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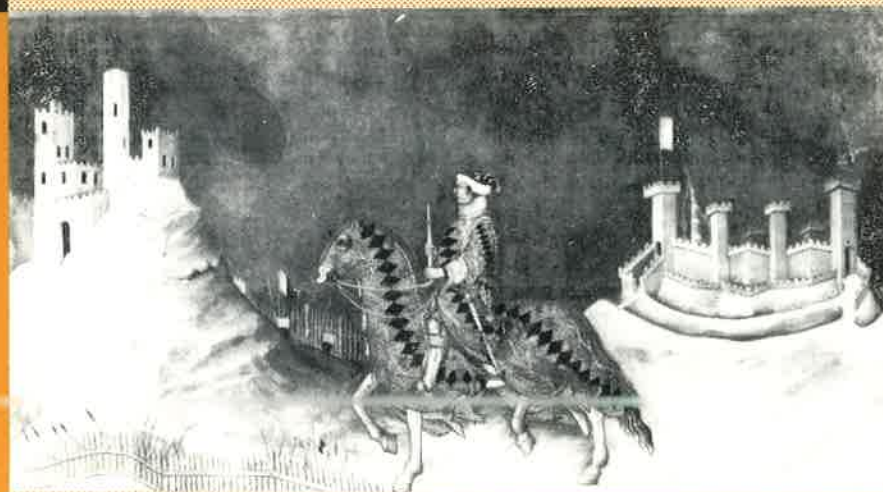
QUADERNI DEL DIPARTIMENTO DI ECONOMIA POLITICA

Nicola Dimitri

ASYMPOTIC CONSENSUS WITH PARTITIONS



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ASYMPTOTIC CONSENSUS WITH PARTITIONS[*]

1 Introduction

The understanding of how people come to have the same opinion,or why they disagree about certain issues that are part of their economic lives represents a most important aspect in economic theory.In fact,for example, it is well known that differences of opinions can explain speculative trades.

The body of the present state of art on this subject can certainly be found in the research contributions which formalize and discuss the concept of *common knowledge* . Since the seminal paper by Aumann (1976) there have been many efforts studying the problem of how *consensus* , about for example the probability that a certain event would occur, could be explained.

In particular, Aumann himself proved that if in a finite state space agents are endowed with an information partition structure, and with the same prior probability density over the possible states of the world, they cannot *agree to disagree* when their posteriors are common knowledge. So, the outcome of his work is indeed *consensus* about the updated probabilities in spite of the differences on individual information partitions[1]. This path

[*] I should like to acknowledge the very useful comments I had on an earlier version of this paper by Frank Hahn , Giancarlo Marini, Piermario Pacini, Stefano Vannucci and Carlo Zappia . The usual disclaimers apply.

[1] In other words, we have here that consensus emerges out of some appropriate assumptions ,among which that of common knowledge of

breaking paper gave origin to a line of research where the result was both generalized to different information structures [2] and applied to theoretical economic contexts [3].

It should be noticed that these models, where consensus is the main object of study and where it is shown under what conditions could be achieved, are somehow *static* models .In fact , the construction concerns a single experiment context after which various mechanisms may enter the picture in order to obtain consensus. For example, in Geanakoplos-Polemarchakis (1982) a state of the world realizes and then two agents repeatedly exchange[4] their posterior probabilities .Eventually they reveal to each other what sets of their information partitions have been observed ; in doing so they end up sharing the same probabilistic evaluation about the relevant event.

A similar type of mechanism is assumed in McKelvey-Page (1986) and Nielsen-Brandenburger-Geanakoplos-McKelvey-Page (1990) where rather than exchanging posteriors people observe an aggregate statistic of the posteriors, which is revised sequentially in such a way that eventually consensus emerges.

Throughout this paper I shall assume information partitions and a finite state space; I shall take however a different perspective

posteriors plays a key role. In this setting differences on how information is imparted have no importance and people agree about the values of their posteriors even though originally they observe different subsets of the state space.

[2] See,among others Geanakoplos-Polemarchakis (1982), Cave (1983), Nielsen (1984), Bacharach (1985), Brandenburger-Dekel (1987), Shin (1987),Bergin (1989), Rubinstein-Wolinsky (1990), Samet (1990).

[3] See,for instance Milgrom-Stokey (1982), McKelvey-Page (1986), Monderer-Samet (1989), Rubinstein (1989), Binmore-Brandenburger (1990).

[4] Without cheating on the value of the posteriors that they communicate.At each round these probabilities are revised on the basis of the information received from the other agent.

from that of the above models by considering a *learning process*[5] which unfolds over time. The work will explore whether or not two individuals agree on the probability of a specified event when there is no communication [6], namely when they are not imparted information from the other agent.Furthermore,the paper will study the case where agents in order to form a *reliable* probabilistic measure about a certain event of interest, observe and elaborate the outcomes of a sequence of experiments.

Many of the models in the literature start with the hypothesis that information partitions are common knowledge among agents but this assumption is certainly very demanding. One of the main goals of this paper is in fact to obtain conditions for consensus even when the other individual's partition is not supposed to be known.

The learning processes I shall consider are very simple because the idea is just to highlight a few examples in which repeated experiments can lead to consensus, without assuming partitions to be common knowledge and when agents *do not communicate*. It will be shown that sequential experiments may in fact substitute the lack of additional information that individuals might receive from the other agent.

In the following section I shall set up the necessary notation together with a very simple model which is presented in order to have a benchmark case to refer to. Section three discusses at lenght an adaptive learning process proposed with the aim of considering a feedback between nature and individuals. In the

[5] In the following section I shall define in detail the *rules* (*schemes*) which are meant to represent learning processes.

[6] There may be various reasons to justify such an assumption: they can't, they do not want to to avoid revealing each other what they know etc.

fourth part we analyze the role played by subjective and objective probabilities. In the fifth section we relax the assumption of independent experiments and we show a convergence result in a Markovian system. Finally, in section six we conclude by discussing some of the issues raised in the paper.

2 The Basic Model

Before discussing the model of this section I shall write down some motivation for the analysis that will follow. Consider for example two individuals who decide to meet at a particular time in the future ,where they agree on designing a contract which will be based upon the future occurrence of a specified random event.The agents will then be interested in *learning* something about the probability of that event before the meeting would take place; what I imagine they will do is to observe a sequence of posterior probabilities that will be elaborated according to the *learning scheme* adopted.

When the time of the meeting will come they will use the outcome of their learning processes as the probability for the relevant event. In the paper I assume ,as an approximation, that agents agree on designing the contract after a *sufficiently long* period ;under this assumption the analysis will make use of some standard results from the asymptotic theory of probability. A rationale for such an hypothesis could be given by supposing that agents prefer to wait long enough before meeting, in order to

obtain a more reliable probabilistic measure of the relevant event.

It is important to notice that the issue I am interested in is that of having some understanding on how probabilistic opinions may or may not turn out to be the same when the learning processes are *only* based upon individual observations of the experiments, and *not* on information exchanged between agents. Moreover, the paper does not address the issue of what happens when agents are designing the contract; namely, I am not dealing with the problem concerning the use of the learnt probability.

Let us start now the analysis with some notation and definitions.

Assume W to be the finite state space, W its cardinality, w a generic state and $P(W)$ the associated power set. Furthermore, suppose there are 2 individuals, each endowed with an information partition of the state space given by Γ_i where $i=1,2$[7]. Moreover assume that $P_t(w)>0$ for every $w\in W$,where $P_t(A)$ is the common prior probability[8], at time t ,with $t=0,1,\ldots$,for any non empty set $A\in P(W)$[9].

[7] The implications of assuming an information partition structure are discussed at lenght in Binmore-Brandenburger (1990).
[8] Common between agents. This is basically the celebrated Harsanyi's doctrine ; such an assumption is made in order to compare our results with those of the literature where this hypothesis plays an important role (see e.g. Aumann (1976)).Notice that later on this assumption will be partially relaxed.
[9] I am assuming here identity between subjective and objective probabilities. However,as we mentioned in the introduction, some problems concerning this issue will be discussed at lenght in section three.

When at time t state w realizes let $P_t(A/\Gamma_{i(t)}(w))$ be the posterior probability which agent i observes ,given her information partition[10].

To begin with let us give the definition of *learning rule* (*scheme*) by which a learning process is characterized.

***Definition 1** A learning rule (scheme) L is any statistic calculated on the basis of the outcomes observed* .

The above statement is certainly rather general but ,at the same time, fairly precise;recall that the only possible outcomes are represented by the observed sets of the individuals' partitions. Having defined what a learning rule is I can now state the following.

***Definition 2** We say that asymptotic consensus is achieved if the learning rules used by the two agents both converge with probability one (almost surely or strongly) to the same constant*

It is very important here to stress that even though agents are supposed to know their own partitions I *do not* assume that individuals know each others' partitions; what is more, I do not even need to work with agents knowing that other individuals have partitions since I shall only analyze the long run behaviour of the learning rule for each agent separately .

[10] The notation $\Gamma_{i(t)}(w)$ indicates the set of agent i's partition at time t which contains the state of the world that realizes at time t,for example w. On the basis of that observation she updates her prior probability through Bayes' formula . However, since the posterior is a random variable (finitely discrete, being W finite) from now on we choose to drop w to indicate the realization of one of the sets which form the information partition .Of course the randomness of the posterior is induced by that of $\Gamma_{i(t)}$ which in turn is generated by that of w .Needless to say that the posterior at time t is calculated from the prior at time t.

The characteristics of the stochastic process defining the system are not supposed to be known by agents ; indeed, if they were known individuals could understand immediately how the learning scheme adopted will eventually behave and they would not need to elaborate the outcomes obtained from repeated experiments.

The above hypotheses play a key role and represent a crucial difference with ,for example , Aumann's model[11] where partitions are assumed to be common knowledge to start with. To appreciate the importance of the previous statement it should suffice to recall how controversial this hypothesis has been in the literature[12].

Let us start now with an extremely simple learning process ; however, before defining the scheme we shall need to state some further assumptions.

Suppose the stochastic process is made of independent experiments, that agents have the same partition over time[13] and that priors are common and such that $P_t(A)=P_0(A)$ for every t[14]. This means that the posteriors are independent and identically distributed (i.i.d.) finitely discrete random variables, with

[11] See Aumann (1976). Remind that according to Definition 2 asymptotic consensus will be obtained when both learning rules converge to the same constant; it is important to notice that in principle the schemes could be different but ,in any case, with the same time limit. This of course may be seen as a limitation of this approach with respect to the *static* approach.

[12] See for instance Brandenburger-Dekel (1987).

[13] More precisely, partitions vary in general between agents but each agent has the same partition for every t.

[14] In other words,we are assuming that priors are independent of time and equal to the prior at time zero.Of course the posterior of an agent, for example the ith, will be of the type $P_0(A/\Gamma_{i(t)})$, which indicates that it is calculated by mean of the prior at time 0. From now on,to keep the notation as simple as possible, we shall write $P(A)$ instead of $P_0(A)$ and $P(A/B)$ instead of $P_0(A/B)$ for A and B subsets of W.

cardinality of the domain depending upon the number of sets individuals' partitions.

Suppose both agents take, as learning rule for the probability of an event $A\epsilon P(W)$, the time average of the observed posteriors which at time T we indicate with $L_T(A)$. The scheme for agent i, calculated at time T, would then be defined by

$$L_T(A)= \Sigma_t\ P(A/\Gamma_{i(t)})/T \qquad t=0,1,..,T \quad ; \quad T>0$$

Having stated the above assumptions we have now the following result.

Proposition 1 *As T tends to infinity $L_T(A)$ tends to $P(A)$ with probability one.*

Proof Let's first recall that the expected value of the posteriors $P(A/\Gamma_{i(t)})$ is given by $P(A)$ for *any* t and *any* partition[15]. In fact

$$E(P(A/\Gamma_{i(t)}))=\Sigma_k\ P(A/\Gamma_{ik})P(\Gamma_{ik})=\Sigma_k\ P(A\cap\Gamma_{ik})=P(A) \qquad (1)$$

where $k=1,..,I$ and I is the number of sets Γ_{ik} forming the partition Γ_i.

Since the posteriors are i.i.d. random variables ,from a version of the strong law of large numbers[16] we immediately obtain convergence with probability one of $L_T(A)$ to $P(A)$.■

In other words ; given the particular assumptions of this model, agents' probabilities will almost surely converge to the prior $P(A)$ which is assumed to be the same for all individuals. Asymptotic consensus emerges independently of how information gets imparted to agents, namely independently of information partitions!

[15] This holds evidently for both individuals.
[16] See ,for example,Laha-Rohatgi (1979)

In this model the lack of communication between agents does not prevent them from reaching consensus.

At this stage four points need to be underlined:

- first of all it is very important to stress the role played by the independency of experiments, although convergence can be shown to occur even with a certain degree of correlation[17];
- the result is crucially based upon the expected value of the posterior probability being independent of the partition considered; in fact, the same property does not in general hold for moments of higher order;
- the hypothesis of random variables identically distributed over time (for the same agent) does not necessarily play a crucial part as long as we retain independency. We shall discuss this point below;
- the model is particular in various senses; both agents adopt the same learning rule which does not change as time passes, the state space remains unaltered as time unfolds, priors are common and never revised. However,later on in this section and in the following ones we shall be able to see that asymptotic consensus can also be obtained when some of these assumptions are weakened. In particular, section four will deal with an *adaptive learning* process which produces a continuum of convergent learning schemes.

One should also note that even though $P(A)$ seems to represent the natural candidate for almost sure convergence it is not however the only possible 'limit point' of the learning processes ;as a matter of fact in section five I show that after having

[17] See section five.

dropped the independency assumption it can still be possible to obtain strong convergence[18] but not necessarily to $P(A)$.

Let us now start relaxing some of the hypotheses made in order to test the robustness of the above result .

To begin with suppose that the experiments are independent but that for at least one individual the information partition may change[19] with time ;in other terms, assume that the posteriors are no longer identically distributed. If everything else remains the same we have now the following result

Corollary 1 If agent i 's information partition changes with time $L_T(A)$ *converges to* $P(A)$ *with probability one.*

Proof Since W is finite it is immediate to check that the variance of the posterior will always be bounded by a positive real, say C[20], independent of the partition considered.

Given that the expected value is invariant with respect to the partition chosen a version of the strong law of large numbers[21] guarantees that in order to have almost sure convergence of $L_T(A)$ to $P(A)$ it is sufficient to check the convergence of the following deterministic series

$$\Sigma_t\ Var(P(A|\Gamma_{i(t)}))/t^2 \qquad t=1,2,... \qquad (2)$$

[18] As it was mentioned earlier 'strong' convergence stands for 'with probability one' or 'almost sure' convergence.

[19] I am not interested here in how and why they change.

[20] In particular the variance can never exceed $C=P(A)(1-P(A))$ which does not depend upon the partition chosen. In fact $Var\ (P(A|\Gamma_{i(t)}))=EP(A|\Gamma_{i(t)})^2-P(A)^2\leq EP(A|\Gamma_{i(t)})-P(A)^2=P(A)(1-P(A))$

[21] For independent but not identically distributed random variables.

But from what we have previously said each term of the above series is bounded above by C/t^2 and since variances are positive $L_T(A)$ converges with probability one to $P(A)$[22].∎

Once again it is important to point out the role played by the invariance property (with respect to the partition selected) characterizing the expected value of the posterior ; it does not matter whether or not the information structure changes with time because that expected value remains unaltered. What is more , partitions would not matter even if they were to change randomly !

Before going on to further generalizations it is crucial to stop now for a moment and dwell on an important aspect that we have not yet discussed in detail .

3 Subjective and Objective Probabilities

So far we have not distinguished between *subjective* and *objective* probabilities ; however, the separation of the two cases can indeed be matter of substance as far as our models are concerened.

[22] Notice that this version of the strong law of large numbers says that the time average of the posteriors converges strongly to the limit (if any) of the time average of the expected values. But since for any t, and any partition, the expected value of the posterior random variable is $P(A)$, the time average of the expected values is trivially constant and equal to the prior $P(A)$.

Assume in fact now that at time t the objective prior probability for the event A would be given by $\pi_t(A)>0$[23] ,where π and P are in general different. In this case *partitions could matter*; in fact the expected value of the posterior calculated by mean of the objective probabilities,for the *ith* agent and any t, will be given by

$$E(P(A|\Gamma_{i(t)}))=\Sigma_k P(A|\Gamma_{ik})\pi(\Gamma_{ik}) \qquad (3)$$

with $k=1,\ldots,I$, which in general is different among individuals depending upon how information is imparted. Furthermore, (3) will also be different from $P(A)$[24] . As a result in case of i.i.d. experiments ,and when both agents use for example $L_T(A)$, asymptotic convergence in general would occur about different constants since (1) in this case may no longer apply [25].

So, if all individuals share the same probabilities, but they are all wrong[26], consensus in general *will not* arise .

The above conclusion may seem at first counter intuitive but the result should not be surprising for the models discussed in the previous section . If for some reason agents share the same mistaken probabilities,but they do not revise them on the basis of what they observe, the *mistake* would not be corrected . To possibly solve this problem one should explore the implications of considering a feedback between the system and the subjective

[23] I assume that the objective probability is strictly greater than zero for all subsets of W. Moreover, suppose that $\pi_t(A)=\pi_0(A)=\pi(A)$

[24] Recall that the stochastic process is governed by the objective prior probabilities while the subjective priors determine the values of the posterior.

[25] Unless by chance the expected values of all agents would be equal.

[26] Later on I shall discuss this point in greater detail .

probabilities; next section will discuss this point within a specific context.

4 An Adaptive Learning Rule

In order to understand something about the implications of a feedback between the system and agents' probabilities consider now the following model[27].

Suppose that at time t ,after the state of the world realizes,agent i (and analogously the other) calculates the following convex combination of her subjective prior and posterior

$$P^i_t(A)=\alpha_i P^i_{t-1}(A)+(1-\alpha_i)P_{t-1}(A|\Gamma_{i(t-1)}) \qquad 0\leq\alpha_i\leq 1 \qquad (4)$$

where $t=1,2..$, to have a probabilistic evaluation for event A[28], where $P^i_0(A)=P(A)$. The idea behind the above expression is that agent i ,after having observed her own posterior, may not necessarily change her mind completely on the basis of this observation combining prior and posterior in a somewhat *cautionary* manner. Moreover, suppose that priors are revised for each t according to the following principle

$$P^i_t(A)=P^i_t(A)$$

namely such that the convex combination would also represent the updated prior probability. Such a simple scheme can account for a feedback between system and subjective probabilities since *nature*

[27] In this section I assume that agents' partitions do not change with time.

[28] I shall use the notation $P^i_t(A)$ to indicate agent i's subjective prior at time t.It is necessary to introduce the superscript for the generic agent i since her prior depends on α_i.

determines the state of the world which in turn changes subjective priors etc.

We also assume that objective priors $\pi_t(A)$ are independent of time and equal to $P(A)$; all this amounts to say that agents' subjective priors are right at time zero but that afterwards they depart from the *correct* probability according to the rule described by (4).

Furthermore ,suppose experiments are independent and that the learning rule adopted by agent i (and by the other) is given by

$$L^i{}_T(A)=\Sigma_t\ P^i{}_t(A)/T=\Sigma_t\ P^i{}_t(A)/T \qquad t=0,1..,T \quad ,T>0 \tag{5}$$

We can now state the following preliminary lemma

Lemma 1 *If $0<\alpha_i\leq 1$ we have $P_t(A/\Gamma_{i(t)})=P(A/\Gamma_{i(t)})$ while for $\alpha_i=0$ we have $P_t(A/\Gamma_{i(t)})=P(A/\Gamma_{i(0)})$ when $\Gamma_{i(t)}=\Gamma_{i(0)}$ and $P_t(A/\Gamma_{i(t)})=P(A)$ when $\Gamma_{i(t)}$ is different from $\Gamma_{i(0)}$ for all t and i*

Proof The first part is immediate; in fact when $0<\alpha_i\leq 1$ we have that

$$P_t(A/\Gamma_{i(t)})=P^i{}_t(A\cap\Gamma_{i(t)})/P^i{}_t(\Gamma_{i(t)})$$

which, from (4) is equal to

$$\frac{\alpha_i P^i{}_{t-1}(A\cap\Gamma_{i(t)})+(1-\alpha_i)P_{t-1}(A\cap\Gamma_{i(t)}/\Gamma_{i(t-1)})}{\alpha_i P^i{}_{t-1}(\Gamma_{i(t)})+(1-\alpha_i)P_{t-1}(\Gamma_{i(t)}/\Gamma_{i(t-1)})} \tag{6}$$

But it is easy to see that (6) is equal to $P_{t-1}(A/\Gamma_{i(t-1)})$ for $\Gamma_{i(t-1)}=\Gamma_{i(t)}$ and to $P_{t-1}(A/\Gamma_{i(t)})$ when $\Gamma_{i(t-1)}$ is different from $\Gamma_{i(t)}$. Since this holds for any t and i the first part is proved.

Secondly , when $\alpha_i=0$ we have that

$P^i{}_t(A)=P_{t-1}(A/\Gamma_{i(t-1)})==P(A/\Gamma_{i(0)})$ for $\Gamma_{i(t-1)}=\Gamma_{i(0)}$ and $P^i{}_t(A)=P(A)$ otherwise, with $t=1,2...$

In fact ,the posterior at time t is given by

$$P_t(A/\Gamma_{i(t)})=P^i{}_t(A\cap\Gamma_{i(t)})/P^i{}_t(\Gamma_{i(t)})$$

which in turn is equal to

$$P_{t-1}(A\cap\Gamma_{i(t)}/\Gamma_{i(t-1)})/P_{t-1}(\Gamma_{i(t)}/\Gamma_{i(t-1)})$$

But from the above it is trivial to deduce that

$$P_t(A/\Gamma_{i(t)})=P_{t-1}(A/\Gamma_{i(t)}\cap\Gamma_{i(t-1)}) \tag{7}$$

Expression (7) leads us immediately to the conclusion of the second part. It is however necessary to point out that we are here assuming *regular probabilities*[11]■

The consequences of the first part of Lemma 1 are very important. When $0<\alpha_i\leq 1$,and according to (4) subjective priors change at each t, the posteriors take values as if objective and subjective priors would coincide ! Since the expected value of $P^i{}_t(A)$ calculated by mean of the objective probabilities is given by $P(A)$[12] for each t and i,namely $P^i{}_t(A)$ and $L^i{}_T(A)$ are unbiased estimates of $P(A)$, it would seem reasonable to expect that asymptotically $L^i{}_T(A)$ could converge strongly to the prior at time zero .

Next result confirms the intuition establishing such convergence.

Proposition 2 *Suppose $0<\alpha_i\leq 1$; then as T tends to infinity $L^i{}_T(A)$ tends to $P(A)$ with probability one.*

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[11] See for instance Brandenburger-Dekel (1987). More specifically, suppose we are interested in a certain conditional probability $P(A/B)$,where $P(B)=0$,then regularity entails $P(A/B)=P(A)$; in other words,the information brought about by the probability zero event is ignored.

[12] In fact $EP^i{}_t(A)=\alpha_i EP^i{}_{t-1}(A)+(1-\alpha_i)EP(A/\Gamma_{i(t)})=$
$=\alpha_i EP^i{}_{t-1}(A)+(1-\alpha_i)P(A)$ *so that*
$EP^i{}_t(A)=(\alpha_i)^t P(A)+P(A)(1+\alpha_i+..+\alpha_i{}^{(t-1)})(1-\alpha_i)=P(A)$
since $EP^i{}_0(A)=P(A)$.Notice that such result holds for $\alpha_i\in[0,1]$.As a matter of fact when $\alpha_i=0$ we have $EP^i{}_t(A)=P(A)$ for $t=0$ and $EP^i{}_t(A)=P(A/\Gamma_{(0)})$ for each $t>0$, since in the latter case it is $P^i{}_t(A)=P(A/\Gamma_{(0)})$ with probability one.

Proof If $\alpha_i=1$ the proposition is proved immediately since $P^i_t(A)=P(A)$ for all t.

Consider now the case when α_i belongs to the open interval $(0,1)$.

Adding up term by term ,for $t=1,\dots,T$, both sides of (7) and dividing by T we obtain the following equality

$$L^i_T(A)-P(A)/T=[\alpha_i L^i_{T-1}(A)+(1-\alpha_i)L_{T-1}(A)](T-1)/T \qquad (8)$$

Suppose first that $L^i_T(A)$ asymptotically tends to a constant, say $L^*(A)$, with probability one. Then eventually the above expression becomes

$$L^*(A)=\alpha_i L^*(A)+(1-\alpha_i)P(A) \qquad (9)$$

since the experiments are independent and $L_{T-1}(A)$ tends to $P(A)$ almost surely. From (9) we immediately obtain that $L^*(A)$ is given by

$$L^*(A)=P(A)$$

Suppose instead that $L^i_T(A)$ does not tend to a constant strongly. Then considering what we said above, there exists a large enough T^* such that for all $T\geq T^*$ (8) takes the following form almost surely

$$L^i_T(A)=\alpha_i L^i_{T-1}(A)+(1-\alpha_i)P(A) \qquad (10)$$

From (10) we have that for large enough T $L^i_T(A)$ is deterministic with probability one so that its long run almost sure behaviour can be obtained by solving it as a first order difference equation. Taking some $\underline{T}\geq T^*$ and solving recursively, from (10) we obtain that

$$L^i_{\underline{T}+t}(A)=(\alpha_i)^t L^i_{\underline{T}}(A)+P(A)(1-(\alpha_i)^t)$$

which, as t tends to infinity, tends to $P(A)$ with probability one contradicting the initial hypothesis .■

So $L^i_T(A)$ must converge strongly to a constant and this can only be $P(A)$.

Adaptive learning of the type I have analyzed does not prevent the two agents from achieving asymptotic consensus; the interesting point is that the final result is independent of the absolute value of α_i. In fact, Proposition 3 says that in the long run individual differences in the value of that parameter are irrelevant as far as strong convergence to $P(A)$ is concerned (when $\alpha_i \in (0,1]$); namely, there is a continuum of possible convergent adaptive rules.

However, when $\alpha_i=0$ asymptotic consensus in general would not transpire; the following corollary formalizes this statement

Corollary 2 Suppose $\alpha_i=0$;then as T tends to infinity $L^i_T(A)$ tends to $P(A|\Gamma_{i(0)})$ with probability one.

Proof Immediate.

The observation at time zero will be crucial in this case for the learning process implying , in general, that asymptotic consensus does not emerge.

5 The Case of Dependent Experiments

In this section I shall work exclusively with $Lr(A)$ bearing in mind that the analysis for $L^i r(A)$ could be carried on analogously. Moreover, I assume that information partitions do not change with time and that both agents use $Lr(A)$.

Consider now the presence of correlated experiments in order to explore the existence of a role for dependent random trials on the possibility of achieving asymptotic consensus.

Let us take the particular case where the sequence of experiments is chacterized by a first order correlation, in other words suppose we have a (stationary) Markov Chain. It is not my goal here to dwell on the theory of Markov Chains at lenght by considering all the possible alternative behaviours that the system can have in a finite state space[11] .What I shall do would in fact be to concentrate myself on the specific situation where W is partitioned in two sets; one is an irreducible subset of positive recurrent states while the other is made of transient states. To make things easier suppose there are no periodic states[12] ; moreover ,let $\pi_t(j)$ $j=1,..,W$, be the objective probability[13] that at time t state j would occur and $\pi(j,h)$ $j,h=1,..,W$ the probabilty that if the system is in state j it would go in one step to state h.

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[11] When the state space is finite there must be at least one positive recurrent state where the stationary distribution is concentrated. There may be however more than one set of positive recurrent states which partition ,together with the transient states, the state space. In the paper I for simplicity assume that there is only one irreducible set of positive recurrent states; in the general case the analysis would be similar to the one I am proposing. For an introducion to Markov Chains see Karlin-Taylor (1975).

[12] The analysis would not basically vary if one allows for the presence of periodic states.

[13] From now on I shall always use π for objective and P for subjective probabilities.I also assume that subjective priors are common and constant over time.

Let $\pi(t)=(\pi_t(1),..,\pi_t(W))$ be the vector of absolute probabilities at time t and Ω is the WxW matrix of transition probabilities .

It is well known that in matrix terms the evolution of the absolute probabilities will be given by

$$\pi(t-1)\Omega=\pi(t) \qquad t=0,1,.. \tag{11}$$

It is then immediate to see that we can also write

$$\pi(t)=\pi(0)\Omega^t \tag{12}$$

Let O be the partition of the state space on the basis of the nature of its elements with R being the set of positive recurrent states and Z that of transient states; notice that R is closed, namely once is reached it will never be left.

It is a well known result in the theory of Markov Chains that the *stationary* distribution concentrates only on R and that in the long run the system will end up in R with probability one, independently of its initial position.

The expected value of the posteriors varies for finite t. However, when the columns of the nth power of the transition matrix asymptotically converge to the stationary/steady-state distribution the expected value of the chain becomes constant; since eventually the latter will be equal to the expected value calculated by means of the stationary distribution our analysis will only deal with the long-run features of the stochastic process.

Let $\underline{\pi}$ denote the vector of stationary probabilities; this will be given by

$$\underline{\pi}= (\underline{\pi}(1),..,\underline{\pi}(r),0,..,0)$$

where r is the cardinality of R and where the number of zeros is given by $W-r$,namely the cardinality of Z[34] .

Asymptotically, the expected value of the subjective posterior will be given by

$$E(P(A|\Gamma_{i(t)}))= \Sigma_k\ P(A|\Gamma_{ik})\underline{\pi}(\Gamma_{ik}) \qquad k=1,..,I \tag{13}$$

Nothing guarantees that agents' learning rules will converge with probability one to the same value because of the following two reasons;

a) the objective stationary distribution is in general different from subjective priors and in principle (we have seen this previously) a formula analogous to (1) does not hold

b) although the stationary distribution is a function of the common priors R[35] ,being a subset of W, may not be partitioned by agents' information structures as they are originally given; again this implies that a result analogous to (1) would not hold.

Nevertheless,in spite of points a) and b) there is a case where it is possible to produce a result that guarantees, under appropriate conditions, strong convergence to a constant. However, before stating the proposition I need first to set up some further notation; let M be the *meet*[36] of agents' partitions and $\sigma(M)$ the σ-field generated by M[37]. Then we have the following[38]

[34] There is no loss of generality in assuming that the first r states are positive recurrent and that the remaining $W-r$ are transient .Notice that stationarity of the distribution implies independency of time.Remind that the stationary distribution satisfies the following equation $\underline{\pi}=\underline{\pi}\Omega^n$ for any positive integer n.
[35] It is important to recall that the stationary distribution is obtained by mean of the transition matrix.
[36] The meet of the partitions is defined as their *finest common coarsening*.
[37] Namely, the collection of sets closed with respect to union and intersection (always empty) and complementation of the sets forming the meet.
[38] Notice that in what follows we set $P_t(A|R)=P(A|R)$ as well as $P_t(A)=P(A)$

Proposition 3 Let $\underline{\pi}(A)=P(A|R)$ for each subset A of W and suppose $R\epsilon\sigma(M)$; then $L_T(A)$ converges to $P(A|R)$ with probability one.

Proof Let Γ_{is} $s=1,..,S$ the sets of agent i's partition which have a non empty intersection with R. Taking as objective probability the stationary distribution we obtain the following expected value of the posterior

$$E(P(A|\Gamma_{i(t)}))= \Sigma_s\ P(A|\Gamma_{is})P(\Gamma_{is}|R) \qquad s=1,..,S \tag{14}$$

But (14) is equal to

$$\Sigma_s\ P(A|\Gamma_{is})P(\Gamma_{is}\cap R)/P(R) = \Sigma_s\ P(A|\Gamma_{is})P(\Gamma_{is})/P(R) \tag{15}$$

since R is a member of $\sigma(M)$ and so either is a member of M or the union of some of its sets[39] ; this indeed implies that each Γ_{is} is contained in R. But the right hand side of (15) is equal to

$$\Sigma_s P(A\cap\Gamma_{is})/P(R) = P(U_s(A\cap\Gamma_{is})/P(R) = P(A\cap R)/P(R) = P(A|R)$$

with $s=1,..,S$.

We know that there must exist a large enought T, say T^* ,such that for all $T>T^*$ the system is absorbed in R (with probability one) and the absolute probability converges to the stationary distribution. Then, from that T^* onward the Markov Chain becomes,with probability one, an ergodic stationary process ; the asymptotic behaviour of $L_T(A)$ is consequently governed by a version of the strong ergodic theorem which in our case guarantees strong convergence to $P(A|R)$[40].■

So for the particular case where the stationary distribution equals the conditional (with respect to R) subjective probability, asymptotic consensus arises about $P(A|R)$ itself.

[39] Recall that R can not be empty.
[40] See Karlin-Taylor (1975). For the convergence result it is not possible to use here the strong law of large numbers because experiments are correlated.

Notice that a peculiar Markov Chain would be the one in which all states are positive recurrent; in this case,under the assumptions of Proposition 3, there is no partitioning of the state space and we have strong convergence to $P(A)$.

6 Conclusions

In this paper I have obtained conditions for asymptotic probabilistic consensus to emerge when agents adopt some simple learning rules. The aim of the work has been to test whether or not *individual* learning could lead to consensus. I was able to identify in the coincidence between subjective and objective probabilities a point which seems to play a crucial part in guaranteeing asymptotic consensus.

Though somehow extreme, the case where no agent knows the objective probability represents an interesting benchmark for drawing a distinction between learning rules. When individuals do not know the objective prior, and when they do not revise their subjective priors on the basis of what they observe, asymptotic consensus may not arise. However, even when subjective priors are different both between agents and from the objective prior we have seen an adaptive learning scheme which may lead to consensus. Learning rules seem to be distinguishible on whether or not they incorporate a feedback between nature and individuals. The case of adaptive learning that was presented entails a more robust

analysis since allows the possibility for differential prior probabilities.

The work deals exclusively with a finite state space, with information partition structures and excludes the possibility of communication between agents; furthermore in much of the analysis common priors are assumed in order to compare our results with some of the most important contributions of the literature.

A fundamental issue which must be highlighted is that we show how consensus can emerge without assuming the *a priori* common knowledge of individual partitions .

An agenda for further reasearch should include work on the speed of convergence for the learning rules proposed, on higher order correlated experiments and possibly a treatment under alternative information structures.

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