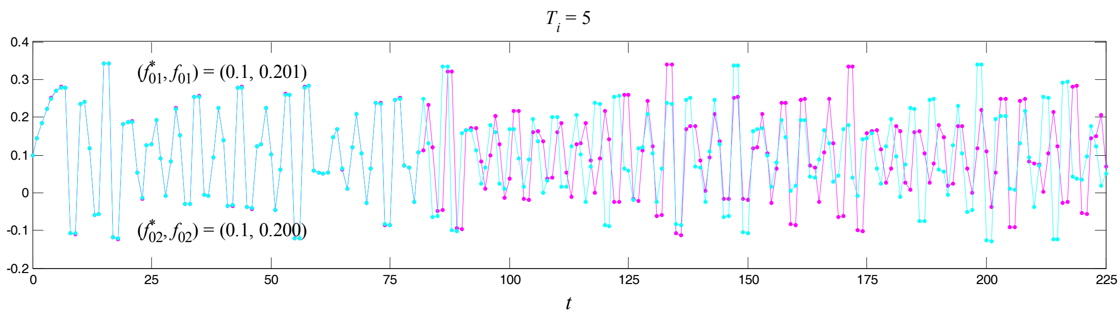

Fig. 10. *Evidence of the property of sensitive dependence to initial conditions for the case of Fig. 9*

6. Conclusions

We have shown in this paper that a simple mechanism of heterogeneous expectations may clarify the role of expectations in financial fluctuations. In particular our analysis confirms and clarifies the crucial role of expectations within the core of the FIH. The interaction between the realised liquidity index and the expected solvency index depends in a precise and explicit way on the mechanism of expectations formation. This mechanism is specified in this paper in a simple way to produce a dynamic behaviour of the variables consistent with the crucial implications of the FIH. The hypotheses underlying the expectations dynamics in the model are rooted in a vision of the economic behaviour under strong uncertainty strictly related to that of Minsky and Keynes, although updated and formalised in the light of recent advances in complex dynamics, decision theory and behavioural economics.

The analysis developed in this paper may be generalised in many directions. First, we can take account of the heterogeneity of agents by giving different individual weights to the two mechanisms of expectations formation and by assuming different attitudes on the part of agents. Second, we can endogenise the shifts of the margin of safety. Furthermore, we can make explicit the relationship between financial and real variables.

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