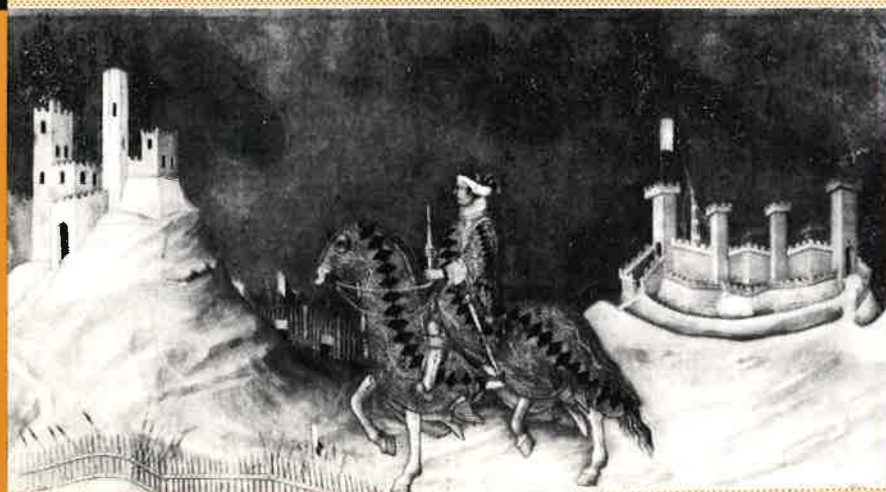
103

UNIVERSITÀ DEGLI STUDI DI SIENA

QUADERNI DEL DIPARTIMENTO
DI ECONOMIA POLITICA

Stefano Vannucci

ON THE CLOSURE OPERATORS OF AN EFFECTIVITY FUNCTION



QUADERNI DEL DIPARTIMENTO DI ECONOMIA POLITICA

Redazione: Dipartimento di Economia Politica, Piazza S. Francesco, 17 - 53100 Siena - tel.0577/298620

La Redazione ottempera agli obblighi previsti dall'Art.1 del D.L.L. 31.8.45 n.660

Le richieste di copie della presente pubblicazione dovranno essere indirizzate alla Redazione

I Quaderni del Dipartimento di Economia Politica dell'Università di Siena vengono pubblicati come servizio atto a favorire la tempestiva divulgazione di ricerche scientifiche originali, siano esse in forma provvisoria o definitiva. I Quaderni vengono regolarmente inviati a tutti gli istituti e dipartimenti italiani, a carattere economico, nonché a numerosi docenti e ricercatori universitari. Vengono altresì inviati ad enti e personalità italiane ed estere. L'accesso ai Quaderni è approvato dal Comitato scientifico, sentito il parere di un referee.

(Dal 1979 a tutto il 1987 la denominazione della presente Collana è stata: "Quaderni dell'Istituto di Economia". La numerazione progressiva rimane tuttavia la stessa).

* A preliminary version of this paper was presented, under the same title, at the Fifth Italian Meeting on Game Theory and its Applications (Cagliari, October 1988). The research summarized in the article was started while the author was visiting the Faculty of Economics and Politics of the University of Cambridge, U.K. . Financial support by the CNR is also gratefully acknowledged.

Stefano Vannucci

ON THE CLOSURE OPERATORS OF AN EFFECTIVITY FUNCTION

Siena, giugno 1990

Stefano Vannucci is "Ricercatore" in the Department of Economics of the University of Siena.

1. Introduction.

An effectivity function (EF) models the allocation of power among coalitions under a given game form.

EFs were first introduced in a strategic social choice setting, to deal with implementation problems (see Moulin,Peleg(1982)).

Therefore, the underlying sets of the players and of the alternatives were at first taken to be finite (but see Ichiishi(1986) for a fairly general survey including results on EFs of extensive game forms).

Necessary and sufficient conditions for core stability and implementability in strong Nash equilibrium of EFs defined over finite sets have been established (see Keiding(1985) and Moulin,Peleg(1982) respectively).

Applications in a sensible economic setting and/or to general game forms have provided motivation for some of these results to be extended to the case of infinite sets of alternatives and/or players(see Abdou(1987 a, 1988), Abdou,Mertens(1989); see also Rosenthal(1972) and Andjiga,Moulen(1989) for some even more general extensions of the EF concept).

Topological EFs have also been defined in this context whenever the set of alternatives is a topological space (see Abdou (1987 b)).An EF is said to be topological in that sense if it happens to be "consistent" with the topological structure of the underlying alternative space.

The aim of this note is to point out that EFs may also induce a topology in a natural way over the sets of all families of coalitions and issues, respectively.

This task is accomplished by pointing out that a Galois connection can be attached in a natural way to any EF .

Therefore a pair of closure operators are attached to any EF .

Some simple properties of the closure operators of an EF are established.

In particular, it is shown that:

i) the cardinality of the set of closed families of issues is a sensible, if rough, measure of the "smoothness" of the power allocation induced by the EF - taking its minimum value, 3, on simple EFs .

ii) Lack of specialized decision tasks turns out to be a sufficient condition in order to ensure that a nonsimple monotonic EF has topological closure operators .

2. Definitions and results .

Throughout this paper we shall use the following notation: for any set X, $P(X)$ will indicate its power set, and 2^X the set of all non empty subsets of X .

Let N and A be two non empty sets, called the set of **players** and the set of **alternatives**, respectively.

We also assume that both $|N|>2$ and $|A|>2$, in order to avoid trivialities.

Furthermore, subsets of N are also called **coalitions**; subsets of A, **issues**.

An **effectivity function** (EF) on N and A is a function $f:P(N) \longrightarrow P(P(A))$ s.t.

(EF 1) $f(\emptyset)=\emptyset$

(EF 2) for any $S \subset N$, $\emptyset \notin f(S)$

(EF 3) for any $B \subset A$, $B \in f(N)$

(EF 4) for any $S \subset N$, $A \in f(S)$.

We denote by **EF(N,A)** the set of all effectivity functions on N and A.

Effectivity functions may be classified according to the sharpness of the power allocation among coalitions they represent.

Thus, an EF f is said to be **simple** iff a set W of non empty coalitions exists s.t.: i) $W \neq \emptyset, W \neq P(2^N)$; ii) $f(S)=2^A$ whenever $S \in W$ and $f(S)=\{A\}$ otherwise.

The next ingredients for our analysis are closure operators and Galois connections.

A **closure operator** over a preordered set $(X,\leq)$ is a function $k:X \longrightarrow X$ s.t.

k i) for any $x \in X$, $x \leq k(x)$

k ii) for any $x,y \in X$, if $x \leq y$ then $k(x) \leq k(y)$

k iii) for any $x \in X$, $k(k(x)) \leq k(x)$.

The elements belonging to $k(X)$ are called **k-closed** .

We denote the set of all closure operators of a preordered set $(X,\leq)$ by $C((X,\leq))$ -or simply $C(X)$, whenever the relevant preordering is clearly identifiable from context.

Let the relevant preordered set be ($P(X),\subset$) for some arbitrary set X. Then a closure operator k is said to be **normal** iff $k(\emptyset)=\emptyset$. It is said to be **topological** (or Kuratowski) iff it is normal and **additive**, i.e. such that $k(Y \cup Z)=k(Y) \cup k(Z)$, for any $Y,Z \subset X$.

A **Galois connection** -between two preordered sets $\langle X,\leq_1 \rangle, \langle Y,\leq_2 \rangle$ - is a pair of functions $g:X \longrightarrow Y$, $h:Y \longrightarrow X$ s.t. :

i) both g and h are monotonic -i.e. for any $x_1,x_2 \in X$, $y_1,y_2 \in Y$, $x_1 \leq_1 x_2$ implies $g(x_1) \leq_2 g(x_2)$ and $y_1 \leq_2 y_2$ implies $h(y_1) \leq_1 h(y_2)$;

ii) for any pair $(x,y) \in X \times Y$, $y \leq_2 g(x)$ iff $h(y) \leq_1 x$.

The following are well-known facts about Galois connections:

a) the composition functions $h \circ g: X \to X$, $g \circ h: Y \to Y$ are closure operators;

b) whenever the relevant $(X, \leq)$ is a complete lattice– i.e. for any subset of X both a $\leq$- greatest lower bound and a $\leq$-least upper bound exist– the k-closed elements of X also form a complete lattice w.r.t. set inclusion;

c) for any binary relation $R \subset X \times Y$ we can define a Galois connection (g_R, h_R) between $(P(X), \subset)$ and $(P(Y), \subset)$ by posing–for any $B \in P(X)$, $C \in P(Y)$:

$$g_R(B) = \{y \in Y \ / \text{for all } b \in B: (b,y) \in R\}$$

$$h_R(C) = \{x \in X \ / \text{for all } c \in C: (x,c) \in R\}.$$

(g_R, h_R) is also referred to as the classical Galois connection of R.

We now adopt the terminology of Abdou(1987b) and introduce the following :

Definition : A pseudo-effectivity function (PEF) on N and A is a function $f: P(N) \to P(P(A))$ satisfying (EF 1) and (EF 2).

It can be easily shown that the following proposition holds true:

Proposition 1 :Let f be a PEF on N and A. Let us define two functions $K_f: P(P(N)) \to P(P(N))$, $K^*_f: P(P(A)) \to P(P(A))$ as follows:

$K_f(\sigma) = \{S \subset N / B \in f(S)$ for any $B \subset A$ s.t. $B \in f(T)$ for all $T \in \sigma\}$ for any $\sigma \subset 2^N$;

$K^*_f(\alpha) = \{B \subset A / B \in f(S)$ for any $S \subseteq N$ s.t. $C \in f(S)$ for all $C \in \alpha\}$ for any $\alpha \subset 2^A$.

Then, both K_f and K^*_f are normal closure operators w.r.t. set inclusion.

Thus, in particular, K_f (K^*_f) is topological iff

$K_f(\sigma_1 \cup \sigma_2) \subset K_f(\sigma_1) \cup K_f(\sigma_2)$, for any $\sigma_1, \sigma_2 \subset 2^N$

($K^*_f(\alpha_1 \cup \alpha_2) \subset K^*_f(\alpha_1) \cup K^*_f(\alpha_2)$, for any $\alpha_1, \alpha_2 \subset 2^A$).

Proof. Let us notice that PEF(N,A) is bijective to the set PER(N,A) of binary relations $R \subset P(N) \times P(A)$ s.t. $(S,B) \notin R$ if $\emptyset \in \{S,B\}$.

The natural bijection is, obviously, $F(f) = \{(S,B) \in P(N) \times P(A) / B \in f(S)\}$.

It is immediately seen that $K_f = h_{F(f)} \circ g_{F(f)}$, $K^*_f = g_{F(f)} \circ h_{F(f)}$ where $(g_{F(f)}, h_{F(f)})$ is the classical Galois connection attached to $F(f)$.

Therefore, by virtue of the property c) of Galois connections pointed out above, K_f and K^*_f are actually closure operators. The same conclusion can be obtained, of course, by checking that both K_f and K^*_f satisfy the properties of a closure operator.

Furthermore, $\{B \subset A / B \in f(S)$ for all $S \in \emptyset\} = P(A)$.

Thus, $K_f(\emptyset) = \{S \subset N / B \in f(S)$ for any $B \subset A\} = \emptyset$, by EF2.

Similarly, $K^*_f(\emptyset) = \{B \subset A / B \in f(S)$ for any $S \subset N\} = \emptyset$, by EF1 and EF2. ∎

Corollary 1. Let f, K_f, K^*_f be defined as in Proposition 1. Then the set of closed families $K_f(P(P(N)))$, $K^*_f(P(P(A)))$ turn out to be complete lattices (w.r.t. the usual set-theoretic join and meet operations).

Moreover, $|K_f(P(P(N)))| = |K^*_f(P(P(A)))|$.

Definition . An EF f on N and A is said to be regular, or proper, iff for any $S \in 2^N$, $B \in 2^A$: $B \in f(S)$ implies $A - B \notin f(N-S)$.

Definition. An EF f on N and A is said to be **reduced** iff $\{B\subset A/\ B\in f(S)$ for any $S\subset N\}=\{A\}$.

Remark 1. If f is a regular $EF(N,A)$, then it can be easily seen that $\emptyset, K^*_f(\{A\}), 2^A$ are three distinct K^*_f-closed families of issues, whereas $\emptyset, K_f(\{N\}), 2^N$ are three distinct K_f-closed families of coalitions. Moreover, if f is a reduced EF between N and A, then $K^*_f(\{A\})=\{A\}$. Also, as we shall see in a moment, the cardinalities of such sets of closed families, regularity, reduction, and "simplicity" of an EF are strictly related.

Proposition 2. Let $f \in EF(N,A)$. Then : i) f is regular and simple entail $|K_f(P(2^N))|=|K^*_f(P(2^A))|=3$; ii) f is reduced and $|K_f(P(2^N))|=|K^*_f(P(2^A))|=3$ entail that f is simple.

Proof. i) Let us assume that f is regular and simple. Hence, a non empty set W of coalitions- $W\subsetneq P(N)$- exists s.t. $f(S)=2^A$ if $S\in W$, and $f(S)=\{A\}$ otherwise. Let us consider now a non empty set of coalitions, σ. By the definition of K_f and our hypothesis, it turns out that we only need consider two basic cases: either a) σ does contain a coalition $S'\in 2^N-W$ or b) $\sigma\subset W$.
If a) is the case, then-obviously- $K_f(\sigma)=2^N$.
If b) is the case instead it is easily seen that $K_f(\sigma)=W$.
Thus, $K_f(P(2^N))=\{\emptyset, W, 2^N\}$.
Turning now to K^*_f, let us consider a non empty set of issues, α. Here again we need only consider two cases.

If α reduces to $\{A\}$ then obviously $K^*_f(\{A\})=\{A\}$. Otherwise, it is immediate to check that $K^*_f(\alpha)=2^A$. Therefore, $K^*_f(P(P(A)))=\{\emptyset,\{A\},2^A\}$.
ii) Let us suppose that $|K_f(P(2^N))|=|K^*_f(P(2^A))|=3$ while f being reduced but not simple. Then, for some $B_1, B_2 \in 2^A-\{A\}$, and $S\in 2^N$ the following holds true: $B_1\in f(S)$ and $B_2 \notin f(S)$. It follows that $B_2\notin K^*_f(\{B_1\})$.
Hence $K^*_f(\{B_1\})\neq 2^A$.
Furthermore, by reduction, $K^*_f(\{A\})=\{A\}\neq K^*_f(\{B_1\})$. Obviously, $K^*_f(\{B_1\})\neq\emptyset$. Therefore $|K^*_f(P(2^A))|>3$, a contradiction. ∎

Corollary 2. Let f be a reduced regular EF on N and A. Then, f is simple iff $|K_f(P(2^N))|=|K^*_f(P(2^A))|=3$. Moreover, if f is not simple then $|K_f(P(2^N))|=|K^*_f(P(2^A))|>3$.

Remark 2. Proposition 2 and Corollary 2 tell us that both $|K_f(P(P(N))|$ and $|K^*_f(P(P(A))|$ belong to the interval $(3, \min\{|P(P(N))|, |P(P(A))|\})$.
Thus, as claimed before, the cardinalities of the sets of closed families of coalitions (and issues) represent a measure of the "smoothness" of the power allocations induced by different EFs.

Remark 3. It should be noticed that $EF(N,A)$ is bijective to the subset of $PER(N,A)$ (see the proof of Proposition 2) consisting of relations $R\subset P(N)\times P(A)$ s.t. $\{(N,B)/\ \emptyset\neq B\subset A\}\subset R$ and $\{(S,A)/\emptyset\neq S\subset N\}\subset R$ (the natural bijection being, obviously, the function F occurring in the proof of Proposition 1). Let us call it $ER(N,A)$.
It is easily checked that $ER(N,A)$ is bijective to the set $\{g_{F(f)}: P(P(N))\longrightarrow P(P(A))\ /\ f\in EF(N,A)\}$

(see the definitions above and recall that ,by definition, $g_{F(f)}(\sigma)=\bigcap_{S\in\sigma}f(S)$,for any $\sigma\subset P(N)$).
Similarly, EF(N,A) turns out to be bijective to the set $\{ h_{F(f)}:P(P(A))\longrightarrow P(P(N))$ / $f \in EF(N,A) \}$.
Finally,it is worth mentioning the following fact :
for any $R\subset P(N)\times P(A)$, $R \in ER(N,A)$ iff $R^{-1}\in ER(A,N)$.
As a result ,it should be clear the connection between a theory of duality of EFs in our own framework and the theory of duality presented in Peleg (1984).
The two of them turn out to be equivalent, actually.
In fact, for any given $f \in EF(N,A)$, Peleg's dual of f is the restriction of $h_{F(f)}$ to singleton-families of issues, which in turn essentially coincides with $h_{F(f)}$.
However,it seems to us that the present framework makes it a bit clearer how the duality theory of EFs merges into general residuation theory .

Let us now turn to topology-inducing EFs .

Example 1. Let f be a regular simple EF on N and A , i.e. a representation of order |A| of a proper simple game.
Then, bearing in mind Proposition 2 i) and its proof , it is easily seen that both K_f and K^*_f are U-additive .
Hence,they are topological closure operators.

Remark 4. By definition, a "nice" EF does represent an efficient, fairly symmetric (hence nontrivial, i.e. nondictatorial) and strategically stable collective decision procedure.
Obviously, a decision procedure may or may not be strategically stable,depending on the relevant equilibrium concept.

In fact, "nice" simple EFs happen to be consistent with a noncooperative environment ,perhaps under a complete information assumption (see Dutta(1984)), much less so with a cooperative one.
By contrast, "nice" nonsimple EFs are known to exist in a cooperative environment (see Moulin,Peleg(1982)) .
An outstanding example is the class of maximal additive EFs which are known to be core-stable, implementable w.r.t. strong Nash equilibrium (see Moulin,Peleg(1982)) , and strongly core-stable (i.e. enjoying a kind of nonmanipulability property : see Demange(1987)).
Thus, it is worth asking whether any "nice" nonsimple EF has a topological closure operator .

We can get a hint from the following example of a "nice" EF which is not (fully) topology-inducing .

Example 2 . Let $N=\{1,2,..,n\}$, $n\geq 3$, $A=\{a_o,a_1,...,a_n\}$.
Let us define
$f'(\{i\})=\{B\subset A/\ A\setminus\{x_o\}\subset B\ \}\ U\ \{\ B\subset A\ /\ A\setminus\{a_i\}\ \}$,for any $i\in N$,
and
$f'(\{S\})= U_{i\in S}[f'(\{i\})]$, for any $S\subset N$.
f' may be interpreted as a very simple example of a minimally libertarian decision procedure .
It can be easily checked that f' is nucleus-stable (see Holzman(1987) for a definition of such a notion ,and for details).
Hence, f' is core-stable and implementable w.r.t. strong Nash equilibrium .
Now, K_f is not a topological closure operator .
In fact, for any $i,j \in N$,$i\neq j$, $K_f(\{i\})=\{S\subset N/\ \ i\in S\ \}$, whereas $K_f(\{i\},\{j\})= 2^N$.

Remark 5. The key feature of f' is that different players and coalitions are endowed with very specialized decision tasks .

The area of decision-making power any pair of them share reduces to $A\setminus\{a_o\}$, which belongs to the effectivity domain of any coalition.

Hence, K_f fails to satisfy U-additivity and, as a consequence, it is not topological.

By contrast, the simple EFs mentioned in example 1 do represent neutral collective decision procedures (i.e. treat the alternatives symmetrically) .

Thus, one is tempted to conclude that the topology-inducing properties of an EF may be related in some way to symmetry properties.

This will be made more precise in the next proposition.

Some more definitions are in order here .

Definition. Let f be an EF on N and A . f is said to be **N-monotonic** (**A-monotonic**) iff for any $S,T\subset N$ and $B\subset A$: $B \in f(S)$ and $S\subset T$ entail $B \in f(T)$ (for any $B,C\subset A$ and $S\subset N$: $B \in f(S)$ and $B\subset C$ entail $C \in f(S)$) .

Definition. Let f be an EF on N and A . f is said to be **anonymous** iff for any $S,T\subset N$: $|S|=|T|$ entails $f(S)=f(T)$.

Definition. Let f be an EF on N and A . f is said to be **neutral** iff for any $B,C\subset A$ and $S\subset N$: $|B|=|C|$ entails ($B \in f(S)$ iff $C \in f(S)$).

Proposition 3 . Let f be an EF on N and A , and K_f, K^*_f its own closure operators (see proposition 1 above).

Then the following holds true :

if f is anonymous and N-monotonic then K^*_f is topological .

Dually, if f is neutral and A-monotonic , then K_f is topological.

Proof. Let f be an anonymous N-monotonic EF. Then we can define the function $k^*_f : P(P(A)) \longrightarrow \{0,1,\dots,n\}$ by the following rule : for any $\beta\subset P(A)$,

$k^*_f(\beta) = \inf\{\, |S| \,/\, B \in f(S) \text{ for any } B \in \beta \,\}$.

It follows that , for any $\beta\subset P(A)$:

$K^*_f(\beta) = \{\, C\subset A \,/\, C \in f(S) \text{ for any } S\subset N \text{ s.t. } |S| = k_f(\beta) \,\}$.

Now, for any $\beta_1,\beta_2\subset P(A)$,

$k^*_f(\beta_1\cup\beta_2) = \max\{\, k^*_f(\beta_1), k^*_f(\beta_2) \,\}$

(by anonymity and N-monotonicity).

Therefore $K^*_f(\beta_1\cup\beta_2) = K^*_f(\beta_1) \cup K^*_f(\beta_2)$.

The dual thesis follows from a similar argument which is left to the reader. ∎

Remark 6. Clearly the conditions singled out by proposition 3 are not necessary for the topology-inducing property . In fact, we know that any simple f has a topological K^*_f (see example 1).

Remark 7 . The attentive reader will notice that the definition of k^*_f implicitly assumes finiteness of N .

However, the argument can be easily recasted for infinite N (and A) , provided that N (and A) are embedded in a suitable measure space .

Remark 8. Under the hypotheses of proposition 3 the topologies generated by K_f and/or K^*_f have "few" open sets. In fact, they are not even T_0 (as a consequence of neutrality and anonymity, respectively).
In order to obtain T_0 spaces one has to consider some natural quotient spaces, namely the "type"-spaces of an EF.
This is made precise in the sequel.

Definition. Let f be a PEF on N and A (see the definition above). Then, for any coalition $S\subset N$ (issue $B\subset A$) the type of S (of B, respectively) in f is a function $t_{f,S}:K_f[P^2(N)]\longrightarrow\{0,1\}$ (a function $t^*_{f,B}:K^*_f[P^2(A)]\longrightarrow\{0,1\}$), defined by the following rules:

$$t_{f,S}(\sigma)=\begin{cases}1 \text{ if } S\in\sigma \\ \\ 0 \text{ otherwise}\end{cases} , \quad \text{for any } \sigma\in K_f[P^2(N)]$$

$$\left(t^*_{f,B}(\beta)=\begin{cases}1 \text{ if } B\in\beta \\ \\ 0 \text{ otherwise}\end{cases} , \quad \text{for any } \beta\in K^*_f[P^2(A)]\right).$$

Definition. Let f be an EF on N and A . We define two equivalence relations $R_f\subset P(N)\times P(N)$, $R^*_f\subset P(A)\times P(A)$ by the following rules: for any $S,T\subset N$ and $B,C\subset A$
$(S,T)\in R_f$ iff $t_{f,S}=t_{f,T}$,and
$(B,C)\in R^*_f$ iff $t^*_{f,B}=t^*_{f,C}$.

Proposition 4 . Let f be an EF on N and A, and R_f, R^*_f as in the definition above. Then the following holds true:
i) if f is anonymous and N-monotonic then the quotient space $P(A)/R^*_f$ induced by K^*_f is T_0 ;
ii) dually, if f is neutral and A-monotonic then the quotient space $P(N)/R_f$ induced by K_f is T_0 .

Proof. i) Let us consider two distinct points of $P(A)/R^*_f$, say $[B],[C]$. Thus, by definition, $t^*_{f,B}=t^*_{f,C}$, i.e. without loss of generality
$t^*_{f,B}(\beta)=1$ and $t^*_{f,C}(\beta)=0$ for some $\beta\in K^*_f[P(P(A))]$.
By N-monotonicity and anonymity, it follows that $k^*_f([B]) < k^*_f([C])$ (where k^*_f is defined as in the proof of proposition 3).
Next, for any $[D]\in P(A)/R^*_f$ we define the sets
$D\uparrow=\{\,[D']\in P(A)/R^*_f \ / \ k^*_f([D'])\le k^*_f([D])\,\}$.
It is straightforward to check that, under our hypotheses, the closed sets of $P(A)/R^*_f$ turn out to be sets of that form, namely each of them is equal to $[D]\uparrow$ for some $[D]\in P(A)/R^*_f$.
Hence, the thesis follows since obviously $[B]\in[B]\uparrow$, whereas $[C]\notin[B]\uparrow$.
ii) the proof of ii) parallels the previous one and is therefore omitted. ∎

Remark 9. It should be noticed that whenever the relevant EF f is simple , its "type space" $P(N)/R_f$ (endowed with the quotient topology induced by K_f) turns out to be homeomorphic to the Sierpinski space $(\{0,1\}, \{\emptyset,\{1\},\{0,1\}\})$.

3. Extensions to general coalitional game forms .

It is worth mentioning that the foregoing analysis is amenable to further generalizations and extensions.

In fact, closure operators with the properties mentioned in proposition 1 can be defined for any coalitional game form ,provided that the following hold true:

i) the relevant coalitional and outcome structures are preordered sets with a minimum (say $< @,\leq,0 >$ and $<X,\leq',0'>$,respectively).

ii) the coalitional game form is representable by a correspondence $R\subset @xX$ s.t. a) $R(0)=\emptyset$,b) $R^{-1}(0')=\emptyset$.

The foregoing requirements i)-ii) amount to a generalization of pseudo-effectivity functions (see the definition above): let us call them pseudo general coalitional game forms (PGCGF).

Extending the notion of an effectivity function to a general coalitional game-theoretical setting requires some more structure.

Namely, both the coalitional structure and the outcome structure must be preordered sets with a minimum and a maximum, say $<@,\leq,0,1>$ and $<X,\leq',0',1'>$, respectively.

Then, we may define a general coalitional game form (GCGF) as follows:

a GCGF is a correspondence $R\subset @xX$ s.t. a) $R(0)=\emptyset$ b) $R^{-1}(0')=\emptyset$; c) $R(1)=X$; d) $R^{-1}(1')=@$.

Proposition 1 can be easily extended to PGCGFs.

Propositions 2,3,4 can also be extended to GCGFs,but this task would obviously require a lot of adaptations from previous definitions.

Hence,the details are left as a topic for another paper.

We conclude with two examples of GCGFs.

1) Constitutional game forms (CGFs).

Given an universal set N of conceivable players and an universal set A of conceivable outcomes , a CGF f specifies for any coalition S , set T of opponents to S

and feasible outcome set B , the set of all subsets of B which S has the power to veto.

Thus ,we may define the relevant GCGF as follows:

$@=\{ (S,T) / S\subset N, T\subset N, S\cap T=\emptyset \}$, preordered by the correspondence $<<$ defined as the lexicographic preordering induced by (c,c^{-1}) (i.e. $(S_1,T_1)<<(S_2,T_2)$ iff $S_1 cS_2$ or $S_1=S_2$ and T_2cT_1 , for any $(S_1,T_1),(S_2,T_2)\in @$).

Clearly, $(\emptyset,N)$ and $(N,\emptyset)$ are the minimum and the maximum of $< @,<< >$,respectively.

For any $(S,T)\in @$ the members of $N\setminus(S\cup T)$ are meant to represent "unconcerned" players .

Similarly ,

$X=\{ (B,C) / B\subset A, C\subset A , B\cap C=\emptyset \}$ is preordered by $<<'$ ($(B_1,C_1)<<'(B_2,C_2)$ iff $B_1 cB_2$ or $B_1=B_2$ and $C_1 cC_2$).

Hence, $(\emptyset,\emptyset)$ and $(A,\emptyset)$ are the minimum and the maximum of $< X,<<' >$,respectively .

For any $(B,C)\in X$, $B\cup C$ is meant to represent the set of feasible outcomes ,and C the set of rejected outcomes (given the feasible set $B\cup C$) .

Obviously $((S,T),(B,C)) \in R$ iff $(B,C) \in f((S,T))$,for any $((S,T)x(B,C)) \in @xX$.

It is worth mentioning that CGFs generalize both Andjiga-Moulen's "games in constitutional form" and Blau-Brown's "directed sums of simple games" (or "semisimple games":see Packel (1981)).

2) Partitional game forms (PGFs).

Given an universal set N of players and an universal set A of outcomes, a PGF f specifies for any coalition S as embedded in a coalition structure π the set of all subsets B of A s.t. (S,π) can veto outcomes in $A\setminus B$.

The GCGF attached to such a PGF is defined as follows.

$@=\{ (S,\pi)$ / a set $\{S_i\}_{i\in I}$ exists s.t. :i) $S_i \subset N$ for any $i\in I$; ii) $\cup S_i=N$ and $S_i \cap S_j=\emptyset$ for any $i\neq j$ iii) $S=S_j$ for some $j\in I$ $\}$

(i.e. @ is the set of embedded coalitions).

@ is preordered by the covering relation <<* as defined by the following rule (see Myerson(1977)) : for any (S_1,π_1), $(S_2,\pi_2)\in @$

$(S_1,\pi_1) <<^* (S_2,\pi_2)$ iff $S_1 \subset S_2$ and $\pi_1 \perp \pi_2 = \pi_1$

(where $\pi_1 \perp \pi_2 = \{ S_1 \cap S_2$ / $S_1 \in \pi_1$ and $S_2 \in \pi_2 \}$).

Hence the minimum of the coalition structure is $(\emptyset,\{\emptyset,\{i\}_{i\in N})$ whereas the maximum is $(N,\{\emptyset,N\})$.

The outcome preordering $< X,\leq' >$ is defined as follows: $X=P(A)$, and $\leq'=\subset$,the set theoretic inclusion correspondence .

The correspondence $R\subset @\times X$ is defined as in the previous example: namely, $((S,\pi),B) \in R$ iff $B \in f((S,\pi))$.

Moreover,it is also worth noticing here that PGFs can be further generalized by specifying the outcome preordering along the lines of the OGF case as presented above.

4. Conclusions.

In recent years, a growing concern for institutional design problems and for incomplete information situations has provided motivation for analysing strategic interdependence in terms of game forms as opposed to games.

The main result of this paper consists in showing that EFs and more general coalitional game forms do have a fairly rich order-theoretic structure.

In fact ,our preliminary analysis shows that under suitable symmetry or simplicity properties , such an

order-theoretic structure is rich enough to induce an "intrinsic" topological structure on the relevant coalitional game form.

References

ABDOU J. (1987a): Stable Effectivity Functions with an Infinity of Players and Alternatives. Journal of Mathematical Economics 16,291-295.

ABDOU J. (1987b) .: Stability of Topological Effectivity Functions. Social Choice and Welfare 4 ,163-171 .

ABDOU J. (1988) : Neutral Veto Correspondences with a Continuum of Alternatives. International Journal of Game Theory 17,135-164.

ABDOU J.,J.F.MERTENS (1989):Correlated Effectivity Functions. Economics Letters 30,97-101 .

ANDJIGA N.G.,J.MOULEN (1989): Necessary and Sufficient Conditions for l-Stability of Games in Constitutional Form. International Journal of Game Theory 18, 91-110.

DEMANGE G. (1987): Nonmanipulable Cores. Econometrica 55,1057-1074.

DUTTA B. (1984): Effectivity Functions and Acceptable Game Forms. Econometrica 52,1151-1166.

HOLZMAN R.(1987): Sub-Core Solutions to the Problem of Strong Implementation. International Journal of Game Theory 16, 263-289.

ICHIISHI T.(1986): The Effectivity Function Approach to the Core, in W.Hildenbrand,A.Mas-Colell (eds.)

Contributions to Mathematical Economics in Honor of G.Debreu. Amsterdam ,North Holland, 268-293.

KEIDING H. (1985): Necessary and Sufficient Conditions for Stability of Effectivity Functions. International Journal of Game Theory 14,93-101.

MOULIN H.,B.PELEG (1982): Cores of Effectivity Functions and Implementation Theory. Journal of Mathematical Economics 10,115-145.

MYERSON R.B. (1977): Values of Games in Partition Function Form. International Journal of Game Theory 6,23-31.

PACKEL E.W. (1981): Semisimple Games,Enactment Power and Generalized Power Indices. International Journal of Game Theory 10,137-146 .

PELEG B. (1984): Core Stability and Duality of Effectivity Functions , in G.Hammer,D.Pallaschke(eds.) Selected Topics in Operations Research and Mathematical Economics. New York Heidelberg Berlin ,Springer Verlag , 272-287 .

ROSENTHAL R.W.(1972): Cooperative Games in Effectiveness Form . Journal of Economic Theory 5,88-101.

Elenco dei Quaderni pubblicati

n.1 (febbraio 1979) MASSIMO DI MATTEO, Alcune considerazioni sui concetti di lavoro produttivo e improduttivo in Marx
n.2 (marzo 1979) MARIA L.RUIZ, Mercati oligopolistici e scambi internazionali di manufatti. Alcune ipotesi e un'applicazione all'Italia
n.3 (maggio 1979) DOMENICO MARIO NUTI, Le contraddizioni delle economie socialiste: una interpretazione marxista
n.4 (giugno 1979) ALESSANDRO VERCELLI, Equilibrio e dinamica del sistema economico-semantica dei linguaggi formalizzati e modello keynesiano
n.5 (settembre 1979) A. RONCAGLIA - M. TONVERONACHI, Monetaristi e neokeynesiani: due scuole o una?
n.6 (dicembre 1979) NERI SALVADORI, Mutamento dei metodi di produzione e produzione congiunta
n.7 (gennaio 1980) GIUSEPPE DELLA TORRE, La struttura del sistema finanziario italiano: considerazioni in margine ad un'indagine sull'evoluzione quantitativa nel dopoguerra (1948-1978).
n.8 (gennaio 1980) AGOSTINO D'ERCOLE, Ruolo della moneta ed impostazione antiquantitativa in Marx: una nota
n.9 (novembre 1980) GIULIO CIFARELLI, The natural rate of unemployment with rational expectations hypothesis. Some problems of estimation
n.10 (dicembre 1980) SILVANO VICARELLI, Note su ammortamenti, rimpiazzi e tasso di crescita
n.10 bis (aprile 1981) LIONELLO F. PUNZO, Does the standard system exist?
n.11 (marzo 1982) SANDRO GRONCHI, A meaningful sufficient condition for the uniqueness of the internal rate of return
n.12 (giugno 1982) FABIO PETRI, Some implications of money creation in a growing economy
n.13 (settembre 1982) RUGGERO PALADINI, Da Cournot all'oligopolio: aspetti dei processi concorrenziali
n.14 (ottobre 1982) SANDRO GRONCHI, A Generalized internal rate of return depending on the cost of capital
n.15 (novembre 1982) FABIO PETRI, The Patinkin controversy revisited
n.16 (dicembre 1982) MARINELLA TERRASI BALESTRIERI, La dinamica della localizzazione industriale: aspetti teorici e analisi empirica
n.17 (gennaio 1983) FABIO PETRI, The connection between Say's law and the theory of the rate of interest in Ricardo
n.18 (gennaio 1983) GIULIO CIFARELLI, Inflation and output in Italy: a rational expectations interpretation
n.19 (gennaio 1983) MASSIMO DI MATTEO, Monetary conditions in a classical growth cycle
n.20 (marzo 1983) M. DI MATTEO - M.L. RUIZ, Effetti dell'interdipendenza tra paesi produttori di petrolio e paesi industrializzati: un'analisi macrodinamica
n.21 (marzo 1983) ANTONIO CRISTOFARO, La base imponibile dell'IRPEF: un'analisi empirica
n.22 (gennaio 1984) FLAVIO CASPRINI, L'efficienza del mercato dei cambi. Analisi teorica e verifica empirica
n.23 (febbraio 1984) PIETRO PUCCINELLI, Imprese e mercato nelle economie socialiste: due approcci alternativi

n.24 (febbraio 1984) BRUNO MICONI, Potere prezzi e distribuzione in economie mercantili caratterizzate da diverse relazioni sociali
n.25 (aprile 1984) SANDRO GRONCHI, On investment criteria based on the internal rate of return
n.26 (maggio 1984) SANDRO GRONCHI, On Karmel's criterion for optimal truncation
n.27 (giugno 1984) SANDRO GRONCHI, On truncation "theorems"
n.28 (ottobre 1984) LIONELLO F. PUNZO, La matematica di Sraffa
n.29 (dicembre 1984) ANTONELLA STIRATI, Women's work in economic development process
n.30 (gennaio 1985) GIULIO CIFARELLI, The natural rate of unemployment and rational expectation hypotheses: some empirical tests
n.31 (gennaio 1985) SIMONETTA BOTARELLI, Alcuni aspetti della concentrazione dei redditi nel Comune di Siena
n.32 (febbraio 1985) FOSCO GIOVANNONI, Alcune considerazioni metodologiche sulla riforma di un sistema tributario
n.33 (febbraio 1985) SIMONETTA BOTARELLI, Ineguaglianza dei redditi personali a livello comunale
n.34 (marzo 1985) IAN STEEDMAN, Produced inputs and tax incidence theory
n.35 (aprile 1985) RICHARD GOODWIN, Prelude to a reconstruction of economic theory. A critique of Sraffa
n.36 (aprile 1985) MICHIO MORISHIMA, Classical, neoclassical and Keynesian in the Leontief world
n.37 (aprile 1985) SECONDO TARDITI, Analisi delle politiche settoriali: prezzi e redditi nel settore agroalimentare
n.38 (maggio 1985) PIETRO BOD, Sui punti fissi di applicazioni isotoniche
n.39 (giugno 1985) STEFANO VANNUCCI, Schemi di gioco simmetrici e stabili e teoremi di possibilità per scelte collettive
n.40 (luglio 1985) RICHARD GOODWIN, The use of gradient dynamics in linear general disequilibrium theory
n.41 (agosto 1985) M. MORISHIMA and T. SAWA, Expectations and the life span of the regime.
n.42 (settembre 1985) ALESSANDRO VERCELLI, Keynes, Schumpeter, Marx and the structural instability of capitalism
n.43 (ottobre 1985) ALESSANDRO VERCELLI, Money and production in Schumpeter and Keynes: two dichotomies
n.44 (novembre 1985) MARCO LONZI, Aspetti matematici nella ricerca di condizioni di unicità per il Tasso Interno di Rendimento
n.45 (dicembre 1985) NIKOLAUS K.A. LAUFER, Theoretical foundations of a new money supply hypothesis for the FRG.
n.46 (gennaio 1986) ENRICO ZAGHINI, Una dimostrazione alternativa dell'esistenza di equilibri in un modello di accumulazione pura
n.47 (febbraio 1986) ALESSANDRO VERCELLI, Structural stability and the epistemology of change: a critical appraisal
n.48 (marzo 1986) JOHN HICKS, Rational behaviour: observation or assumption?
n.49 (aprile 1986) DOMENICO MARIO NUTI, Merger conditions and the measurement of disequilibrium in labour-managed economies
n.50 (maggio 1986) SUSAN SENIOR NELLO, Un'applicazione della Public Choise Theory alla questione della riforma della Politica Agricola Comune della CEE
n.51 (maggio 1986) SERENA SORDI, Some notes on the second version of Kalecki's Business Cycle Theory

n.52 (giugno 1986) PIERANGELO GAREGNANI, The classical theory of wage and the role of demand schedules in the determination of relative prices
n.53 (luglio 1986) FRANCO BIRARDI, Crescita, sviluppo ed evoluzione: l'affermazione dei nessi strutturali sistematici e la combinazione di scienza, tecnologia ed economica nelle prime industrializzazioni
n.54 (agosto 1986) M. AMENDOLA - J.L. GAFFARD, Innovation as creation of technology: a sequential model
n.55 (settembre 1986) GIULIO CIFARELLI, Exchange rates, market efficiency and "news". Model specification and econometric identification
n.56 (settembre 1986) ALESSANDRO VERCELLI, Technological flexibility, financial fragility and the recent revival of Schumpeterian entrepreneurship
n.57 (ottobre 1986) FABIO PETRI, The meaning of the Cambridge controversies on capital theory: an attempt at communication
n.58 (ottobre 1986) DANIELE CHECCHI, The international coordination of counter-cyclical policies
n.59 (novembre 1986) ALESSANDRO VERCELLI, Probabilistic causality and economic models: Suppes, Keynes, Granger
n.60 (dicembre 1986) LIONELLO F. PUNZO, J. Von Neumann and K. Menger's mathematical colloquium
n.61 (dicembre 1986) MICHAEL BACHARACH, Three scenarios for fulfilled expectations
n.62 (gennaio 1987) J. ESTEBAN and T. MILLAN, Competitive equilibria and the core of overlapping generations economies
n.63 (gennaio 1987) ROBERT W. CLOWER, New directions for keynesian economics
n.64 (febbraio 1987) MARCO P. TUCCI, A simple introduction to flexible functional forms and consumer behaviour theory
n.65 (marzo 1987) NICOLA DIMITRI, Generalizations of some continuos time epidemic models
n.66 (aprile 1987) MASSIMO DI MATTEO, Goodwin and the Evolution of a Capitalistic Economy: An Afterthought
n.67 (maggio 1987) MASSIMO DI MATTEO, Early Discussions on Long Waves
n.68 (giugno 1987) MASSIMO DI MATTEO, Warranted, Natural, and Actual Rates of Growth: Reflections of a Perplex
n.69 (settembre 1987) LUISA MONTUSCHI, Inward-Looking development and import substitution in the Argentine Economy 1950-1980
n.70 (dicembre 1987) GIANCARLO GANDOLFO and PIETRO CARLO PADOAN, The Mark V version of the Italian continuos time model
n.71 (dicembre 1987) SILVANO VICARELLI, Impatience to consume, the utility of wealth and the rate of interest
n.72 (gennaio 1988) OMAR CHISARI, Worker's assets, foreign debt and the dynamics of an economy
n.73 (febbraio 1988) STEFANO VANNUCCI, On exactly strong-Nash-implementable power allocations
n.74 (marzo 1988) PIER LUIGI LANDI, Aspetti economici della società anonima della strada ferrata Empoli-Siena o Centrale Toscana fino al 1859
n.75 (aprile 1988) FRANCO BIRARDI, Generating the space of physical characteristics by means of elements which are discrete and limited over time (A note on innovation in chemical technologies)
n.76 (maggio 1988) LILIA COSTABILE, Politica monetaria, debito pubblico e tasso di interesse

n.77 (giugno 1988) GIUSEPPE DELLA TORRE, L'evoluzione dei sistemi finanziari nell'approccio contabile di Raymond Goldsmith. Alcune considerazioni analitiche
n.78 (luglio 1988) FRANCESCO FARINA, Fattori tecnologici e transazionali nella divisione interindustriale del lavoro
n.79 (ottobre 1988) FRANCESCO FARINA, Gli obiettivi della Banca d'Italia e i tassi d'interesse sul debito pubblico
n.80 (novembre 1988) GIULIO CIFARELLI, La razionalità del tasso di cambio a termine e le "notizie": una analisi empirica
n.81 (dicembre 1988) MAURO CAMINATI, Cyclical growth in a long-period perspective
n.82 (gennaio 1989) LUIGI LUINI, Crescita esterna dell'impresa e teorie economiche
n.83 (marzo 1989) LUIGI M. TOMASINI, A Scheme to Allocate Wastewater Rights
n.84 (aprile 1989) STEFANO VANNUCCI, On Consistency Properties of some Strongly Implementable Social Choice Rules with Endogenous Agenda Formation
n.85 (maggio 1989) AGOSTINO D'ERCOLE, Economia e Moneta in Schumpeter
n.86 (maggio 1989) CHARLES GOODHART, What do Central Banks Do?
n.87 (giugno 1989) PAOLO DEL GIOVANE, Foreign Firms' Behavior in the United States Market during the Dollar Cycle
n.88 (giugno 1989) GÉRARD DUMÉNIL and DOMINIQUE LÉVY, The classical legacy and beyond
n.89 (luglio 1989) RICCARDO FIORITO, Inventory accounting and profit evaluation
n.90 (settembre 1989) NICOLA DIMITRI, On the robustness of the rational expectations hypothesis
n.91 (settembre 1989) PIERMARIO PACINI, Expectations, learning and equilibrium in a non-walrasian model: a constructive approach
n.92 (ottobre 1989) AGOSTINO D'ERCOLE, Sulla teoria monetaria del tasso di interesse di Schumpeter
n.93 (ottobre 1989) UGO PAGANO, Firms, co-ordination and disequilibrium
n.94 (novembre 1989) UGO PAGANO, Determinazione endogena dei diritti di proprietà in una economia di mercato
n.95 (dicembre 1989) UGO PAGANO, Property rights, asset specificity, and the division of labour under alternative capitalist relations
n.96 (dicembre 1989) MICHIO MORISHIMA, Wage differentials in Japan: 1958-1985
n.97 (gennaio 1990) PIERPAOLO PIERANI - PIER LUIGI RIZZI, An econometric analysis of the olive oil market in Italy
n.98 (febbraio 1990) ENRICO GIOVANNINI, Squilibri settoriali e dinamica macroeconomica di breve periodo: il caso dell'industria italiana
n.99 (maggio 1990) GIULIO CIFARELLI Sui determinanti degli errori di previsione a termine nel mercato dei cambi
n.100 (maggio 1990) EDITED BY MASSIMO DI MATTEO Celebrating R.M. Goodwin's 75 birthday
n.101 (maggio 1990) OMAR O. CHISARI - JOSE' M. FANELLI Three-gap models, optimal growth and the economic dynamics of highly indebted countries
n.102 (giugno 1990) FRANCESCO FARINA Italy's monetary policy inside the european monetary system

Alla presente Collana "gialla" di quaderni il Dipartimento di Economia Politica affianca una Collana "celeste" di monografie. I numeri pubblicati a tutt'oggi sono i seguenti:

1) GIUSEPPE DELLA TORRE, Istituzioni creditizie ed Accumulazione del Capitale in Italia (1948-81), 1984.

2) SANDRO GRONCHI, Tasso Interno di Rendimento e Valutazione dei Progetti: una Analisi Teorica, 1984.

3) ALDINO MONTI, La Proprietà Immobiliare a Bologna in Età Napoleonica (1797-1810), 1984.

4) GIULIO CIFARELLI, Equilibrium and Disequilibrium Interpretations of Inflation, 1985.