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NICOLA DIMITRI

**ON THE NOTION OF INDEPENDENCE
BETWEEN EVENTS WITH NON-ADDITIVE
PROBABILITIES**



***QUADERNI DEL DIPARTIMENTO
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Nicola Dimitri

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Abstract

We complement the work by Gilboa-Schmeidler (1993) and axiomatize two notions of independence between events with non-additive probabilities. It is found that among the properties entailed by the independence relations, only commutativity seems to be somehow "robust" with respect to lack of additivity.

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1. Introduction *

In recent years the expected utility (EU) paradigm has been challenged by several theories attempting to capture individuals' behaviour which proved to be inconsistent with respect to some of the main axioms underlying EU. Among these alternative proposals there have been formalizations (notably by Gilboa (1987), Schmeidler (1989) and Gilboa-Schmeidler (1989)) of a decision maker's preferences, that could provide an explanation to the Ellsberg's Paradox findings. In particular Schmeidler (1989), within the Anscombe-Aumann framework (horse race - roulette lotteries), weakens the Independence Axiom and allows it to hold only for Comonotonic acts¹ rather than for all acts. By doing so he is able to obtain a representation theorem for a decision maker's preferences under uncertainty, in terms of the Choquet Integral of the utility function. The word *uncertainty*, as opposed to *risk*, has a specific meaning in this context inherited from Ellsberg's experiment, which refers to agents' lack of information on the probabilities "governing" the relevant random mechanism.

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The Choquet Integral Representation of a function was proposed by Choquet (1955)² when the integrating measure is a normalized capacity, namely a monotonic, not necessarily additive, set function $v(\cdot)$ such that $v(\emptyset)=0$ and $v(S)=1$. Schmeidler's representation then seemed attractive to formalize decision problems under uncertainty since, for example, super-additivity quite intuitively captures agents' lack of reliable information about the probabilistic law of the random process at issue.

As for the case of additive probabilities, a main theoretical problem is to establish the way capacities should be updated in the face of new evidence. The already well known Dempster-Shafer (DS) updating rule (Dempster 1967-1968, Shafer 1976) was recently axiomatized by Gilboa-Schmeidler (1993), who showed it to be the non-additive measure associated with updated preferences, over acts, representable by both a non-additive unique prior and multiple priors. More specifically, they propose a generalized Bayesian updating type of scheme for the former and a Maximum Likelihood principle for the latter, which allow them the following two characterizations of DS: i) the rule is so justified also in terms of the second statistical updating principle and ii) can be seen related to the preferences of a pessimistic agent (uncertainty averse), namely an agent who thinks that in the unrealized event the best possible outcome would have obtained.

While the work by Gilboa-Schmeidler provides an axiomatic presentation of DS, namely of a plausible updating scheme when probabilities are non-additive, a closely related theoretical issue, the concept of independence between events does not seem (to the best of our knowledge) to have yet been treated axiomatically.

With additive probabilities Bayes' rule suggests a natural criterion for formulating a notion of independence, whenever conditional and unconditional (marginal) probabilities coincide; if instead probabilities are non-additive matters seem, at least in principle, less clear-cut. For instance in the most widely explored case of non-additivity, that of super-additivity, "upper and lower probabilities" appear (Dempster 1967, Wasserman 1990) and it is not immediate which of the two one should refer to when formulating an independence criterion³.

This paper building on, and complementing, the work by Gilboa-Schmeidler (1993) aims at representing a preliminary and exploratory endeavour for the provision of a notion of independence between events when the updating rules are DS and the standard Bayesian one. The latter will be indicated with GS since it was characterized by Gilboa-Schmeidler themselves as the updating non-additive measure related to the preferences of an optimistic individual, namely a person who thinks that in the unrealized event the worst possible outcome would have obtained. The note is confined to the discussion of the concept of independence, and will leave

untouched the analysis of, for example, its decision theoretic implications which will be pursued in separate work. The paper has the following structure. In Section 2 we propose two notions of independence and explore some properties of the binary relation, defined on the power set of the state space, entailed by them. In Section 3 after a further analysis of those properties, at the light of a simple alternative characterization of DS and GS, we also investigate commutativity. Section 4 concludes the work.

2. Definitions of Independence and their Properties

We begin by setting the formal framework of the model. Let S be the state space and 2^S the associated power set. A normalized capacity is defined to be a set function $v(\cdot)$ with the following properties

- i) $v: 2^S \rightarrow [0, 1]$
- ii) $v(\emptyset) = 0$
- iii) $v(S) = 1$
- iv) $A \subseteq B$ implies $v(A) \leq v(B)$ for any $A, B \in 2^S$

Uncertainty aversion, the conceptual realm within which DS fails, is formalized by convex capacities identifiable as follows

$$v(A \cup B) + v(A \cap B) \geq v(A) + v(B) \quad (1)$$

Uncertainty seeking, which accommodates the GS updating rule, is represented instead by concave capacities specified according to the following definition

$$v(A \cup B) + v(A \cap B) \leq v(A) + v(B) \quad (2)$$

It is worth anticipating however that the properties of the independence relation (defined on 2^S) induced by the notions proposed will not be related to (1) and (2) and, in general, to the structure of the capacity.

If in (1) A and B are disjoint, for example a partition of the state space, super-additivity of $v(\cdot)$ emerges; analogously, in (2) we would have sub-additivity.

Let us formulate now the updating rules: the Dempster-Shafer formula of updating $v_{DS}(A|B)$ is defined by the following expression:

$$v_{DS}(A|B) = [v(A \cup B^c) - v(B^c)] / [1 - v(B^c)] \quad \text{for } v(B^c) < 1$$

and

$$v_{DS}(A|B) = v^*(A|B) \quad \text{for } v(B^c) = 1$$

where the superscript c indicates the complementary event and $v^*(\cdot|B)$ is a normalized capacity.

The Gilboa-Schmeidler rule of updating $v_{GS}(A|B)$ is defined as follows:

$$v_{GS}(A|B) = v(A \cap B) / v(B) \quad \text{for } v(B) > 0$$

and

$$v_{GS}(A|B) = v'(A|B) \quad \text{for } v(B)=0$$

where $v'(\cdot|B)$ is a normalized capacity.

The full specification of DS and GS is obviously made in order to have well defined updating schemes even when conditioning on an event which entails a null denominator. In the sequel we shall see that some of the properties characterizing the independence relation are valid in this case too.

We are now ready to introduce and discuss the notion of independence; however, before doing so it might be useful to introduce a terminology which could be of some help in justifying the criteria selected. Consider event $A \in 2^S$; then, for any such event, (see e.g. Shafer (1976), Wasserman (1990)) we define the *Belief Function* $B_e(A)$ and the *Plausibility Function* $P_l(A)$ as follows⁴:

$$B_e(A) = v(A) ; P_l(A) = 1 - v(A^c) \quad (3)$$

Evidently, in case of super-additivity we have $P_l(A) \geq B_e(A)$ while for sub-additivity we have $P_l(A) \leq B_e(A)$ the equality holding, for each A , iff v is additive. So, for example, with uncertainty aversion plausibility can be interpreted as the upper probability while belief as the lower probability for event A .

It would seem natural, in analogy with the case of additivity, to define independence by setting the

conditional measure $v(A|B)$ equal to an appropriate "marginal" (non-additive) probability. The two functions in (3) seem both to be reasonable candidates as "marginals" to equalize to $v(A|B)$, and this is indeed what we shall be doing.

Independence between events induces a binary relation σ on 2^S ; in particular, $A\sigma B$ reads "event A is independent of event B ". We shall be specifically interested in the following properties of σ :

- a) $A\sigma B$ iff $B\sigma A$ (symmetry)
- b) $A\sigma B$ iff $B^c\sigma A^c$ (complementarity)
- c) $A\sigma B$ and $A\sigma C$ imply $A\sigma(B \cap C)$ (commutativity)

Below we give the first definition:

Definition 1 Event A is said to be B_e -independent⁵ of B if $v(A|B) = v(A)$.

B_e -independence entails a binary relation σ_b which satisfies the following properties:

Fact 1 σ_b satisfies complementarity for DS and symmetry for GS.

As is well known, with additive probabilities Bayes rule induces an independence relation satisfying both

symmetry and complementarity; with non-additive probabilities instead, B_e -independence separates the two properties which are respectively imputed to GS and DS.

An interesting implication of the above fact is the following. Consider the quantity $|r(A)| = |1 - v(A) - v(A^C)|$ which can be interpreted as a measure of the attitude (aversion vs propensity) towards uncertainty⁶ at A , for an individual whose preferences over acts are representable through $v(\cdot)$. Then, if we write $|r_B(A)| = |1 - v(A|B) - v(A^C|B)|$ and interpret it as the conditional (on B) coefficient of uncertainty attitude at A (for both DS and GS) A being B_e -independent of B is not a sufficient condition for the "amount" of uncertainty aversion/propensity to be the same after B is obtained; i.e. for $r(A)$ to be equal to $r_B(A)$ ⁷.

Consider now the following alternative definition.

Definition 2 Event A is said to be P_1 -independent⁸ of event B if A^C is B_e -independent of B ; namely $v(A^C|B) = v(A^C)$.

The proposal is evidently suggested by the criterion of having the conditional plausibility, namely $1 - v(A^C|B)$, be equal to the unconditional one. As before, we define σ_p to be the independence relation induced by P_1 -independence. Given Fact 1 it is immediate that the following result holds.

Fact 2 σ_p satisfies symmetry for DS and complementarity for GS.

The two facts together manifest an interesting underlying "symmetric" phenomenon characterizing these definitions; the properties at issue, namely symmetry and complementarity, are "inverted" over the updating rules according to the notion of independence selected. Again, this sheds further light on Bayes rule for additive measures, in so far as it reconciles the two facts by satisfying both properties simultaneously. Since GS is indeed Bayes rule⁹, the main role in separating the two properties over the two updating schemes seems to be played by non-additivity.

Analogous considerations, to the ones we put forward earlier, concerning the conditional measure of uncertainty attitude $r_B(A)$ can be provided also for the case of P_1 -independence. Obviously, if A is both P_1 -independent and B_e -independent of B then the conditional coefficient of uncertainty attitude will be equal to the unconditional one (for both DS and GS).

3. An Alternative Characterization of Independence

In order to have a better understanding of Facts 1 and

2 (and help the investigation of commutativity), in this section we introduce and discuss a simple alternative (non decision theoretic) characterization of DS and GS, and consequently of the independence relation, in terms of a pair of pseudo-Bayesian updating schemes. Consider the following definition¹⁰.

Definition 3 We define $P_1(A|B) = P_1(A \cap B) / P_1(B)$ and $B_e(A|B) = B_e(A \cap B) / B_e(B)$ to be, respectively, the plausibility and the belief of event A conditional on event B.

Then, the fact below is immediately verifiable

Fact 3 The functional equations

$$P_1(A^C|B) = 1 - v(A|B) \quad (*)$$

and

$$B_e(A|B) = v(A|B) \quad (**)$$

are solved in $v(A|B)$, respectively, by DS and GS

The above result shows a further, interesting, conceptual common denominator underlying DS and GS. Indeed, they are both obtainable from Bayesian-like updating schemes, in such a way that the former hinges on conditional plausibilities and the latter on conditional beliefs.

Take Definition 1; if that is the case equation (*) entails that

$$P_1(A^C|B) = P_1(A^C) \quad \text{and} \quad P_1(A^C \cap B) = P_1(A^C)P_1(B)$$

and we also deduce

$$P_1(B|A^C) = P_1(B) = 1 - v(B^C) = 1 - v_{DS}(B^C|A^C)$$

Considering (**), a similar reasoning suggests immediately that whenever Definition 1 applies we have

$$B_e(A|B) = B_e(A) \quad \text{and} \quad B_e(A \cap B) = B_e(A)B_e(B)$$

which entails

$$B_e(B|A) = B_e(B) = v(B) = v_{GS}(B|A)$$

Whenever event A is B_e -independent of B, the two pseudo-Bayesian updating criteria induce a product factorization of the joint plausibility and belief (in close analogy with the additive case). However, the events appearing in the factorizations are not the same, and so the "symmetry"¹¹ that springs out of such product factorizations is of different nature. More explicitly, if we consider plausibilities the "symmetric phenomenon" emerging concerns A^C and B whilst for beliefs A and B are the relevant ones.

When Definition 2 applies, product factorization concerns the joint plausibility of A and B and the joint belief of A^C and B.

The above considerations allow us to formulate the following general statement, which holds independently on whether the function is a plausibility or a belief function.

Fact 4 i) The product factorization concerns events A and B if and only if the independence relation induced by the relevant updating rule¹² satisfies symmetry.

ii) The product factorization concerns events A^c and B if and only if the independence relation induced by the relevant updating rule satisfies complementarity.

We now pass to consider commutativity, for the analysis of which the following preliminary result will be useful (Gilboa-Schmeidler, 1993).

Fact 5 For both DS and GS is

$$v(A|D)|_{v(A|B)} = v(A|B \cap D) = v(A|B)|_{v(A|D)}$$

whenever A, B and D in 2^S are such that no relevant denominator, in the formula, is null.

where $v(A|D)|_{v(A|B)}$ is equal to

$$v_{GS}(A \cap D|B) / v_{GS}(D|B)$$

for GS and to

$$[v_{DS}(A \cup D^c|B) - v_{DS}(D^c|B)] / [1 - v_{DS}(D^c|B)]$$

for DS. In other words, $v(A|D)|_{v(A|B)}$ is the conditional measure of A given D , calculated using the conditional measure $v(\cdot|B)$ rather than $v(\cdot)$ ¹³. Non-additivity does not prevent commutativity to keep holding for the updating rules under consideration (as it indeed happens for Bayes rule with additive probabilities); moreover, quite interestingly

the result is true regardless of the structure of the capacity.

To provide some sufficient conditions for commutativity to be valid for the independence relation as well more definitions would be in order; in particular the following concerns GS.

Definition 4 For GS we say that events A and D are conditionally B_e -independent of B , with A, B and D in 2^S , if $v_{GS}(A \cap D|B) = v_{GS}(A|B)v_{GS}(D|B)$. In this case we write $(A, D)\sigma_{b(c)B}$

The above definition is analogous to the one for additive probabilities¹⁴ and in our context could be reformulated, to give a specific characterization, by stating that

$$B_e(A \cap D|B) = B_e(A|B)B_e(D|B)$$

namely that the updated belief of the intersection is factorized as the product of the updated beliefs of the marginals¹⁵. We can now state the following result which is given under the assumptions underlying Fact 5.

Fact 6 Consider GS; if $(A, D)\sigma_{b(c)B}$ and $(A, B)\sigma_{b(c)D}$ then σ_b commutes.

Hence, it is possible to characterize commutativity as relying not only on B_e -independence but also on conditional B_e -independence (in analogy with the case of additivity). It is simple to check that a similar result, after having provided the appropriate notion of conditional P_1 -independence, could be given by substituting belief with plausibility.

Considering now DS we can formulate the definition below.

Definition 5 For DS we say that events A and D are conditionally B_e -independent of B , with A, B and D in 2^S , if

$$v_{DS}(AUD^C|B) - v_{DS}(D^C|B) = v_{DS}(A|B)[1 - v_{DS}(D^C|B)]$$

In this case we write $(A, D)\sigma_{b(C)}B$.

A specific characterization of conditional B_e -independence in this context would then be

$$P_1(D|B) - P_1(A^C \cap D|B) = [1 - P_1(A^C|B)]P_1(D|B)$$

namely

$$P_1(A^C \cap D|B) = P_1(A^C|B)P_1(D|B)$$

a condition analogous to that of the additive case too. The characterizations of conditional B_e -independence for GS and DS have a rather interesting and intuitive interpretation since, as Fact 3 suggests, GS and DS hinge on beliefs and plausibilities. The final result, still under the assumptions behind Fact 3, can now be stated

Fact 7 Consider DS; if $(A, D)\sigma_{b(C)}B$ and $(A, B)\sigma_{b(C)}D$ then σ_b commutes.

As for GS one could replace belief with plausibility to formulate a similar proposition.

4. Conclusions

In this note we have proposed two notions of independence between events, when probabilities are non-additive. The conceptual approach undertaken has been similar to the case of additivity, where independence is defined through Bayes rule when conditional and marginal probabilities are equal. The notions proposed build on, and complement, the work by Gilboa-Schmeidler (1993) who recently axiomatized two remarkable updating rules (Dempster-Shafer and Bayes) when capacities replace standard additive probabilities.

Our main findings are the following. On the one hand, when probabilities are non-additive the independence relation between events (regardless of the definition adopted) does not satisfy, simultaneously, symmetry and complementarity as it happens instead when additive probabilities are updated through Bayes rule. On the other hand, sufficient conditions for commutativity of the

independence relation to hold can be provided in close analogy with the case of additive measures; this would seem to suggest that, unlike the other two properties, commutativity exhibits a form of "robustness" with respect to lack of additivity.

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NOTES

- 1) Roughly speaking, two acts $f, g \in F$ (where F is the set of finite acts on which a preference relation $\geq$ is defined) are said to be Comonotonic if for no pair of states $s, t \in S$ (where S is the state space) is $f(s) > f(t)$ and $g(t) > g(s)$. For more on comonotonicity as related to non-expected utility models see also Wakker (1990).
- 2) See also Schmeidler (1986).
- 3) An analogous ambiguity would arise with sub-additivity.
- 4) It must be underlined that our terminology is a generalization of the one prevailing in the literature. More specifically Shafer (1976), for example, defines a Belief Function to be such that $B_e(A) \leq P_1(A)$; instead, we shall allow the converse as being possible without changing names to the mathematical functions.
- 5) The B_e in front of independence stands for Belief.
- 6) For the case of super-additivity see Dow-Werlang (1992).
- 7) It is worth reminding that with additive probabilities DS and GS coincide with Bayes rule, and $r(A) = 0 = r_B(A)$ for any pair of events A and B in the power set.
- 8) The P_1 in front of independence stands for Plausibility.
- 9) It is important to specify that GS is Bayes rule as "applied" to beliefs; in Section 2 we shall elaborate more on this point and show that a similar characterization holds for DS too.
- 10) In what follows, to simplify the exposition, we are assuming that denominators are never null; it is

straightforward to extend the analysis to cover the case of null denominators.

- 11) Not to be confused with the property of *symmetry* characterizing the independence relation.
- 12) Of course, the relevant updating rules are DS for plausibilities and GS for beliefs.
- 13) $v(A|B) | v(A|D)$ is defined analogously.
- 14) See, for example, Grimmett-Stirzaker (1991).
- 15) Conditional (updated) beliefs and plausibilities will be formally defined in Section 3.

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