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**Fabio Petri**

**The Patinkin Controversy  
Revisited**



*Facoltà di Scienze Economiche e Bancarie*  
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# CONTENTS

## PART I : THE CONTROVERSY UNTIL 1958

I - Introduction (§§ 1-2)	pag. 3
II- The Meaning of the Classical Dichotomy (§§ 3-12)	" 9
III-Hicks (§§ 13-19)	" 27
IV- Lange, Patinkin, Valavanis (§§ 20-23)	" 37
V - Archibald and Lipsey (§§ 24-26)	" 48

## PART II / THE ROLE OF THE STATIONARY STATE ASSUMPTION

VI- Criticism of the 'Full Equilibrium' Assumption (§§ 27-28)	" 55
VII-Capital Theory and Full Equilibrium (§29)	" 59
VIII-The Stationary State Assumption (§§ 30-33)	" 62
IX -A Rebuttal of Liviatan and of Ball and Bodkin (§§ 34-35)	" 70

## APPENDIX : WALRAS, THE CLASSICAL DICHOTOMY, AND PATINKIN

REFERENCES	" 91
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PART I

THE CONTROVERSY UNTIL 1958

## I - Introduction(\*)

1. The purpose of the present paper is to reappraise the evolution of the so-called Patinkin controversy, or "Classical Dichotomy" controversy, which started with the works on monetary theory and Say's Law by Lange (1942) and Patinkin (1948, 1949, 1950-51, 1951, 1954, 1965) and continued well into the 1960's.

This reappraisal is motivated by the recently achieved clearer understanding of the radical change undergone by the notion of equilibrium in the 1930's and 1940's, when Hicks's "week" (a short-period, neo-Walrasian general equilibrium) became the generally adopted framework for value theory, displacing the traditional concern with long-period equilibrium (1).

(\*) I am grateful to Professors P. Garegnani, C. Gnesutta, B. Jossa, A. Vercelli, for their comments on a previous draft. I have also benefitted from discussing parts of this work in a seminar of the Consiglio Nazionale delle Ricerche group on "Foundations of Economic Theory" in April 1981. As Robertson put it: "But the path of true knowledge is not always plainly marked; and for my strayings into the fields of error I must accept full blame". Financial help from the University of Siena is gratefully acknowledged.

(1) See Hicks (1946, ch.9). The change was from a notion of general equilibrium, in which the composition of capital is not a datum but rather adapts itself to the composition of output so as to yield a uniform rate of profit on the supply price of the capital goods (and prices are what Marshall called "long-period normal prices"), to a notion of general equilibrium determined on the basis of given endowments of each kind of capital goods. Garegnani (1976) has distinguished the two notions by calling them "long-period" and "short-period" general equilibrium, respectively. I prefer to use the term "neo-Walrasian" for the latter

It has been convincingly argued that the distinction between these two notions of equilibrium is crucial in order correctly to appraise: a) the evolution of value theory since the 1940's; b) the meaning and relevance of the so-called "Cambridge criticism" of neo-classical capital theory; and also c) much of the debate on the interpretation of Keynes (Garegnani 1976; Petri 1978; Eatwell 1979). I will argue here that the same distinction is also the key to an understanding of the evolution of the "Patinkin controversy". Its very birth, I will argue, was due to an insufficient awareness of the change, indicated above, in the notion of equilibrium, and of its implications for monetary theory. It will emerge that, when the once commonplace notion of long-period general equilibrium (LPGE) and the method based on that notion are fully understood, then Lange's criticism of traditional monetary theory is easily perceived to be misplaced; and even Archibald and Lipsey's fundamental contribution reveals some important deficiencies.

The direct aims of the present paper are three. First, it intends to adduce one further proof of the importance of the above-mentioned change to a neo-Walrasian notion of general equilibrium (NWGE), by showing how, when that change is understood, the evolution of such a confusing controversy becomes comprehensible. Second, it intends to clear

... (1) notion of equilibrium in order not to favour its mistaken identification with Marshall's or Keynes's different conceptions of the short period (on which see Garegnani 1976, p.37, note 25, and p.38; note 28. The simplest way to highlight the difference is to notice that in Marshall as well as in Keynes, even in the short period the amounts of each kind of circulating capital goods are not given: see, e.g., Keynes (1936, p. 124)). For more extended discussions see Garegnani (1976, 1979b); Petri (1978); Eatwell (1979); Milgate (1983); Petri (1982).

up a number of points which to this day remain confused in the "Patinkin controversy", particularly about shifts of meaning of the term "demand for money". Third, it will hopefully contribute to re-establish a correct understanding of the traditional method of analysis, based upon "long-period positions" (2). This understanding, not very common nowadays, appears to be necessary in order fully to appreciate the limitations of NWGE theories and the reasons in favour of the proposed return to a (renewed) classical approach to the theory of value and distribution: a fourth, indirect aim of this paper is to help establish a better communication between the parties debating on these last, fundamental issues.

2. The paper is organised as follows. Section II posits the basic analytical distinction, by pointing out that the equilibria, to which the term "Classical Dichotomy" can be correctly applied, are equilibria in which, as required by the concept of LPGE, the initial distribution of the total money endowment among the agents is not among the data, but rather among the variables to be determined endogenously (the analogy being immediate with the composition of the initial endowment of capital goods, which in these equilibria is

(2) "Long-period positions" is the term suggested by Garegnani (1976) to cover both the marginalist (or "neoclassical") LPGE, and the corresponding (but quite different, owing to the different theory behind it) notion of a situation where prices are "natural prices" or "prices of production", to be found in the approach of Ricardo, Marx and other classical authors: see below, § 3 and note (5).



a variable too, rather than being among the data as in NWGE) (3). Section III shows that Hicks's analysis of the demand for money in Value and Capital inadvertently introduced the first departures from the traditional treatment; and that, from the lack of a clear consciousness of the changes thus introduced, there resulted some contradictions in Hicks's own analysis.

In Section IV, Lange's contribution is shown to have brought Hicks's approach to its logical outcome. The specific inconsistency charge, advanced by Lange against the "Classical Dichotomy", is shown to depend on the fact that he implicitly takes the initial money endowment of each agent as given, but without realizing that this was not the case with the theories he criticizes: from the same misunderstanding he also derives a mistaken interpretation of the meaning of the traditional balanced-budget constraint. Patinkin is then shown to have added nothing of substance to Lange's criticism on this subject-matter(4).

It is then argued that Lange's and Patinkin's confusions were not correctly perceived even by their critics. Attention is specifically given to the distinction, advanced by Valavanis (1955), between two different meanings of "demand for money". Against those, like Patinkin (1965) or

(3) See Petri (1978).

(4) Patinkin's interpretation of Keynes will not be discussed here; an implicit criticism is contained in the alternative interpretation advanced by the authors cited in note 1; see especially Garegnani (1976, 1979a).

Hahn (1960), who contemptuously dismissed that distinction, it is argued that Valavanis was on the right track, although he partly shared his opponents' confusions, a thing which made it difficult to understand what he was trying to argue.

In Section V it is argued that the contribution by Archibald and Lipsey (1958, 1960), although correct on its basic point, shows an incomplete grasp of the method of traditional analysis; its deficiencies in this respect opened the way to the subsequent criticisms by Ball and Bodkin (1960) and by Liviatan (1965).

These criticisms are discussed in a separate half (Part II) of this article, because they raise issues which are largely independent of monetary-theory questions, being rather connected with the role of the stationary-state assumption in marginalist analyses. After presenting these criticisms in Section VI, it is argued in Sections VII and VIII that the issues at stake actually concern "real", rather than "monetary", neoclassical theory, and that there would be little to object to the method of analysis adopted by marginalist authors (consisting of successive approximations to reality, starting from the stationary state), were it not for the inconsistencies of their "real" theory (capital theory in particular). On this basis, in Section IX it is shown that Liviatan's and Ball and Bodkin's criticisms are misplaced.

An Appendix discusses Walras's theory of money, in order to show that there too, as in his theory of capital,

Walras was, one might say, half-way in between the LPGE and the later NWGE frameworks. Patinkin (1965) is shown to have been guilty of some confusions on this issue too. The reconstruction of Walras's determination of the price level clarifies one possible origin of Lange's confusion.

## II - The Meaning of the Classical Dichotomy

3. The object of discussion in the Patinkin controversy was the logical consistency of the "Classical Dichotomy". The definition of this term was usually given in highly formalized terms: a system of equations, purporting to determine both relative and absolute equilibrium prices (i.e. both relative equilibrium prices, and the equilibrium price level, or value of money), was said to embody the Classical Dichotomy if the equations determining the relative prices could be solved independently of the "money equation", the latter equation being necessary only to determine the price level. The economic meaning was anyway sufficiently clear: systems of equations embodying the Classical Dichotomy reflect the theory that equilibrium relative prices are independent of the quantity of money; *ceteris paribus* the latter (via the "money equation") only determines the price level (in the way indicated by the Quantity Theory of Money).

It must be said at the outset that the word "Classical" was used in the controversy in a sense more akin to that of Keynes than to that of Marx, i.e. as referring essentially to the marginalist tradition, without grasping the radical difference on the theory of value and distribution between that approach and the "classical" (in Marx's sense) approach of Smith and Ricardo (and Marx), a difference which has been clarified by Sraffa, Garegnani and others (6).

(5) Sraffa (1951, 1960) is not self-explanatory and an illustration of the relevance of this work is rather to be found in Meek (1976), Eatwell (1977, 1979) Garegnani (1977), Garegnani and Petri (1982).



Thus, the equations determining relative prices considered in the controversy were always derived from the marginalist theoretical framework; and here too, LPGE will always mean the marginalist long period.

In so far as the Classical Dichotomy simply expresses the quantity theory of money, it is of course applicable also to a number of classical authors; e.g. to Ricardo, who also thought that both relative prices and quantities produced are determined independently of the quantity of money, the latter only determining the price level. But the evolution of the "Classical Dichotomy" controversy was strictly tied to the marginalist specification of the forces which determine relative prices, and to the evolution of marginalist theory in modelling these forces. It is only within the marginalist framework that the shifts of meaning were possible, which gave rise to the confusions from which the controversy was born. The marginalist theory of value and distribution will therefore be the framework behind the following discussion. (Occasionally, "neoclassical" will be used as synonymous with "marginalist" - a concession to the predominant, although incorrect, terminology). (6)

4. The theory of LPGE tries to describe what Marshall would have called the "long-period normal" (and Wicksell simply the "equilibrium") behaviour of the economic agents, i.e.

(6) See Petri (1978, p. 246, note 1).

that behaviour which would be observed if the economy were in a state of "tranquillity": this term is used here to indicate a state of rest of the forces which make for the uniformity of the profit rate (i.e. of the rate of return over the supply prices of capital goods). (7)

The analysis of LPGE was generally conducted under the hypothesis of stationariness. The meaning of this hypothesis will be discussed in Part II of this essay. For the moment, it can be seen as a convenient simplification, which, besides, allows one to avoid problems connected with the non-constancy of the price level if the quantity of money remains the same while the volume of real transactions does not. (8)

5. The stationary state described by Marshall or Wicksell or Pigou etc. did not assume the actual existence of an auctioneer (or an equivalent mechanism) who gets every disequilibrium sorted out before the period's trading and production commence; it purported to depict how a real economy would look like if stationary. Exchanges therefore were not assumed to be agreed upon all at the same time, but rather

(7) See, e.g., Robertson (1957-59: vol. I, p. 95; vol. II, p. 11; vol. 3, p. 9); Marshall (1961, Bk.V., ch. V); Garegnani (1976, Sec. 5).

(8) Variations in the quantity of money, on the other side, in order to be adequately treated, would require the discussion of problems (e.g. seignorage from money creation, see Petri (1983)), which must be left aside here. Also for simplicity, it will be implicitly assumed throughout that money is of the outside variety only: see Patinkin (1965) pp. 277-8 for a discussion of a "dichotomous" system obtained by assuming all money is inside money.

to be at different moments; and to be, typically, of goods or services etc. against money.

In order to put the central issue in the starkest terms, I will assume money to consist exclusively of inconvertible paper notes - no deposit money, no promissory notes used as money, etc.

Let us further assume that each agent's behaviour follows a pattern which repeats itself indefinitely. Associated with it, there will be a function which represents the amount of money held by the agent  $i$  as a function of time. Let us indicate it as  $h_i(t)$ . The graph of this function will also show a pattern which repeats itself again and again. A standard example is the worker who receives his wage at the beginning of the period and spends it in a continuous flow until he finishes it exactly an instant before the beginning of the next period: the graph of  $h_i(t)$  has then the well-known saw-like shape reproduced in Fig. 1.

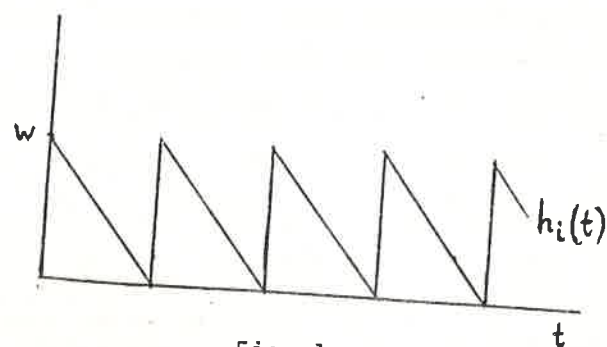


Fig. 1

Clearly, behind the pattern of the graph of  $h_i(t)$  there is an optimization performed by the agent; but it is not the purpose of this paper to discuss this process in any detail. In order to illustrate the fundamental difference between the long-period analysis and Lange's model, and the associated differences in the meaning attached to "demand for money", it suffices to assume that money is only held for transaction purposes, and that the rate of interest is zero (so idle money is not converted into bonds - this would give rise to the inventory-theoretic problems analyzed eg. by Baumol (1952), Tobin (1956), Clower and Howitt (1978)). Then  $h_i(t)$  will represent the minimum money balance an agent needs for her transactions, which will be given once the real equilibrium transaction of the agent, their time pattern, and money prices, are all given (Ellis 1937-38); and its graph will reach the value zero at least once in each "cycle" of the agent. (The example of Fig. 1 satisfies these assumptions). This minimum money balance  $h_i(t)$  might be called the agent's "demand for money" (9): and the sum of all agents' demands for money at given moment gives the total demand for money at that moment (but see below, § 10). It should be noticed, though, that here the word "demand" is, strictly, improper:  $h_i(t)$  is not an amount of money demanded, by offering something in exchange, as in the usual case of demand for goods; it is simply a pre-requisite of equilibrium behaviour.

(9) Thus Hicks (1935) means by demand for money the amount of money an agent chooses to hold at a point of time.

6. If the agent held at any moment more money than shown by this graph, then his behaviour would not be the one determined by the real equations; knowing he had more money than needed for the stationary behaviour, he could for instance afford some extra consumption for one or more periods. If he held less money than shown by the graph, he would find it impossible to follow the stationary behaviour. In either case, the agent's behaviour would not repeat itself unaltered period after period, excess demands would not all be zero at the equilibrium prices, and the other agents too would abandon equilibrium behaviour. The economy would be in disequilibrium.

It follows that the determination of a long-period equilibrium cannot include, among the data of the economy, the distribution of the total money stock among the various agents; the money endowment of each agent (both at each moment and on average) must result from the equilibrium real behaviour of the agent facing given (money) prices; it must be that amount which is a pre-requisite for that equilibrium behaviour to be actually carried through. (10)

7. This is why in the traditional formulations of authors adhering to the Classical Dichotomy the equations determining the equilibrium behaviour e.g. of a consumer do not depend

(10) The analogy is evident with the treatment of the composition of capital, which must result from the equilibrium condition of a uniform rate of profit, see above, note 1 and § 2, and below, Part II, § 28.

on his money endowment: the equilibrium behaviour to be determined is long-period equilibrium behaviour, and this presupposes that the money endowment has adapted itself to the requirements of that behaviour. So equilibrium behaviour, since it requires the money balances to be the "right" ones, does not depend on them but rather determines them.

An implication, not grasped e.g. by Lange, is the following: from the fact that the agents' excess demand functions for goods (including services) depend only on relative prices it does not follow that it is sufficient to look at relative prices to establish whether there is equilibrium in the markets for goods. Those relative prices, which solve the equations expressing equilibrium on the goods markets, are necessary but not sufficient conditions for equilibrium on those markets.

Equilibrium requires in addition: 1) that the price level be such as will make the total demand for money equal to the economy's money endowment; 2) that the economy's money endowment be distributed among the agents in the proportions required by each agent's equilibrium behaviour.

This is because the agents' excess demand functions for goods and services are derived with an implicit hypothesis behind them that the agents always have the "right" money balances. Thus, even if only the second one of those two conditions is not satisfied, then, even if relative prices are the equilibrium ones, still some agents will find it not optimal, or impossible, to behave as equilibrium would require, their behaviour will not be the one indicated



by the "real" equations, and there will be disequilibrium in the actual economy. (11)

8. Less, but by no means no, attention was normally dedicated to the second of those two conditions (12). Anyway, the existence of a tendency of the distribution of the total money endowment among the agents towards the equilibrium one could be derived from the same mechanism usually thought to be at work (if, on average, money balances were greater or smaller than the price level required) to push the price level towards the equilibrium level, i.e. the mechanism based on the "real balance effect":

"Let us suppose that for some reason or other commodity prices rise while the stock of money remains unchanged, or that the stock of money is diminished while prices remain temporarily unchanged. The cash balances will gradually appear to be too small... I can rely on a higher level of receipts in the future. But meanwhile I run the risk of being unable to meet my obligations punctually, and at best I may easily be forced by shortage of ready money to forgo some purchase that would otherwise have been profitable. I therefore seek to enlarge my balance... through a reduction

(11) See also below, § 25.

(12) The discussions of "forced savings" show a clear awareness of the re-distributive effects, with possible permanent further effects, of initial non-equilibrium distributions of money: see, e.g., Wicksell (1935, p. 165), Hayek (1932), Saulnier (1938, pp. 63-73, 147, 243-4). Also see Fisher (1922), p. 166. These effects were discussed at a second stage of the analysis, see below, Part II, Section VIII.

in my demand for goods and services, or through an increase in the supply of my own commodity... the universal reduction in demand and increase in supply of commodities will necessarily bring about a continuous fall in all prices. This can only cease when prices have fallen to the level at which the cash balances are regarded as adequate". (Wicksell 1936, pp. 39-40)(13).

9. The logic of LPGE, then appears to require the following procedure. For each economic agent, if absolute (money) prices are parametrically given, "real" theory will give us the long-period equilibrium behaviour of the agent (14), as a function of relative prices only; if the time pattern of the equilibrium transactions of the agent is also given,

(13) See also Wicksell 1935, p. 64; Fisher 1922, pp. 153-4; and obviously Pigou 1943. The possibility of permanent effects of an initial non-equilibrium distribution of money was not denied, see note 8 above.

(14) Relative prices, though, cannot be arbitrarily given, because, as shown by Sraffa (1925, 1926), a competitive (marginalist) equilibrium requires that there are constant returns to scale or else that "all productive enterprises have already reached the limit beyond which a further increase in the scale of production will no longer yield any advantage" (Wicksell 1934, p. 126); but then unless price equals minimum average cost the dimension of firms is either zero or infinite. The set of acceptable relative price vectors is given by the condition that the rate of profits must be the same in all sectors; it is represented by the set of relative price vectors associated with the possible values of the rate of profits on the "wage-profit frontier", see Sraffa (1960, chs. 6, 12); Lippi (1980, ch. 1); Schefold (1976). Salvadori (1979) shows that in some cases of joint production with choice of techniques there are problems in defining the wage-profit frontier whose implications are still to be explored.

of the agent or to an integer multiple of it. In the example of Fig. 1, clearly,  $d_i = \frac{1}{2} w$ . What was said at the end of § 5 about the peculiar meaning of the word "demand" applies to  $d_i$  too.

This preference for an average concept of the required money balances derived from the belief that

"Each man's adjustment is, of course, somewhat rough, and dependent largely on the accident of the moment; but, in the long run and for a large number of people, the average rate of turnover, or what amounts to the same thing, the average time money remains in the same hands, will be very closely determined" (Fisher 1922, pp.152-3). (15)

In other words, it was believed that, so long as one was clear that one was speaking about averages over time and over many individuals, it was not necessary, for the validity of the quantity-theoretic determination of the price level, to assume that the economy actually was in equilibrium; the belief in equilibrium as a centre of gravitation meant that accidental or stochastic deviations from average behaviour would anyway sufficiently compensate one another as between individuals and over time. This also explains

(15) Also see Wicksell (1936, p.41; 1935, p.62); Pigou (1917-18, p.53; 1942, p.76); Fisher (1922), p.80. It is noteworthy that most studies of the individual's transaction demand for money still concern themselves with the average demand (see, for a survey, Nagatani 1978, ch.4).



the comparatively little space traditionally dedicated to discussing the individual's "demand for money" equation in comparison with the economy-wide equation. What was considered important was rather the average resulting from the differences between individuals, an average which, as the above quotation shows, was believed to be more stable than the single individuals' behaviour. Such average demand for money for the economy as a whole - let us indicate it with the symbol  $D$  - was, obviously, in terms of the Fisherine  $MV=PT$ , given by  $D \equiv PT/V$ .

Anyway, this total money demand  $D$  could have been rigorously derived from the individuals' demands, simply by summing over all  $d_i$ 's. And, with the additional assumption that all intended transactions did "match", i.e. that to every intended sale there corresponded an intended purchase (as would have been the case if relative prices were equilibrium prices), if the various  $h_i(t)$ 's were all well-defined then the same result would have been obtained by summing over all  $h_i(t)$ 's calculated at the same instant, no matter which instant were chosen, because the deviations of the various  $h_i(t)$ 's from the respective  $d_i$ 's would, under that assumption, sum up to zero.

Equilibrium then required that the total money demand thus obtained be equal to the (given) quantity of money  $\bar{M}$  existing in the economy: this equation  $D = \bar{M}$  established the price level. Although this is a well-known subject, it may be worthwhile to spell out the argument. It follows, from what has been said above about the logic of LPGE, that

if all prices vary in the same proportion  $D$  must vary in that proportion too. If then one defines  $D_R \equiv \frac{D}{P}$  (where  $P$  is an index of the price level) as the total demand for real balances, this is given once relative prices and real transactions are given, and the equation  $\bar{M} = PD_R$  determines the equilibrium price level (16).

11. When one enlarges the picture, introducing bonds etc., and one considers further motives for holding money besides the transactions motive, as, e.g., in the so-called Cambridge approach (17), then the desired money balance (at each moment as well as on the average) will also result from, e.g., the choices of the agent between money and bonds, and therefore will depend on the rate of interest too; but it remains true that, in order to obtain a repetitive behaviour pattern of each agent, the money balance held on average as well as at each moment by an agent must adjust itself, and its equilibrium value will therefore depend upon the "real" behaviour and wealth of the agent (18).

(16)  $D_R$  is the demand for "real balances" and, ceteris paribus, is independent of the price level. The equivalent expression in the Fisherine equation  $MV=PT$  is  $T/V$ . This can be seen as a weighted average of each agent's personal  $T/V$  ratio, where  $V$  is what Valavanis (1955) called "personalized velocity of money".

(17) See, e.g., Pigou (1917-18).

(18) "... let us suppose that the inhabitants of a country... find it just worth their while to keep by them on the average ready purchasing power to the extent of a tenth part of their annual income, together with a fiftieth part of their property..." (Marshall 1923, p.33). Also see Fisher (1922, p.152).

minimum of formalization consider an exchange economy without bonds. If there is one agent, the agent includes production periods (say, from  $t=0$  to  $t=T$ ). The excess demands for goods  $j$  are  $\bar{x}_{j,t}$  ( $j=s, \dots, n$ ;  $t=0, \dots, T$ ). Let  $p_{j,t}/p_1$  be the relative prices. Let  $e$  be the vector of (given) endowments of goods. Then one can assume that  $e$  also depends on

analysis of the money balance, which - assuming the money supply is constant - will depend, in a way that can be specified, on the transaction prices. Let  $h_t$  stand for the money balance at time  $t$ . One must have an instant before each  $t$  at which his behaviour be the one that can be written:

of equal length for all

agents - one adds the excess demands of all agents for each good, one obtains the total excess demand functions for goods, which depend on relative prices only; the conditions imposing that these excess demands be zero will determine relative prices. The  $h_t$ 's will allow the derivation of  $D$ , the total money, and the last equation

$$D = \bar{M}$$

(or its Fisherian or Cambridge variants) will determine the price level. This is the (Neo) Classical Dichotomy as traditionally intended.

Nowadays, the spread of the habit of thinking in terms of Hicksian "weeks" or Arrow-Debreu intertemporal equilibria may make it difficult to grasp the economic logic behind these equations. But it is not difficult to establish the connection between them and the modes of representing the agents' choices more fashionable nowadays. It is nowadays customary to imagine an agent at the beginning of a Walrasian "tâtonnement" or Edgeworthian recontracting, having his initial money balance among the data determining his decisions. Now, independently of the hypothesis of tâtonnements or recontractings which bring equilibrium about without "false price" tradings, it is reasonable to assume that the plans of an agent, at any given moment, are influenced by the money balances he holds at that moment, relative to money prices.

Let us therefore consider an agent formulating his plans of sales and purchases, as functions of expected money prices

and of his given initial money balance. Let us assume his optimizing decisions can be expressed via excess demand functions  $e_{jt}$  for commodity  $j=1, \dots, n$  at time  $t=0, \dots, \bar{t}$ . These will entail certain money balances  $m_t$  ( $t=0, \dots, \bar{t}$ ) at the end of transaction period  $t$ . Let  $\bar{m}$  be the initial money balance;  $p$  is defined as before;  $m$  is the vector of  $m_t$ 's;  $e$  is the vector of  $e_{jt}$ 's. Then we have

$$(3) \quad e = e(p, \bar{m})$$

$$(4) \quad m = m(p, \bar{m})$$

where the two groups of equations are connected by:

$$(5) \quad m_t = \bar{m} - \sum_{t=0}^{\bar{t}} \sum_{j=1}^n e_{jt} p_{jt}$$

How does one pass from these equations to the long-period ones, (1)-(2)?

Let us assume, first of all, and crucially, that  $\bar{m}$  is a variable rather than a datum. Secondly, let us add to equations (3)-(5) the additional condition

$$(6) \quad \sum_{t=0}^{\bar{t}} \sum_{j=1}^n e_{jt} p_{jt} = 0$$

i.e. the traditional balanced-budget constraint (and in order for this to make sense, let us assume that the "cycle" of the agent is  $\bar{t}+1$  periods long).

Since by assumption  $e$  depends on the initial money balance, for given  $p$  one can consider  $e$  as function of  $\bar{m}$  only:

$$e = f_{(p)}(\bar{m})$$

$$m = \varphi_{(p)}(\bar{m})$$

To satisfy equation (6), one must look for an initial money balance  $\bar{m}^*$  such that  $m_{\bar{t}}^* = \varphi_{\bar{t}(p)}(\bar{m}^*) = \bar{m}^*$ . In fact,  $m_{\bar{t}} = \bar{m}$  is equivalent to the balanced-budget constraint, owing to Walras's law for the individual. Such an  $\bar{m}^*$  was traditionally assumed to exist (under stationary real conditions), and this authorized the imposition of the balance-budget constraint; and as our purpose here is explanatory only, let us accept this assumption. Obviously  $\bar{m}^*$  is a function of  $p$ : let us write

$$\bar{m}^* = \psi(p)$$

Assume now that in (3) the money endowment  $\bar{m}$  varies as given by  $\psi$ . Then one can write new excess demand functions which depend only on  $p$ :

$$(7) \quad e' = e(p, \psi(p)) = e'(p).$$

These are nothing else than the  $e'$  of equation (1), (19). It is easy, in fact, to demonstrate that they are

(19) Thus, contrary to the opinion of Samuelson (1968, p.187; 1972, p. 286) it is perfectly correct and adequate, within the LPGE framework to write down excess demand functions for goods which depend on relative prices only, and then determine the price level via a Fisherian or Cambridge equation (the system Samuelson calls A', B'). Samuelson's 1968 article is an unhappy attempt at reconciling the contending parties. He is not clear on whether the initial distribution of the money endowment should be treated as an unknown or a datum in "dichotomous" models: he

stration appears unnecessary: it, under the balanced-budget assumption set of the agent does change in the same proportioning vector in that set does

than  $m_{t-1}^*$ ; and  $h_0 \equiv \bar{m}^*$ .

### III - Hicks

13. It should be clear, from what has been said above, that the Classical Dichotomy only implies that equilibrium relative prices can be determined independently of the quantity of money and of its distribution among the agents. No one would have denied e.g. that if, in a stationary economy, consumer A loses a bet with consumer B and transfers some part of his (equilibrium) endowment of money to B, then the equilibrium would most probably be disturbed (20), and there would be some at least temporary effect on relative prices.

It was only natural, then, that the Classical Dichotomy should have been given up when the initial money endowments of each agent was included among the data of the equilibrium: which is what Patinkin did.

This radical departure from the established tradition was made possible by the shift to a concept of temporary general equilibrium.

Among the pioneering contributions to that shift, Hicks's Value and Capital was probably (21) the most influ-

like (including, apparently, their money endowments, see p. 184) so single consumer, whose initial money to the economy's total endowment. unsatisfactory and Patinkin's account (see here, below, Section IV) are long period is also misunderstood Samuelson is able to agree with sues at stake.

(20) Patinkin (1965) assumes for most of his book that consumers' indifference maps are such that equilibrium in this case would not be disturbed; but this is only done in order better to concentrate on the aggregate real balance effect.

(21) The story of the spread of NWGE is still to be written. Milgate (1979) discusses the origin of the concept in Hayek and Lindahl (Hicks, by his own admission, was greatly influenced by both); but a history of the enthusiasms and resistances caused by the NWGE approach after Value and Capital is still lacking. It is interesting, anyway, to notice that some resistance came from highly respected theoreticians, e.g. Leontief (1936-7), p.92, note.



data determining the equilibrium for the "week". (22)

(22) The fact that, in any actual market economy, "false-price" transactions and productions cannot be taken to be absent makes the length of the "week" very difficult to decide upon, if one wants to use temporary equilibrium analysis as a basis to explain and predict the tendencies of actual economies. Even granting the "stability" of the temporary equilibria (but see below, Part II, p. 60, note 54; and Petri 1978, p. 256, on the peculiar meaning of "stability" in this context), the following unhappy dilemma appears inescapable. The shorter the "week", the more problematic it is to assume that the actual market processes of trial and error will get the economy to a position reasonably close to an equilibrium. The longer the "week" (or the sequence of "weeks"), the greater the danger that the behaviour of the actual economy comes to diverge considerably, as the end of the "week" approaches, from the theoretically determined equilibrium path owing to a "cumulation of errors" (see Garegnani 1976, p. 38, note 30). Hicks's definition of the "week as that period of time during which variations in prices can be neglected" (Hicks 1946, p. 123) appears to refer to an economy already in (temporary) equilibrium, otherwise prices could not be assumed to remain unchanged for any length of time however short, so it is of no help. The two horns of this dilemma have their common roots in the "impermanence" (Garegnani 1976) of some of the data of a temporary equilibrium, namely the composition of capital and expectations; also see below, Part II, note 48. It would seem that a necessary pre-requisite, for the opinion that such difficulties do not undermine all analyses based on temporary equilibria and their sequences, is the belief that such sequences of "weeks" do not systematically diverge from the actual (disequilibrium) path of an economy: and this seems to require a belief that in the sequence of "weeks" the same long-period forces of tendencies come to dominate as in the actual, disequilibrium processes (see below, note 36). But then: a) the notion of temporary equilibrium would still appear to be superfluous, because one could directly study, in traditional fashion, the long-period forces, the long-period position corresponding to the full working-out of their effects, and then the varied (disequilibrium) processes of gravitation around and towards it, thus dispensing with unreal assumptions about auctioneers, absence of "false-price" transactions and the like; b) an analysis of the long-period forces would anyway be inescapable, in order to support the abovementioned belief in the (qualitative, at least) "correctness" of sequences of "weeks": and on this ground, the "Cambridge criticism" could no longer be eluded (see Part II, §§ 28 and 29).



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also see Hicks 1965, p.73),  
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6-7.

over the agent's "cycle" as a pre-requisite for equilibrium behaviour. Hicks, on the contrary, in *Value and Capital* appears to use "demand for money" to mean the amount of money an agents wants to hold at the end of the "week". Thus, when discussing the case of an individual who "plans to spend, in every future week, the same amount as he receives," Hicks concludes: "Then, if he is perfectly confident that he can carry out his plan, his demand for money will be nil. All the money he receives will be paid out again at once; he will need to keep over from one week to another no money balance at all to finance his transactions " (Hicks 1946, p. 240), (24). Thus, "demand for money" here seems to mean

(24) It deserves notice that Hicks (1946) is apparently unable to conceive of the need for money for transaction purposes during the week. In order to conceive of the transaction demand for money at all, he appears to think that one must assume that "receipts...come in, not every week, but, say, every fourth week; then, even if receipts and expenditures balanced over the four weeks taken together, the money balance could only fall to zero in the week just before the month's receipts were due to come in" (Hicks 1946, p. 240). Also, it appears to have escaped him that it is not possible that, if goods and services are exchanged against money, all agents end up the week with zero money balances even if "receipts come in every week" and each agent "plans to spend, in every future week, the same amount as he receives". Perhaps this slip was due to the difficulty of conceiving the transactions within the "week" as actually requiring money, once the fiction of the auctioneer (or some equivalent assumption) is accepted: it is not clear, then, why exchanges could not be all simultaneous and in kind, the only money transfers being those associated with the changes in the agents' money endowments between the beginning and the end of the "week", since equilibrium has been established before the exchanges started, and then the same institution which has brought equilibrium about could very naturally function as a clearing house (see for considerations pointing in the same direction, Clower 1967 pp. 202-204;

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148, note 28, interprets  
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derives from it that, if there is equilibrium on the markets for goods and securities, then the variations in the cash balances ("net acquisitions of cash" in Hicks's terminology) of the agents must necessarily compensate one another. His conclusion is that therefore:

"To say that the net acquisition of money by trading is zero, taken over the whole community, is the same thing as to say that the demand for money equals the supply of money. Consequently, if there is equilibrium in the markets for goods and services, and in the market for loans, there must also be equilibrium in the market for money" (ibid., p. 157).

The confusions which characterize the subsequent controversy have their roots in this passage. The "money equation", which establishes the equilibrium in the money market, is according to Hicks the equation which establishes that in the economy as a whole the "net acquisitions of money" of all agents must compensate one another. Let us see where this conception brings Hicks into contradiction.

17. Hicks proceeds to argue that any one of the  $n+1$  equations establishing the equilibrium for the  $n-1$  goods, for money, and for securities, being derivable from other ones, can be eliminated. For instance, one can eliminate the "money equation":

"If we decide to eliminate the money equation, then we can think of prices and interest being determined on the markets for goods and services, and the market for loans; the money equation becomes completely otiose, having nothing to tell us". (ibid., p. 158)

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whatever the price level (Lange 1942) starts off precisely from here).

Hicks's mistake appears to lie in believing that his "money equation" is the same thing as a Fisherine or Cambridge equation. In LPGE analysis, the fact that Hicks's "money equation" is satisfied, i.e. that in the aggregate the desired net acquisition of cash of the agents is zero (27), still tells us nothing about the relationship between the "supply" and the "demand" for money in the traditional sense; i.e. it tells us nothing on whether the amount of

(27) In LPGE, this result does not require that the balanced-budget constraint holds, if the length of the time period is chosen arbitrarily. In the LPGE framework, it is perfectly legitimate to define the "excess demand for money" or "net acquisition of cash" of an agent  $i$  between two dates  $t_1$  and  $t_2$  as given by  $h_i(t_2) - h_i(t_1)$ : it will depend (given the "shape" of  $h_i(t)$ ) on the points of the "cycle" of the agent where  $t_1$  and  $t_2$  fall. In general,  $h_i(t_2) - h_i(t_1)$  will be zero only if  $t_1$  and  $t_2$  correspond to the same point in the "cycle" of the agent. One can very well consider a period of such length that, for some agents,  $h_i(t_2) - h_i(t_1) \neq 0$ ; still, so long as each sale entering the budget of an agent appears as a purchase in the budget of some other agent (as e.g. is the case if relative prices are the equilibrium ones), this will only mean that the increase in the required balances of some agents from  $t_1$  to  $t_2$  will be exactly balanced by the decrease in the required balances of the other agents. The sum total of required balances of all agents, so long as the italicized condition holds, will be given as a function of absolute prices, and strictly proportional to the price level, independently of the choice of  $t_1$  and  $t_2$ ; and the aggregate "net acquisition of cash" will still be identically zero, although it will not be zero for individual agents, and will remain zero if all prices vary in the same proportion. This comes from the fact that the individual agent's "net acquisition of cash" is the required one: as the amount of money with which an agent must end the period varies with the price level, the amount of money with which the agent must start the period (a variable too) varies in the same proportion.

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#### IV - Lange, Patinkin, Valvanis

20. Lange (1942) added to Hicks's confusions (29) by introducing a peculiar interpretation of Say's Law and of its consequences for price theory, and a further change in the meaning of "demand for money".

By Say's Law Lange means the following. Define  $M$  as "the total increase of cash balances (in excess of a possible increase of the quantity of money) desired by all individuals" (Lange 1942, p.152). This - apart from the possibility of variations within the period of the total quantity of money in the economy, a complication which may here be left aside - is the same as Hicks's aggregate "net acquisition of money". As was said above, by Walras's Law this will be equal to the aggregate value of total excess supplies on all other markets: and it will be zero only if the latter is zero. Lange takes Say's Law to mean the assumption that  $M=0$  always, i.e. whatever the relative as well as absolute prices. He then proceeds to writes:

(29) Lange's article was published three years after the first edition of Hicks's book and makes explicit reference to the latter, but a first version of it (apparently very similar in contents) had been read at an Econometric Society meeting in 1938 (see the summary in *Econometrica*, vol 7, 2, April 1939, pp. 172-173). Research would therefore be needed to establish priority of conception, or possibly previous exchanges of ideas between Lange and Hicks. For our purposes, what is important is that most readers of Lange's article had probably read Hicks's book already, and therefore had his temporary equilibrium general framework in mind as the basic scheme within which to interpret Lange's statements: and Lange's references to Hicks prove that he too had that framework in mind, whether arrived at independently or not.

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mean, for instance, that even finding a billion dollars' treasure chest would not alter the money expenditure of a street beggar (Lange's aggregative definition of Say's Law does not exclude the possibility that the beggar's increased expenditure be exactly offset by someone else's decreased expenditure; but this appears even more absurd since there is no mechanism tying people's decisions so as to bring such a result about), or that even a hundredfold increase in all prices would prevent no one from carrying on his real transaction as before, in spite of the fact that his money balances have not increased (31).

It is unnecessary to show here in detail how wrong the attribution of this hypothesis to Say, Ricardo etc., is (32). Rather, it is useful to trace the root of the mistake: although Lange always remains at the level of all individuals "taken together", it would seem that his mistake lies in interpreting Say's Law as consisting of the introduction of a balanced-budget constraint upon individual choices simultaneously with the assumption that the individuals' initial

(31) Clower's specification of the budget constraint in Clower (1967), which imposes that the agent's intended purchases cannot exceed the value of his initial money endowment, can be seen as reflecting, in a crude way, the consciousness of the difficulty of carrying out an increasing volume of monetary transactions always with the same initial money endowment. As a constraint on equilibrium behaviour, it therefore makes sense only in a temporary equilibrium framework. It is very crude anyway since it excludes the possibility that some of the purchases within the period be paid with money obtained from sales within the period.

(32) Becker and Baumol (1952) were quick to point it out; but they were not as clear on the meaning of traditional analysis, see below, § 22.



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Lange's confusion can be  
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essentially superfluous mathematics (34). The purpose of  
his efforts was to show that "Say's Law" and a Cambridge-  
type equation imply contradictory statements about the  
behaviour of the demand for money. This he achieved by  
showing that "Say's Law" implies that the demand for money  
is homogeneous of degree zero in absolute prices (i.e. does  
not change at all if only the price level changes), while  
a Cambridge-type equation implies that the demand for money  
is homogeneous of degree one in absolute prices (i.e. changes  
in the same proportion as the price level). As has been  
shown, Lange (1942, pp.163-167) had already said it all.

22. It is clear from the above that the adoption of the  
Hicksian framework had prevented Lange from grasping the  
meaning of the "demand for money" which appears in a  
Fisherine or Cambridge equation. Lange's definition of the  
demand for (and supply of) money is different even from  
Hicks's, and dominated the subsequent controversy. Treating  
money simply as the  $n$  - th commodity in a system of  $n$   
demand=supply equilibrium equations of the type  $D_i(p_1, p_2, \dots, p_n)$   
 $= S_i(p_1, p_2, \dots, p_n)$  (the price of money being  $p_n=1$ ), he defi-  
nes (Lange 1941, p. 150) the demand for money as

$$D_n = \sum_{i=1}^{n-1} p_i S_i$$

(34) Patinkin admits it in Patinkin (1951), p. 139, note 15.

to do with my D of §10).  
by Patinkin: "the only way  
selling goods, hence the  
with the supply of all goods"  
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f Patinkin in the subsequent

f (1950) accepted Patinkin's  
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indicate the demand for money in  
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ckman (1950); and Patinkin (1951).  
For Leontief, see Leontief (1950).  
d to refute Lange's demonstration  
in the "Classical Dichotomy" (as  
success (see Johnson 1962, p.21)  
no wrote down an identity as if it  
66 in Lange (1942) reads

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defines  $\Delta M$  and should therefore have  
e anyway below, § 23, on where the  
and Patinkin.

ceive?) Patinkin's shift in the meaning of "demand for money"  
nor - in spite of their historical perspective - the shift  
in the notion of equilibrium. Their criticisms were only aimed  
at showing that the "Classics probably never held views like  
those ascribed to them" (Becker and Baumol 1952, p. 355-6),  
but only to conclude that Patinkin's own equilibrium model  
(with initial money endowments among the data of each agent's  
decisions) was closer than the "Classical Dichotomy" model  
to what the 'Classics' meant (36).

23. The case with Valavanis (1955) is more interesting and  
deserves careful examination. Valavanis started by repeating  
Hickman's argument that, from a purely formal viewpoint,  
a Cambridge-type equation is not inconsistent with demand-  
and-supply equations embodying Say's Law in Lange's sense  
and hence determining relative prices only; nor is inconsistency  
created by the addition to these equations of Patinkin's

$D_{n+1} = S_{n+1}$  "money equation", i.e. of the equation stating  
that the demand for money (as defined by Lange) must equal  
the supply of money (again, as defined by Lange), an equation  
that Valavanis appropriately defined the "mirror-image equa-  
tion for money" because it does no more than mirror the money

(36) Later, Baumol (1960) claimed that he had always been clear on the  
distinction between the two notions of equilibrium and the different  
treatment of the money endowment associated with them. His 1952 contri-  
bution does not support that claim; and even in the 1960 article there  
are traces of confusion (see below, note 41).

nst money. This equation, ge nature, is necessarily demand equations for goods is not determinate yet; hich determines the price system nor contradict any

ckman (1950), but did not e of inconsistency because Lange, unclear in Patinkin (1951), is about a sup- ons concerning individual side, the aggregate demand these depend on relative d budget constraint which, ation, implies individuals itial money balances), and pe equation (which implies th their money balances). kin required showing that lies that the agents' in- ; but Valavanis failed to at he was not very clear

"the Cambridge equation lly consistent with, the /alavanis 1955, p.355) but

his distinction between the two (37) did not focus on the difference between the meaning of "demand for money" in the two equations. After reproducing Lange's definition of the (mirror-image) demand for, and supply of, money as (Valavanis 1955, p.353):

$$D_m = \sum_{i=1}^n p_i S_i, \quad S_m = \sum_{i=1}^n p_i D_i$$

(i.e. exactly as in Lange except for an inessential change in the numbering of commodities, money being now the  $n+1$ -th commodity, with index  $m$ ), Valavanis proceeds to write the Cambridge-type equation as (ibid. p.355).

$$MV = \sum_{i=1}^n p_i D_i$$

Here the right-hand side is Lange's definition of the (mirror-image) "supply of money"  $S_m$ . Now, this is a correct writing of a Cambridge-type equation only if the demands for goods which appear on the right-hand side are derived from a LPGE framework, i.e. from agent-choice problems which do not include among the data the initial holdings of money (see

(37) He wrote: "Neither equation explains market behaviour in the sense in which equations apply to commodities. The mirror-image equation is nothing but a reflection of the fact that barter operates through a monetary medium. The Cambridge equation is nothing but a technological restriction on the rate at which money can move. There is no contact between the two" (Valavanis 1955, p. 357). This is not wrong (although one might dispute the "technological restriction" nature of the Cambridge equation), see above in the text, § 18, but one grasps what it means only if one already has grasped the other distinctions, about the notion of equilibrium and the meaning of "demand for money".

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 ret them as derived from  
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 assumption that the initial  
 runner 1951, p.170) (39).  
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claim (Brunner, 1951, p. 171)  
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 , p. 358). He proceeds to con-  
 ial stock of cash ■ and a given  
 a "little Cambridge equation"  
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 this cash can service" (ibid.):  
 ks make even clearer, he follows  
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 ding  $d_i$  (see §10 above), except  
 rather from the value of the  
 ed velocity of money". It is not  
 level of average money balances  
 librium behaviour.

Valavanis as someone who had advanced "the obscure contention  
 ... that money, alone of all goods, must have two equations  
 (termed by Valavanis the "mirror image" and "technological  
 restriction", respectively) to describe its behaviour"  
 (Patinkin 1965, p. 629, note 45; also see Hahn 1960, p.40).

as at the beginning), which they also call simply equilibrium (40).

They correctly contend that the "Classical Dichotomy" only applies to comparative statics exercises where situations are compared in each of which all agents are in full equilibrium. Since in full equilibrium the balanced-budget constraint holds, the individual's demands and supplies of goods in full equilibrium depend on relative prices only, and therefore so do the aggregate demands and supplies. And there is no formal inconsistency in adding a Cambridge equation to determine the price level.

They must then answer the charge that this addition of a Cambridge equation is illegitimate because, as they put it in their own words:

"assume that the system is in equilibrium; double all money prices; since relative prices are unaffected, the goods markets are still in equilibrium; thus either the system leaves money prices undetermined, in the absence of a stock demand for money, or, if there is a stock demand, there are two inconsistent excess demand functions" (Archibald and Lipsey 1958, p.11).

(40) This terminology suggests by itself that a sequence of temporary equilibria was thought of as (normally) converging towards a "full equilibrium". The same belief appears to be shared, in relation to the "real" forces determining relative prices and the composition of capital, by the numerous economists who, while admitting that NWGE heterogeneous-capital models are the only "logically consistent" versions of neoclassical theory, still proceed to use what is to all effects traditional LPGE capital theory for macro-economic problems; most monetarists fall into this category, see e.g., Parkin and Bade (1982), pp. 259 ff.; needless to say, a justification of this procedure is wanting. See above, note 22: and below, Part II, note 49.



that "the model has no  
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for which Walras's Law holds (see above, §18). If all LPGE  
excess demands for goods are zero then the "mirror-image"  
excess demand for money is also zero but we still do not  
know whether the price level is compatible with the quantity  
of money existing in the economy. If not (and this will show  
up in the Cambridge equation), then in the actual economy  
people will not be behaving as the "real" equations would  
tell us, because, as Archibald and Lipsey themselves write  
in their reply to their critics:

"the goods market equations are only supposed  
to tell us what individuals do if their bal-  
ances are in equilibrium; if we arbitrarily  
double money prices, their balances are not  
in equilibrium, and the goods equations are  
not designed to tell us what happens"  
(Archibald and Lipsey 1960, p.53).

But even in this case Walras's Law does hold for the behav-  
iour shown by the goods equations. (42)

Perhaps the consciousness that, traditionally, demand  
for money of an individual meant his average necessary money  
balances would have avoided their confusion on this point.

(42) Thus, it is not accurate to state, as Archibald and Lipsey do, that  
"out of equilibrium, the system does not make economic sense" (ibid.,  
p.16). Systems of equations can make sense in a variety of ways. What  
is true is that, if the price level is out of equilibrium, the goods  
market equations do not describe actual planned behaviour; but the con-  
clusion we can derive from the system, that equilibrium behaviour would  
require, at the given price level, total money balances greater  
(or smaller) than the actual total money endowment of the economy, does  
make sense. More generally, static equations are only supposed to tell  
us, when they are not satisfied, the existence and direction of  
disequilibrium, and this is shown, for the problem of the price level,  
by the fact that the Cambridge or FishAine equation is not satisfied.

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## PART II.

### THE ROLE OF THE STATIONARY STATE ASSUMPTION

# VI - Criticisms of the 'Full Equilibrium' assumption

27. In this second part of the present paper, I will discuss two criticisms advanced against Archibald and Lipsey, both of which aim at concluding that "long-run analysis...is actually restricted to a very special...case" and cannot be extended to deal with economies where "savings can also be held in the form of an interest-yielding asset (say, the bond)" or to "a production economy with investment" (Patinkin, 1965, pp. 58-59).

One criticism (Liviatan 1965) can be summarized as follows. Even in a pure exchange economy, if people can transfer consumption intertemporally by holding bonds, then it may well be that a long - run equilibrium for the individual (in the sense of a "weekly" equilibrium which, if established, will repeat itself unaltered "week" after "week") does not exist; it may be that

"the individual will make use of his interest earnings on savings to generate continuous growth in his levels of wealth, income, and consumption. Correspondingly, he will generally not consume all of the windfall gain represented by the real-balance effect, but will devote part of it to increase his savings and thereby generate a permanent upward shift in the planned growth-path of his wealth holdings including wealth in the form of money. Thus the demand for money in this case will not be independent of the price level" (Patinkin 1965, p. 59).

The other criticism - the one which prompted Archibald and Lipsey's concession to Patinkin quoted in Part 1, § 26

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economies therefore made it necessary to abandon the static method in favour of his own 'dynamic' (i.e. temporary-equilibrium) approach. But Hicks's argument rested on a confusion between two conceptions of the stationary state ( see below § 29); the deeper reasons behind the shift to temporary equilibrium - reasons partly hidden by the misplaced concentration on the issue of stationariness - were rather connected with the difficulties of the treatment of capital as a "factor of production" in a long-period framework (45); were it not for these difficulties, then there would be little to criticize in the traditional method, as will be argued below.

I will now try to show that, even for the questions discussed by Archibald and Lipsey, the issues at stake are fundamentally about "real", rather than monetary, problems, and boil down in the end to questions about capital theory, rather than about stationariness. The demonstration will be in two stages. I will first argue that there are problems - connected with the treatment of capital - with the meaningfulness of Archibald and Lipsey's "full equilibrium", independently of the existence of net accumulation. I will then proceed to argue that, if those problems could be surmounted, then the existence of accumulation would not render the notion of "full equilibrium" - or rather, of LPGE - useless, contrary to the criticisms summarized above. Those criticisms will then appear to rest on an inability to grasp the possibility, afforded within marginalist theory (leaving

(45) See Hicks (1946), pp. 116-19; Garegnani (1976), pp. 30-36.

## VII - Capital Theory and Full Equilibrium

29. Archibald and Lipsey's model with production is the same as Patinkin's Part II model, which like most "Keynesian" models of that period is a one-good model, thus circumventing all capital-theory problems. Patinkin, though, believed that this model was a legitimate way of 'summarizing', so to speak, a many-goods (and heterogeneous-capital) economy (Patinkin 1965, p. 199). I will leave aside here the question of whether this is true for a temporary-equilibrium model, the only model Patinkin considers for the production economy, having already dismissed the possibility of discussing the long period equilibrium of the economy with production (on the basis of the criticisms of Liviatan and Ball and Bodkin).

What I want to notice here is that the acceptance of those criticisms allowed both Patinkin and Archibald and Lipsey to avoid a capital-theory problem, which would have confronted them independently of the existence of net accumulation, if they had tried to discuss the existence of, or tendency towards, "full equilibrium" for any economy with heterogeneous capital goods.

The problem is the following. Even if the aggregate propensity to save were zero (46), still the "real" data cannot all be assumed to be the same "week" after "week", because

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(46) - A rigorous definition of a zero aggregate propensity to net savings for disequilibrium situations is itself ultimately impossible, see Garegnani (1970), p. 425, note 2.



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Archibald and Lipsey appears to be indefensible, because of capital theory problems, even apart from the complications introduced by the absence of stationariness. If one believes, as one should, that value theory should be based on the study of long-period positions, then a new theory of value, which need not treat capital as a "factor of productions", is needed (48).

(48) This new theory can be provided by the classical approach to value and distribution (still often misunderstood nowadays), improved by the contributions of Sraffa and others: for references, see note 5 above, Part I, p . On why value theory appears to need, as its foundation, the determination of "long - period positions", see Garegnani (1976-1979b); also here, Part I, notes 18 and 19, and Petri (1978); the reasons there advanced appear to be further strengthened by various considerations relevant to monetary theory: temporary equilibria with money need introduce one further group of data lacking a sufficient degree of permanence: the initial distribution of the total money endowment among the agents; the assumption of the existence of the auctioneer (or of equivalent mechanisms makes it problematic to grasp the fundamental transactional, medium-of-exchange role of money; on the other side, the fruitfulness in monetary theory of the traditional concern with average tendencies (see above, Part I, § 10) appears undeniable. Probably, not the least reason of the popularity of Patinkin's book was the fact that his macro-economic analysis, although formally couched in temporary equilibrium terms and therefore in line with the trend in value theory at the time, was actually based on such "simplifying" assumptions as to make its comparative static results essentially indistinguishable from those which could have been reached via the traditional method, thus probably preserving, in the eyes of many, that relevance to real-world analysis which is often found lacking in Arrow-Debreu models. Patinkin assumes in fact a single-product world, and further assumes, for most of his analysis, no "redistribution effects" (homothetic indifference maps): the lack of permanence of some data of temporary equilibrium is thus made to disappear, or rendered irrelevant.

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ch. 5, e sp. p. 306; also Rob- onary equilibrium is what Hicks volved in the traditional LPGE. escape route is followed by

independently of the existence, or not, of variations in the endowments of "factors of production" (50). Once the operation of these forces had been grasped, then the results of the analysis could be modified, in a successive approximation, to take into account the forces making for alterations of the factor endowments.

The legitimacy of such a procedure was based on the (reasonable) belief that the variations in factor endowments were generally sufficiently slow as not to endanger the tendency towards an equilibrium of the forces examined under the assumption of (static) stationariness, i.e., in essence, of the forces making for the equilibrium between demand and supply of both goods and factors. This belief was only the form taken in the neoclassical, or marginalist, approach by the older, and more general, belief in the usefulness of starting from the study of "long - period positions", i.e.

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(50) Some examples of these forces and mechanisms considered in marginalist theory, and affecting "real" variables, are: tendency towards the uniformity of the rate of profits over the supply price of capital goods (this force is present not only in marginalist theory); tendency, of decreases in the relative remuneration of a factor, to cause technical changes which increase its relative employment; mechanism of "indirect" factor substitution through changes in output composition owing to changes in consumer choices when changes in relative factor remunerations after relative product prices; tendency to full employment of all factors, resulting from the joint operation of the previous tendencies; etc. An example from the field of monetary theory (this, too, not present only in marginalist authors) is the quantity - theoretic tendency of prices to increase with the quantity of money: see below, § 33.

stant or causes it to increase or to diminish, according as it is just equal to, or greater or smaller than, the flow of capital consumption.

Neoclassical long-period theory is based upon the belief that, given the technological possibilities and the amounts employed of the other factors, the amount of "capital" (treated as a single, homogeneous factor of production) which it will be found most convenient to employ with given amounts of the other factors is a decreasing function of the interest rate.

Let us then consider an economy initially in stationary equilibrium where, suddenly, e.g. because the propensity to save has increased, the rate of interest decreases. The demand for "capital" will go up; and consequently investment too; but - if the interest rate remains at the new, lower level - until the quantity of "capital" becomes the one appropriate to the new level of the interest rate (and to the existing amounts of the other factors) the economy cannot be in equilibrium, because either the other factors are not fully employed, or the entrepreneurs are not in equilibrium, because they are not using the factors in the optimal proportions defined by the new interest rate (55).

Their attempts to reach the optimal factor proportions, together with the tendency towards the full employment of

(55) The "form" of fixed capital will also not be the one best adapted to the new prices associated with the new interest rate, see Garegnani (1978), pp. 346-348.

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32. Reference to the (static) stationary equilibrium, it could have been argued by neoclassical LPGE authors, is nonetheless useful in that it gives us the situation from which the economy is deviating (the stationary state appropriate to the existing quantity of capital) and the situation towards which the economy is tending (the stationary state appropriate to the new level of the rate of interest); the equilibrium distributive variables and relative prices could furthermore be expected to differ relatively little as between the two situations, owing to the implicitly assumed high elasticity of the demand for capital to the rate of interest, (59) and to the relative slowness of the changes in the relative amounts of the factors: the rate of interest would therefore need to change relatively little in order to bring net investment to absorb the eventual net savings; the slowness and graduality of the change made it possible, therefore, to assume that the stationary equilibrium corresponding to the given factor endowments was a good first approximation to the average value of the actual prices and quantities (except, of course, for the determination of investment).

The successive approximation to the analysis of real, non-stationary economies was thus clearly delineated: it was to include, for instance, an analysis of the forces causing

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(59) An assumption necessary for the plausibility of the marginalist theory of distribution, see Garegnani and Petri (1982), pp. 758-759.

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the effect of an increase in the quantity of money assuming a constant output, obtaining thus - under the traditional hypotheses of the quantity theory - a proportional increase in all prices; having thus isolated this mechanism, one can then proceed to inquire whether in fact output will remain constant: at this point, if e.g. full utilization of productive capacity is assumed (as was done not only by neoclassical economists but also by Ricardo), the considerations about "forced savings" etc. can be introduced, and the results concerning the price level variation can then be appropriately qualified (62). The effects of decreases in the quantity of money can be analogously studied in two stages, introducing in the second stage possible effects on output, bankruptcies, etc. In both cases, the second stage of the analysis - one can notice - rests upon the clarity about the basic quantity-theory mechanism achieved in the first stage. Analogous considerations apply to anti-quantity-theory analyses: the reader should have no difficulty in thinking up examples.

(62) E.g. one can consider changes in the volume of transactions and in the velocity of money, due to redistributive effects and to changes in the composition of output, owing to the different propensities to hoard of different classes, and to the different degree of vertical integration of different industries.



and Bodkin

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mined would then be a secular equilibrium), but rather be-  
cause it could be plausibly argued that, in the aggregate,  
the effect on relative prices of the slow variation in the  
distribution of wealth could be safely neglected (except  
perhaps for the analysis of secular trends) (63).

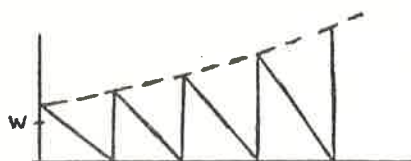
35; Before concluding, it may be worthwhile to stress that  
the assumption of stationariness embodied in "Classical  
Dichotomy" models of the LPGE tradition appears to be moti-  
vated by the neoclassical analysis of real forces; no cor-  
responding need for a stationary state assumption appears  
to be implied by the analysis of monetary forces behind the  
"Classical Dichotomy", i.e. by the Quantity Theory of Money  
(a theory much older than neoclassical "real" analysis,  
and utilized by authors who, like e.g. Ricardo, had no need  
of a stationariness hypothesis to determine relative prices).  
On this score, a few observations may suffice.

(63) The analysis of these secular effects could be left for a successive  
stage of the analysis of accumulation. If there is no accumulation in  
the economy, then relative prices could only be (secularly) affected by  
the redistribution of wealth, if the latter caused changes in the compo-  
sition of demand. Such changes, if only due to the differences in  
individual saving propensities, could in all plausibility taken to be  
so slow as to make the assumption of constant relative prices (equal to  
those determined under a static stationariness assumption) a satisfactory  
one (except perhaps for secular analysis). This is not the case with net  
accumulation, which in the neoclassical approach must be due to a not  
irrelevant divergence of the rate of interest from its (static) station-  
ary level; thus, the complications introduced by overall net accumulation  
(which oblige to analyze the situation as one of disequilibrium, see  
above, § 31) do not appear to arise, or to arise to a similar degree,  
if there is only redistribution of wealth among the individuals.

that determination had been indicated under the assumption of stationariness. But the considerations just advanced for the treatment of output can be repeated here; and they are further strengthened by considering the only secondary importance of what is happening to the single individual as compared with the importance of the average (see the quote from Fisher, above, §10).

Anyway, the extension of the traditional analysis of the individual's transaction "demand" for money to a non-stationary situation would not present great difficulties. In fact, the crucial element in the determination of that demand in the stationary state was not the idea that transaction balance must be, at the end of the "cycle" of the agent, the same as at the beginning; but was rather the idea that the transaction balances must be the minimum ones which allow the agent to carry on his (given) transactions. This idea is generalizable to a non-stationary pattern of transactions. All that is necessary is that a relationship be given between the transaction balances at the end, and those at the beginning of the period under consideration. Two examples of such a relationship have been considered already. The first was the stationary case, with the time period of the analysis as long as the "cycle" of the agent: then the relationship was one of equality. The second was the stationary case, with a time period of different length from an integer multiple of the length of the "cycle" of the agent: then the relationship was a complex one, deri-

will be well defined.



"Fig.2"

So the worker's average need, or "demand" over any period of time, for transaction balances will be well - defined as well, the only difference from the stationary case being that it will not be a constant but a function of time.

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## APPENDIX

WALRAS, THE CLASSICAL DICHOTOMY, AND PATINKIN

## I

In the transition from the traditional method, based upon the study of long - period positions, to the method based on neo - Walrasian equilibria, Walras occupies a crucial role. The peculiarity of his position in this respect, which justifies its description as "half-way in between" the LPGE and the temporary NWGE frameworks, can be described as follows: he seems not to have initially realized that, in formulating the systems of equations characterizing the equilibrium, he was including among the data some magnitudes which were incompatible with what he wanted to determine, i.e., a long-period equilibrium. These magnitudes are the endowments of each type of capital goods. As has been shown elsewhere (1), the inclusion of these magnitudes among the data of the equilibrium makes the system of equations in general devoid of solutions, a fact of which Walras himself became, in time, conscious: in the 4th edn. of the Elements, there appears the admission that

" If we suppose that old fixed capital goods proper... are already found in the economy in quantities  $Q_k, Q_k, Q_k, \dots$  respectively... it is not at all certain that the amount of savings  $E$  will be adequate for the manufacture of new fixed capital goods proper in just such quantities as will satisfy the last 1 equations of the above system. In an economy like the one we have imagined, which establishes its economic equilibrium *ab ovo*, it is probable that there would be no equality of rates of net income. Nor would such an equality be likely to exist in an

(1) See Garegnani (1960); Petri (1978); Tosato (1969); De Vivo (1976).



## II

I have argued above in the text that, analogously as for capital goods, the long - period and the neo -Walrasian frameworks differ in their treatment of the distribution of the total money endowment among the agents at the beginning of the period; this distribution is exogenously given in the neo-Walrasian framework, while it is endogenously determined in the long-period framework; and I have further argued that the balanced-budget constraint assumption only makes sense in the long-period framework, and that in fact this framework was behind that assumption.

The peculiarity of Walras is that, at least in the 4th edition of the *Elements*, he takes as given the distribution of the total money endowment among consumers (not among all agents; on this more presently); and that he also assumes the balanced-budget constraint holds. Walras does therefore make the assumption that each consumer wants to end up the period with the same amount of money he had started with - the assumption which corresponds, at the individual's level, to what Lange would later call Say's Law (2): further, as I shall presently illustrate, in his model entrepreneurs too end the period with exactly the same amount of money they started with (zero); so Walras does assume Say's Law in Lange's sense. In spite of this, the price level in Walras is not indeterminate, owing to the peculiar model he envisages.

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(2) On the contrary, it does not seem that Walras ever wrote down the relationship defined "Walras' Law" by Lange. This will be easier to understand, once his overall model with money will have been illustrated.

use it as working balances, i.e. to buy with it the services and circulating capital goods they need for their firms to carry on the productive activity of that period (4). The "encaisse désirée" of an entrepreneur is determined in a manner analogous to that for consumers. At the end of the period, these bonds are repaid, with interest, to the consumers, and, since the entrepreneurs by assumption make zero net profits (5), all money goes back to the consumers. Monetary equilibrium requires that the consumers' "total effective offer of money" be equal to the total "encaisse désirée" of entrepreneurs (6). Since the first decreases and the second increases with increases of the price level, the equilibrium price level will be uniquely determined.

Patinkin, in his discussion of Walras's theory of money, appears to be a prisoner of his own framework. He nowhere makes clear that, in Walras, the demand for money

(4) I agree with Kuenne when he writes, on this topic: "The cash is not hired to purchase new capital goods: that is done in the securities market, not the promissory note market". Kuenne (1963) p. 315.

(5) This assumption, again, appears to derive from, and to be justified only in, the long-period equilibrium framework. In any period short enough to make the assumption of given composition of capital even remotely plausible, many fixed plants would not in practice be traded and hence would earn a residual quasi-rent which would appear as enterprise's profits (see Petri (1978) p. 255 note 13) and there would be no guarantee that these are just sufficient to repay interest on past debts. If Walras's assumption that capital goods are rented to firms were maintained, then realistically one would have to take into account the existence of fixed rent commitments entered into in previous periods, and there would again be no guarantee that these were the ones appropriate to ensure "ni profit ni perte" to the firms.

(6) See equation (10) in Walras (1954) o. 323.

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(1954), pp. 543-4.

"the equation of monetary circulation, when money is not a commodity, comes very close, in reality, to falling outside the system of equations of (general) economic equilibrium. If we first suppose (general) economic equilibrium to be established, then the equation of monetary circulation would be solved almost without any groping" (Walras 1954, p. 327). Patinkin, with some caution, interprets this passage as showing that Walras had "no objection in principle" to a dichotomization of the pricing process, and was therefore liable to fall under Lange's criticism (Patinkin 1965, p. 561). But in fact Walras's model, owing to the peculiar way the price level is determined, does not fall under Lange's criticism, as was made clear above.

The previous discussion of his model makes it also easy to understand the above quotation, where Walras states that relative prices and the price level are not separately determined (7), but that the influence of the second on the equations determining the first is of minor importance. His point can be put as follows.

The demand equations of a consumer are by assumption constrained by his income, which includes the interest earned on his "quantity of money effectively offered"  $o_u$ .

(7) In his well-known article on Walras's monetary theory, Marget (1931) fails to notice this point, as well as, more generally, the radical departure from the traditional method introduced by the inclusion of the individuals' money endowments among the data of the equilibrium and the incompatibility between this treatment of the money endowment and the balanced-budget constraint.

tion (i.e. the assumption that each agent will end up the period with the same amount of money he started with) is not made less unacceptable by the fact that Walras, by constructing a model in which the demand and supply on the market for short-term loans (to be used by entrepreneurs as working balances) depends on the price level, is able to determine the latter. Walras is clear that the initial money endowments are "random" (Walras 1954, p. 318), i.e. of any magnitude. It is then absurd to exclude the possibility that individuals may want to start the following period with a different amount of money balances (8).

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(8) Kuenne (1963, p. 317) argues that the structure of Walras's model "also allows us to abstract from the short-term problems of building up or reducing cash balances to desired levels over a succession of weeks in the Marshallian tradition. The consumer and the entrepreneur buy the cash balance services they desire, and therefore are capable of reaching an equilibrium for the long term quite quickly". Interesting as they are for their interpretation of Walras as trying to describe a long-period equilibrium, these lines are not convincing. A consumer may well want not to have to borrow cash any more, and thus proceed to build up his cash balances: there is nothing in Walras's model which may justify excluding this choice (the meaning of "long term equilibrium" in these sentences is anyway far from clear, reflecting the dominant confusion among neo-Walrasian theorists as to the use of this and cognate expressions). Kuenne does anyway conclude, in accord with Lange and Patinkin, that the balanced-budget constraint (Say's Identity) should be given up; nor is this surprising, in view of his neo-Walrasian value theory. But, having refused, as shown by the quotation just discussed, the gist of Lange's criticism, the reason he offers for his rejection of the balanced-budget constraint is a new, and I think, unpersuasive one. His argument runs as follows (Kuenne 1963 pp. 320-322). The balanced budget constraint, Kuenne argues, prevents the existence of an economic mechanism to establish i, the rate of interest on money lent for one period.

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that a similar confusion may lie behind his treatment of money endowments. Or, possibly, having started, with his treatment of capital in the 4th edn., to drift away from LPGE towards his concept of "equilibrium ab ovo", he ended up by finding it natural that the individuals' money endowment should be among the data too. Anyway, his lack of realization of the incompatibility of this assumption with the balanced-budget constraint is there to show how much he was still thinking in traditional terms.

The inherent instability deriving to Walras's monetary construction by the contradictory assumptions embodied in it, an instability which may explain why Pareto went back to the traditional analysis of the price level in terms of velocity of circulation etc. (e.g. Pareto 1909, ch.5, §27, p. 297), while Hicks and Lange dropped the balanced-budget constraint (9), can also be seen from a different angle. In order to determine the demand and the supply on the promissory notes market, the price level must be known already. But in fact it cannot be known (because goods and services will be sold afterwards) unless one includes, as an indispensable element of the picture, the existence of groping "actually done with" tickets; and establishing sim-

(9) Walras appears therefore to be the only author who made both the assumptions which Lange thought typical of traditional analysis, and contradictory; and he appears to have done so only because of his inability to grasp how what was implicit in the traditional treatment. Ironically, he nonetheless found a way to make the price level determinate.



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