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PRICE SYSTEMS,
BARGAINING THEORY AND SOCIAL RELATIONS

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0. INTRODUCTION AND MAIN RESULTS. [1, 2]

Standard results in economic analysis are the multiplicity of walrasian and non walrasian equilibria in general economic equilibrium (GEE) systems, and indeterminacy of the profit rate in production prices (PP) systems. Therefore, neither, respectively, the intertemporal GEE approach in its walrasian and non-walrasian versions, nor the PP approach provide *the* price system that would rule an economy. Rather they supply the general structure of each of those price systems.

This article deals with the problem of the selection of each of those price systems (and so with the problem of equilibrium selection in the case of GEE). We stress that our price systems are always taken separately: the problem concerns which GEE price among GEE price systems and which PP among PP price systems will be selected in their respective field.

We approach the problem of the determination of the price system through the framework offered by bargaining game theory. This choice is suggested by the fact that forming an economy under different price systems, in the PP and GEE approach respectively, would give different advantages to different people and coalitions. This is the kind of situation bargaining theory deals with. In particular, we use bargaining theory in the Nash-Harsanyi version, as axiomatized by Hart (1985), see § II.2.1..

In our analysis we give a part to the social relations of production by which people are related to each other (see § II.1.). These were very relevant at the time of classical economists and Marx. They are notably absent, at least explicitly, in the modern version of PP (e.g. Sraffa (1960)). In the neoclassical approach they are somehow hidden in the "initial distribution of endowments" and, so to speak, left there to sleep. Here they are considered more explicitly, both in PP and GEE model.

The first part of the article deals with the definitions of economies, of price systems and their existence. Different social relations and the game theoretic framework are dealt with in the second part. Part 3 presents the problem of forming an economy and the main results for each of the two cases under study: PP and GEE. The main results consist in the following findings:

- The bargaining game offers a solution about *the* system of prices which will be established in the respective fields of PP and GEE.

[1]) This paper is still work in progress. However, it presents in a compact and coordinated way a sequence of results that we believe interesting, some of which have received and some other will receive wider treatment separatedly elsewhere. The authors benefited from suggestions and comments by Profs. F.H. Hahn and M. Lippi. The usual disclaimers apply.

[2]) The article has been written jointly. Its ideas and the way of exposing them have been at length discussed by the authors. For practical reasons Part I and III.2. are attributed to P.M. Pacini and Part II and III.3. to B. Miconi.

- In the case of PP, this result entails the solution of Marx's transformation problem.

- Due to the continuity of the utility function, GEE analysis deals very inade-quately with the necessity, as opposed to the possibility, of particular exchange and production activities and hence with different production relations as defined in this article.

I - MARKET ECONOMIES AND SYSTEMS OF PRICES.

I.1. Economies.

We first describe the basic framework, then we distinguish between classical and neo-classical economies and attribute some general and common features to both of them.

I.1.1. Generalities.

In our economy there are H people indexed by h and F firms indexed by f. There are K private goods and services indexed by k. z_h is an allocation of commodities and labour for person h. Z_h is their set, $Z_h \subseteq \mathbb{R}^K_+$.

For each h, we define ***minimal survival*** bundles of goods as points in Z_h satisfying the following

Definition. A vector $b_h \in Z_h$ is a survival vector if consumer h survives for $z'_h \in b_h + \mathbb{R}^K_+$, but he cannot survive for $z''_h \in b_h + \mathbb{R}^K_{--}$.

These vectors form the set B_h. Let us call $C_h = \bigcup_{b_h \in B_h} \{b_h + \mathbb{R}^K_+\}$ the closed set of points guaranteeing at least survival. In other words, if we add positive amounts of commodities to the points in the set B_h, the new set C_h still allows survival.

Comment. The definition above can well apply to any vector of goods which are considered to be of some basic importance. Although we will stick to the notion of physical survival, we could have substituted it with any other standard historically and socially defined as minimal.

We assume that the labour input for the economy as a whole, is positive whenever production activities are performed. We also assume that there is an amount of production goods in the economies which are capable, right from the first period of production, if manned by the maximum labour services belonging to the h individuals to produce vector of goods such that h-1 of the individuals, can at least survive on this vector of goods, with the h-th having just more than that.

I.1.2. Classical economy.

A classical economy - i.e. an economy analyzed in terms of production prices, referring to the work of classical economists - is indicated here by

$$\mathcal{S}((Z_h,\mathfrak{U}_h),(\alpha_h),Y)$$

where $\mathfrak{U}_h$ (on which see § III.2.1.) is an ordinal function, that we baptize the *usefulness* function giving individual valuations of the bundles of goods and labour in Z_h. α_h is a vector representing the proportion of means of production that individual h possesses in each single sector. The means of production actually possessed are left unspecified and depend on the possible positions taken by the system in terms of physical vectors of net products and means of production. Y represents the set of available productive techniques (supposedly linear) for the system.

I.1.3. Neoclassical economy.

A neoclassical (private) economy -i.e. an economy analyzed in terms of Walrasian (WGEE) and Non-Walrasian (NWGEE) general equilibrium concepts for the system of prices- is indicated by

$$\mathcal{S}((Z_h,U_h,\overline{z}_h),(\theta_{hf}),(Y_f))$$

where U_h is an ordinal utility function representing individual preferences over consumption alternatives in Z_h, $\overline{z}_h$ are initial endowments comprising consumption goods and labour and θ_{hf} h's share in firm f. Each firm is completely owned by consumers and chooses production plans in the production set Y_f.

I.2. Market economies.

A classical or neoclassical market economy is characterized by the corresponding $\mathcal{S}$ described above and a vector of prices p, with a commodity (i.e. a good in a market economy) or a combination of them taken as numeraire. These prices are such that nobody participating only in the circuit of exchange, which in principle is open to every participant of the economy, can end up, exchanging at those prices, with more than the commodities he had started from.

I.2.1. Classical market economies: production prices.

A market economy is a PP market economy if there is a pair (p,r) such that the following expression holds

$$(yp + lw)(1+r) = Bp$$

where $p > 0$, $0 \leq r < r^{max}$ is the rate of profit that capitalists earn on their means of production, w is the wage rate that workers receive, $\overline{y} = [y,l] \in Y$ is the linear production coefficient matrix chosen according to some rule (ours will be specified later in § II.2.), where y is the matrix of coefficients for

material inputs and each entry of l specifies the amount of simple labour employed in the corresponding industry, obtained via the reduction of whatever complex labour exists to simple labour. B is the semipositive output matrix.

I.2.2. Neoclassical market economies: equilibrium prices.

A market economy is a neoclassical market economy if individual actions are compatible on the market at the associated vector of prices p. Such situations are distinguished into Walrasian and Non-Walrasian equilibria, depending on the absence (presence respectively) of an explicit quantity rationing mechanism.

I.2.2.1. Walrasian equilibria.

A neoclassical market economy is a Walrasian equilibrium (WGEE) when the triplet (p^*,z^*,y^*), where we take $\sum_{k=1}^{K} p_k^* = 1$, satisfies the following requirements

A) For every f, y_f^* maximizes profits p^*y_f on Y_f;

B) For every h, z_h^* maximizes $U_h(z_h)$ over the budget set

$$\{ z_h \in Z_h | \ p^*z_h \leq p^*\overline{z}_h + \sum_{f=1}^{F} \theta_{hf} p^* y_f^* \};$$

C) $\sum_{h=1}^{H} z_h^* - \sum_{f=1}^{F} y_f^* \leq \sum_{h=1}^{H} \overline{z}_h$.

Let E^W be the set of Walrasian equilibria for this market economy.

I.2.2.2. Non-Walrasian equilibria.

A neoclassical market economy is a Non-Walrasian equilibrium (NWGEE) for a given $\overline{p}$, $\overline{p} > 0$, $\sum_{k=1}^{K} \overline{p}_k = 1$, and a given rationing scheme $\rho(.,.)$, whose properties are examined later in I.3.2.2., if there is a pair (z^*,y^*) such that

a) for every f, y_f^* maximizes profits $\overline{p}y_f$, $y_f \in Y_f$, given quantity constraints

$$|y_{fk}| \leq |\rho_{fk}(z,y)|, \quad y_{fk}\cdot\rho_{fk}(z,y) \geq 0 \qquad \forall\ k;$$

b) for every h, z_h^* maximizes $U_h(z_h)$ over the budget set

$$\{ z_h \in Z_h | \ \overline{p}z_h \leq \overline{p}\overline{z}_h + \sum_{f=1}^{F} \theta_{hf}\overline{p}\rho_{hf}(z,y) \}$$

given quantity constraints $z_{hk} \leq \rho_{hk}(z,y)$, $(z_{hk}-\overline{z}_{hk})\cdot(\rho_{hk}(z,y)-\overline{z}_{hk}) \geq 0 \ \ \forall k$;

c) $\sum_{h=1}^{H} \rho_h(z^*,y^*) + \sum_{f=1}^{F} \rho_f(z^*,y^*) = \sum_{h=1}^{H} \overline{z}_h$.

Let E^{NW} be the set of Non-Walrasian equilibria for this market economy.

I.3. Existence of the system of prices.

We now list the assumptions and results concerning the existence of the system of prices.

I.3.1. Classical market economies: existence of production prices.

We add to the description of the economies of § I.1.1. the following:

A1) y is semi-positive, vital, indecomposable and giving rise to simple production.

Assuming simple production implies that the matrix B is the identity matrix. As it is well known, under our assumptions and *independently* of U_h, there exists a production price system such that if $r' > r''$ ($r' < r''$ respectively), then $w' < w''$ ($w' > w''$ respectively) and, in any numeraire, the lower (higher) wage can buy a lower (higher) amount of net products (e.g., see Lippi (1979)).

I.3.2. Neoclassical economies. Existence of equilibria.

In order for equilibria to exist, we pose the following assumptions that are common to both the Walrasian and Non-Walrasian case.

A2) In the first period, $\bar{z}_{hk} > 0$, for every h, k.

A3) For every h, $U_h(.)$ is continuous, increasing and strictly quasi-concave.

A4) For each firm f, the production set satisfies the standard properties of irre-versibility and free disposal.

I.3.2.1. Walrasian equilibria.

As it is well known, under our assumptions and A2)-A4), there exists at least a Walrasian equilibrium ($E^W \neq \emptyset$). In fact, our assumptions are specifications suitable for the present case of the

usual ones sufficient for the existence proposition (see Debreu (1959)). In particular, the assumption of free disposal in A4) rules out negative prices, so that only $p \geq 0$ can result.

I.3.2.2. Non-walrasian equilibria.

In so far as Non-Walrasian equilibria are concerned, we assume:

A5) $\bar{p} \notin E^W$.

In this case we are concerned with price systems outside the walrasian equilibrium price systems. In order to guarantee the existence of at least a Non-Walrasian equilibrium, we make the following assumptions about the rationing schemes:

A6) a) $\rho_h(z,y)$, $\rho_f(z,y)$ are continuous functions of demands and supplies.

b) *Voluntarity of exchange:* $|y_{fk}| \leq |\rho_{fk}(z,y)|$, $y_{fk} \cdot \rho_{fk}(z,y) \geq 0 \quad \forall f,k$;

$z_{hk} \leq \rho_{hk}(z,y)$, $(z_{hk} - \bar{z}_{hk}) \cdot (\rho_{hk}(z,y) - \bar{z}_{hk}) \geq 0 \quad \forall h,k$.

c) *Orderly Markets:* $(\rho_{hk}(z,y) - z_{hk}) \cdot (\rho_{ik}(z,y) - z_{ik}) \geq 0, \quad h \neq i$;

$(\rho_{fk}(z,y) - z_{fk}) \cdot (\rho_{gk}(z,y) - z_{gk}) \geq 0, \quad f \neq g$.

d) Rationing schemes are nonmanipulable and are intended to clear the market.

A6) means that (b) nobody is forced to change the side of the market or to exchange (produce) more than he wants and that (c) there cannot be demanders and suppliers simultaneously rationed on the same market. (b) and (c) together imply that the short side of the market is always satisfied.

Under our assumptions and A2)-A6), we can prove the existence of a Non-Walrasian equilibrium ($E^{NW} \neq \emptyset$). (e.g., see Drèze (1975), Benassy (1975)).

II - SOCIAL RELATIONS AND BARGAINING THEORY.

II.1. Social relations.

As mentioned in § 0., we consider the social structure defined in terms of the relations by which people are linked among themselves. This will affect substantially the results presented later.

In the definition of different social relations, also called production relations, a crucial role is traditionally played by the capacity of a group of people to produce survival. To analyze this point, we introduce all possible partitions of the grand coalition N in subcoalitions S that the H people could form. For each partition, we then analyze the capacity of coalitions to produce at least survival commodities vectors for their participants.

We call the following situations *non-capitalistic*:

1) All partitions of N can produce at least survival.
2) There is no partition of N, apart the grand coalition, which can produce at least survival for all its participants.

In case 1), everybody, isolated or in a coalition, can survive. It corresponds to the canonical case of simple artisans' production of Smith and Marx. In case 2) nobody can survive, apart from the grand coalition. It can be considered as the specular situation of case 1).

Apart from the cases that we will see in § III.2.3.1. (for the classical economy), we call *capitalistic*, any other situation not included above.

II.2. Bargaining: The Harsanyi solution.

In the previous situations, there are non-trivial ways in which people can agree to divide the advantages of cooperation, by forming a market economy. Such an agreement will be presented as the Harsanyi's solution to a n-people cooperative bargaining game (see part III.). In this section, we formally present the Harsanyi solution as axiomatized by Hart (1985) and we give some comments on the characteristics and structure of the game in the two cases of social relations seen above.

II.2.1. The axiomatization of Harsanyi solution.

Following Hart's axiomatization, we define a game as an ordered pair (N,V) where N is a finite set of players and V is a set valued function (characteristic function) that assigns to every $S \subset N$ a

subset V(S) of $\mathbb{R}^s$ ($s = \#S$). In the following, $\partial V(S)$ denotes the boundary of coalition S payoff set, ψ denotes a solution function possibly multivalued, V W and U are arbitrary games, and $U_{T,c}$ is a game in which each coalition can arbitrarily divide among its members an amount c if it contains all the players of T, and nothing if it does not, $1_T \in \mathbb{R}^s$ ($s \geq t = \#T$) an s-dimentional non-negative vector such that $1^i_T = \underset{\sim}{1}$ if $i \in T$ and $1^i_T = 0$ if $i \in S$ and $i \notin T$ (see Hart (1985), pg. 1298-1299). We assume:

B1) *Efficiency*: $\psi(V) \subset \partial V$.

B2) *Scale covariance*: $\psi(\lambda V) = \lambda\psi(V)$, for all $\lambda \in \mathbb{R}^N_{++}$.

B3) *Conditional additivity*: if $U = W + V$, then $\psi(U) \supset [\psi(W) + \psi(V)] \cap \partial U$.

B4) *Independence of irrelevant alternatives*: if $V \subset W$ then $\psi(V) \supset \psi(W) \cap V$.

B5) *Unanimity games*: for every non-empty coalition $T \subset N$, and every real number c, define $z \equiv z_{T,c} \in \prod_{S \subset N} \mathbb{R}^s$ by $z_S = \left[\frac{c}{\#T}\right] 1_T$ if $S \supset T$ and $z_S = 0$ otherwise; then $\psi(U_{T,c}) = \{z\}$.

B6) *Zero-inessential games*: if $0 \in \partial V$ then, $0 \in \psi(V)$.

Under these axioms, there exists a solution function $\Gamma(V)$ that Hart proves to be the Harsanyi solution for a game, if

C1) For every $S \subset N$, the set V(S) is (a) a non-empty subset of $\mathbb{R}^s$; (b) closed; (c) convex; and (d) comprehensive, i.e., $x \in \partial V(S)$ and $x \geq y$ imply $y \in V(S)$.

C2) The set V(N) is moreover (a) smooth, i.e. V(N) has a unique supporting hyper-plane at each point of its boundary $\partial V(N)$; and (b) non-level, i.e. $x, y \in \partial V(N)$ and $x \geq y$ imply $x = y$.

In words, assumptions B1)-B6) impose the following characteristics on the solution of the game: whenever it exists, it must lie on the frontier of the game (B1) and is invariant to linear rescaling (B2) and to variations in irrelevant characteristics of the game (B4). Provided that a game can be decomposed into subgames, the solution of the former can be looked at as the sum of the payoffs obtained by playing separately the latters, whenever such a sum is also efficient for the original game (B3). Advantages obtained by a coalition are equally split among its members, for all coalitions that are formed (B5)[3]. Finally, if threat points are on the boundary of the game, they themselves are the solutions of the game, which is then irrelevant (B6). Whenever the above conditions are met provided that all the points below the frontier belong to the payoff set (C1(d)) and the frontier is smooth, admitting neither kinks nor flat portions (C2), a solution is proved to exist; in the two players case, the payoff space of the game must be of the type depicted in the following page:

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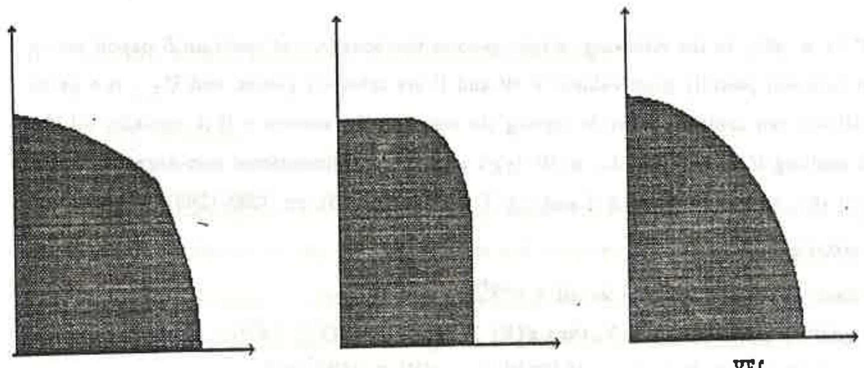
3) This simply means that *what can be gained within* a coalition is equally divided among its members. To obtain the final payoff levels, the initial threat points and the necessary rescaling must be also considered so that the equality of results in a coalition does not imply the equality of individual positions in terms of their preference scales.



NO NO YES

II.2.2. Comments.

The Harsanyi solution of the game features two characteristics which are important from our point of view, linking the contradictory looking phenomena of equality and power. 1) The participants in the game (either single players or players in coalitions) consider themselves to be equal: hence the symmetrical property of the solution of the game in relation to the initial status quo points (see B5). 2) Through the intricacies of coalition formation, coalitions try to give their members threat points which are as advantageous or powerful as possible in comparison to other coalitions, which, at their turn, try to dismantle the threat point of the first coalition and build powerful threat points for themselves.

In relation to the game, there is an important difference depending on whether people are embedded in non-capitalistic or capitalistic production relations. As we have already seen (see § II.1.), in the first instance, there is no possibility on behalf of any coalition to form advantageous threat points against any other coalition, in relation to the capacity of being able to reach or not to reach survival vectors. In capitalistic production relations, on the contrary, such a possibility exists.

III. MARKET ECONOMIES AS THE RESULT OF THE GAME.

III.1. The forming of a market economy.

In this part, we want to consider market economies, run by one of the price systems seen in the first part -taken separately, PP on one side and WGEE-NWGEE price systems together on the other side- as a result of an agreement among H people not already embedded in it, i.e. as a game.

We suppose that people are related among themselves by one of the social relations defined in § II.1.. These social relations are relevant for the game since they affect the structure of coalition formation and determine the initial threat points. As usual, a *threat point* is defined as a binding commitment of a player to perform the action that gives him the higher payoff in the lack of an agreement. In the present context, it is easily identified as the maximum utility (see § III.3.1.) or maximum usefulness (see § III.2.1.) that he, isolatedly, can get.

Starting from these initial conditions, people will play a cooperative game:

- In the case of classical economies, they consider all possible coalitions among themselves and all possible PP that can be enforced within each coalitional structure;
- In the case of a neoclassical economy, they consider all possible coalitions among themselves and all possible neoclassical market equilibria (corresponding to either Walrasian or Non-Walrasian equilibria with rationing schemes) that can be enforced within each coalitional structure.

In both cases, we also admit that, in any coalition, people can ex ante agree on redistributing their initial endowments. We call this *freely entered redistributions.*

In either case, they will reach an agreement on which price system, under which possible coalition, will be chosen and hence which market economy to form.

III.2. Market economies as the result of the game: Classical economies.

In dealing with the classical economy, we introduce the usefulness function and we interpret accordingly the payoff space of the game. The solution of the bargaining game is then presented for both non-capitalistic and capitalistic social relations and finally the transformation problem is dealt with.

III.2.1. The usefulness function.

We observe that, at least implicitly, classical economists admitted some basic and simplified preference structures on the commodity space Z_h as the following ones:

I) Bundles of commodities that guarantee survival are preferred to those that cannot do that.

II) Provided that a bundle of commodities guarantees survival, commodities and leisure time are substitutes.

III) Provided that a bundle of commodities guarantees survival, greater amount of commodities (greater leisure time) is ceteris paribus preferred to smaller one.

In relation to point II), classical economists also considered that the "toil and trouble" of the working activities linked with the production of any given level of commodities, equal or above subsistence, is the same for every h[4].

Proceeding from this basic "preference" structure, we *postulate* the existence of a function (the usefulness function $\mathfrak{U}_h$) defined on the commodity space Z_h, respecting such principles as minimal requirements. In particular, we impose the following restrictions

A7) For every h, the usefulness function $\mathfrak{U}_h$ is:
- a) Right-continuous;
- b) Discontinuous at the survival level;
- c) Increasing and quasiconcave above survival[5].

We will take both preferences and usefulness function in relation to the classics' use of the category of labour value. As to preferences, nothing changes by replacing the words "bundle(s) of commodities" with the words "amount of labour contained in the (bundles) of commodities". As to the usefulness function, this obtains only the transformation of its domain from the commodity space Z_h to the set (the positive orthant of $\mathbb{R}^2$) formed by the pairs (γ_h, ℓ_h) where, for each h, the first coordinate refers to the amount of labour content and the second to performed simple labour[6].

[4]) This assumption can be said to go back to A. Smith (A. Smith (1776), ed. Skinner, pag. 45, 50). Classical economists would not have accepted that differences in preferences in relation to labour activities would have given advantages to lazier people who have higher valuation of their leisure time and hence higher threats points.

[5]) Another instance of a right-continuous utility function used in economics is in Dasgupta-Maskin (1985).

[6]) It is clear that each pair (γ_h, ℓ_h) can correspond to more $z_h \in Z_h$. We do not care about which vector z_h will be preferred among those corresponding to a (eventually chosen) pair (γ_h, ℓ_h); this is a second step choice for which, in this approach, we are not given any criterion. Such a choice then lies outside the theory.

As to the quantities γ_h and ℓ_h, some conventions are in order: since quantities are measured per unit time of production, we set equal to 1 the maximum working time and $\gamma_h = 1$ will indicate the maximum quantity of labour that can be embodied in the commodities produced efficiently working for the unit time of production. Accordingly, $\gamma_h^{ms} < 1$ will indicate the minimum percentage of the unit of labour content that guarantees survival to h, and ℓ_h^{ms} the percentage of the unitary time period necessary to produce efficiently γ_h^{ms} with the chosen technology. We have to make a further side assumption: γ_h^{ms} is to be interpreted as a physiological condition of identical people and hence can be made independent of h. From I) above and the form of $\mathfrak{U}_h$ it follows that any $\gamma_h > \gamma^{ms}$ guarantees survival.

III.2.1.1. Usefulness function: our case.

Apart a technical condition that will be dealt with in § III.2.1.2., the conditions listed in A7) are all that is needed in order to determine the results to follow: any function that respects them is equally good in order to support the qualitative conclusions of the model[7]. We will adopt as a preference measure the following function

$$\mathfrak{U}_h(\gamma_h, \ell_h):\ \mathbb{R}^2 \to \mathbb{R}_+:\ (\gamma_h, \ell_h) \mapsto \chi(\gamma_h - \gamma^{ms}) \cdot \left((1-\ell_h) + \left[\frac{\ell^{ms}}{\tilde{\gamma}_h}\right] \cdot \gamma_h \right)$$

where $\chi(x)$ stands for a step function such that $\chi(x) = 1$ if $x \geq 0$ and $\chi(x) = 0$ if $x < 0$. $\tilde{\gamma}_h$ is a subjective parameter making $\mathfrak{U}_h(\gamma^{ms}, 0) = \mathfrak{U}_h(\gamma^{ms} + \tilde{\gamma}_h, \ell^{ms})$, which illustrates for this case the amount of labour content that makes the situation in which one does not work and survives indifferent to that in which he works to produce subsistence and receives that quantity beyond subsistence.

According to A7), $\mathfrak{U}_h(\gamma_h, \ell_h)$ is a right-continuous function with a discontinuity at the subsistence level. It satisfies the cardinality property. Moreover, it represents the trade off between contained and performed labour on which, in order to make it obey the further point about "toil and trouble" mentioned in the previous paragraph, it is sufficient to pose

A8) For every h, $\tilde{\gamma}_h = \tilde{\gamma}$, for a given $\tilde{\gamma}$.

In other words, the working effort ℓ^{ms} requires a compensation $\tilde{\gamma}$, equal for every h, beyond subsistence to make this situation indifferent to $(\gamma^{ms}, 0)$[8].

[7]) Any difference in the functional form will result in a difference of the numerical values of variables, but will not affect substantially the working of the model.

[8]) In neoclassical terms the rate of substitution between the two situations of (a) full leisure time and subsistence and (b) least working activity needed to produce subsistence and an increase measured by $\tilde{\gamma}$ beyond subsistence is the same for each h. Obviously the theory remains silent on the material content of goods to produce such a situation.

To give a numerical example, in a very particular situation, one could take the case in which $\ell^{ms} = \widetilde{\gamma} = \gamma^{ms}$, for every h. In this case, some relevant point receive a numerically simple valuation: for example, $\mathfrak{U}(0,0) = 0$, $\mathfrak{U}(0,1) = 2$ and $\mathfrak{U}(\gamma^{ms},\ell^{ms}) = \mathfrak{U}(1,1) = 1$.

III.2.1.2. Usefulness function: further conventions.

This paragraph deals with rather technical points. They are due to the fact that given right-continuity, the usefulness function is not immediately applicable to the formation of the payoff space, as described in § II.2. In order to overcome such a problem, we take for every h a continuous function $\Upsilon_h(.)$ built as follows:

i) for each $x \in C_h$, $\Upsilon_h(x) = \mathfrak{U}_h(x)$

ii) for each $x \notin C_h$, $\Upsilon_h(x) = t\cdot\mathfrak{U}_h(a) + (1-t)\cdot\mathfrak{U}_h(b)$ where a and b belong to the lower boundary of $\mathbb{R}^2$ and C_h respectively and are the closest points (in terms of the Euclidean distance) such that $x = t\cdot a + (1-t)\cdot b$, $0 \leq t \leq 1$.

Since $\mathfrak{U}_h(.)$ is cardinal, so is Υ_h.

In other words, in constructing Υ_h, we leave unaltered the characteristics above survival (point i), while each point below survival is looked at as a convex combination between properly chosen above and below survival positions. Its valuation is then taken to be the convex combination, with the same weights, of the usefulness levels of the two constituents points.

The use of $\Upsilon_h(.)$ in the game is subject to the following *conventions*: Suppose that a certain level k of "usefulness" measured by Υ_h is given to h at the solution point. Then three cases are possible:

(a) $\{x|\Upsilon_h(x)=k\} \subset C_h$; this means that a solution has been found in the continuous part of the proper usefulness function.

(b) $\{x|\Upsilon_h(x)=k\}$ is not contained in C_h but $\{x|\Upsilon_h(x)=k\} \cap C_h \neq \emptyset$; this means that the solution can be achieved in the continuous part of the usefulness function; in this case, only the alternatives in $\{x|\Upsilon_h(x)=k\} \cap C_h$ are considered (in neoclassical terms the certain equivalent alternatives).

(c) $\{x|\Upsilon_h(x)=k\} \cap C_h = \emptyset$; this means that the solution, as it is, cannot give subsistence to h.

In other words, by means of $\Upsilon_h(.)$, one can assess if allocations are such that h definitely survives (case (a)) or he does not (case (c)). In case (b), subsistence can be obtained either as a certain event or as a gamble involving a position of non-survival and a one strictly above survival: if this case verifies, then only the certain opportunities are taken.

III.2.2. The pay-off space.

By means of Υ_h we can consruct the pay-off space of the Harsanyi game. In particular, we interpret V(S) in the following way: it is the set of S-vectors describing the level of usefulness Υ_h that each h in S can get from each pair (γ,ℓ) in the set determined by the chosen technology $\bar{y}$, whenever S forms. It remains understood that in case (γ,ℓ) does not guarantees subsistence, the valuations given by the function of previous paragraph are used together with the specified conventions. However, the conditions of smoothness and non-levelness (see C2, § II.2.1.) may not be satisfied on the boundary of the payoff space. In this case, we follow Harsanyi's suggestion, and we solve the game by arbitrarily approximating it with a sequence of games with smooth and non-level payoff for which solutions exist. The frontier of the game will then be the limiting frontier and the solution of the game the limit of the sequence of solutions[9].

III.2.3. The solution of the game for different production relations.

Let us at last apply the series of tools and concepts that we have been developing so far, in order to solve the game whose content is the following: given the usefulness function, technology and a distribution of means of production as giving rise to the social relations seen in § II.1. (to which we add, in the framework of a classical economy, two more cases in § III.2.3.1.), people produce and consume. These activities, which always entails the renewal of the means of production, as usual in this kind of model, can take place with no necessity of exchanges on the market, so that no PP system will exist. Alternatively, people can agree to exchange their work and/or their products at some PP that would be the results of our game.

In this approach we could think of our game as a way of passing from initial to final positions. In the classical economy, this takes the form of a function $T\circ g_y^{--}$: $\mathbb{R}^{HK} \rightarrow \mathbb{R}^{K}$: $\alpha \mapsto T(g_y^{--}(\alpha))$ which is interpreted as meaning that once a technology y is chosen, a vector (of "labour values") $g_y^{--}(\alpha)$ is obtained for all the h's as function of the initial distribution α of means of production. These (labour values) $g_y^{--}(\alpha)$ are transformed (T(.)) into a corresponding vector of production prices and hence into a whole set of possible net vectors z_y^{--}, for each h.

The solution will be dealt with respectively in non-capitalistic and capitalistic social relations.

9) We are in debt for the proof of this point to S. Vannucci.

III.2.3.1. Non-Capitalistic production relations.

We first add two more non-capitalistic cases to the classical economy:

3) There is a partition of N in which every coalition would at least reach survival and such that all the members of the coalitions prefer or are indifferent according to their usefulness function to be in the coalitions of this partition rather than to be in any other possible partition, apart, possibly, the grand coalition.

4) Any situation in which the grand coalition can be divided in a subcoalition S, able to reach survival, and the coalition of remaining agents R which is not able to grant survival to its members and also has the following properties:
   a) the members of S|s are unable to survive, where S|s means the coalition of the members of S but s.
   b) R|r+s is unable to survive, where R|r+s means the coalitions formed by R|r and s.
   c) any r ∈ R can substitute individually any s in S.
   d) the members of S|s are indifferent between forming S or S|s+r
   e) any r will receive more, according to their usefulness function, in the coalition S+R, whenever it will form, than what he receives by partecipating in S|s+r.

Cases 3) and 4) describe respectively the case in which the forming of coalitions that can survive cannot be "objected to successfully" (blocked) by any coalition, in such a way that some coalitions might not survive, and the case in which any survival coalition can be "objected to successfully" (blocked) by any other non-surviving coalition, whereas this does not obviously apply to the grand coalition. As a very short comment, we notice that in these last non-capitalistic cases if somebody does not survive he will have the capacity of blocking any surviving coalitions.

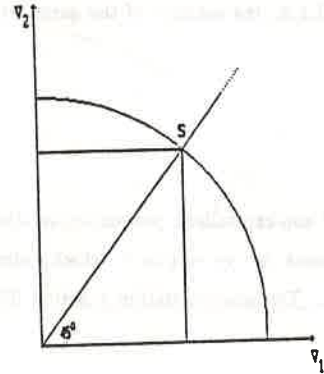
Let us now consider the possible agreement of our H people in non-capitalistic production relations. The situation can be reduced to a game with just two players. This can be done because we have supposed that people have the same usefulness function; in the particular non-capitalistic case, they have also the same initial threat points.

III.2.3.1.1. The solution of the game.

First of all, the matrix of production coefficients y will be chosen according to axiom B1, so that the most efficient technique will be always employed. Secondly, one can notice that, since people are identical, the frontier of the game is symmetric and, given the form of the usefulness function, it is also linear.

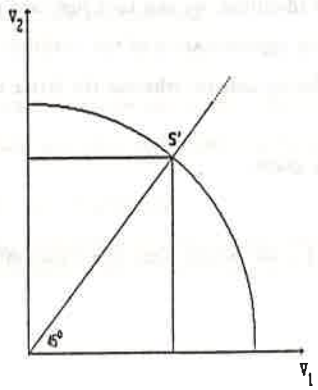
As to the content of the game, despite it does not change in the four cases of non-capitalistic production relations, the results are more clearly seen if we represent the problem separately for case 1) on one hand and 2) 3) 4) on the other.

In case 1), the pure artisans situation, both agents are able to produce survival. Moreover, following the definition of threat point as in § III.1., we have that the situation in which the players stick to their threat points (*status quo*) is also efficient and hence lies on the frontier of the game. This implies that we are in presence of an irrelevant game in which the status quo is also the final solution (see § II.2.). With respect to the properties of the solution, we have that the threat points (and hence the payoffs of the solution) are identical, since the individuals are identical; in turn, this implies that the solution lies at the intersection of the frontier and the 45⁰ line running through the origin, as the following figure shows



On the basis of the assumed usefulness function, point S corresponds to the usefulness levels ($(1+\frac{\ell^{ms}}{\tilde{\gamma}})/2$, $(1+\frac{\ell^{ms}}{\tilde{\gamma}})/2$) for the players. One can further notice that, given the symmetry and linearity of the frontier, this solution point corresponds, for each player, to the midpoint between the least and the most favourable outcome.

In the other three cases, the players will have the status quo at the non-survival level, 0 in our case. In this game, since there are no power positions to enforce against the other and since an agreement guaranteeing survival is preferred by everyone to the initial situation, it follows that all possible advantages of cooperation will be fully exploited and the consequent gains will be equally divided. In terms of our game, this translates into the fact that, starting from the initial non-survival position, players will choose the point on the frontier which will give to each of them the midpoint between the least and the most favourable outcome, S' in the following figure:



The solution point S* corresponds to the point S of figure 2 for the non-capitalistic situation, and as such, it guarantees survival to the players. This solution is then the same as in case 1), although now the game is not irrelevant.

In the particular case of § III.2.1.1. in which $\ell^{ms} = \tilde{\gamma} = \gamma^{ms}$, the solution of the game corresponds to the point (1,1). Simple calculations can show that, in case 1), this is also the status quo position, according to the previous analysis.

In terms of the function of § III.1.3., the solution of the game will be indicated by the mapping $g_{\tilde{\gamma}}^{**}(\alpha)$.

III.2.3.1.2. The price system.

In the above seen four cases of non-capitalistic production relations, if there exist exchanges on the market, a price system corresponds to $y\gamma + l = \gamma$ which, choosing w=1 as numeraire, is proportional to $yp + l = p$ of § I.2.1.. The transformation function T(.) of § III.2.3. is, in this case, the identity map.

III.2.3.2. Capitalistic production relations.

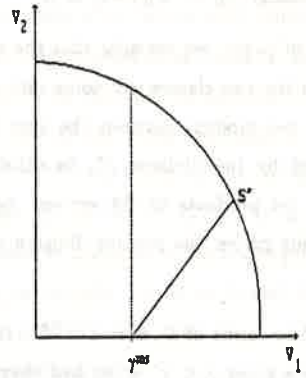
Given capitalistic production relations, i.e outside the four cases of non-capitalistic production relations above seen, some well determined individuals, isolated or in coalition, can survive, whereas others will not be able to do so, and will have no way of blocking the coalition capable of surviving. Owners then have at least survival as threat point whereas people belonging to the proletariat have lower threat points. In relation to this initial situation, there are advantages to be taken if the proletarians working with the means of productions of the owners, so that the proletarians will become wage workers, would reach at least survival, and if the owners in such a situation, employing wage workers and thus becoming capitalists, would get a higher Υ than they would do producing by themselves.

As in the previous case, the game can be reduced to a two persons game: in fact, since individual preferences are supposed identical, we can take just two people, one as representative of the coalition of owners and the other as representative of the coalition of non-owners. They would differ as to their threat points, the former being able to, whereas the latter unable to, guarantee himself survival.

III.2.3.2.1. The solution of the game.

Parallel to § III.2.3.1.1., we notice that the most efficient technique of production will be

chosen, according to axiom B1, a choice different from the one followed in the literature[10], and that the frontier of the game is symmetric and linear. The bargaining will then take place in the part of the payoff space guaranteeing at least the status quo positions to the players: *in this region*, the division of advantages will be in the mid-point between the least favorable and the most advantageous outcome for both owners and non-owners[11]. The following figure shows this situation:



To return to the previous numerical example where $\ell^{ms} = \tilde{\gamma} = \gamma^{ms}$, the situation is the following: owners have threat point at value 1, while non-owners at value 0. These two values limit the bargaining region, in which the best outcomes of the owner and non-owner are 2 and 1 respectively. The solution will be found at the pair of usefulness levels for the owners and proletarians repectively, (1.5, 0.5). These, for example, can be obtained from a situation in which owners of means of production and wage workers are allocated (0.5, 0) and (0.5, 1) respectively, where, as usual, the first number indicates the labour content of obtained commodities and the second labour performed. In fact we will always assume that in these cases the work to produce those commodities is performed only by the wage workers.

In terms of the function of § III.1.3., the solution of the game will be indicated by $g_{\tilde{\gamma}}^{**}(.)$.

III.2.3.3.2. The transformation problem and the price system.

The solution of the game will determine how net social product is divided between the two classes, when wage workers perform working activities and capitalists do not. Let us call σ^* the ratio of

10) This is usually the one that given w maximizes r.

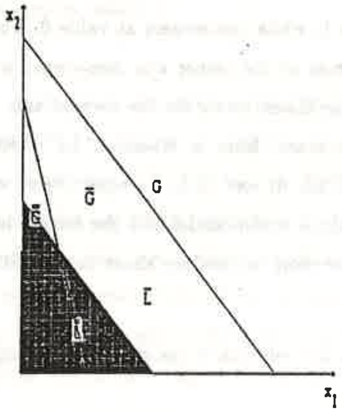
11) We suppose that if there are proletarians who would not survive under this solution, they just disappear from the scene, being burried with their means of production, if any.

the net social product accruing to capitalist to that accruing to the working class[12]. σ^* then determines the division of all possible commodity vectors in terms of their labour content. between the two classes.

In order to pass from this solution to the system of prices, we suggest the following procedure. We notice that, as a rule, in capitalist production, the production of commodities have to be equally profitable in relation to the means of production of each capitalist, by which they are produced. Prices, then, have to satisfy the above seen expression $(yp + lw)(1+r) = p$.

In order to reach such a system of prices, we consider that the exchange, at those prices, of all the possible commodity vectors of which the two classes will come into possession can only change the physical content of the division of the net product between the two classes but not the proportion accruing to each of them as determined by the division σ^*. In other words, whichever accounting system we use to measure all possible net products of the system, be it the one related to labour embodied, or the one related to production prices, the relative division per unit time of production are to remain the same.

Because of the properties of PP in terms of $0 < r < r^{max}$, there is only one division of all possible net products that corresponds to a given $0 < \sigma^* < \infty$ and obeys this rule. This last one would then determine univocally our PP system[13]. In geometrical terms, such a procedure is illustrated by the following figure:



[12]) When usefulness is separable, as in our case, σ^* corresponds to the ratio of the respective usefulness levels at the solution point once the usefulness deriving from leisure time has been subtracted.

[13]) Even if we do not want to pursue this line at length it could be interesting to note that in our transformation procedure we follow rather closely Marx way of approaching the problem. We obtain, in fact, for all possible positions of the system considered together, for variable capital and wages, surplus value and profits, net product in value and prices terms, the respective constancy of their numerical values, independently of the numeraire. In the transformation problem, as treated so far, this result has been excluded by considering a particular position of the system. The generalizing character of the value cathegory would have better suggested not to confuse the whole of the possible cases, to which, as just stated, the value cathegory has to be refered to, with any possible particular case.

The above considerations offer us the determination of the PP system along the following more formal lines: Let us call $\mathbb{R}^m_+$ the space of commodities in which we represent the net products of the system (labour excluded). Since the maximum scale of the system is determined by the finite length of the production period, we have that the set of "possible" net products is also limited: we will call this set G. Let us define on $\mathbb{R}^m_+$ the function $l(x)$ that associates to each x in $\mathbb{R}^m_+$ the labour content of the commodities in x. Finally, on the basis of $l(x)$, we can compute the number $\mathcal{L}(G) = \int \ldots \int_G l(x)\, dx_1 \ldots dx_2$, that represents the labour content of the whole commodities in G. Then, we observe that there is a unique way to contruct symmetrically inward G to a smaller cone $\bar{G}$, such that the ratio of the labour content of the commodities in $\bar{G}$ to that of the commodities in $\bar{\bar{G}} = G/\bar{G}$ is equal to σ^*, i.e. $\mathcal{L}(\bar{\bar{G}})/\mathcal{L}(\bar{G}) = \sigma^*$.[14] Consider now all the cones generated by the wage hyperplanes $w = pz$, each w corresponding to an r, $0 < r < r^{max}$. For every w, the corresponding wage hyperplane splits G in a subcone $\bar{W}$ and its complement in G, $\bar{\bar{W}} = G/\bar{W}$. It is easily seen that the number $n(w) = \mathcal{L}(\bar{\bar{W}})/\mathcal{L}(\bar{W})$ (i.e. the ratio of labour contents of the commodities in $\bar{W}$ and $\bar{\bar{W}}$) varies inversely and monotonically with w, starting at infinity for $w = 0$ and decreasing to 0 for $w = w^{max}$. Given monotonicity, there is a unique $w = w^*$ such that $n(w^*) = \sigma^*$. Let r^* be the corresponding rate of profit and p the corresponding system of prices.

The production price system would then be $(yp+lw)(1+r^*) = p$.

In terms of our correspondence, the transformation function that takes initial data (α) into final results will be written as $T(g_{ij}^{**}(\alpha))$, where $g_{ij}^{**}(\alpha)$ are allocations in terms of labour content as determined by the solution of the game. Final allocations in terms of physical commodities will then be indicated with $z_{ij}^{**} = T(g_{ij}^{**}(\alpha))$.

III.3. Market economies as the result of the game: Neoclassical economies.

In dealing with neoclassical economies, we proceed as in the previous case, substituting the usefulness function with the utility function and reinterpreting the payoff space according to the case under study. We will describe the limited validity of this approach to the problem we are dealing with. In relation to our analysis, with reference to the field of exchange, some examples are then set forth for clarification.

[14]) The numbers $\mathcal{L}(\bar{\bar{G}})$ and $\mathcal{L}(\bar{G})$ (as well as $\mathcal{L}(\bar{\bar{W}})$ and $\mathcal{L}(\bar{W})$ later) are computed in the same way as $\mathcal{L}(G)$, i.e. as the integrals of $l(x)$ taken over $\bar{\bar{G}}$ and $\bar{G}$ respectively ($\bar{\bar{W}}$ and $\bar{W}$ respectively), and hence they represent the labour content of the whole commodities in the respective sets.

III.3.1. Ordinal and cardinal utility.

A utility function is assumed to exist with the properties described in A3). In so far as it corresponds to a preference order relation, it needs only to describe an ordinal ranking, so that it is unique up to any monotonically increasing transformation. As it is well known this function can be made cardinal, that is unique up to a linear transformation: adopting the Von Neumann-Morgenstern axioms the existence proof for a continuous cardinal utility function is standard (see Von Neumann-Morgenstern (1944)).

III.3.2. The payoff space.

In the case of the use of a cardinal utility function $U_h(.)$, we interpret the V(S) for the neoclassical economy as the set of utility payoffs that each $h \in S$ will get in each possible Walrasian and Non-Walrasian equilibrium positions. If we admit that all those allocations can be achieved with different probabilities, we then obtain the convex hull in utilities of these allocations that we use as our convex payoff space. Each point of the payoff then corresponds to forming a market economy ruled by a WGEE or NWGEE price systems (or a probabilistic combination of them) and, if necessary, rationing schemes, belonging respectively to E^{W} and E^{NW}. Following this construction, we expect all WGEE, because of their Pareto optimality, to lie on the frontier of the game[15]. We could also expect that every point of the frontier, is a WGEE, since all the points on the frontier can be obtained as EEG through redistributions of initial endowments (second Welfare theorem). However, in our case it could well happen that not all the redistributions will be freely entered. This is a consequence of the fact that the gains in efficiency obtained through such redistributions could be offset, for some actors of the game, by the losses in terms of the new threat points obtained following such redistributions. In these cases, these last ones would not be freely entered and it could happen, as a consequence, that a NWGEE lies on the pay-off frontier.

With respect to Hart's conditions of § II.2., in relation to the above build pay-off, our game satisfies non-emptiness, closedness, convexity and comprehensiveness (as in C1). However, the conditions of smoothness and non-levelness may not be satisfied on the boundary of the payoff space. As in the case of classical economies, we follow Harsanyi's suggestion and solve the game by arbitrarily approximating it with a sequence of games with smooth and non-level payoff for which solutions exist. The frontier of the game will then be the limiting frontier and the solution of the game the limit of the sequence of solutions[16]. We conclude that the game is suitable for the application of the Harsanyi solution

[15]) See however Kehoe (1985), where a WGEE is Pareto inferior to a linear combination of other WGEE.

III.3.3. The solution of the game for different social relations.

In the framework of GEE prices, the game will assume the following content. Given preferences, technology and the initial endowments of goods, distributed as to satisfy the social relations of § II.1., people could agree to exchange their work and/or their products at some walrasian or non-walrasian GEE prices. These GEE prices would then be the result of the game and we could think of it as a way of passing from initial to final positions. This takes the form of a correspondence $f^{**}(\bar{z},\theta)$: $\mathbb{R}^{H(K+F)} \to \mathbb{R}^{H(K+F)}$: $(\bar{z},\theta) \mapsto (z^{**},\theta^{**})$ which is interpreted as meaning that a vector of prices $p^{**} \in E^{W} \cup E^{NW}$ (and if necessary rationing schemes $\rho(.,.)$), is chosen given $(\bar{z},\theta)$ and, consequently, allocations (z^{**},θ^{**}) are obtained, where θ^{**} may differ from the original θ due to the possibility of freely entered redistributions.

The solution of the game, in a neoclassical market economy, will be dealt with respectively in non-capitalistic and capitalistic social relations.

III.3.3.1. Non-capitalistic production relations.

Given the initial endowments such as to characterize the non-capitalistic production relations of § III.1., Harsanyi solution can be applied to the game. According to the rules of the game and to the intricacies of coalition building based on the preferences of the H agents, a solution will be determined on the frontier of the payoff of the game.

Since the frontier of the game is generally non-linear, and given the possible presence of Non-Walrasian equilibria on it (which, given the quantity constraints on individuals, gives incentives to private redistributions thus eliminating the simple bargaining condition, see Harsanyi (1977), pg. 259), multiple solutions cannot be ruled out. In this case, we apply the second step procedure of Harsanyi (see Harsanyi (1977), pg. 261) for the determination of a unique outcome. In this step, each agent takes as his own starting position the least favourable outcome among those obtained in the first step and collectively all the agents play a simple Nash bargaining game.

In terms of our correspondence the solution (z^{**},θ^{**}) of the game will be indicated by the mapping $f^{**}(.,.)$, defined on the initial $(\bar{z},\theta)$.

III.3.3.2. Capitalistic production relations.

We can repeat word by word what has already been said in the last paragraph, only changing non-capitalistic to capitalistic.

[16]) See note 8.

III.3.4. The limits of validity of neo-classical analysis.

Contrarywise to the case of classical market economies (see § III.2.3.1 and § III.3.3.2.), we have treated non-capitalistic and capitalistic production relations in equal terms, the difference being reduced to the difference in the distribution of initial endowments. The continuity of preferences *omogenize* the two situation and allows the substitutability between above and below survival possibilities. The same utility can in fact be achieved either if h works and survives or if h has a very high level of leisure time that compensate a so tiny consumption that he cannot survive (so that h has, really, after all, a lot of leisure time). This also explain why we have not added here the two cases of § III.2.3.1.. Survival and not survival situations can be equally well preferred, indifferent or dispreferred. One then has that:

(1) Nothing prevents WGEE or NWGEE to correspond to some *ghost equilibria*, not in the sense that such equilibria would not exist, but in the sense that at those allocation people cannot survive but still act on the market of goods. It has to be noticed that, due to the continuity of the utility function, GEE positions might take place for whatever level of initial endowments.

(2) *Even on the strong assumption of ignoring the problem of ghost equilibria*, neoclassical analysis is not able to recognize in social relations as defined in this article, a distinguishing mark of the characteristics of the system, outside a relatively unimportant different initial endowments of goods for the people of the economy.

In general the whole GEE analysis, based on a continuous utility function, both in its walrasian and non-walrasian versions, is unable to analyze systems in which producing and obtaining survival commodities is a *must* for the participants in the economy; it remains simply an optional[17]. More generally (see Comment in § I.1.1.), it is never able to cope meaningfully with the achievement of whatever standard (not expressed in utility terms) socially reckoned as minimal.

Given this situation, one can preserve validity to GEE analysis only outside production relations as defined in this article, by admitting that survival commodities, once recognized as such, are supposed to exist outside the theory.

This. in turn, implies that one has to suppose that the origin of the consumption set of the actors, as the minimal point guaranteeing such standards. In our case, (survival) bundles of commodities would be ensured to everybody for *each period* of an intertemporal walrasian or non-walrasian equilibrium, as if in these economies (survival) commodities fell from the sky or were produced by unmanned robots.

[17]) As the only aside of the article, we notice that being the obtaining and producing of necessities an optional rather than a necessity it follows that employment must also have, in this line of analysis, the same characteristics. Unemployment theories built on GEE analysis do not seem to be very promising.

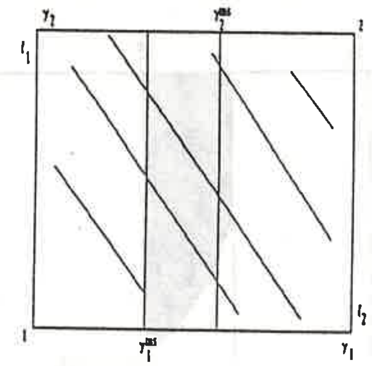
III.3.5. Some simple examples in the field of exchange.

The above results come out clearly in the field of exchange, in which GEE analysis is supposed to be particularly strong. The utility function can well attach a lower index to a situation in which agents can reach survival compared with a situation in which they do not reach survival, retaining a large amount of non-survival commodities (e..g. free time). The importance of exchanges of non-survival for survival commodities is ignored by the analysis, even when its possible achievement (but not its necessary achievement) could be analyzed.

We will now show some examples to illustrate this. We define the consumption set for all the participants in the economy as the set of pairs (γ,ℓ) in $\mathbf{R}^2_+$ where ℓ is labour time and γ represents the only good of the system[18]. For simplicity, consumers are supposed to be identical:

- In point 1), we will consider the advantages of exchanging wage labour for goods. It will be shown that subsistence for both participants is possible but no better than no-subsistence. In order to do this, we take a simple linear utility function that presents the feature of substitutability of survival and non-survival situations.
- In point 2), also in the case of a non-linear utility function, its continuity will determine the possibility of existence of ghost equilibria.
- In point 3), we illustrate the difference between the opportunity for producing and exchanging labour in a classical economy, as given in this article, and a neoclassical economy.

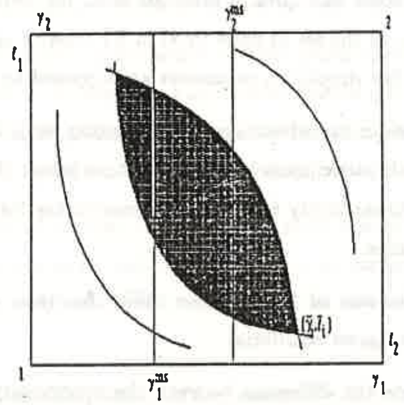
1) Let us now choose a unit time of production just as to make subsistence a proportion strictly less than the production performed in that time span. Given the utility function $U(\gamma,\ell) = (1-\ell)+2\gamma$, we have linear and parallel indifference curves with slope -2, that can be represented in an Edgworth box as follows



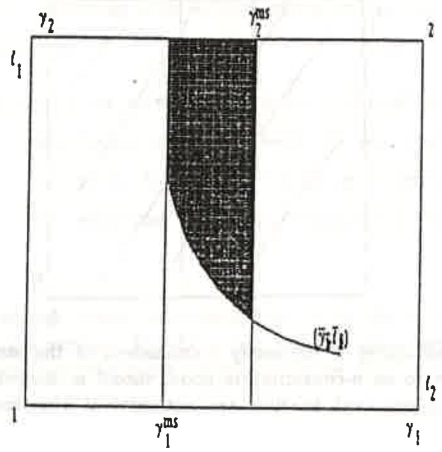
[18]) The conclusion above is obviously independent of the simplicity of the one commodity model used. The extension to an n-consumption goods model is immediate allowing for the possibility of indifference curves comprising both survival and not-survival situations.

Whatever the level of subsistence, there will be a part of the lens (i.e. the region of the box of not worse positions for both players, in this case the thick line above) which assigns less than subsistence to at least one of the participants in the exchange. If subsistence level is low enough both player have the opportunity to survive, notwithstanding the fact that such positions are indifferent to some other in which either of them is not able to survive.

2) With respect to the second point, the linearity of the utility function is a simple mean that does not affect the validity of the results. In fact, whenever a nonlinear utility function is chosen, it will affect only the form of the lens as in the following figure:



Now there are advantages to be taken by the people of the box in all the interior part of the lens. Contrary to what has been seen in case 1) above, there is now the possibility for both people to be better off reaching subsistence. Let us introduce the core of such exchange economy. There is no need in principle for the positions in the core to lie completely above subsistence for both players, so that there could be ghost equilibria.



3) Here, we compare the situation arising with a standard utility function with the (ficticious) case of a right-continuous utility function, discontinuous at the subsistence level. Could the utility function be discontinuous in the same way as our usefulness function was, all points in the dashed part of the above figure would represent mutually advantageous potential agreements. This part generally differs from the corresponding part in figure 7 when the utility functions were continuous. There are different opportunities of exchanges between wage labour and goods in a classical capitalist market economy in which all exchanges would lie above subsistence, from the case of exchanges between wage labour and goods in the neoclassical market economy, in which, due to the continuous utility functions used in GEE analysis, exchanges could also lie below subsistence.

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