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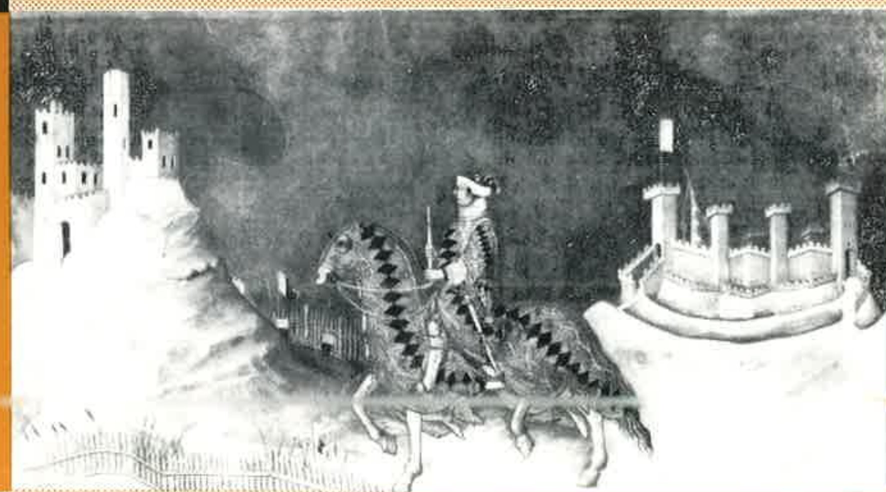
QUADERNI DEL DIPARTIMENTO
DI ECONOMIA POLITICA

Stefano Vannucci

ON STABLE STANDARDS OF BEHAVIOUR



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Stefano Vannucci

ON STABLE STANDARDS OF BEHAVIOUR

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ABSTRACT. VNM-like stable standards of behaviour (SSBs) are analysed. It is shown that: i) the vNM-stable sets of certain abstract games are the ergodic sets (other) abstract games; ii) the capacity of a simple game is not essentially affected from variations in the notion of implementation or in the choice of the relevant specification of a SSB. Generalizations of the theory of social situations are also suggested.

1. INTRODUCTION

The last few years have seen a remarkable growth of interest in von Neumann-Morgenstern stable standards of behaviour and related solution concepts.

This is partly due to the increasing popularity of the view holding that the relevance of stable standars of behaviour (henceforth, SSBs), is by no means confined to the realm of cooperative games.

In fact, several promising applications of (some version of) SSBs to noncooperative games have been proposed (see Schotter (1986), Kaneko (1987), and most notably Greenberg (1989a, 1989b, 1990)).

The present paper is devoted to some further analysis of SSBs. Some miscellaneous results are presented.

The paper is organized as follows. The second section (which draws heavily on Vannucci (1990)) analyses the relation between von Neumann-Morgenstern (vNM) stable sets and ergodic sets in the general setting of abstract games. Since ergodic sets a general notion of dynamic stability for abstract games, the results presented here may help to clarify the relationship of vNM-stability to dynamic stability. Moreover, that analysis makes it possible to single out a further set of conditions ensuring existence and uniqueness of vNM-stable sets. The third section is devoted to efficiency properties of SSBs as well as to implementation theory in SSBs (several variants od SSBs are taken into consideration). The fourth section points out some connections between Greenberg's «social situations», SSBs, and automata theory leading to a generalization of «social situations».

2. VNM-STABLE SETS AND ERGODIC SETS

An (exact) **abstract game** -or 1-digraph- is a pair (X, R), where X is an arbitrary nonempty set (the state space) and $R \subseteq X \times X$ is a binary relation on X, the dominance relation (i.e. $(x,y) \in R$ iff x «dominates» y).

The following standard notation for basic set-theoretic operations on subsets of X and on binary relations defined over X will be used: we shall denote by o the partial binary composition operation, by $(.)^k$ the k-th (composition) exponential operator (for any positive integer k), by $(.)^0$ the complementation operator (with respect to the relevant universal set, i.e. X, or the universal relation X x X), by $\cap$ the binary meet operation, by $\cup$ the binary join operation, by $(.)^i$ the inversion operator, by I the identity relation, by □ (.) the source (or dominion) operator, by (.) □ the target (or codominion) operator.

Also, we shall denote by P(R) (I(R)) the asymmetric (symmetric) component of R, and by T(R) the transitive closure of R, namely.

$P(R) = R \cap (R^i)^c$, $I(R) = R \cap R^i$, $T(R) = \cup^{\infty}_{k1} R^k$

A **vNM-stable set** of an abstract game (X,R) is a set $S \subseteq X$ enjoying the following properties:

Internal Stability (IS): $S \times S \subseteq R^c$ (i.e. $S \subseteq ((R \cap S \times X)\,\square\,)^c$);

External Stability (ES): $((R \cap S \times X)\,\square\,)^c \subseteq S$.

Equivalently, a vNM-stable set is a fixed point of the set function $f^c{}_R = (\cap(R,(.)xX))\,\square\,)^c$, as defined on $\mathcal{P}(X)$, the power set of X.

Thus, a vNM-stable set of an abstract game is a set of states S s.t. no states in S are dominated by other states in S, whereas any state not in S is dominated by some state in S. We shall denote by **vNM-solution** of (X,R) the set of its vNM-stable sets.

The original vNM-stability conditions have been both strenghtened and relaxed in several alternative ways (see e.g. Harsanyi (1979), Roth (1976), Lucas (1977), Schmidt, Ströhlein (1985)).

We shall take into consideration the following variants of the original vNM stability requirements:

Strict Internal Stability (SIS): $S \times S \subseteq (T(R))^c$;

Self Protection (SP): $S \subseteq (f^c{}_R)^2\,(S)$;

Esclusion (EX): $(f^c{}_R)^2\,(S) \subseteq S$.

Strictly stable sets are subsets of X satisfyng SIS and ES: see Harsanyi (1974). **Semi-stable sets** are subsets of X satisfying IS and SP: see Berge (1979), Schmidt, Ströhlein (1985). **Substable sets** are subsets of X satisfying IS, SP, EX: see Roth (1976, 1984). **Strictly semistable sets** and **strictly substable sets** can also be defined in a natural way by strenghtening their internal stability requirement to SIS.

An **ergodic set** of an abstract game (X,R) is a set $S \subseteq X$ of states satisfying:

Persistency (PS): $S^c \times S \subseteq R^c$

PS-Minimality (PSM): $S \subseteq S'$ for any nonempty $S' \subseteq X$ which satisfies PS.

The **ergodic solution** of (X,R) is the join of its ergodic sets.

The ideas underlying ergodic sets arise from Markov chain theory, as opposed to the distinctly game-theoretical origins of vNM-stable sets. In fact, ergodic sets embody a very general criterion of (local) dynamic stability: an ergodic set is a set of states which will not be left, whenever entered.

An alternative definition of ergodic sets has been also proposed, sometimes under a different label (see Shenoy (1979); see also Packel (1981), Gilboa, Matsui (1991)). We shall see in a moment that the two proposed definitions are in fact equivalent.

Following Shenoy (1979), we define a (nontrivial) **elementary dynamic solution** of the abstract game (X,R) as a (nonempty) set $S \subseteq X$ satisfyng:

Persistency* (PS*): $S^c \times S \subseteq (T(R))^c$;

Communication (C): $S \times S \subseteq T(R)$.

Proposition 2.1. - Let (X,R) be an abstract game, and $S \subseteq X$. Then, S satisfies Persistency* iff it satisfies Persistency. Moreover, S is a (nonempty) ergodic set of (X,R) iff it is a (nonempty) elementary dynamic solution of (X,R).

Proof. It is easily checked that the empty set is both an ergodic set and an elementary dynamic solution. Hence, we may assume $S \neq \emptyset$. Obviously, if S satisfies Persistency* then it also satisfies Persistency. Conversely, let S satisfys Persistency while violating Persistency*. Thus, $(y,x) \in T(R)$ for some $y \in S^c$, $x \in S$. Hence -by definition of T(R)- $S \cup S^c = X$ entails $(y',x') \in R$ for some $y' \in S^c$, $x' \in S$, a contradiction.

Let us now assume that S is an ergodic set of (X,R) which violates Communication. Then $(x,y) \notin T(R)$ for some $x,y \in S$. Let us denote by $T(R)_S(y)$ the T(R)-upper contour of y in S, i.e.

$T(R)_S(y)=(y) = \{ z \in S \,/\, (z,y) \in T(R) \}$.

Since $x \notin T(R)_S(y)$, $T(R)_S(y) \cup \{y\} \subset S$. Moreover, $(w,z) \notin T(R)$, for any $w \in T(R)_S(y))^c$, $z \in T(R)_S(y)$.In fact, suppose to the contrary that $(w,z) \in T(R)$ for some w,z as defined above. Then, $(w,z) \in T(R)$, whence $(w,z) \in T(R)_S(y)$, a contradiction. As a consequence $T(R)_S(y)$

satisfies Persistency, contradicting PS-Minimality of S.

Conversely, let S be an elementary dynamic solution of (X,R) which does not satisfy PS-minimality.

Then, a nonempty $S' \subseteq X$ exists such that $(S')^c x S' \subseteq R^c$ and $S' \subset S$. In particular, $(y,x) \in R^c$ for any $y \in S \cup (S')^c$. However, $(y,x) \in S \cup (S')^c x S'$ entails $(x,y) \in T(R)$, since-by hypothesis-S satisfies Communication. It follows that $y' \in (S')^c, x' \in S'$ exist such that $(y',x') \in R$: a contradiction ❑.

Remark 2.1. The foregoing proposition clearly implies that i) the ergodic sets of a game are paiwise disjoint; ii) $T(R) \cup I$ is complete then (X,R) admits not more than one nonempty ergodic set.

Remark 2.2. Since «cyclically stable sets», as defined by Gilboa, Matsui (1991), amount (as it is easily checked) to a noncooperative specification of elementary dynamic solutions, it follows -by Proposition 2.1.- that they are also a specification of ergodic sets.

Remark 2.3. In a remarkable paper, Kalai, Schmeidler (1977) prove that the ergodic solution E of an abstract game (X,R)- i.e. the join of its ergodic sets, which they call the «admissible set»- is characterized by the following condition:

Admissibility (AD) $(ExE) \cap T(R) \subseteq (ExE) \cap T(R)^i$.

It follows that, whenever a game (X,R) has a unique ergodic set, {AD} , {PS,PSM}, and {PS*,C} turn out to be three equivalent sets of restrictions over subsets of X.

Alternative -and more complex- characterizations of the ergodic solution of a game are offered in Schwartz (1972) (who works with a finite asymmetric (X,R)) and in Kim, Roush (1979) (see also Deb (1977)).

Moreover, Kalai, Schmeidler (1977) provides several cooperative and noncooperative specifications of ergodic solutions. The ergodic solution of an abstract game has also been studied in a specialized voting setting where R denotes a suitably defined social preference relation (see, among others, Kalai, Pazner, Schmeidler (1976), Campbell (1980), Bordes (1983)).

The following general relation between vNM-stable sets and ergodic sets holds true.

Proposition 2.2. Let (x,R) be an abstract game s.t. T(R) I is complete. Then, the following statements are equivalent:

i) $A \subseteq X$ is a nonempty ergodic set of (X,R);

ii) $A \subseteq X$ is a vNM-stable set of (X,P(T(R)));

iii) $A \subseteq X$ is a strictly stable set of (X,P(T(R))).

Proof. Let A be a nonempty ergodic set of (X,R). Thus, for any $x,y \in A$ $(x,y) \in T(R)$ and $(y,x) \in T(r)$, by Communication and Proposition 2.1. Therefore, $(x,y) \notin P(T(R))$, whence Internal Stability of A in (X,R) follows. Furthermore, T(R) is obviously transitive which entails that P(T(R)) is transitive as well. Hence T(P(T(K)))=P(T(R)). As a result, A also satisfies Strict Internal Stability in (X,P(T(R))). Let us now consider any pair $x \in A$, $y \in A^c$. Obviously, $(y,x) \notin T(R)$ (by Proposition 2.1 and Persistency of A). Hence $(x,y) \in T(R)$ (by completeness of $T(R) \cup I$). It follows that $(x,y) \in P(T(R))$, which entails External Stability of A in (X,P(T(R)).

Conversely, let A be a vNM-stable set of (X,P(T(R))) (hence a strictly stable set as well, since P(T(R)) is transitive). Obviously A is nonempty, by External Stability. Moreover, for any $x \in A$, $y \in A^c$ $(y,x) \in T(R)$ implies $(z,x) \in P(T(R))$ for some $z \in A$, by External Stability of A and transitivity of T(R), contradicting Internal Stability of A. Hence A satisfies Persistency in (X,R), by Proposition 2.1. The proof that A satisfies PS-Minimality is also by contradiction. For, suppose it does not. Then a nonempty set $B \subset A$ exists which satisfies Persistency in (X,R). As a consequence, $A \cap B^c$ is nonempty, and $(x,y) \notin T(R)$ for any $x \in A \cap B^c$, $y \in B$ (by Proposition 2.1. and Per-

sistency of B in (X,R)). On the other hand, Internal Stability of A in (X,P(T(R))) entails $(y,x) \notin P(T(R)$ whence $(y,x) \notin T(R)$, a contradiction. ❑

Remark 2.4. An equivalent statement of proposition 2.2. in the language of «arrows-and-diagrams» is the following (where Γ denotes the class of all abstract games Γ_{tc} denotes the class of all abstract games (X,R) such that $T(R) \cup I$ is complete, Set denotes the class of all sets):

i) for any selection $F_s: \Gamma \to$ Set of vNM-stable sets a selection $F_e: \Gamma_{tc} \to$ Set of (nonempty) ergodic sets exists s.t. the square diagram below commutes:

$$\begin{tikzcd} \Gamma_{tc} \arrow[r, dashed, "F_e"] \arrow[d, "(\mathrm{Id}, ^{\mathrm{coi}})"'] & \mathrm{Set} \\ \Gamma_{tc} \times \Gamma \arrow[r, "\cap"] & \Gamma \arrow[u, "F_s"'] \end{tikzcd}$$

ii) for any selection F_e: $\Gamma_{tc} \to$ Set of nonempty ergodic sets a selection F_s : $\Gamma \to$ Set of vNM-stable sets exists s.t. the square diagram below commutes:

$$\begin{tikzcd} \Gamma_{tc} \arrow[r, "F_e"] \arrow[d, "(Id, coi)"'] & Set \\ \Gamma_{tc} \times \Gamma \arrow[r, "\cap"] & \Gamma \arrow[u, dashed, "F_g"'] \end{tikzcd}$$

Remark 2.5. It is worth noticing that, in particular, Proposition 2.2 holds true whenever the abstract game (X,R) is a tournament (i.e. R∪I is antisymmetric and complete).

The relevance of Proposition 2.2. to vNM-stable sets it twofold. First, it makes it possible to relate vNM-stable sets to a notion of local dynamic stability, as embodied in ergodic sets. Second, it provides a new condition ensuring the existence and uniqueness of vNM-stable sets. These two points are made precise below.

Let (X,R) be an abstract game. Then an (X,R)-**compatible dynamic process** (or, simply, an **R-dynamic process**) is an X-valued sequence $(x_i)_{i\in N}$ such that, for any $i\in N$, $(x_{i+1},x_i)\in R\cup I$ (x_0 is the starting point of such a dynamic process) Let us now consider a topology τ on X. A set S⊆X is **locally** (X,R,τ)-**stable** iff for any τ-neighborhood T of S a τ-neighborhood U of S exists s.t. for any x∈U, and any R-dynamic process $(x_i)_{i\in N}$, x_0=x implies $x_i\in T$ for every i∈N. Moreover, a set

S⊆X is **R-invariant** iff q(R∩XxS)⊆S. Obviously, if S is a locally (X,R,τ)-stable set and S∈τ then S is R-invariant as well. On the other hand, it is easily checked that a set S⊆X is persistent (ergodic) in (X,R) iff it is an R-invariant (a minimal R-invariant) set. In particular, if τ is the discrete topology on X, then a set S⊆X is a locally (X,R,τ)-stable set iff it satisfies Persistency in (X,R). Since S⊆X satisfies Persistency in (X,R) implies that S⊆X satisfies Persistency in any (X,R′) s.t. R′⊆R, the following corollary obtains:

Corollary 2.1. Let (X,R) be an abstract game s.t. T(R)∪I is complete, and τ the discrete topology on X. Then any vNM-stable of the abstract game (X,P(T(R))) is a locally (X,R,τ)-stable set.

Remark 2.6. The converse of the foregoing corollary does not hold true. In fact, the empty set is trivially locally (X,R,τ)-stable w.r.t. any abstract game (X,R) when τ is the discrete topology on X. However, the empty set cannot possibly be vNM-stable, by External Stability. A more interesting class of counterexamples is the following: consider an infinite abstract game (X,R′) where R′ satisfies the following (consistent) properties: i) R′ is a strict partial order (i.e. it is asymmetric and transitive); ii) R′ is not quasi-complete (i.e. {(x,y),(y,x)} ∩ R′=∅, for some distinct x,y∈X); iii) R′∩YxY is a complete ordering of Y without a maximum, for some infinite $Y \subseteq X$; iv) R' admits maximal elements (of course, the R'-maximal elements are in Y^c). Then the core of (X,R'), i.e. the set of R'-maximal states, is both nonempty and locally (X,R',τ)-stable (when τ is the discrete topology on X) but it is not a vNM-stable set of (X,R') (it fails to satisfy External Stability, due to the existence of Y). On the other hand, the well-known Szpilrajn Theorem (requiring the Axiom of Choice) ensures that (X,R') does satisfy the hypothesis of Proposition 2.2. (and of Corollary 2.1.).

Corollary 2.2. Let (X,R,τ) be a compact abstract game (i.e. $(X,R,)$ is an abstract game, and (X,τ) is a compact topological space). Also, suppose that $T(R) \cup I$ is complete. Then,

i) If for any $x \in X$ the R-lower contour of x (i.e. the set $\{y \in X/(x,y) \in R\}$) is τ-open then $(X,P(T(R))$ has a unique vNM-stable set S, which is also strictly stable (namely, S, is the unique nonempty ergodic set of (X,R)).

ii) If for any $x \in X$ the R-upper contour of x (i.e. the set $\{y \in X/(y,x) \in R\}$) is τ-closed and, moreover, R is transitive, then $(X,P(T(R)))$ has a unique vNM-stable set S, which is also strictly stable (namely, S is the unique nonempty ergodic set of (X,R)).

Proof. i) From Proposition 2.2, Remark 2.1 ii), and Theorem 2 in Kalai, Schmeidler (1977).

ii) From Proposition 2.2, Remark 2.1 ii), and Theorem 3 in Kalai, Schmeidler (1977).□

Remark 2.7. A well-known condition ensuring existence and uniqueness of vNM-stable sets (which may not be strictly stable, however) is **Strict Acyclicity** of (X,R), i.e.: $R^k = \emptyset$ for some positive integer k (see von Neumann, Morgenstern (1953)). It should also be recalled that, under finiteness of X, Strict Acyclicity reduces to **Acyclicity**, namely $R^k \cap I = \emptyset$ for any positive integer k.

Another, more obvious, condition ensuring existence and uniqueness of vNM-stable sets is (X,R) being a strict order with a (necessarily unique) maximum x (in that case the unique (strictly!) vNM-stable set of (X,R) is $\{x\}$): see, again, von Neumann, Morgenstern (1953).

To see that both Strict Acyclicity and **Strict Order cum Maximum** are in fact independent from the condition singled out by Corollary 2.2., consider the ensuing examples.

Let [0,1] be the real unit interval, endowed with the canonical strict order >, and n a positive integer. Let us consider the following binary relations:

i) $R = \{ (x,y) \in [0,1]^2 / \{x,y\} \subseteq [n-1/n,1]$ or $x<y$ and

$\{x,y\} \subseteq [k-1/n,k/n]$ for some $k<n$ $\}$.

Obviously, T(R) is complete, and P(T(R)) is a strict order, but not a quasi-complete one (because $T(R) \cap I^c$ has a nonempty symmetric component);

ii) Take the two following restrictions of $>$: $>_1 \; = \; > \cap [0,a]^2$, $>_2 = > \cap (a,1]^2$ where $a \in (0,1)$, and define $R' = >_1 \cup >_2$. Notice that R′ is neither strictly acyclic nor transitive, and $T(R) \cup I$ is complete. Next take the topology (u(R)) having the set of R-lower sets

$$u(R) = \{ \{ y \in [0,1] \,/\, (x,y) \in R \} \,/\, x \in [0,1] \}$$

as its basis ($\tau(u(R'))$ is defined in a similar way by means of R′-lower sets)).

It can be shown that both $([0,1],\tau(u(R)))$ and $([0,1],\tau(u(R')))$ are compact.

The set $[n-1/n,1]$ is the only nonempty ergodic set of ([0,1],R) and the only (strictly) vNM-stable set of ([0,1],P(T(R))), whereas the singleton $\{1\}$ is the only nonempty ergodic set of ([0,1],R′) and the only (strictly) vNM-stable set of ([0,1],P(T(R′))).

To check that neither Strict Acyclicity nor Strict Order cum Maximum entail the hypothesis of Corollary 2.2 i) just consider any triple (X,R,τ) s.t. the abstract game (X,R) is a finite quasi-completely strictly ordered set and τ is the indiscrete topology on X (i.e. $\tau = \{\emptyset, X\}$).

Remark 2.8. Notice that in case τ is the discrete topology Corollary 2.2 i) simply says that the only ergodic set of an abstract (X,R) s.t. X is finite and $T(R) \cup I$ is complete is also the only strictly stable set of (X,P(T(R))) (recall that if τ is the discrete topology then (X,τ) is compact if and only if X is finite).

The next example shows that Proposition 2.2 does not hold true if $T(R) \cup I$ fails to be complete.

Example 2.1.. Let $X = \{x,y,z,w,u,t\}$, $R = \{ (x,y),(y,z),(z,w),(u,t) \}$. The unique (strictly) vNM-stable set of (X,P(T(R)) is $\{x,u\}$, whereas (X,R) has two nonempty ergodic sets $\{x\},\{y\}$ (here, the unique vNM-stable set of (X,P(T(R))) equals the join of the ergodic sets, or the ergodic solution, of (X,R)).

Also, replacing in Proposition 2.2. P(T(R)) either with T(P(R) or with a complete T(R) (or even with a complete R) turns the proposition into an invalid statement, as the following example shows:

Example 2.2. Let $X = \{x,y,z,w\}$, $R = \{ (x,y),(y,z),(z,w),(w,z) \}$. Notice that $T(R) \cup I$ is complete. Moreover, R, is asymmetric (whence, T(P(R)=T(R)) and quasi-complete (i.e. a tournament). The only nonempty ergodic set (hence the ergodic solution) of (X,R) is the whole state space X: (X,R) is ergodically chaotic, or irreducible. On the other hand, it can be easily checked that: i) (X,T(R)) has no vNM-stable sets; ii) the set of vNM-stable sets of (X,P(T(R))) is the set of singletons $\{\{x\} / x \in X\}$; ii) (X,R) has two vNM-stable sets, namely $\{x,z\}$ and $\{y,w\}$.

3. STABLE STANDARDS OF BEHAVIOUR AND IMPLEMENTATION THEORY

Brodly speaking, implementation theory deals with some consistency-tests between sets of criteria of good performance for systems of social interaction and theories of rational strategic behaviour, as embodied in certain game-theoretical solution concepts (those tests are in fact conditional upon certain general assumptions about the possible, or admissible, combinations of the relevant individual characteristics). This section is devoted to some versions of a particular implementation problem (namely, obtaining infinite capacities for certain power structures) with (some version of) stable standards of behaviour as the relevant solution concept.

The following proposition has some import on subsequent analysis.

Proposition 3.1. Let (X,R) be a strictly acyclic abstract game. Then (X,R) has a unique sub-stable set.

Proof. By contradiction. If follows from the well-known result by von Neumann, Morgenstern (1953, chpt. XII) that (X,R) has a unique vNM-stable set, say S. Obviously, S is sub-stable as well. Let us now suppose that S′ is another sub-stable set of (X.R), $S' \neq S$. Then, it must be the case that S′ satisfies $S' \subseteq f^c_R(S')$ (i.e. Internal Stability), $S' = f^c_R(f^c_R(S'))$ (i.e. Self-Protection plus Exclusion) and $x \in f^c_R S' \cap (S')^c$ for some $x \in X$. Hence, by Exclusion, a $y \in f^c_R(S')$ exists such that $(y,x) \in R$. But then $y \in f^c_R S' \cap (S')^c$ as well (otherwise): contradiction). By induction, we obtain an infinite path in (X,R) contradicting its strict acyclicity. ❑

Remark 3.1. Strict Acyclicity does not ensure the existence of a strictly sub-stable set (since, as we know, it doesn't entail existence of strictly stable sets). On the other hand, if (X,R) is a strict order cum maximum, then its (unique) maximum constitutes the unique strictly stable set and the unique strictly sub-stable set of (X,R).

A few more definitions are in order here. A **grand coalitional game form (gcgf)** is an ordered pair (N,A) of nonempty sets (where N denotes the set of players, and A the set of alternatives).

An **effectivity function (EF)** for a gcgf (N,A) is a function $f: \mathbf{P}(N) \to \mathbf{P}^2(A)$ s.t.

EF i): $f(\emptyset)=\emptyset$; EF ii) $\emptyset \notin f(S)$ for any $S \subseteq N$; EF iii): $B \in f(N)$ for any nonempty $B \subseteq A$; EF iv): $A \in f(S)$ for any nonempty $S \subseteq N$. Moreover, an EF for (N,A) is **A-monotonic** if for any $B,C \subseteq A$ and $S \subseteq N$ $B \in f(S)$ and $B \subseteq C$ entail $C \in f(S)$, and **simple** if a nonempty set $W \subseteq \mathbf{P}(N)$ exists s.t. for any $S \subseteq N$ $f(S)=\mathbf{P}(A) \setminus \{\emptyset\}$ if $S \in W$ and $f(S)=\{A\}$ otherwise ((N,W) is in fact a simple game, namely the simple game of f).

Now, let $\mathfrak{R}(A)$ be the set of all weakly acyclic binary relations on A (i.e. relations r A x A s.t. their asymmetric component does not cycle, i.e. for any positive integer k $(P(r)^k \subseteq I^c)$, and $\mathcal{O}(A)$ the set of all strict (partial) orders (i.e. transitive and asymmetric binary relations) on A. For any $S \subseteq N$, a **weakly acyclic** (respectively, **strictly ordered**) **S-profile** on a is a function $r_S: s \to \mathfrak{R}(A)$ (respectively, a functions $r_S: s \to O(A)$. As usual, such S-profiles are to be interpreted as preference profiles of players in S. N-èrofiles (either weakly acyclic or strictly ordered) are also called profiles (with no coalitional prefiz).

The **(protective) abstract game** attached to an EF f for (N,A) at a (weakly acyclic or strictly ordered) S-profile r_S on A (where $S \subseteq N$) is $(\mathbf{P}(X),R)$ where the dominance relation $R=R(f,r_S))$ is defined as for any $B,C \subseteq A$ $(B,C) \in R$ iff a pair $(x,T) \in C \times \mathbf{P}(S)$ exists s.t.: a) $(y,x,) \in \bigcap_{i \in \mathcal{S}} r_{\mathcal{S}}(i)$, $y \neq x$ for any $y \in B$, and b) $B \in f(T)$.

Moreover, for any (weakly acyclic or strictly ordered) profile r_N on A, the (weakly) **Pareto efficient** set at r_N is

$$PO(r_N)=\{x \in A \text{ / no } y \in A \text{ exists s.t. } (y,x) \in \bigcap_{i \in N} P(r_N(i))\}$$

(in fact, if r_N is a strictly ordered profile then $PO(r_N)$ is also the (strongly) Pareto efficient set of r_N).

The following proposition holds true.

Proposition 3.2. Let f be an EF for (N,X), r_N a strictly ordered profile on X, $(\mathbf{P}(X),R(f,r_N))$ the protective abstract game induced by f at r_N, $\mathcal{S} \subseteq \mathbf{P}(X)$ a vNM-stable set of $(\mathbf{P}(X),R(f,r_N))$. Then $\cup \mathcal{S} \subseteq PO(r_N)$.

Proof. By contradiction. Let $\mathcal{S} \subseteq \mathbf{P}(X)$ be a vNM-stable set of $(\mathbf{P}(X),R(f,r_N))$, such that for some $y \in Y \in \mathcal{S}$ and $x \in X$ and for any $i \in N$, $(x,y) \in r_N(i)$. Since $\{x\} \in f(N)$ -by EF iii)- it follows that $(\{x\},Y) \in R(f,r_N)$ (by definition of $R(f,r_N)$). Therefore, Internal Stability of $\mathcal{S}$ implies that $\{x\} \notin \mathcal{S}$. Hence, by External Stability of $\mathcal{S}$,$(Z,\{x\}) \in R(f,r_N)$ for some $Z \in \mathcal{S}$, i.e. for some $Z \subseteq X$ and $T \subseteq N$: $Z \subseteq f(T) \cap \mathcal{S}$ and $(z,x) \in r_N(i)$ for any $i \in T$. It follows that $(z,y) \in r_N(i)$ for any $z \in Z$ and any $i \in T$, by transitivity of $r_N(i)$. Therefore $(Z,Y) \in R(f,r_N)$, contradicting Internal Stability of $\mathcal{S}$.□

Corollary 3.1. Let f be an EF for (N,X), r_N a strictly ordered profile on X, $(\mathbf{P}(X),R(f,r_N))$ the abstract game induced by f at r_N, and $\mathcal{S}$ a sub-stable set of $(\mathbf{P}(X),R(f,r_N))$. Then, $(\mathbf{P}(X),R(f,r_N))$ is strictly acyclic entails $\cup\mathcal{S}\subseteq PO(r_N)$.

Proof. From Propositions 3.1, 3.2 and from the fact that a vNM-stable set of a game is also a sub-stable set of the same game.□

Remark 3.2. Proposition 3.2. and Corollary 3.1. imply that vNM-stable, strictly stable sets and, under strict acyclicity, sub-stable sets are a refinement of (Pareto) efficiency, when applied to (protective) abstract games induced by an EF at a linear profile. As a result, an extension of Nakamura's core-capacity analysis (to be defined below) to SSBs does make sense.

Let us now turn to SSB-capacity theory for simple games.

A **simple game** on a nonempty finite set N of players is a pair g=(N,W). where $W\subseteq\mathbf{P}(N)$ is a nonempty set (W is to be interpreted as the set of winning coalitions). A simple game is **monotonic** if $S\in W$ and $S\subseteq T\subseteq N$ entail $T\in W$ (in the following we shall confine ourselves to monotonic simple games).

A simple game is **proper** if for any $S\subseteq N$ $S\in W$ entails $S^c\notin W$, and **weak** if $\cap W\neq 0$.

The **Nakamura number** $\nu(W)$ of a simple game (N,W) is defined by

the following rules:

$\nu(W)=\min\{\#W'/W'\subseteq W \text{ and } \cap W'\neq\emptyset\}$ if (N,W) is nonweak and $\nu(W)=\infty$ otherwise.

Notice that a simple game on N can be attached in a natural way to any EF f=(N,Y) on the same set of players, namely g(f)=(N,W(f)) where $W(f)=\{S\subseteq N / Z\in f(S) \text{ for any nonempty } Z\subseteq Y\}$.

A **solution concept** for a class Γ of games is a correspondence $\mathcal{E}:\Gamma\to\to X[\Gamma]$ mapping each game $G\in\Gamma$ to its «outcome set» X(g) (the «outcome set» of games in Γ may consist of states, strategy profiles, or outcomes proper, according to the format of such games).

Let $\mathcal{E}$ be a solution concept for the class $\Gamma=\Gamma(N,\mathcal{L})$ of (protective) EF-induced abstract games $(\mathbf{P}(Y),R(f,r_N))$ where f is an EF for (N,Y) and r_N is a strictly ordered (preference) profile on Y. Then. the **$(\mathcal{E},\mathcal{O})$-capacity** of a simple game g=(N,W) is the nonnegative integer number $C(g;\mathcal{E},\mathcal{O})$ defined as follows:

$$C(g;\varepsilon,\sigma)=\begin{cases}\max\{\#Y / Y \text{ is a finite set and for any set Z s.t. } \#Z\leq\#Y \text{ a simple EF f for (N,Z) and an f-induced protective abstract game form } (\mathbf{P}(Z),R(f,.)) \text{ evist s.t. for any } r_N\in(\mathcal{O}(Z))^N:\ \emptyset\neq\cup\ \mathcal{E}(\mathbf{P}(Z),R(f,r_N))\subseteq PO(r_N), \text{ and } W(f)=W\}\\ \infty \text{ if such a maximum does not exist}\end{cases}$$

Remark 3.2. - The capacity of a simple game g is a measure of the maximum size-complexity of collective decision problems which can be solved in an efficient way by means of a decision procedure which allocate the (winning) decision power for its induced protective abstract games as prescribed by g, under the assumptions that: i) any admissible (say, weakly acyclic or strictly ordered) preference profile (and only admissible preference profiles) may obtain, and ii) $\mathcal{E}$ is the «right» theory of rational strategic behaviour for the problems under consideration.

The interest in capacity theory is mainly grounded on institutional design problems, whence the prominent role of Pareto efficiency ensues. The essential role of (Pareto) efficiency also explains why the somewhat cumbersone-albeit more general -notion of a (protective) abstract game form has been introduced. In fact, a simpler, and prima facie better, alternative would be analysing implementation by (full) **coalitional game forms**- i.e. triples (N,Y,V) with attached simple game (N,W(V)), where $V:\mathbf{P}(N)\rightarrow\mathbf{P}(Y)$ is s.t. $V(\emptyset)=\emptyset$ and for any $y\in Y$ an $S\subseteq N$ exists with $y\in V(S)$, and $W(V)=\{S\subseteq N \,/\, V(S)=Y\}$. The latter is indeed (essentially) Nakamura's approach in his own analysis of the capacity of simple games with respect to the core (see the observations below). The problem is, in this framework vNM-stable sets need not ensure even weak (Pareto) efficiency of outcomes, so that extending capacity analysis to SSBs in that setting is pointless.

The notion of capacity of a simple game was first introduced and studied by Peleg (see Peleg (1978, 1984)) for direct games in strategic form while taking exact coalitional (or strong) Nash equilibrium as the relevant solution concept. A fairly complex theory obtains: weak (monotonic, proper) simple games turn out to have an infinite capacity, and nonweak (monotonic, proper) simple games a bounded one, with different classes of nonweak (monotonic, proper) simple games having different upper capacity bounds (see Peleg (1984), Holzman (1986)). Essentially the same problem -with a different terminology- has been studied by Nakamura for (full) coalitional games, taking the core as the relevant solution concept (see Nakamura (1979)). Nakamura shows that the core-capacity of a (monotonic, proper) simple game g=(N,W) on the weakly acyclic domain is infinite g is weak, whereas it is $\nu(W)$-1 if g is nonweak (notice that the theorem holds true for the whole acyclic domain, i.e. the individual preference relations are not required to be quasi-complete).

It goes without saying that the details of Peleg's and Nakamura's definitions are different from the foregoing definition of the capacity of a simple game, due to the different formats of the games involved: the essentials are, however, the same.

The following results show that Nakamura's theorem cannot be essentially improved upon by taking stable standards of behaviour as the relevant solution concept (and shifting to protective abstract games).

Proposition 3.3. Let g=(N,W) be a monotonic proper simple game, $\Gamma=\Gamma(N,\mathcal{O})$ the class of EF-induced protective abstract games with EF-player set N and strictly ordered preferences, and $\mathcal{E}$ a selection of vNM-stable sets, as defined on Γ. Then $C(g;\mathcal{E},\mathcal{O})=\infty$ if g is weak, and $\nu(W)-1 \leq C(g;\mathcal{E},\sigma) \leq \nu(W)$ if g is non-weak. Moreover, $C(g;\mathcal{E},\mathcal{O})=\nu(W)-1$ whenever $\nu(W)$ is odd.

Proof. Let us denote by f(W) the simple and X-monotonic EF for (N,X) induced by W (through the obvious definition). Obviously, W(f(W))=W. Next, consider the protective abstract game $(\mathbf{P}(X),R(f(W),r_N))$, for any strict profile $r_N\in(\mathcal{O}(X))^N$. Now, the abstract game $(X,R'(f(W),r_N))$ (where (x,y) $R'\in$iff $x,y\in X$ and $(x,y)\in\cap_{i\in T}r_N(i)$ for some $T\in W$) is asymmetric, by properness of g. It follows that the main result in Ferejohn, McKelvey (1983) applies, to the effect that for any $r_N\in(\mathcal{O}(X))^N$ either $(X,R(f(W),r_N))$ is acyclic or #X is the length of the shortest cycle of $(X,R(f(w),r_N))$, and is even. Also, it is easily seen that $(X,R'(f(W),r_N)$ cycles only if $(\mathbf{P}(X),R(f(W),r_N))$ cycles as well (if $(x_i)_{i\leq k}$ is a cycle of the former game, a cycle of the latter game may be made up by ignoring any x_i s.t. $x_{i-1}=x_i$ and taking the

elements x_i of such a reduced cycle of length $h\leq k$ to form the sequence $(\{x_j\})_{j\leq h})$.

Now, suppose that at profile $r_N\in(\mathcal{O}(X))^N$ the abstract game $(\mathbf{P}(X),R(f(W),r_N))$ is acyclic. Then, by the well-known von Neumann-Morgenstern result mentioned above, such a game has a unique vNM-stable set. Moreover, if g is weak then acyclicity of $(\mathbf{P}(X),R(f(W),R_N))$ follows from weak acyclicity of $r_N(i)$, for any $i\in N$, in view of asymmetry of $R(f(W),r_N)$. Thus, Proposition 3.2 above implies that $C(g;\mathcal{E},\mathcal{O})=\infty$ for any weak g.

On the other hand, if g is nonweak then the Nakamura Theorem mentioned above entails that $(X,R'(f(W),r_N))$ is not acyclic at every profile $r_N\in(\mathcal{O}(X))^N$, unless $\#X<\nu(W)$. Hence, $(\mathbf{P}(X),R(f(W),r_N)$ also cycles at some profile $r_N\in(\mathcal{O}(X))^N$ iff $\#X\geq\nu(W)$. It follows that $C(g;\mathcal{E},\mathcal{O})\geq\nu(W)-1$. Now, suppose $C(g;\mathcal{E},\mathcal{O})\geq\nu(W)$ for some nonweak g=(N.W). Then, -by definition of $C(g;\mathcal{E},\mathcal{O})$ for any X s.t. $\nu(W)\leq\#X\leq C(g;\mathcal{E},\mathcal{O})$, $(\mathbf{P}(X),R(f(W),r_N))$ is cyclic at some $r_N\in(\mathcal{O}(X))^N$, and #X is even (and the length of the shortest cycle of $(X,R'(f(W),r_N))$. This is, obviously, absurd if $\nu(W)$ is odd, and entails $\nu(W)=C(g;\mathcal{E},\mathcal{O})$ if $\nu(W)$ is even. It follows that in case g is nonweak

$\nu(W)-1\leq C(g;\mathcal{E},\mathcal{O})\leq\nu(W)$, and $C(g;\mathcal{E},\mathcal{O})=\nu(W)-1$ if $\nu(W)$ is odd. ❑

Proposition 3.4.. Let $g=(N,W)$ be a monotonic proper simple game, Γ the class of EF-induced protective abstract games with players set N and strictly ordered preferences, and $\mathcal{E}$ a selection of sub-stable sets, as defined on Γ. Then $C(g;\mathcal{E},\mathcal{O})=\infty$ if g is weak, and $\nu(W)-1\leq C(g;\mathcal{E},\mathcal{O})\leq\nu(W)-1$, ($C(g;\mathcal{E},\mathcal{O})=\nu(W)-1$ if $\nu(W)$ is odd) if g is nonweak.

Proof. First, notice that at any profile $r_N\in(\mathcal{O}(X))^N$, $Core(\mathbf{P}(X),R(f(W),r_N))=f^c{}_R(\mathbf{P}(X))$. Therefore, for any sub-stable set $\mathcal{S}$ of $(\mathbf{P}(X),R(f(W),r_N))$, $Core(\mathbf{P}(X),R(f(W),r_N)\subseteq f^c{}_R(f^c{}_R(S))$, whence $Core(\mathbf{P}(X),R(f(W),r_N))\subseteq\mathcal{S}$, by Exclusion. It follows -by Corollary 3.1. above and the Nakamura Theorem (see Remark 3.2.)- that $C(g;\mathcal{E},\mathcal{O})\geq\nu(W)-1$, and $C(g;\mathcal{E},\mathcal{O})=\infty$ if g is weak.

Now, suppose $C(g;\mathcal{E},\mathcal{O})\geq\nu(W)$. Then for any Y s.t. $\#Y\leq\nu(W)$, and any profile $r_N\in(\mathcal{S}(Y))^N)$, $\cup\mathcal{E}((\mathbf{P}(Y),R(f(W),r_N)))\neq\emptyset$ i.e. a nonempty $\mathcal{S}\subseteq\mathbf{P}(X)$ exists which is also a sub-stable set of $(\mathbf{P}(Y),R(f(W),r_N))$ (where f(W) is a suitable simple EF for (N,Y)). Hence, $\mathcal{S}$ is a semi-stable set of $(\mathbf{P}(Y),R(f(W),r_N))$ (see the definition above, in Section2).

Now, take a set X s.t. $\#X=\nu(W)$. It follows that for any $Z\subseteq X$ and any simple EF f(W) for (N,Z) a nonempty semi-stable set $\mathcal{S}$ of $(\mathbf{P}(Z),R(f(W),r_N))$ exists. However, it can be easily shown that if (Y,R) is a finite abstract game s.t. all of its nonempty subgames have a

nonempty semi-stable set, then (Y,R) has a vNM-stable set: in fact, any maximal semi-stable set of (X,R) is also vNM-stable (see Berge (1979), p. 311, who credits this result to V. Neumann Lara). As a consequence, $(\mathbf{P}(X),R(f(W),r_N))$ must have a vNM-stable set. Therefore, by virtue of Proposition 3.2, $C(g;\mathcal{E},\mathcal{O})\leq C(g;\mathcal{E}',\mathcal{O})$, where $\mathcal{E}'$ is a suitable selection of vNM-stable sets as defined on Γ. Hence the thesis follows from Proposition 3.3. above. ❑

As mentioned in the Introduction, a major extension of the scope of vNM-like SSBs has been recently provided by Greenberg's «theory of social situations» (see Greenberg (1989b, 1990). A **social situation** is a pair $S=(\Gamma,\delta)$ where Γ is a nonempty set of objects called «positions», and δ is a correspondence to be defined below, the «inducement correspondence». A **position** $G\in\Gamma$ is an array $(N(G),X(G),(u_i(G))_{i\in N(G)})$, where N(G) is the player set, X(G) is the outcome set of G, and $u_i(G):X(G)\rightarrow\mathbf{R}$ is the real-valued utility function of player $i\in N(G)$ at G. Thus, a position amounts to a grand coalitional game with representable completely preordered preferences.

The **inducement correspondence** δ maps each triple (G,x,S) s.t. $G\in\Gamma, x\in X(G)$, $S\subseteq N(G)$ to a subset of Γ, to be interpreted as the set of positions that coalition S is able to induce (directly) from G whenever x is the «proposed» (or «status quo») outcome (as a matter of fact, Greenberg imposes a further condition upon δ, to the effect that the player set of any induced position must include the «inducing» coalition: this seems to us unduly restrictive and is therefore omitted).

A **standard of behaviour** for a social situation $S=(\Gamma,\delta)$ is a correspondance $\sigma:\Gamma\rightarrow\rightarrow X[\Gamma]$ mapping each position $G\in\Gamma$ to a set $\sigma(G)\subseteq x(g)$ of its outcomes (the solution-outcomes of position G).

VNM-like stability properties of standards of behaviour are defined by suitable parametric dominance relations as follows.

The **optimistic dominion** of a standard of behaviour σ for $S=(\Gamma,\delta)$ at position $G\in\Gamma$ is

$OD(\sigma,G)=\{x\in X(G)$ / $T\subseteq N(G)$, $H\in\delta(G,x,T)$ and $y\in\sigma(H)$ exist s.t. $T\subseteq N(H)$ and $u_i(H)(y)>u_i(G)(x)$ for any $i\in T\}$.

An **optimistically (vNM-)stable standard of behaviour (OSSB)** of a social situation $S=(\Gamma,\delta)$ is a standard of behaviour $\sigma:\Gamma\rightarrow\rightarrow X[\Gamma]$ for S s.t. for any $G\in\Gamma$ $\sigma(G)=X(G)\cap(OD(\sigma,G))^c$.

Similary, the **protective dominion** of a standard of behaviour σ for $S=(\Gamma,\delta)$ at position $G\in\Gamma$ is

$PD(\sigma,G)=\{x\in X(G)$ / $T\subseteq N(G)$, $H\in\delta(G,x,T)$ exist s.t. $T\subseteq N(H)$, $\sigma(H)\neq\emptyset$ and $u_i(H)(y)>u_i(G)(x)$ for any $i\in T$ and $y\in\sigma(H)\}$.

Thus, a **protectively (vNM-)stable standard of behaviour (PSSB)** og a social situation $S=(\Gamma,\delta)$ is a standard of behaviour $\sigma:\Gamma\rightarrow\rightarrow X[\Gamma]$ for S s.t. for any $G\in\Gamma$ $\sigma(G)=X(G)\cap(PD(\sigma,G))^c$.

As reported in Greenberg's book (see Greenberg (1990), Theorem 4.5), it has been shown by B. Shitovitz that a standard of behaviour σ for $S=(\Gamma,\delta)$ is an OSSB of S iff its graph

$$gr(\sigma)=\{(G,x)/G\in\Gamma \text{ and } x\in\sigma(G)\}$$

is a vNM-stable set of the abstract game $(X(S),R(S))$ where $X(S)=\{(G,x)/G\in\Gamma$ and $x\in X(G)\}$ and for any $G,H\in\Gamma$, $x\in G$, $y\in H$ $((H,y),(G,x))\in R(S)$ iff for some $T\subseteq N(G)\cap N(H)$ both $H\in\delta(G,x,T)$ and $u_i(H)(y)>u_i(G)(x)$ for any $i\in T$ hold true.

On the other hand, a standard of behaviour σ_A for S can be attached in a natural way to any $A\subseteq X(S)$ (namely, for any

$$G\in\Gamma \,/\, \sigma_A(G)=\{x\in X(G) \,/\, (G,x)\in A\}).$$

Hence, the foregoing result suggests a straightforward extension of social situation analysis to other plausible standards of behaviour, for example **optimistically sub-stable standards of behaviour (OsSSBs)** or **optimistically ergodic standards of behaviour (OESBs)** (see section 2).

Remark 3.3. Shitovitz's equivalence result does not transfer automatically to PSSBs. However, a partial analog of that result for PSSBs can be proven by using an auxiliary correspondence $F:X(S)\rightarrow\rightarrow X(S)$ (see again Greenberg (1990), Theorem 11.8.4., who credits this result to U. Rothblum). This result may be stated as follows: if $S=(\Gamma,\delta)$ is s.t. for any $G,H\in\Gamma$, $S,T\subseteq N(G),(S\neq T)$, $x\in X(G)$, $H\in\delta(G,x,S)$ implies $H\notin\delta(G,x,T)$, then a standard of behaviour σ for **S** is a PSSB of **S** iff its graph is a vNM-stable set of the abstract game $(X(S),R^*(S))$ where $X(S)$ is as defined above, and for any $G,H\in\Gamma$, $x\in G$, $y\in H$ $((H,y),(G,x))\in R^*(S)$ iff $(H,y)\in A$ for some $A\subseteq X(S)$, where $\mathfrak{R}(S)\subseteq \mathbf{P}(X(S))xX(S)$ is an auxiliary relation defined as follows: for any $A\subseteq X(S),(G,x)\in X(S)$ $(A,(G,x))\in\mathfrak{R}(S)$ iff a pair $(H',z)\in A$ exists s.t. $((H',w),(G,x))\in R(S)$ for any $w\in X(H')$.

It should come as no surprise, at this point, that if a social situations setting is introduced and some vNM-like stable standard of behaviour (either of the optimistic or of the protective variety) is used as the relevant solution concept, then the corresponding capacity of a monotonic proper simple game behaves (essentially) as with the vNM-like solution concepts analysed previously. This claim is made precise in the sequel.

Unfortunately, the exact statement of that claim requires some more definitions, mostly borrowed (with slight modifications) from Greenberg (1990), which are now to be recalled.

A **position-form** $\mathcal{G}$ is a grand coalitional game form, i.e. a pair $(N(\mathcal{G}),X(\mathcal{G}))$ consisting of a player set $N(\mathcal{G})$ and an outcome set $X(\mathcal{G})$.

A **situation-form** is a pair $S^*=(\Gamma^*,\delta^*)$, where Γ^* is a set of position-

forms and δ^* is a correspondence mapping each triple (G',x,T) s.t. $G'\in\Gamma^*,T\subseteq N(G')$ and $x\in X(G')$ to a subset of Γ^*.

Given an «universal» player set N, an **N-situation-form** is a situation-form $S^*=(\Gamma^*,\delta^*)$ satisfyng the following requirements: i) an «universal» set $X(N)$ exists s.t. $(N,X(N))\in\Gamma^*$ and for any $G'\in\Gamma^*$ both $N(G')\subseteq N$ and $X(G')\subseteq X(N)$; ii) for any $G'\in\Gamma^*$, $x\in X(G')$, $T\subseteq N(G')$ if $H'\in\delta^*(G',x,T)$ then $N(H')\subseteq N(G')$ and $X(H')\subseteq X(G')$.

Moreover, a **social decision procedure** for a position-form (N,A) is a triple $\Pi=(\Gamma^*,\delta^*,\pi)$, where (Γ^*,δ^*) is a situation-form for N and $\pi:X(N)\rightarrow A$ is an outcome-function.

Now, let $\mathcal{L}(A)$ be the set of all linear orderings (i.e. transitive, complete, antisymmetric binary relations) on X, and $(\mathcal{L}(A))^N$ the set of all N-profiles of linear orderings on A. Then, the social situation $S=S(S^*(N),\mathcal{L}(A))$ induced by a N-situation-form $S^*=((\Gamma^*,\delta^*)$ on $(\mathcal{L}(A))^N$ is defined as follows:

$S=(\Gamma,\delta)$, where $\Gamma=\{(N(G'),X(G'),((u_i(G'))_{i\in N(G')}))\ /\ G'\in\Gamma^*$, and for any $i\in N(G')$ $u_i(G'):X(G')\rightarrow\mathbf{R}$ is defined as $u_i(G')=u_i o\pi_{|X(G')}$ where u_i is an arbitrary 1-1 real-valued utility function on A$\}$

and for any $G=(N(G'),X(G'),\ ((u_i(G'))_{i\in N(G')}))\in\Gamma$, $x\in X(G')$ and $T\subseteq N(G'))$: $\delta(G,x,T)=\{(H',((u_i(H'))_{i\in N(H')}))/H'\in\delta^*(G',x,T)\}$.

A social choice correspondence (SCC) for (N,A) on $(\mathcal{L}(A))^N$ is a correspondence $F:(\mathcal{L}(A))^N \to\to A$ s.t.

i) for any $r_N \in (\mathcal{L}(x))^N$, $F(r_N) \neq \emptyset$, and ii) for any $x \in X$ a $r_N \in (\mathcal{L}(X))^N$ exists with $F(r_N)=\{x\}$. A simple game $g(F)=(N,W(F))$ can be attached to any such SCC by the following definition:

$W(F)=\{T \subseteq N$ / for any $x \in A$ and any $r_N \in (\mathcal{L}(A))^N$: if $(x,y) \in r_N(i)$ for any $i \in T$ and any $y \in A$ then $F(r_N)=\{x\}\ \}$.

A SCC F for (N,A) on $(\mathcal{L}(A))^N$ is **OSSB-implementable** iff a social decision procedure $\Pi=(\Gamma^*,\delta^*,\pi)$ for (N,A) exists s.t.: i) the (Γ^*,δ^*)-induced social situation on $(\mathcal{L}(A))^N$, namely $S=S((\Gamma^*,\delta^*),\mathcal{L}(A))$, has a unique OSSB σ; ii) for any $r_N \in (\mathcal{L}(A))^N$ $F(r_N)=\pi[\sigma(G_N)]$, where $G_N=(N,X(N),((v_i)_{i \in N}))$ and for any $i \in N$, $v_i=u_i o\pi$ for some real-valued utility function $u_i:A \to \mathbf{R}$ which represents the linear ordering $r_N(i)$. Similary, one may define **PSSB-implementable, OsSSB-implementable** and **PsSS-implementable SCCs** by requiring that S has a unique PSSB, OsSSB, or PsSSB, respectively.

The **(OSSB,$\mathcal{L}$)-capacity** of a (monotonic, proper) simple game g=(N,W), to be written C(g;OSSB,$\mathcal{L}$), can also be defined. Namely

C(g;OSSB,$\mathcal{L}$)= max {#Y / Y is a finite set and for any Z, if #Z≤#Y then a SCC F for (N,Z) on $(\mathcal{L}(Z))^N$ exists s.t. $F(r_N) \subseteq PO(r_N)$ for any $r_N \in (\mathcal{L}(Z))^N$ F is OSSB-implementable, and W(F)=W}

∞ if such a maximum does not exist

(**(PSSB,$\mathcal{L}$)-capacity,(OsSSB,$\mathcal{L}$)-capacity, (PsSSB,$\mathcal{L}$)-capacity** are defined in a similar way by substituting the PSSB-, OsSSB-, PsSSB-implementability requirement, respectively, for the OSSB-implementability requirement).

Remark 3.4. It can be proven (see Greenberg (1990), Theorem 10.1.2,) that a SCC F for (N,A) on $(\mathcal{L}(A))^N$ is OSSB-implementable - and PSSB-implementable as well- iff satisfies the well-known **Strong Positive Association (SPA)** property, namely:

(SPA) for any $a \in A$, and for any $r_N,q_N \in (\mathcal{L}(A))^N$, if $a \in F(r_N)$ and for any $i \in N$, $b \in A$ $(a,b) \in r_N(i)$ implies $(a,b) \in q_N(i)$ then $b \in F(q_N)$.

Moreover, it turns out that if the SCC F satisfies SPA then at any linear profile (unique) OSSB and the (unique) PSSB of the social situation induced by any social decision procedure which OSSB-implements F (or PSSB-implements F) do coincide (see again Greenberg (1990), Theorem 10.1.2).

We are now ready to state and prove the announced results on capacities of simple games with respect to vNM-like stable standards of behaviour, which are summarized in the following proposition.

Proposition 3.5. Let g=(N,W) be a monotonic proper simple game: Then,

i) $C(g;OSSB,\mathcal{L})=C(g;PSSB,\mathcal{L})=C(g;OSSB,\mathcal{L})=C(g;PSSB,\mathcal{L})=\infty$ if g is weak,

ii) $C(g;OSSB,\mathcal{L})=C(g;PSSB,\mathcal{L})=C(g;OSSB,\mathcal{L})=C(g;PSSB,\mathcal{L})=$ $=\nu(W)-1$ if g is non-weak.

Proof. Let us consider the Core(g,.)-correspondence for (N,X) on $(\mathcal{L}(X))^N$, namely, for any $r_N\in(\mathcal{L}(X))^N$

$$Core(g,r_N)=\{x\in X \,/\, \text{no } T\in W \text{ and } y\in X \text{ exist s.t. } (y,x)\in\cap_{i\in T}r_N(i)\}.$$

Obviously, $Core(g,r_N)\subseteq PO(r_N)$, for any $r_N\in(\mathcal{L}(X))^N$. Also, it is easily checked that Core(g,.) satisfies SPA, and for any (finite) set s.t. $\#X<\nu(W)$ and for any profile $r_N\in(\mathcal{L}(X)^N$, $Core(g,r_N)\neq\emptyset$, whence

Core(g,.) is a well defined SCC for (N,X) on $(\mathcal{L}(X))^N$). Furthermore, W(Core(g,.))=W. (In fact, Core(g,.) is a tight representation of g i.e. the «natural» simple games attached to it coincide: see Peleg (1984), Theorem 3.2.5. and Remark 3.2.6 for the relevant details). By virtue of the Greenberg result mentioned above (see Remark 3.4 above), this implies

$C(g;OSSB,\mathcal{L})=C(g;PSSB,\mathcal{L})=\infty$ if g is weak, and

$C(g;OSSB,\mathcal{L})=C(g;PSSB,\mathcal{L})\geq\nu(W)-1$ if g is non-weak.

We now show that for any non-weak g

$$(*)\qquad C(g;OSSB,\mathcal{L})=C(g;PSSB,\mathcal{L})\leq\nu(W)-1.$$

To begin with, let us define the Generalized Condorcet Winner of a proper simple game g=(N,W) at profile $r_N\in\mathcal{L}(X))^N$ -to be written $GCW(g,r_N)$- as follows:

$$GCW(g,r_N)=\{x\in X \,/\, \text{for any } y\in X\setminus\{x\} \text{ an } S\in W \text{ exists s.t. } (x,y)\in r_N(i) \text{ for any } i\in S\}.$$

Obviously, for any $r_N\in(\mathcal{L}(X))^N$, $\#GCW(g,r_N)\leq 1$, (and $GCW(g,r_N)=Core(g,r_N)$ whenever $GCW(g,r_N)\neq\emptyset$).

Next, let us consider a SCC F for (N,X) on $(\mathcal{L}(X))^N$ such that W(F)=W (where g=(N,W) is any nonweak monotonic proper simple game, and $\#X\geq\nu(W)$). We want to show that F cannot possibly satisfy SPA: in view of the Greenberg result mentioned above this suffices to establish the inequality (*).

First, notice that W(F)=W entails $F(r_N)=GCW(g,r_N)$ at any $r_N \in (\mathcal{L}(X)^N$ s.t. $GCW(g,r_N) \neq \emptyset$.

On the other hand, $GCW(r_N)=\emptyset$ at the canonycal «cyclic» profile r_N defined as follows. Take the finite sets $X'=\{x_1,...,x_{\nu(W)}\} \subseteq X$, and $\{T_1,...,T_{\nu(W)}\} \subseteq W$ s.t. $\cap W'=\emptyset$. Then endow the index set $H=\{1,...,\nu(W)\}$ with the structure of the (additive) cyclic group of order $\nu(W)$, and, for any $i \in N$ define $r_N(i)$ by the following rules:

$(x_{j+k},x_{j+k+1}) \in r_N(i)$, $k=1,...,\nu(W)-1$, where $j=j(i) \in H$ is s.t. $i \notin T(j)$, and $X'x(X \setminus X') \subseteq r_N(i)$. Let us suppose w.l.o.g. that $x_{j+k} \in F(r_N)$, where $j=j(i')$.

Then define a new profile r'_N as follows:

$(x_{j+k-1},x) \in r'_N(i)$ for any $x \in X$ and $i \in N$ s.t. $r_N(i) \neq r_N(i')$,

$(x_{j+k-1},x) \in r'_N(i)$ for any $x \in X \setminus \{x_{j+k}\}$ and $i \in N$ s.t. $r_N(i)=r_N(i')$, and

$(x,y) \in r_N(i)$ if and only if $(x,y) \in r'_N(i)$ for any $i \in N$ and for any $x,y \in X$ s.t. $x_{j+k-1} \notin \{x,y\}$.

Notice that $j(i)=j(i')$ entails both $r_N(i)=r_N(i')$, and $r'_N(i)=r'_N(i')$.

Since $(x_{j+k},x_{j+k-1}) \in r_N(i)$ only if $r_N(i)=r_N(i')$, r'_N preserves the position of x_{j+k} with respect to r_N : it follows that if F satisfies SPA then $x_{j+k} \in F(r'_N)$. However, it can be easily checked that

$GCW(r'_N)=\{x_{j+k-1}\}$, a contradiction. This is so, because

$(x_{j+k},x_{j+k-1}) \in r_N(i)$ if and only if $r_N(i)=r_N(i')$ (i.e. $j(i)=j(i')$), and $i' \notin T(j(i'))$ entail that for any $i \in T(j(i'))$ $r_N(i)=r_N(i')$, whence

$(x_{j+k-1},x_{j+k}) \in r'_N(i)$. Moreover, for any $y=x_{j+k}$, $(x_{j+k-1},y) \in r_N(i)$ for any

$i \in N$.

The "sub-stable" part of the proposition results from the following observations.

Let us consider the «universal» procedure employed by Greenberg (see Greenberg (1990), chpt. 10) to show that any SCC F for (N,X) which satisfies SPA is OSSB-and -PSSB-implementable (on a linearly ordered domain), namely

$$\Pi=(\Gamma^*,\delta^*\pi),$$

where $\Gamma^*=\{g(N)\} \cup \{g(i,x) / i \in N, x \in \mathcal{X}\}$,

$\mathcal{X}=\{(r_N,x) / r_N \in (\mathcal{L}(X))^X, x \in F(r_N)$,

$g(N)=(N(g(N)),\mathcal{X}(g(N)))=(N,\mathcal{X})$,

$g(i,x)=(N(g(i,x)),\mathcal{X}(g(i,x)))=(\{i\},\{x\})$,

$\delta^*(g(N),(r,x),\{i\})=\{g(i,x) \ / \ \mathcal{X}=(r'_N,y)$ s.t. $(x,y) \in r_N(i)\}$, whereas $\delta^*(g,x,T)=\emptyset$ otherwise, and

$\pi:\mathcal{X} \rightarrow X$ is defined by the rule $\pi((r_N,x))=x$, for any $(r_N,x) \in \mathcal{X}$.

The 1-digraph attached to (Γ^*,δ^*), namely $(\mathcal{G}^*,\mathcal{A}^*)$, with set of arcs $\mathcal{A}^*=\{(g,g') / g,g' \in \Gamma^*$ and $g \in \delta^*(g',x,T)$ for some $x \in \mathcal{X}(g')$ and $T \subseteq N(g')\}$ is obviously strictly acyclic.

Then, for any $r_N \in (\mathcal{L}(X))_N$, take the situation $S=S(((\Gamma^*,\delta^*,\pi),r_N)$ induced by (Γ^*,δ^*,π) at r_N, and the abstract game (1-digraph) (X(S),R(S)) induced by S, i.e.

$(\{(g,x) / g \in \Gamma^* \ x \in \Re(g)\}, R(S))$, where $((g,x),(g',x')) \in R(S)$ if and only if a $T \subseteq N(g')$ exists such that $g \in \delta^*(g',x,T)$ and $(\pi(x),\pi(x')) \in (r_N)$ for any $i \in T$.

Therefore $R(S)=\mathcal{A}^* \cap \mathfrak{R}(r_N)$ where $((g,x),(g',x')) \in \mathfrak{R}(r_N)$ iff a $T \subseteq N(g')$ exists such that $(\pi(x),\pi(x')) \in r_N(i)$ for any $i \in T$. Hence R(S) is strictly acyclic by virtue of strict acyclicity of $\mathcal{A}^*$. As a result, Proposition 3.1. applies, to the effect that the abstract game (X(S),R(S)) has a unique sub-stable set, which is bound to coincide with its unique vNM-stable set, and is therefore-by the Shitovitz result mentioned above (see Remark 3.3.)- the graph of the (unique) OSSB (and of the unique PSSB) of S. Thus, the identities of Proposition 3.5. concerning sub-stable standards of behaviour follow from their respective counterparts concerning vNM-stable standards of behaviour, whose validity has just been shown. ❑

Remark 3.5.. Proposition 3.5. shows that substituting sub-stable standards of behaviour for Greenberg's vNM-stable standards does not affect implementability by social situations as far as the capacity theory of simple games is concerned. The proof also suggests similar results for other variants of stable standards of behaviour: the details, however, will not be pursued here.

4. SOCIAL SITUATIONS AND AUTOMATA THEORY

A by-product of the result presented in the previous sections is the previous sections is the suggestion that the theory of social situations can be generalized in a natural way by allowing for variants of vNM-stable standards of behaviour. This section is devoted to the task of pointing out some connections between social situations and automata, to the effect of suggesting another natural extension of the former notion.

To begin with let us introduce two basic notions from automata theory.

A **relational Medvedev-automaton** is an array (S,I,d), where S is a set (the set of states), I is set (the set of inputs), and $d \subseteq S x I x S$ is a partial correspondence (the nondeterminastic dynamics mapping some state-input combinations to set of of states).

A **relational Moore-automaton** is an array (S,I,d,O,h) where (S,I,d) is a relational Medvedev-automaton, O is a set (the output set), and $h \subseteq S \times O$ is a partial correspondence (the output correspondence).

The following fact is easily checked:

Fact i) Any social situation $S=(\Gamma,\delta)$ is (up to a natural bijection) a relational Medvedev-automaton. ii) Let σ be a standard of behaviour for the social situation $S=(\Gamma,\delta)$. Then (Γ,δ,σ) is (up to a natural bijection) a relational Moore-automaton.

Proof. i) Put $S=\Gamma$, $I=X[\Gamma]x\mathbf{P}(N)$, $d=\delta$

ii) Define S, I, **d** as under i), and put $O=X[\Gamma]$, $h=\sigma$. ❑

Since a well-developed theory of Medvedev- and Moore- automata is at hand (see e.g. Ehrig, Kiermeier, Kreowski, Kühnel (1974)) the foregoing observation might prove to be useful for further developments and extensions of the theory of social situations.

In particular, if stochastic Moore-automata (and, accordingly, a stochastic variant of social situations) are introduced, then (standard) stochastic games are easily- and most naturally- represented as stochastic social situations.

Namely, let us define a **stochastic social situation** as a pair $S'=(\Gamma',\delta')$ where Γ' is a set of games (say, normal form games $G_j=(N,(S_{ji})_{i\in N},(u_{ji})_{i\in N})$, $j\in J$) and δ' a function mapping each pair consisting of a game $G_j\in\Gamma'$ and a strategy profile $s\in x_{i\in N}S_{ji}$ to a probability distribution on Γ'. Now, this also amounts to the (standard) definition of a stochastic game (usually, Γ' is assumed to be finite: see e.g. Owen (1982)). This observation suggests a further analysis of stochastic games through stable standards of behaviour (this analysis will be the topic of a subsequent paper).

More generally, the foregoing facts suggest that the «right» generalization of a social situation in Greenberg's sense might be the notion of a Moore-automaton with a set of states consisting of games.

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