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Some Implications of Money Creation in a Growing Economy

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Introduction

If P and V are constant, the Fisherine identity MV ≡ PQ implies that, if Q is growing, M must be growing too. This paper examines some little-noticed implications of this fact (1). Contrary to what the title might suggest, its main concern is not with the theory of inflation (the price level is assumed constant) nor with the debates around monetarism or rational expectations, but rather - as is made clear in the last section - with certain problems with Say's Law. But as a byproduct, I obtain some results about the nature of V and the theory of growth, which appear to be interesting in their own right.

The finiteness of V implies that, on average, there is a lag between receiving money from a transaction and then spending it again. The existence of this lag, when coupled with money creation, causes V to depend not only on the time structure, synchronization etc. of transactions, but also on the growth rate. This is discussed in Section II, where

(1) My interest in this subject was aroused in 1972 by the lectures of Professor Bruno Trezza, then at Siena, out of which grew a book, B. Trezza (1975), Economia e Moneta, Bologna, Il Mulino. A first draft of the present paper was read at two seminars, at the University of Cambridge in November 1977 and at the University of Siena in January 1979. The present version owes much to the comments I received on those occasions, as well as to the careful comments on a second draft from Professors Michael Krüger, Ruggero Paladini and Mario Tonveronachi, to whom all goes my gratitude. The usual caveat applies as to the remaining mistakes.

it is shown that this fact imposes some limitations on the analyses based on $\dot{M} = \dot{P}+\dot{Q}$.

In Section III it is shown that the Kaldor-Pasinetti results on the connection between profit rate and growth rate in steady growth are modified in an interesting way when account is taken of the existence of that lag; it becomes possible, for instance, that the growth rate of a capitalist economy be greater than the profit rate even when the workers do not save; it is even possible to have a positive steady growth rate without inflation although the society's propensity to consume is one. Section IV briefly discusses the connection between the results of the previous Section and standard dynamic multiplier analysis.

In Section V the argument of Section I about the necessity, in certain cases, of a "seignorage" from money creation in order to have a sufficient level of aggregate demand is used to criticize one aspect of Ricardo's justification of Say's Law.

I

Assume constant money prices (inclusive of money wages). Let h be the average time interval a unit of money remains idle between two successive transactions. In other words, money received at time t from a payment is, on average, expended again at time t+h. (It is not possible to put h=1/V for reasons which will become clear below). Assume h is constant.

The total expenditure at time t from money received from previous payments equals total expenditure at time t-h. Let B_t represent the money value of total transactions, and assume $B_t = B_o e^{gt}$, where g is the growth rate. Then at time t expenditure out of money received from previous payments is $B_{t-h} = B_o e^{g(t-h)}$ and if g is positive it is insufficient for all the payments. If for simplicity we assume that expenditures (i.e. transactions) are only for the purchase of produced goods or productive services (no purely financial transactions, no land purchases, etc.), then the above means that expenditure out of money from previous payments is insufficient to buy the whole product (1). There must be purchases which are not being paid with

(1) Besides Trezza, op. cit. (and B. Trezza, "On the Concept of the Monetary Economy", in AA.VV. (1978), Pioneering Economics - International Essays in Honour of Giovanni Demaria, Padova, CEDAM; B. Trezza, (1979), "Moneta e distribuzione", Studi Economici, n. 8; it is beyond the scope of the present paper to comment on Prof. Trezza's more general theory, developed from that basic idea, but with many aspects

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money obtained from the previous sale of goods or services. (I restrict myself here to a closed economy).

The gap $B_t - B_{t-h}$ is necessarily filled by money creation. If this extra money is fiat money (paper money, or credit) produced with irrelevant costs, then there exists a "seignorage" equal in value to the value of the extra money. If for instance, this extra money is created by the State, this means that the State budget is perennially in deficit, without creating any inflation.

The evidence of an appropriation of part of the social product, deriving only from the right to create money, would not be denied by anybody if it is the State to do it. The thing is less immediately clear if the extra purchasing power is created by the banking system and given to entrepreneurs - a case in which, in fact, many people would refuse to admit there is a "seignorage". It would not be denied that money is created by the banks; but, it would be argued, it is not simply presented as a gift to entrepreneurs: these must give

...(1) or which I cannot agree), also see H. Neisser (1934), "General Overproduction", Journal of Political Economy, also in American Economic Association (1950), "Readings in Business Cycle Theory"); E.E. Hagen, "The Classical Theory of the Level of Output and Employment" in M.G. Mueller, ed. (1971), Readings in Macroeconomics, The Dryden press, Hinsdale, Illinois; A.C. Pigou (1949), Employment and Equilibrium 2nd ed., MacMillan, London, p. 19. I have not found other works where this point is explicitly made; the creation of money is usually discussed only in connection with inflation. For a discussion of related points in connection with Marx's reproduction schemes, see A. D'Ercole (1980), "On Monetary Economy", Economic Notes (2).

it back after a time, so they are not enjoying a "seignorage" (being thus in a very different position from a State creating money; the analogous situation being rather that of a State borrowing from the general public); as to the banks, it is true that, after a while, they receive back the money they have created; but they then proceed to re-lend it; thus they too do not use this newly created money simply to appropriate goods, in the way of a State creating money.

But this reasoning forgets that the money thus continuously created by the banks, precisely because it is then loaned out again and again, in fact is never "given back" to someone outside the capitalist class as a whole; and it allows the capitalist class as a whole (i.e. inclusive of bankers, entrepreneurs, middlemen etc; I am assuming for simplicity that there is only another class, the workers, who neither save nor receive credit) to get hold at every moment of a greater amount of resources that they would have found possible otherwise. The picture can be made clearer as follows. Let us imagine the extra money is created by the State, which uses it to enlarge State-owned firms, i.e. for investment. Then the amount of investment is the same as if the money had been created by banks and lent to entrepreneurs who use it for investment. Therefore, if one speaks of a seignorage from money creation in the case of the State getting hold of investment resources owing to that money creation, it would seem necessary to speak of the capitalist class as a whole enjoying a seignorage from money creation if it is the banks which create it. In fact, entrepreneurs do not really ever give back the newly created money lent to them: they get it immediately re-lent back to them, in order to start again the production cycle. In this case, not only is the capitalist class appropriating the surplus; if this surplus grows, then the capitalist class also presents itself (or some of its members) with a gift: that continuous flow of extra purchasing power which it needs in order to be able to sell the whole product. In Section III it will be shown that this fact does cause some differences relative to a situation, e.g. barter, where this "seignorage" did not exist.

II

Let us assume that actually each unit of money is used every h time units. Assume g is positive, and assume that the newly created money is spent the moment it is created. Then

$$(dM/dt)_t = B_t - B_{t-h} = B_t(1-e^{-gh}) = B_o e^{gt}(1-e^{-gh}).$$

Integrating, one gets $M_t = \frac{1}{g} B_t(1-e^{-gh})+C$, where C is an arbitrary constant. The growth rate of M_t, $(dM/dt)/M = (B_t(1-e^{-gh}))/$ $/(\frac{1}{g} B_t(1-e^{-gh})+C)$, tends to g for $t \to +\infty$; in steady growth it is $M_o = B_o(1-e^{-gh})$, i.e. C=0, and the growth rate of M equals g.

From $B_t \equiv PQ_t$ and from $M_t V \equiv PQ_t$ one gets $V = (ge^{gh})/$ $/(e^{gh}-1)$, which is a function both of h and g, and different from 1/h unless g=0.

(If g=0 the previous expression for V is indeterminate, but $B_t = B_o$ and $M_t = M_o$ imply $V = B_o/M_o$; as the transactions in the unit period equal B_o and the transactionw in a period of length h equal M_o one gets $M_o : h = B_o : 1$, i.e. $V = 1/h$. The same result can be reached by applying L'Hôpital's rule).

Further, $\frac{dV}{dg} = \frac{e^{gh}(e^{gh}-gh-1)}{(e^{gh}-1)^2}$ is positive for $g \geq 0$,

because, by applying L'Hôpital's rule twice one finds that it is equal to 1/2 for g=0; and, as g increases, it remains positive because the denominator is positive and, at the numerator, e^{gh} increases faster than gh. So, comparing steady growth paths, V is an increasing function of g, in spite of the constancy of h.

The economic meaning of these results is not difficult to grasp. Choose the time unit so that h=1, and consider a unitary time period. By assumption, each money unit already in existence at the beginning of the period performs one and only one transaction during the period; its velocity of circulation is 1. But besides these money units, if g is positive, during the period there appear further money units, which are, by assumption, used immediately, but then remain idle (within the period) for less than the entire period; their velocity of circulation is then greater than 1; and V is the weighted average of all the individual velocities. Or put it like this: the transactions during a period of length h=1 equal M+M', where M is the money existing at the beginning and M' the money created during the period; but the average quantity of money during the period is less than M+M', and since the transactions during the period are equal to M+M', it follows that V is greater than 1.

(If the newly created money is expended after a lag z since it is created, then $dM_{t-z}/dt = B_t(1-e^{-gh})$; and in steady growth $M_t = B_t(e^{gh}-1) / ge^{g(h-z)}$. In this case, then, V=1/h can coexist with a positive g, but only by a fluke: z must be such that $h = (e^{gh}-1)/ge^{g(h-z)}$).

(More generally, since the velocity of money V, over the time interval t_2-t_1, is given by

$$V \equiv \frac{\int_{t_1}^{t_2} B_t dt}{\frac{1}{t_2-t_1}\int_{t_1}^{t_2} M_t dt},$$

for constant h one sees that V is a function not only of g but also of how g varies over the period).

The fact that, for constant h, V is a function of g appears to have some interest, because the magnitude which it seems legitimate to consider given by 'institutional factors', e.g. the structure of the synchronization of payments, is rather h than V. Now, a given h implies a constant V only if g is constant. Thus, the equation which is often found in monetary analyses: $\dot{M} = \dot{P}+\dot{Q}$ (where the dots indicate rates of change), and which is based on the supposed constancy of V, is only valid if the growth rate of PQ is constant (until now I have assumed the growth rate of P to be zero, but the previous analysis applies as well with a non-zero growth rate of prices $\underline{i}$: one must only substitute $g+\underline{i}$ to g).

III

Imagine an economy where all net income, once received, is re-spent on consumption goods. This might be the case, for instance, in an economy of self-managed enterprises, where all value added is distributed to the workers and these save nothing. Traditionally, it would be argued that, in such an economy, growth is impossible unless the buying power of the workers is reduced by inflation caused by an expansionary monetary policy (forced savings). But this conclusion forgets that the re-spending of the income received will only happen with a lag. If in the meanwhile the net product has grown, then with constant money prices, the workers will be unable to buy the whole of it, and a part of it will be left over for re-investment.

It may even be that a steady growth is possible. Consider, for instance, the following model. All capital goods are circulating capital goods which produce their product after a period of length m from the moment they are installed (and I assume for simplicity that they are sold and installed the moment they are produced); thus total product at time t, T_t , is the result of gross investment of time t-m, J_{t-m}, with $J_{t-m}/T_t = u$, where u, the capital - total product ratio, is given and less that 1. J_{t-m} must be replaced by depreciation D_t; $J_{t-m}=D_t$. Let Y_t T_t-D_t; then $Y_t=J_{t-m}(1-u)/u$. Let $J_t-D_t\equiv I_t$ represent net investment; let c represent the 'propensity to consume' in the sense of the proportion of money income which is re-spent (with a lag h)

on consumption goods (I will initially not put c=1, in order to have equations which may serve for reference later). Assume now that money creation goes entirely to finance investment expenditure. Then consumption expenditure is given by

$$C_t = cY_{t-h} = ce^{-gh}Y_t,$$ and net investment by

(1) $$I_t = Y_t - C_t = (1-ce^{-gh})Y_t;$$

to this we can add

(2) $$Y_t = J_{t-m}(1-u)/u$$

and, from $J_t = I_t - D_t = I_t + J_{t-m}$ and from the steady growth condition $J_t = J_o e^{gt}$, we derive in addition:

(3) $$I_t = J_{tm}(e^{gm}-1).$$

By substituting from (1) and (2) into (3) one gets

$$I_t = (1-ce^{-gh}) \cdot \frac{1-u}{u} \cdot \frac{I_t}{e^{gm}-1},$$

Eliminating I_t from both sides, indicating for simplicity (1-u)/u as §, putting c=1 as hypothesized, and multiplying both sides by e^{gh}, with simple passages one gets an equation in the single unknown g:

$$e^{g(m+h)} + \S = e^{gh}(\S+1) .$$

To study the existence of solutions it is useful to put this equation in logarithms:

$$\log\,(e^{g(m+h)}+\S) = gh+\log\,(\S+1).$$

This equation always has the solution $g=0$. But it may have another positive solution too. Put

$$y_1 \equiv gh + \log\,(\S+1)$$

$$y_2 \equiv \log\,(e^{g(m+h)}+\S),$$

and draw both on the same diagram as functions of g. The graph of y_2 is (as shown by its first derivative $dy_2/dg = (m+h)\,e^{g(m+h)}/(e^{g(m+h)}+\S)$, which is positive, takes on the value $(m+h)/(1+\S)$ for $g=0$, and increases monotonically towards the limit value of $m+h$ as g tends to $+\infty$) a convex upward-sloping curve which crosses the graph of y_1 at $g=0$ and which tends to take a slope greater than the slope of y_1. Therefore the graph of y_2 will cross again the graph of y_1 in the positive orthant iff its slope is less than the slope of y_1 at $g=0$, i.e. iff $(m+h)/(1+\S)$ h, i.e. iff $\S$ m/h. (This result is interesting because this condition does not appear to exclude, on grounds of empirical total lack of plausibility, the

possible occurrence of a positive solution; it would appear that what is required is a production lag, m, shorter that the expenditure lag h, something which is not impossible, considering that the greatest part of products takes a very short time to be produced. The greater u, the smaller would have to be m relative to h. This extreme case with $c=1$ is anyway only a sharp way to make a point which is relevant in the more general case, discussed at the end of this Section). The two possibilities are shown in Fig. 1; the second solution one gets, if the above inequality holds, is shown as g^*.

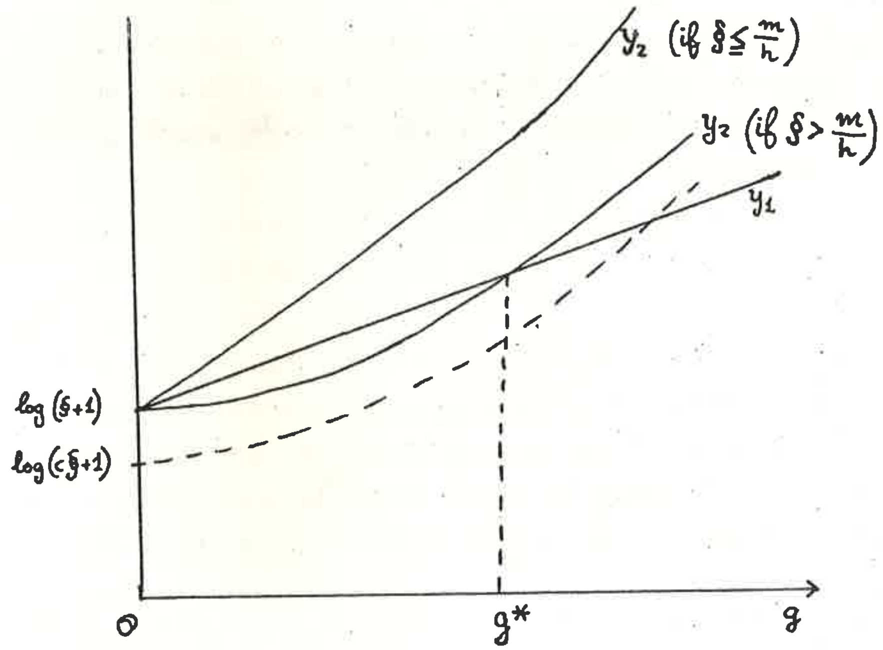

Fig. 1

If c, the propensity to consume, is less than 1, only y_2 is affected, becoming equal to log $(e^{g(m+h)}+c\S)$, which, at g=0, takes on the value log (c§+1), which is less than the value taken by y_1; but its behaviour is still that of a convex upward-sloping curve, tending to take a slope equal to m+h; therefore it always intersects y_1 only once (in the positive orthant). The graph of y_2 in this case is shown in Fig. 1 as a broken curve. It is worth noticing that, if $(m+h)/(1+\S)<h$, as c tends to 1 this unique solution tends to g*, rather than to g=0.

This model also shows that - contrary to the results obtained when no expenditure lag is considered (1)- the growth rate of a capitalist economy may be greater than the profit rate even when the workers do not save. The case just discussed, with c=1, offers an example. Instead of an economy of self-managed enterprises, one can imagine the model to refer to a capitalist economy, where the workers are paid at the end of the productive process, and their wages absorb the entire value added of the enterprise, so that the profit rate is zero.

The reason, to repeat, is that to receive the entire net income is not the same as to buy the entire net income. This is clearly seen if one assumes the wages are paid by the entrepreneurs immediately after selling the product: then if the product is growing, the workers are able to be

(1) See, e.g., L.L. Pasinetti (1974), Growth and Income Distribution, Cambridge, Cambridge University Press, chs. 5 and 6.

paid the entire value added (i.e. the whole product can be sold at costant money prices) only because the do not buy the entire net product whose sale allows them to receive that income.

It might be objected that a capitalist economy with a zero profit rate is an impossibility. But c=1 is not a necessary assumption in order to get a growth rate greater than the profit rate in spite of the absence of workers' savings. Even with c less than one, one still gets that the growth rate in my model is greater than the growth rate in the model one obtains from it by abolishing the expenditure lag. In the latter case the only equation which is affected is equation (1), which becomes $I_t=(1-c)Y_t$, the usual formulation; the steady growth equation determining g then becomes

(4) $$e^{gm} = (1-c)\S+1$$

instead of

(5) $$e^{gm} = (1-ce^{-gh})\S+1.$$

Putting $y_3 \equiv (1-c)\S+1$, $y_4 \equiv e^{gm}$, $Y_5 \equiv (1-ce^{-gh})\S+1$, it is easy to show graphically that g', the intersection of the graphs of y_3 and y_4, i.e. the solution of equation (4), is always to the left of (i.e. smaller than) g", the intersection of y_4 and y_5, i.e. the solution of the equation (5) (see Fig. 2) because

$\frac{dy_5}{dg}$ $\S c h e^{-gh}$ is positive at $g=0$ and decreasing as g increases. Therefore, if $g'=r$, where r is the profit rate, then $g''>r$.

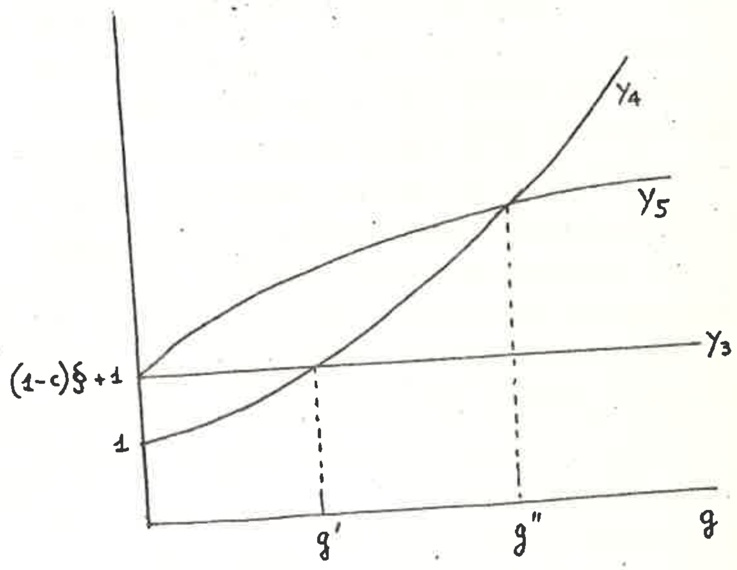

Fig. 2

IV

It may be interesting to show briefly the connection between the above and the theory of the dynamic multiplier.

For this purpose I pass from continuous to discrete time. Assume $Y_t = C_t+I_t$, $C_t = cY_{t-1}$. Then $Y_t = I_t+cI_{t-1} + c^2I_{t-2}+c^3I_{t-3}+\ldots$ If I_t is constant, then $Y_t = I_t/(1-c)$. If I_t grows at the rate g, i.e. $I_t = (1+g)^n I_{t-n}$, then

$$Y_t = I_t\left(1+\frac{c}{1+g}+\frac{c^2}{(1+g)^2}+\frac{c^3}{(1+g)^3}+\ldots\right)=\frac{1}{1-\frac{c}{1+g}}I_t$$

(which is smaller than $I_t/(1-c)$. It is immediate that this latter expression is simply the equivalent, in discrete time, of $I_t = (1-ce^{-gh})Y_t$ (keeping in mind that the above analysis in dicrete time implicity assumes $h=1$). This is as it should be, since the basic element of the discussion of the previous sections, the lag between receiving an income and spending it, is the central element of dynamic multiplier analyses too. Thus Section III of the present paper can be interpreted as an exploration of some little-noticed implications of the hypotheses implicit in standard dynamic multiplier analysis.

The results on steady growth of Section III could be re-formulated in terms of this discrete-time model with $h=1$. A slightly different, more usual formulation (1) might be:

(1) Suggested to me by Prof. Paladini.

$$Y_t = vK_{t-1}$$
$$Y_t = I_t + cY_{t-1}$$
$$K_t = I_t + K_{t-1}$$

which yields the steady-growth condition:

$$g = \frac{I_t}{K_{t-1}} = (1 - \frac{c}{1+g})Y_t/(Y_t/v) = v(1 - \frac{c}{1+g})$$

i.e.

(6) $g^2+g(1-v)-v(1-c) = 0.$

For c=1, this yields the following result. If also v=1, then g=0 is the only solution. If v≠ 1, then equation (6) becomes g(g+1-v) = 0 which has, besides g=0, also the solution g*=v-1, which is positive only if v>1, an unrealistic condition; it is thus confirmed that, for another positive solution g* to be realistically possible, the expenditure lag h must be greater than the production lag m (rather than equal, as here).

V

Ricardo accepts "Say's Law", i.e. the principle that (apart from misadaptations of the composition of production to the composition of the social demand) the total effectual demand is normally sufficient to buy the social product at the natural prices. Accumulation is then determined by savings, i.e. all that is not consumed is invested; and, in fact, central to Ricardo's acceptance of that principle are Smith's arguments against hoarding, which bring Ricardo to conclude that

> The revenue is in all cases spent, but in one case the objects on which it is expended are consumed, and nothing reproduced in the other those objects form a new capital tending to increased production(1).

A more detailed analysis of why Ricardo believes that "the revenue is in all cases spent" has been attempted elsewhere(2). Here it will only be pointed out that the absence of hoarding, i.e. the re-spending of all revenue, may not be sufficient for the equality of total demand and total product at constant prices, for reasons that by now the reader can guess.

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(1) D. Ricardo, Notes on Bentham (1810-11), in id. (1951-73), Collected Works, ed. by P. Sraffa, Cambridge, Cambridge University Press, vol. III, p. 299.

(2) F. Petri (november 1982), The Connection between Say's Law and the Theory of the Rate of Interest in Ricardo, unpublished manuscript.

Ricardo in fact accepts that the velocity of circulation of money is finite and normally fairly stable. He also accepts that normally the growth rate of the economy is positive. He also considers a stable price level as the norm - in the double sense that he both considers various mechanisms which ensure price level stability and, were these mechanisms not to work, he advocates the intervention of the monetary authorities.

The previous analysis then shows that the sole money expenditure coming from revenue from previous sales will be insufficient to buy a growing product. What ensures that the growing product is all sold at constant prices is the existence of money "creation" - of purchases financed with money previously not existing in the nation.

In Keynesian terms, one might re-formulate the problem as follows: Ricardo assumes that, directly or indirectly, the whole revenue is re-spent, i.e. that the propensity to spend out of revenue on consumption, c, and on investment, k, sum up to 1; and he thought that this ensured that any income level would be an equilibrium level.

But if the revenue, out of which the propensity to spend is 1, is revenue which must have been already actually received in monetary form, then the inevitable presence of a lag will have the effect that, if the product is growing, the propensity to spend at a certain moment, relative to the product at that moment, will be less than one; and, to fill in the gap, an additional source of expenditure, besides money from previous sales (internal to the economy), is

necessary.

In an open economy, which does not produce gold, the money "creation" needed to fill the gap $B_t - B_{t-h}$ might be provided by an excess of exports over imports causing an inflow of gold (or, in modern times, an inflow e.g. of dollars, with consequent "seignorage" of the United States, as is well known to all experts of international monetary relations). Other ways are of course an expansion of credit, or a State deficit, or gold production, the latter two sources of extra money needing to be less than $B_t - B_{t-h}$ if there is at work the bank deposit multiplier. The same bank deposit multiplier would ensure that the inflow of gold (or of dollars, etc.) from abroad through a balance-of-trade surplus would have to be less than $B_t - B_{t-h}$. The existence of an expansion of credit, implicit in the increase in the amount of money through the bank deposit multiplier, which is not simply a transfer of income, means that even gold production would not eliminate the phenomena described in Section I. Ricardo does not appear to have seen this problem, and it is understandable why: he had not seen the bank deposit multiplier. He does discuss the need to let the total amount of bank credit expand with the needs of trade, but he does not connect this expansion of credit with the "all revenue is spent" thesis. Clearly, if money "creation" by the private sector, i.e. credit expansion, is needed, growth cannot come only from retained profits or the savings of consumers: the "animal spirits" of entrepreneurs must be such as to induce them to borrow, and not only to borrow the

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