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**A NOTE ON ERGODIC AND VMN-STABLE
SETS OF ABSTRACT GAMES**



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Stefano Vannucci

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Siena, luglio 1990

1. Introduction.

The present note is devoted to the task of clarifying the relation between two different solution concepts for abstract games, namely von Neumann-Morgenstern (vNM)-stable sets and ergodic sets.

VNM-stable sets purport to characterize those states which are robust w.r.t. theory-absorption on the part of the players. Hence, they amount to a quite sophisticated solution concept, which is in a sense an earlier game-theoretical counterpart of equilibrium concepts based on rational expectations.

By contrast, ergodic sets are meant to capture the possible "long run" outcomes of the relevant process, namely the recurrent states under a typically myopic behaviour.

Nevertheless, an underlying structural similarity among the two solution concepts has been sometimes alluded to.

In particular, Packel (1981) suggests that the ergodic solution (i.e. the join of the ergodic sets) of certain games can also be viewed as a vNM-stable set.

The aim of this note is to make precise the relation between vNM-stable sets and ergodic sets.

2. Definitions and main result.

An (exact) abstract game is a pair (X, R) , where X is an arbitrary nonempty set, called the state space, and $R \subset X \times X$ is a binary relation on X , the dominance relation.

We shall denote by $P(R)$ ($I(R)$) the asymmetric (symmetric) component of R , and by $T(R)$ the transitive closure of R . Moreover, for any binary relation $Bc(X \times X)$ we shall denote by iB the inverse relation of B , and by $-B$ the complementary relation of B (w.r.t. $X \times X$).

A von Neumann-Morgenstern (vNM) stable set of an abstract game (X, R) is a set ScX of states enjoying the following properties:

vNM i) (internal stability) $(S \times S) \cap R = \emptyset$.

vNM ii) (external stability) for any $x \in X \setminus S$, $Sx(x) \cap R \neq \emptyset$.

An absolutely stable set of (X, R) is an ScX enjoying external stability and

i') (absolute internal stability) $(S \times S) \cap T(R) = \emptyset$.

Obviously, an absolutely stable set is a vNM-stable set as well. In fact, the notion of an absolutely stable set was introduced by Harsanyi (1974) in order to strengthen the robustness of vNM-stable sets w.r.t. to theory-absorption phenomena (as mentioned above, such robustness lies at the very foundations of vNM solution theory: see von Neumann, Morgenstern (1947) chpts. 4 and 30).

The vNM-solution of a game is the set of its vNM-stable sets.

An ergodic set of an abstract game (X, R) is a nonempty set ScX of states s.t. the following conditions hold true:

E i) (persistence) $((X \setminus S) \times S) \cap R = \emptyset$.

E ii) (minimality) for any nonempty $S' \subset X$, $S' \subset S$ and $S' \neq S$ entail that S' is not persistent (i.e. does not satisfy E i)).

The ergodic solution of (X, R) is the join of its ergodic sets.

The intended meaning of an ergodic set is quite clear: it is a set of states which cannot be left whenever reached.

However, another definition of an ergodic set has been sometimes proposed (see, for example, Packel (1981) who relabels Shenoy's "elementary dynamic solutions" (Shenoy (1979)) as "ergodic sets").

We shall see in a moment that the two proposed definitions are in fact equivalent.

Following Shenoy, we define a nontrivial (i.e. nonempty) elementary dynamic solution of the game (X, R) as a nonempty set ScX satisfying:

DS i) (pseudo-strong persistency) $((X \setminus S) \times S) \cap T(R) = \emptyset$

DS ii) (communication) $(S \times S) \subset T(R)$.

Proposition. Let (X, R) be an abstract game, and ScX a nonempty set of states. Then, S is persistent (i.e. satisfies E i)) iff it satisfies DS i).

Moreover, S is an ergodic set of (X, R) iff it is an elementary dynamic solution of (X, R) .

Proof. Obviously, any set ScX which satisfies DS i) satisfies E i) as well. On the other hand, let S be a persistent set of (X, R) . Let us assume for a moment that S violates DS i).

Thus, for some $y \in X \setminus S$, $x \in S$ $(y, x) \in T(R)$.

Hence, by definition of $T(R)$, $(y', x') \in R$, for some $y' \in X \setminus S$, $x' \in S$, a contradiction.

Let us now assume that S is an ergodic set of (X, R) which violates DS ii). Then $(x, y) \notin T(R)$ for some $x, y \in S$.

Let us denote by $S(y)$ the $T(R)$ -upper contour of y in S , i.e. $S(y) = \{z \in S / (z, y) \in T(R)\}$.

Obviously $x \notin S(y)$, whence $S(y) \cup \{y\}$ is a proper subset of S .

Moreover, for any $z \in S(y)$, $w \in X \setminus S(y)$, $(w, z) \in T(R)$.

In fact, suppose to the contrary that $(w,z) \in T(R)$ for some w,z as defined above. Then $(w,y) \in T(R)$, whence $w \in S(y)$, a contradiction.

As a consequence, $S(y) \cup \{y\}$ is persistent as well, which in turn contradicts the minimality of S (i.e. property Eii)). Conversely, let S be a nontrivial elementary dynamic solution of (X,R) which is not minimal persistent.

Hence, a nonempty set $S' \subset S$ exists s.t. $S' \neq S$ and $(y,x) \notin R$, for any $y \in S \setminus S', x \in S'$. However, $(y,x) \in T(R)$, by DSii). It follows that $y' \in S \setminus S', x' \in S'$ exist s.t. $(y',x') \in R$, by definition of $T(R)$ and DS i) : a contradiction. ■

Remark 1. The foregoing proposition makes it crystal clear that the ergodic sets of a given game are pairwise disjoint.

Remark 2. In a remarkable paper, Kalai, Schmeidler (1977) prove that the ergodic solution E of an abstract game (X,R) (i.e. the join of its ergodic sets, which they call the "admissible set") is characterized by the following condition:

(admissibility): $((X \times E) \cap T(R) \subset (X \times E) \cap i(T(R)))$.

It follows that, whenever a game has a unique ergodic set, "admissibility", "persistency plus minimality" and "persistency plus communication" turn out to be three equivalent sets of restrictions over the power set of X .

More complex alternative characterizations of the ergodic solution of a game are offered in Schwartz (1972) (who works with a finite asymmetric (X,R)) and in Kim, Roush (1979).

The following general relation between vNM-stable sets and ergodic sets holds true.

Theorem. Let (X,R) be an abstract game s.t. $(X,T(R))$ is connected. Then, the following statements are equivalent:

- i) AcX is an ergodic set of (X,R) ;
- ii) AcX is a vNM-stable set of $(X,P(T(R)))$;
- iii) AcX is an absolutely stable set of $(X,P(T(R)))$.

Proof. Let A be an ergodic set of (X,R) . Thus, for any $x,y \in A$, $(x,y) \in T(R)$ and $(y,x) \in T(R)$, by DSii) and the foregoing proposition. Therefore, $(x,y) \in P(T(R))$ whence the internal stability of A follows. Notice: since $T(R)$ is transitive, $P(T(R))$ is transitive as well. Thus, $T(P(T(R))) = P(T(R))$. As a result, A turns out to enjoy absolute internal stability w.r.t. $(X,P(T(R)))$.

Let us now consider any ordered pair $(y,x) \in ((X \setminus A) \times A)$. Obviously, $(y,x) \notin T(R)$ (by E i)). Hence, by connectedness, $(x,y) \in T(R)$. Therefore $(x,y) \in P(T(R))$, which entails external stability.

Conversely, let A be a vNM-stable set of $(X,P(T(R)))$ (hence an absolutely stable set as well, in view of $P(T(R))$'s transitivity). Obviously, A is nonempty, by external stability.

Moreover, for any $x \in A, y \notin A$ $(y,x) \in T(R)$ implies $(z,x) \in P(T(R))$ for some $z \in A$, by external stability of A and transitivity of $T(R)$: a contradiction. Hence A is persistent.

The proof that A satisfies minimality (E ii) is by contradiction as well. Let us assume that a set $B \subset X$ exists s.t. it is both a proper subset of A and an ergodic set of (X,R) .

Let us now consider two states $x \in A \setminus B, y \in B$. It follows that $(x,y) \in T(R)$ by persistency (Ei) of B .

On the other hand, internal stability of A entails $(y,x) \in P(T(R))$, whence $(y,x) \in T(R)$, a contradiction. Q.E.D. ■

Remark 3. It should be clear from the foregoing theorem that an ergodic set A of (X,R) with connected $T(R)$ turns out to be the (nonempty) core of $(X,P(T(R)))$ which is in fact the unique vNM-stable set of $(X,P(T(R)))$. It follows that under the hypothesis of our theorem (X,R) has at most one ergodic set, which - if it exists - is the same as the whole ergodic solution.

It is left to the reader as a simple exercise to check that A is indeed admissible.

Also, if (X,R) is an (asymmetric) strict total ordering then a set $A \subset X$ of states is an ergodic set iff it is an absolutely stable set (or, equivalently, a vNM-stable set) of (X,R) , i.e. the three solutions concepts - as should be expected - coincide.

Furthermore, let us call a game chaotic w.r.t. a given solution concept whenever its corresponding solution is the whole state space.

Then, the theorem implies that (X,R) is ergodically chaotic iff $(X,P(T(R)))$ is vNM-chaotic, provided that $T(R)$ is connected (see McKelvey(1979) for a fairly general chaos theorem).

The following example makes it clear that the theorem does not hold true if $T(R)$ is not connected:

Example 1. Let $X = \{x,y,z,w,u,t\}$, $R = \{(x,y), (y,z), (z,w), (u,t)\}$. $\{x,z,u\}$ is an absolutely stable set of $(X,P(T(R)))$. However, the ergodic sets of $(X,T(R))$ are $\{x\}$ and $\{u\}$ (hence $\{x,u\}$ is the ergodic solution of $(X,T(R))$, and x,u are in fact

absorbant states: see Roberts(1976), or Berge(1979) for a detailed presentation of such graph-theoretic concepts).

Also, replacing $P(T(R))$ with a connected $T(R)$, or even with a connected R turns the theorem into an invalid statement, as the following example shows:

Example 2. Let $X = \{x,y,z,w\}$, $R' = \{(x,y), (y,z), (z,w), (w,x)\}$. Of course, $T(R')$ is connected. On the other hand, X is the only ergodic set (hence the ergodic solution) of (X,R') and $(X,T(R'))$: in graph-theoretic terminology the state space X is, in both cases, irreducible (in game-theoretical terminology, the game is ergodically chaotic). However, neither (X,R') nor $(X,T(R'))$ possess any vNM-stable set.

As a straightforward consequence of the theorem proved above and of previous results due to Kalai and Schmeidler one gets the following existence and uniqueness results for (absolutely) vNM-stable sets:

Corollary. Let (X,R) be s.t. X is a nonempty compact topological space and $T(R)$ is connected.

Then the following statements hold true:

- i) if the lower contours of R (i.e. the sets $\{y \in X / (x,y) \in R\}$, for any $x \in X$) are open, then $(X,P(T(R)))$ has a unique vNM-stable set (which is in fact an absolutely stable set as well): namely, the ergodic set of (X,R) .
- ii) if the upper contours of $T(R)$ are closed, then $(X,P(T(R)))$ has an unique vNM-stable (and absolutely stable) set, namely the ergodic set of $(X,T(R))$.

Proof.i) It follows from theorem 2 in Kalai,Schmeidler(1977) and the theorem above.

ii) It follows from theorem 3 in Kalai,Schmeidler(1977) and the theorem above. ■

Remark 4.The foregoing existence results are indeed general enough to deserve mention, since it is well known that vNM-stable sets may not exist (see Lucas(1977)).

The uniqueness results are perhaps even more relevant. This is so because multiplicity of vNM-stable sets is in fact a quite common occurrence.

On the other hand, uniqueness enhances considerably the plausibility of a vNM-stable set as a solution.

Obviously, a similar claim may be made for any solution concept.

However, uniqueness of vNM-stable sets is a highly valuable property on a more "intrinsic" ground, namely theory-absorption. In fact, a unique vNM-stable set is more robust w.r.t. theory-absorption in that it entails lack of possible ambiguities about the relevant vNM-stable set or "standard of behaviour": it enjoys the "solidity" requirement, according to the terminology of Forman, Laing(1982), which is not the case for multiple vNM-stable sets.

It should also be mentioned here that any finite (X,R) has an ergodic solution (see Schwartz(1972) Kalai,Pazner, Schmeidler (1976), and Kalai,Schmeidler(1977), theorem 1).

As a consequence, whenever $(X,T(R))$ is finite and connected a set of states $A \subset X$ exists s.t.: a) A is the unique ergodic set of (X,R) ; b) A is the core of $(X,P(T(R)))$; c) A is the unique vNM-stable set of $(X,P(T(R)))$; d) A is the unique absolutely stable set of $(X,P(T(R)))$.

3. Concluding remarks.

It is worth stressing again the striking difference between the arguments which motivate the use of vNM-stable sets and ergodic sets, respectively, as a sensible solution concept.

The classical argument advanced by von Neumann and Morgenstern is a paradigm of game-theoretic reasoning.

The von Neumann-Morgenstern line of reasoning may be described as follows: whenever the players know the game they are playing and learn that vNM-stable sets are the relevant solution concept, each state belonging to a vNM-stable set is in a sense "hard to leave". In fact, it turns out that any state z which happens to dominate x cannot be a real threat. This is so because such a state cannot belong to the above mentioned vNM-stable set, by internal stability: hence, whenever implemented it would be replaced by another state y of the given vNM-stable set. However, no coalition effectively supports y against x , including of course those coalitions which could effectively support the move from x to z . It follows that the relevant coalitions lack any positive incentive to implement such a move from x to z , provided that the coalitions which can force z against x can also force y against x .

On the other hand, a state z which does not belong to a given vNM-stable set S is "easy to leave", since it is exposed to an effective threat by a state of S , which in turn cannot be threatened, by the reasoning presented in the foregoing paragraph.

Thus, for each single state which belongs to a vNM-stable set some "stability claim" - if admittedly supported by the whole set - is in fact implied (see von Neumann, Morgenstern(1947))

for the original definition of stable sets; see also Lucas(1977), Lucas, Michaelis(1982), Schmidt, Strohlein(1985) Berge(1979) for critical surveys of problems, ideas and results concerning vNM-stable sets, and Harsanyi(1974), Roth(1976), Forman, Laing(1982) for some interesting attempts to improve and/or generalize the original von Neumann-Morgenstern's argument reported above).

Let us now turn to ergodic sets and solutions.

As a matter of fact, the ideas underlying ergodic sets are not game-theoretic in origin. Rather, they arise from Markov chains and graph theory.

Versions of ergodic sets, under various labels, were introduced in social choice and game theory in the '70s to support transitive closure-maximizing social choice functions (see Schwartz(1972), Kalai, Pazner, Schmeidler(1976), Deb(1977),; see also Campbell(1980), Bordes(1983), and Moulin(1986) for a recent comprehensive treatment of this subject and related matters for the tournament case), and to analyze coalition formation processes (see Shenoy(1979)).

Ergodic solutions have also been shown to provide a general structure which easily specializes to various cooperative and noncooperative game-theoretical solution concepts (see Kalai, Schmeidler(1977); see also Greenberg(1989) for an application of vNM-stable sets in a very similar vein).

As mentioned above, ergodic sets are tailored to capture a sort of "long run" stability under a somewhat myopic behaviour. In particular, no attention is paid to the effects of theory-absorption on the part of the players. Also, no stability claim is implied for the single elements of an ergodic set. In fact, the suggested refinements of the ergodic solution concept typically amount to a (stationary) probability distribution on the set of

states, s.t. its support is the ergodic solution itself (see Packel(1981), Ferejohn, McKelvey, Packel(1984)).

The result presented in this note entails that, in spite of those striking differences w.r.t. theoretical background and motivation, ergodic and vNM solutions entertain a remarkably strict relation under connectedness of the indirect dominance relation.

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