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Ambiguous Money Distribution And The Price Stickiness Phenomenon:
A Rationale From An Ambiguous Rational Expectations Approach

Marcello Basili* and Stefano Dalle Mura**

Abstract

This paper shows that ambiguity – as opposed to risk – may lead to sticky prices even with fully rational agents. Attitude towards ambiguity is assumed, as supported by theoretical literature and experimental evidence, to be asymmetric in the form of ambiguity aversion towards uncertain gains and ambiguity seeking towards losses. In this setting that price stickiness follows a change in the money supply level that does not alter the distribution of money constitutes a self fulfilling expectations equilibrium. That is the average (expected) result, but other interesting cases can occur (price overshooting and an inverse relationship between prices economic activity). Money neutrality remains true in the long run. The main result is carried out in a model where ambiguity concerns firms' ignorance about the relationship between the stock of money and money distribution.

KEYWORDS: Ambiguity, multiple priors, price stickiness

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1. Introduction

According to some empirical studies (for example Greenwald and Stiglitz 1989) price stickiness, that is price elasticity below one following aggregated nominal shocks, is a stylised fact. The theoretical puzzle (Farmer 1992) is how to reconcile that fact with a standard model based on agents' rational choices. The nominal rigidity topic is, of course, an old one, and has already been tackled in many ways: for example by introducing menu costs (Blanchard and Kiyotaky 1987), near rationality (Akerlof and Yellen 1985), multiple equilibria (Cooper and John 1988), staggered contracts (Fisher 1977), information incompleteness (Lucas 1972), and money social externalities (Farmer 1993).

This paper tries to solve this puzzle through an application of ambiguity theory to microfounded macroeconomics. Ambiguity is introduced in a general equilibrium framework, inspired by Lucas's approach focusing on information incompleteness. It will be shown how it is possible to overcome a critique that, on the empirical ground, has been levelled against Lucas's model: that, because of readily available information about nominal aggregates, it fails to explain the persistence of price rigidity which data point to (Romer 1996). In the present paper information incompleteness concerns money distribution, that is a variable which is much more difficult to observe than the aggregated money stock. It is assumed that rational firms face an ambiguous problem when they evaluate the impact of monetary policies on the distribution of nominal endowments across heterogeneous consumers. In this context, the approach of multiple priors is used to model attitude towards ambiguity, that is firms are supposed to be unable to assign a fully reliable additive probability distribution to possible events because such events are ambiguous. Hence firms have multiple additive priors on possible events and their preferences are compatible with either maxmin or maxmax expected utility decision rule.[1]

The aim of this paper is to show that asymmetric attitude of firms towards ambiguity can motivate the observed dynamics of prices and quantities, characterised by both prices stickiness and a temporary positive correlation between money supply and economic activity, following a monetary policy which does not alter the nominal endowment distribution among agents.

In the long run it is still true that money neutrality prevails.

As regards to the possible actual (instead of expected) transition dynamics to the long run steady state an example of the potential of our approach is made. It is shown either that prices can overshoot their long run equilibrium value and that prices and economic activity can move in opposite directions.

The paper is organized as follows. In Section 2 ambiguity and the attitude towards it are introduced as well as the idea of an Ambiguous Rational Expectations approach. Section 3 describes the economic framework. Section 4 presents the temporary equilibrium. The concept of a uniform monetary policy is presented in Section 5. Section 6 characterises expectations within the chosen economy model. Section 7 tackles the problem of how to define a good/bad prospect. Section 8 deals with the relationship between ambiguity and price stickiness in the short run. Section 9 and 10, respectively, analyse the dynamics of money distribution and expectations. Section 11 and 12 deal with the long run equilibrium and the dynamic transition issue. Concluding remarks are in section 13.

[1] Maxmin (maxmax) expected utility postulates that firms with multiple priors consider the least (most) value of expected utility for any act and choose that act for which this least (most) value is greatest.

2. Expectations And Ambiguity

The starting point of this paper is that ambiguity is relevant. It is assumed that agents are not ambiguity neutral and multiple priors are used to model ambiguity by a set of possible priors on the underlying state space. This means, as Ellsberg put it, that “each subject does not know enough about the problem to rule out a number of possible distributions” (Ellsberg 1961). If an agent is ambiguity averse (seeking), then she maximizes the minimum (maximum) expected utility with respect to each probability in the prior set, thus exhibiting *maxmin* (*maxmax*) behaviour.[2]

Despite introducing ambiguity explicitly, we want to stick to the standard rational expectations (SRE) approach as much as possible. We want to retain the idea that expectations must be described as correct “on average”: for any observable variable the expected value taken according to the true economic model must be equal to agents' expectation. When expectations are formed according to a single probability distribution it is immediate to define what SRE means. However, with multiple distributions it is not so obvious.

We propose that the test for expectations rationality, within the framework of multiple distributions, is conduced according to the “average distribution”. A bit more formally: let $\Gamma \equiv \{f(x)\}$ be the set of multiple probability distributions for x. Then, let $F_k(x)\quad k=1..K$ be the cumulate for the single distributions in Γ. The cumulative average function is defined as $\tilde{F}(x) := \frac{\sum_{\Gamma} F_k(x)}{K}$ and $\tilde{f}$ is the relevant probability distribution. We then require that expected values taken according to that average distribution should be coherent with the economy actual outcome.

2 See Gilboa and Schmeidler (1989).

We see the approach to expectations formation just described as entirely coherent with the hypothesis of agents rationality, and we name it the ambiguous rational expectations approach (ARE).

An ambiguity averse (seeking) agent will choose optimising with respect to the distribution which minimizes (maximises) her payoff: it is essential to specify the attitude towards uncertainty which describes agents' behaviour.

In a seminal paper, Kahnemann and Twersky (1979) show that agents' preferences among risky prospects are not linear in probabilities and violate the Expected Utility Theory. Agents tend to overweigh small probabilities and underweight large probabilities. Namely, they transform priors into decision weights which measure both the perceived likelihood of beliefs (diminishing sensitivity-discriminability) and the preference for gambles (attractiveness).[3] Kahnemann and Twersky point out this behaviour within the framework of a peculiar Non-Expected Utility Theory called Prospect Theory. The core of prospect theory as well as its recent generalization-axiomatization called Cumulative Prospect Theory[4] is that agents evaluate possible losses and gains differently. Losses and gains are defined with respect to a reference point or neutral outcome. Agents show both the possibility effect (*lower subadditivity*) and the certainty effect (*upper subadditivity*). Agents make decisions based on changes in their monetary endowments rather than based on total monetary outcomes, thus generating a sign-dependent and rank-dependent expected utility. As a result, agents have inverse-S-shaped utility functions: these are concave for small probabilities and convex for high probabilities. This result is consistent with a concave utility function for gains and a convex utility function for losses.

3 Discriminability and attractiveness, respectively, determine curvature (slope) and elevation (intercept) of the inverse-S-shaped utility function in the Cumulative Expected Utility.
4 Tversky and Kahnemann (1992), Wakker and Tversky (1993), Chateauneuf and Wakker (1999).

Since attitude towards ambiguity is sign-dependent and rank-dependent, it is appropriate to focus on the distinction between good and bad uncertain outcomes, that is gains and losses. We argue that attitude towards ambiguity is asymmetric and depends on which case prevails.

Plenty of experimental evidence suggests asymmetry in the attitude towards ambiguity: Cohen et al. (1985), Einhorn and Hogarth (1990), Gonzales and Wu (1999), Abdellaoui (2000), Bleichrodt and Pinto (2000), Bleichrodt et al. (2001), Kilka and Webber (2001). All these papers reporting experimental studies indicate that agents treat gains and losses differently. A variety of parametric functions are used to elicit individual utility, and probability weighing functions are used to represent their behaviour. The common feature of these experimental studies is that the elicited utility functions satisfy both upper and lower subadditivity and are consistent with the following assumption, on which the rest of the present paper is based:

A0: *firms act according to ambiguity aversion for gains and ambiguity seeking for losses*

Let us define good (bad) prospects in terms of expected changes from the current situation. Since ambiguity attitude depends on the sign of the prospect, that the prospect is, in fact, good or bad is to be determined independently from ambiguity attitude. Therefore, it is needed a way to define "expected values" without accounting for ambiguity attitude. It is proposed to refer again to the average probability distribution $\tilde{f}$. For an ambiguity non neutral individual, expected values of ambiguous events taken according to $\tilde{f}$ can be seen as answering to the question about what is her "objective" expectation not affected by personal factors, one that she would provide to another subject as if she were a sort of "independent advisor". To sum up:

A1: *a good (bad) prospect is one characterized by an "average" expected payoff better (worse) than the current situation.*

3. Economic Framework

The analysis rests on a very simplified economic framework, focusing on the effect on firms' pricing decisions of ambiguity regarding non-observable macroeconomic variables. In particular the assumptions rule out intertemporal optimization problems. It is worth remarking that we still consider a sequence economy whose the per-period structure retains, under full observability, the standard properties (homogeneity of demand and money neutrality).

At any time *t* population is constituted by a fix number $H = \{1 ... h ... H\}$ of people belonging to *H* different families. Each person lives for one period only and at the end of it she is replaced by another identical member of her family. Each agent is both a consumer and a producer (there is also a fix number $H = \{1 ... j ... H\}$ of firms); there is no market for firms' shares. Firms are engaged in monopolistic competition and set prices on the basis of their expectations about market demand. Afterwards demand is observed, goods are produced and sold, and profits are paid: time-*t* firm *j* monetary profits are inherited by the time *t+1* member of the same family *j*. Expectations are assumed to be uniform and common knowledge across firms. Consumers are subject to a cash-in-advance constraint: they must use money for transactions.

To simplify the intertemporal structure of the model it is assumed that living agents have lexicographic preferences: they firstly care about their own utility and only in the second instance care about the next generation utility. Therefore due to the cash-in-advance constraint current generation members as consumers spend all their money endowment (caring for their own utility) while maximising profits as firms (for the next family member welfare).

Let p_j the price charged by firm *j*, q_j^h consumer *h* demand for good *j*, M^h agent *h* money endowment and P (simply set $P \equiv \frac{\sum_j p_j}{H}$) a general price level index that each firm consider independent from her own single price. Demand comes from Cobb-Douglas utility functions: $q_j^h = \frac{\alpha M^h}{p_j}$, since it must be $\sum_j \alpha = 1$ the demand function simplify to $q_j^h = \frac{M^h}{Hp_j}$.

To easily make money distribution relevant we further assume that "ability to spend" is actually affected by some stochastic factor $\theta\phi^h$, where $\theta \in [\underline{\theta}, \overline{\theta}]$ $0 < \underline{\theta} < \overline{\theta} < 1$ is a common i.i.d. random variable and $\phi^h \in [0,1]$ a family specific deterministic scale factor. For the moment, since we concentrate on a static setting, we do not consider the stochastic component, therefore

$$
\begin{aligned}
q_j^h &= \frac{\phi^h M^h}{Hp_j} \\
q_j &= \frac{}{Hp_j} \sum_h \phi^h M^h \equiv \frac{1}{Hp_j} \Omega \qquad (1) \\
p_j q_j &= \frac{\Omega}{H}
\end{aligned}
$$

that is real and nominal demand for god *j* depend on the index of money distribution (Ω).

On the supply side the only input is the family *j* member effort (there is no a labour market). The production technology is such that said y the output level the "cost of effort" function (*c*) exhibits decreasing average costs in some initial range and increasing average costs afterwards: $c = (y-k)^3 + k^3$ with $k > 0$.

4. Temporary Equilibrium

Given the lexicographic preferences each firm maximises the difference between the expected purchasing power of current profit[5] and the cost function:

$$E\left[\frac{p_j q_j}{P_{t+1}}\right]-E\left[(q_j-k)^3+k^3\right]=E\left[\frac{\Omega}{P_{t+1}}\right]-E\left[(q_j-k)^3+k^3\right].$$

In this framework the optimal *j*-price is such that $E[q_j]=k$:

$$p_j^*=\frac{E[\Omega]}{Hk} \tag{2}$$

which, given the assumption of uniform expectations, is the same for each firm.

In the temporary equilibrium of the economy, therefore, the real production of each firm is given by ex-post observed demand:

$$q_j^*=k\frac{\Omega}{E[\Omega]} \tag{3}$$

Since the economy is perfectly symmetric in the following the subscript *j* is generally suppressed.

5. Uniform Monetary Policy

The analysis is devoted to the effects of uniform monetary policies on the equilibrium of the economy. We need a definition of equilibrium. That is depicted by optimizing price setting by firms and self-fulfilling expectations about observable variables.

Many reasons explain why prices do not react proportionally to monetary shocks even if agents are assumed to be rational. These reasons relate to the following situations: a change

[5] Since the current general price level is taken as given by each single firm then this is true for the next period price level.

in the money stock is not (believed to be) once for all; it is not immediately implemented, it is not fully observed; or it does not distribute money in proportion to heterogeneous agents' initial nominal balances (Grandmont 1983). Moreover, the creation of money to finance government spending in real goods (i.e. the construction of a bridge or a highway) changes the endowment of the economy, and therefore the new money is likely to affect relative prices and the real equilibrium. To rule out these issues, in what follows public expenditure is not considered (money is supposed to be directly distributed to consumers) and we assume a specific kind of monetary policy consisting in a once for all, announced, immediately implemented, observed (in its macro features), and proportionally distributed change in the exogenous stock of money. Such a policy is named "uniform monetary policy".

More precisely we assume that the economy is, at time $t = 0$, in a steady state where the money distribution among consumers does not change (it will be proved that such a state exists and is a stable equilibrium). Then a uniform monetary policy, assumed to be "once for all", is implemented at time $t = 1$ with the effect that both $M_1 = \lambda M_0$ and $\forall j \;\; M_1^h = \lambda M_0^h$. The new money level is observed immediately; but firms cannot observe whether the change in money supply affected the distribution of nominal holdings among consumers or not; hence expectations are introduced in the model. It is assumed, furthermore, that past empirical evidence does not support any correlation between changes in the money supply level and changes in the nominal holdings distribution, since monetary authorities always attempt to implement uniform monetary policies and any impact on money distribution is purely stochastic. This issue is formalised in the subsequent assumption A4.

6. Expectations

As shown by (2) and (3) the key variable determining the economy outcome is $E[\Omega]$. Thus it becomes necessary to specify how expectations about it are formulated once $M_1 = \lambda M_0$ is observed (but it is not observed that $\Omega_1 = \lambda\Omega_0$).

In the SRE framework - since it has been supposed that there is not any systematic relationship between changes in the money supply level and changes in money distribution - it would be assumed that expectations are taken, at time $t = 1$, according to some prior distribution such that $E_1\lfloor\Omega_1\rfloor = \lambda\Omega_0$.

In the ARE approach we argue that Ω_1 is an ambiguous variable and $E\lfloor\Omega_1\rfloor$is defined with respect to multiple priors. Adapting the previously used notation, let $\Gamma \equiv \{f(\Omega_1)\}$ be the set of multiple probability distributions for Ω_1.

Further to **A0** and **A1** we also assume:

A2: *if firms were ambiguity neutral, their single probability distribution* $f_k(\Omega_1)$ *would be such that* $E\lfloor\Omega_1 | f_k\rfloor = \lambda\Omega_0$

A3: *an increase (reduction) in the money supply level does not reduce (increase) the initial money endowment of any agent.*

A4: $\tilde{f}$ *is such that* $\tilde{E}\lfloor\Omega_1\rfloor \equiv E\lfloor\Omega_1 | \tilde{f}\rfloor = \lambda\Omega_0$

A2 implies that under ambiguity neutrality the ARE framework corresponds to the SRE one. **A3** means that there is a direct relationship between the money stock and the money distribution index. **A3** also implies that if $\lambda > 1$ ($\lambda < 1$) then $\Omega_0 < \min Support(f_k)\ \forall k$ ($\Omega_0 > \max Support(f_k)\ \forall k$) and, therefore, $E\lfloor\Omega_1 | f_k\rfloor > \Omega_0\ \ \forall k$ ($E\lfloor\Omega_1 | f_k\rfloor < \Omega_0\ \ \forall k$): **A3** rules out pathological cases, namely those characterized by an inverse relationship between money supply level and aggregate nominal demand. **A4** is similar to **A2**, it means that "on average" - and without other information but the aggregate change in money supply - individuals caring for ambiguity still believe in money neutrality; it is coherent with the assumption of no statistical correlation between changes in the money supply level and changes in the nominal holdings distribution. A corollary of **A4** is:

C1: Γ *is such that* $\exists k', k'' : E\lfloor \Omega_1 \mid f_{k'} \rfloor < \lambda \Omega_0 < E\lfloor \Omega_1 \mid f_{k''} \rfloor$

7. Monetary Policy As A Good/Bad News

Consequences of monetary policies are to be measured in terms of expected changes in real profits. What is to be stressed is that we do not investigate if a monetary expansion/contraction is a good/bad news: we just look for consequences of assuming that it is believed by economic agents. For convenience we concentrate on the expansionary case. Assume an increase in *M is believed to be a good news, that is* $\frac{d\tilde{E}[\pi]}{dM} > 0$ where – coherently with **A1** - the expectation refers to the before mentioned average probability distribution and π indicates real profits. Since the relevant variable for profits is $E[\Omega]$ the above expression is to be read as $\frac{\partial \tilde{E}[\pi]}{\partial E[\Omega]} \frac{dE[\Omega]}{dM} \geq 0$. Due to A3, $\frac{dE[\Omega]}{dM} > 0$ therefore for a monetary expansion to be believed to be a good news it must be also believed that $\frac{\partial E[\pi]}{\partial E[\Omega]} \geq 0$. The case in which $\frac{\partial E[\pi]}{\partial E[\Omega]} = 0$ is expected to be zero replicates the SRE approach.

8. Short Run Equilibrium

At time *t=0 t*he economy is at a full information equilibrium:

$$\begin{cases} p_0 = \dfrac{\Omega_0}{kH} \\ q_0 = k \\ \pi_0 \equiv p_0 q_0 = \dfrac{\Omega_0}{H} \\ P_0 = p_0 \\ \eta_0 \equiv \dfrac{\pi_0}{P_0} = k \end{cases}$$

A monetary expansion is observed: $M_1 = \lambda M_0 \quad \lambda > 1$.

Under SRE $E[\Omega_1] = \lambda\Omega_0$ and money is neutral.

On the other hand let us assume ARE and that each firm believes an expansion in the money stock is a good news: then, each firm behave accordingly to ambiguity aversion and pick the probability distribution with the expected value that minimises $E[\pi_1]$, that is, given the supposed positive relationship between $E[\Omega_1]$ and the real profit (see section 7.), the one associated with the minimum $E[\Omega_1]$ and, by C1, $\min E[\Omega_1] \equiv \mu\Omega_0 < \lambda\Omega_0$. Therefore, prices really increases less than proportionally w.r.t. the money stock and real profits increases:

$$\begin{cases} p_1 = \dfrac{\mu\Omega_0}{\kappa H} = \mu p_0 < \lambda p_0 \\ P_1 = \mu P_0 < \lambda P_0 \\ q_1 = k\dfrac{\lambda\Omega_0}{\mu\Omega_0} = k\dfrac{\lambda}{\mu} > q_0 \\ \pi_1 = p_1 q_1 = \dfrac{\Omega_1}{H} = \dfrac{\lambda\Omega_0}{H} = \lambda\pi_0 \\ \eta_1 = \dfrac{\pi_1}{P_1} = \dfrac{\lambda\Omega_0}{H}\dfrac{kH}{\mu\Omega_0} = k\dfrac{\lambda}{\mu} > \eta_0 \end{cases}$$

It remains to investigate the relationship between expected and actual values of observable variables. Under ARE the are multiple distributions associated with many expected values:

from the point of view of each agent it is to be distinguished between what is her expected outcome of the economy and the expectation that is used when an optimisation problem is tackled. The first is not affected by attitude towards uncertainty and reflects the average probability distribution, for example the distribution index of which the expected value is supposed formed (see again A1) according to the *average* distribution: $\tilde{E}[\Omega]$. On the other hand when a variable depends directly on agents maximising choices it is known the they are set according to uncertainty aversion; for example the general price level is not an uncertain variable since it is known that it depends on common agents' ARE and $E[P] = p = \frac{E[\Omega]}{Hk}$ where $E[\Omega]$ now reflects ambiguity aversion/seeking. Accordingly it comes out that observable variables are correctly anticipated:

$$\begin{cases} \tilde{E}[q_1] = \dfrac{\tilde{E}[\Omega_1]}{Hp_1} = \dfrac{\tilde{E}[\Omega_1]}{H}\dfrac{Hk}{\mu\Omega_0} = k\dfrac{\lambda\Omega_0}{\mu\Omega_0}\dfrac{\lambda}{\mu} = k\dfrac{\lambda}{\mu} = q_1 \\ \tilde{E}[\pi_1] = \dfrac{\tilde{E}[\Omega_1]}{H} = \dfrac{\lambda\Omega_0}{H} = \pi_1 \\ \tilde{E}[\eta_1] = \tilde{E}\left[\dfrac{p_1 q_1}{P_1}\right] = \tilde{E}[q_1] = k\dfrac{\lambda}{\mu} = \eta_1 \end{cases}$$

An expansive monetary policy turns out to be a good news matching expectations: that agents believe in the short-run positive real effects of money expansion can be seen as a self-sustaining ARE. To sum up when $\lambda > 1$ ($\lambda < 1$) it emerges that firms act according to some probability distribution such that $E[\Omega_1] < \lambda\Omega_0$ ($E[\Omega_1] > \lambda\Omega_0$) which implies also that $P_1 < \lambda P_0$ ($P_1 > \lambda P_0$), that is it implies short-run price stickiness.

Things do not work in the other way round. If an expansive monetary policy is expected to be a bad news (expected negative $\frac{\partial E[\pi]}{\partial E[\Omega]}$ and reduction of real profits) firms act according to ambiguity seeking, thus pick the probability distribution associated with the higher expected real profit $E[\pi]$. But right because $E[\pi]$ and $E[\Omega]$ is believed to exist a negative relationship this means looking again for the lowest $E\lfloor\Omega_1\rfloor$. Prices as before adjust only partially to the nominal shock, real profit grow and expectations are not fulfilled. To believe a monetary expansion is a bad news cannot be an ARE equilibrium.

This is to say that believing a money expansion is a good news and that it will be non neutral constitutes a self fulfilling *prophecy*.

9. The Dynamics Of Money Distribution

To move from the short run perspective towards the medium/long run, the problem is to investigate the dynamics of the money distribution index and of its expected value. In this section the focus is on the evolution of the money distribution, leaving the analysis of expectation dynamics to the next section.

From now on the stochastic variable θ_t is introduced again. As regards money distribution, the changes in agent *h*'s monetary endowment are given by $M_{t+1}^h = M_t^h - \theta_t \phi_h M_t^h + \pi_{h,t}$ from which, given that $\pi_t = \frac{\theta_t \Omega_t}{H}$ we finally have $\Omega_{t+1} = \left(1 + \theta_t \frac{\sum_h \phi_t^h}{H}\right)\Omega_t - \theta_t \sum_h \left(\phi^h\right)^2 M_t^h$. The dynamics of Ω_{t+1}, therefore, depends not only on Ω_t but also on the other money distribution index $\sum_h \left(\phi^h\right)^2 M_t^h$. To maintain analytics as simple as possible, we linearise $\left(\phi^h\right)^2$ around its mean value $\phi \equiv \frac{\sum_h \phi^h}{H}$ obtaining $\left(\phi^h\right)^2 \cong \phi^2 + 2\phi\left(\phi_h - \phi\right) = -\phi^2 + 2\phi\phi^h$, therefore

$$\Omega_{t+1} = \left(1 - \theta_t \phi\right)\Omega_t + \theta_t \phi^2 M_t \tag{4}$$

Four observations follow from equation (4):

O1. *The dynamics of* Ω_t *has a steady state (for a constant money supply)* $\hat{\Omega} = \phi M$ *which does not depend on* θ *and is proportional to the money supply level.*

O2. *The constant in equation* (4) *is positive. The coefficient is positive and lower than 1. Therefore, despite the value of* θ_t, Ω_t *converges towards* $\hat{\Omega}$ *at any time t.*

O3. *The dynamics of* Ω_t *does not depend on expectations about* Ω_t.

O4. *Following a uniform monetary policy (*$\forall h\ M^h \to \lambda M^h$*),* Ω *jumps to its new steady state value.*

In accordance with **O4**, Ω is always equal to its steady state value (which depends on M). This result is independent of either SRE or ARE. However, although agents can always calculate the new value of $\hat{\Omega}$ after the implementation of any monetary policy (see *Observation 1*), they cannot be sure that $\hat{\Omega}$ is attained immediately, since they do not know whether that policy is uniform or not.

10. The Dynamics Of Expectations

Under both SRE and ARE, the starting point of analysis is some prior distribution: a distribution which is unique and such that $E[\Omega] = \Omega$ under SRE, and one which is multiple and not necessarily such that $E[\Omega] = \Omega$ under ARE. Remarkably, the observable equilibrium value of q (see equation (3)) reveals only $\theta\Omega$, and therefore learning about Ω cannot be immediate. In this way we introduce an endogenous dynamics for expectations.

It turns out that the analysis of the dynamics of $E[\Omega]$ is more complex than that of its true value Ω. In the standard (single prior) approach beliefs are updated following Bayes's rule.

We also refer to that rule in the multiple setting, where each prior, say $\psi_t(\Omega_t)$, is updated separately by using Bayes' rule[6] once the signal $s_t = \theta_t \Omega_t$ has been observed. However, the posterior $f_t(\Omega_t \mid s_t)$ calculated in this way does not constitute a suitable new prior for Ω_{t+1} precisely because of the autonomous dynamics of Ω according to equation (4). In fact, given $f_t(\Omega_t \mid s_t)$, Ω_{t+1} is a function of two (independent) random variables: Ω_t and θ_t. Therefore, the updating of $f_t(\Omega_t)$ is carried out by two steps: the first is the Bayesian updating of (each) $f_t(\Omega_t)$, the second consists in the calculation of the relevant probability distribution of the dependent random variable $\Omega_{t+1} = \zeta(\Omega_t, \theta_t)$ which represents the new prior for Ω_{t+1}.

11. Long Run Equilibrium

In view of our goals, in order to characterise the economy behaviour in the long run it suffices to consider the dynamics of the set of admissible values for Ω_t according to agents' beliefs. Such range is updated through the two-step procedure described above.

The relevant result is summarized in the following proposition, whose proof is in the appendix:

Proposition 1 *Let $\wp_t$ be the support of any of the multiple priors, then* $\lim_{t \to \infty} \wp_t = \{\hat{\Omega}\}$.

Proposition 1 shows that the admissible range Ω_t shrinks to the true value of Ω. That demonstrates that, in the limit, perfect learning is eventually achieved; in other words in the long run a uniform monetary policy is neutral.

[6] We refer to Epstein and Schneider (2001).

12. Transition Dynamics

It is time to analyse what happens to $E_t[\Omega]$ for $t > 1$. The case of an expansionary monetary policy ($\lambda > 1$) is considered; the results relative to $\lambda < 1$ can be deduced by analogy. Bayesian updating induces the variable under analysis to behave like a martingale, namely, the expected revision based on information from future signals is zero (otherwise the non-zero revision would be incorporated in the current expectation of future values). Additionally, our framework features equation (4) as a second source of dynamics. Taken together, the martingale property of Bayesian learning and equation (4) entail that only the effects of equation (4) can be forecasted. In the appendix it is proved that the following Proposition holds true:

Proposition 2 *Under ARE the expected path for Ω_t is such that it converges to the new steady state value $\hat{\Omega}$ from below (above) after an expansive (contractionary) uniform monetary policy.*

Proposition 2 implies that the expected average behaviour of prices after a uniform monetary policy consists of price stickiness and gradual adjustment. The same result does not hold under SRE. Under SRE, since $E_1[\Omega_1] = \lambda\Omega_0 = \hat{\Omega}$, and, therefore (because of equation (4)) $E_1[\Omega_t] = \lambda\Omega_0 = \hat{\Omega}$, money is expected to be neutral in transition as well.

Of course, both under SRE and ARE, the actual path for $E_t[\Omega]$ depends on the realisations of s_t, and this path can be quite different from the "average" one. For example, even if $E_t[\Omega] < \hat{\Omega}$ it need not be the case that $E_{t+1}[\Omega] < \hat{\Omega}$. To motivate this suppose that $\theta_t = \overline{\theta}$, then firms learn that $\Omega \geq s_t$ (where $s_t = \hat{\Omega}$ is the minimum of the new support) and, in accordance with (4) $E_{t+1}[\Omega]$ is no lower than $\hat{\Omega}$ for all posterior distribution. Therefore, no matter what the exact posterior probability functions are, the minimum expected value of Ω cannot be lower than $\hat{\Omega}$. A temporary overshooting result is therefore possible. It is noteworthy that as long as $E_t[\Omega] > \hat{\Omega}$ (again apart form the effects of the disturbance θ) not only P_t overshoots its SRE value, but also q_t is lower than its SRE counterpart. If $E_t[\Omega] > \hat{\Omega}$ firms raise prices too much and, given the true nominal demand, this causes a fall in real demand: raising prices and falling quantities are generated. By analogy it follows that after a restrictive monetary policy it is possible, for $t > 1$, that prices fall more than proportionally w.r.t. the money supply as quantities rise.

13. Concluding Remarks

It has been shown that an asymmetric attitude of firms towards ambiguity is a sufficient condition to generate lasting price stickiness as an average (expected) phenomenon ina a ARE setting. On the basis of a growing body of theoretical literature and a large amount of experimental evidence, it has been assumed that firms are characterized by both lower and upper subadditivity. This assumption is consistent with the hypothesis of ambiguity aversion (seeking) with respect to random gains (losses). This kind of asymmetric attitude is sufficient to determine price stickiness following a nominal uniform monetary policy, since that an expansionary (contractionary) policy represents a prospect of gain (loss) in terms of firms' real profit can be a self-fulfilling belief (in the ambiguous setting).

Although, on average, an expansionary monetary policy is expected to induce a gradual increase in prices as the activity level temporarily goes up, it may also generate a temporary overshooting increase in prices along with falling economic activity. Correspondingly, a contractionary monetary policy is likely to have negative effects on economic activity as prices gradually fall to their new equilibrium level, but a different outcome is also possible, with a “too” sharp reduction in prices, occurring together with increasing economic activity.

APPENDIX-PROOF

Proposition 1

Since the value of $\hat{\Omega}$ is always calculable, it must be $\hat{\Omega} \in \wp_1 = [\underline{\Omega}_1, \overline{\Omega}_1]$ where $\underline{\Omega}_t = min\ \wp_t$ and $\overline{\Omega}_t = max\ \wp_t$. Moreover it must hold that $\underline{\Omega}_1 \geq \min_h \{\phi_h\} M$ and $\overline{\Omega_1} \leq \max_h \{\phi_h\} M$. Then, $\wp_1$ is first updated after the agents' observation of the signal $s_1 = \theta_1 \Omega_1$. The new admissible range, say $\wp_1(s_1) = [\underline{\Omega}_1(s_1), \overline{\Omega}_1(s_1)]$, is such that $\overline{\Omega}_1(s_1) = min\left\{\overline{\Omega}_1, \frac{s_1}{\underline{\theta}}\right\}$ and $\underline{\Omega}_1(s_1) = max\left\{\underline{\Omega}_1, \frac{s_1}{\overline{\theta}}\right\}$.

Without loss of generality we assume $\overline{\Omega}_1 > \frac{s_1}{\underline{\theta}}$ and $\underline{\Omega}_1 < \frac{s_1}{\overline{\theta}}$, therefore $\wp_1(s_1) = [s_1, \overline{\Omega}_1]$.

As for the second step equation (4) is applied to $\wp_1(s_1)$ to obtain a new range, expressed as a function of the (unknown) value[7] of θ_1, given by

$$\wp_2(\theta_1) = \left[(1-\theta_1\phi)\frac{s_1}{\overline{\theta}} + \theta_1\phi^2 M, (1-\theta_1\phi)\frac{s_1}{\underline{\theta}} + \theta_1\phi^2 M \right]$$

In general it is $\wp_t(s_t) = \left[\max_{z \le t}\left\{\frac{s_z}{\overline{\theta}}\right\}, \min_{z \le t}\left\{\frac{s_z}{\underline{\theta}}\right\} \right]$ and

$$\wp_{t+1}(\theta_t) = \left[(1-\theta_t\phi)\max_{z \le t}\left\{\frac{s_z}{\overline{\theta}}\right\} + \theta_t\phi^2 M, (1-\theta_t\phi)\max_{z \le t}\left\{\frac{s_z}{\underline{\theta}}\right\} + \theta_t\phi^2 M \right]$$

With respect to the upper limit of $\wp_{t+1}(\theta_t)$ we observe that the dynamics of $\overline{\Omega}_t$ mimics that of Ω_t (see (4)), which converges to $\hat{\Omega}$. It should be added that $\overline{\Omega}_t$ moves towards $\hat{\Omega}$ from above since $\overline{\Omega}_0 > \hat{\Omega}$ and $\overline{\Omega}_t > \hat{\Omega} \Rightarrow \overline{\Omega}_{t+1} > \hat{\Omega}$.

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7 To fully define the evolution of the range of integration for $\wp_1$, the max and min of $\wp_1(\theta_0)$ can be calculated according to the possible values of θ_0; however, such operation is not needed for our purposes.

With respect to the lower limit, on the one hand $\max_{z \le t} \{s_z\}$ is a time-non-decreasing value, and, on the other, at any time *t*, for given $\max_{z \le t} \{s_z\}$ $\underline{\Omega}_t$ moves towards $\hat{\Omega}$ (from below), due to an analogous argument used for the upper limit.

Proposition 2

Given $E_1[\Omega_1]$, the expectation for Ω_2 formulated at time $t = 1$ is given by:

$E_1[\Omega_2] = (1 - \hat{\theta}\phi)E_1[\Omega_1] + \hat{\theta}\phi^2 M_1$, where $\hat{\theta}$ is the average value of θ. Therefore, $E_1[\Omega_2] > E_1[\Omega_1]$ or $E_1[\Omega_2] < E_1[\Omega_1]$ whether $E_1[\Omega_1] < \hat{\Omega}$ or $E_1[\Omega_1] > \hat{\Omega}$. By iteration we obtain that

$$E_1[\Omega_3] = (1 - \hat{\theta}\phi)E_1[\Omega_2] + \hat{\theta}\phi^2 M_1 = (1 - \hat{\theta}\phi)^2 E_1[\Omega_1] + (1 - \hat{\theta}\phi)\hat{\theta}\phi^2 M_1 + \hat{\theta}\phi^2 M_1$$
$$E_1[\Omega_4] = (1 - \hat{\theta}\phi)E_1[\Omega_3] + \hat{\theta}\phi^2 M_1 = (1 - \hat{\theta}\phi)^3 E_1[\Omega_1] + (1 - \hat{\theta}\phi)^2 \hat{\theta}\phi^2 M_1 + (1 - \hat{\theta}\phi)\hat{\theta}\phi^2 M_1 + \hat{\theta}\phi^2 M_1 = \text{t}$$
....
$$E_1[\Omega_t] = (1 - \hat{\theta}\phi)E_1[\Omega_{t-1}] + \hat{\theta}\phi^2 M_1 = (1 - \hat{\theta}\phi)^{t-1} E_1[\Omega_1] + \hat{\theta}\phi^2 M_1 \sum_{x=0}^{t-2} (1 - \hat{\theta}\phi)^{t-3}$$

herefore, from $E_1[\Omega_t] = (1 - \hat{\theta}\phi)^{t-1} E_1[\Omega_1] + \hat{\theta}\phi^2 M_1 \sum_{x=0}^{t-2} (1 - \hat{\theta}\phi)^{t-3}$ we obtain

$$\lim_{t \to \infty} E_1[\Omega_t] = \phi M_1 = \hat{\Omega}$$

moreover from $E_1[\Omega_t] = (1 - \hat{\theta}\phi)E_1[\Omega_{t-1}] + \hat{\theta}\phi^2 M_1$ it descends that

$E_1[\Omega_t] \le \hat{\Omega} \Rightarrow E_1[\Omega_{t+1}] \le \hat{\Omega}$ and, thus, $E_1[\Omega_1] \le \hat{\Omega} \Rightarrow E_1[\Omega_t] \le \hat{\Omega}$

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