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Monetary Conditions in a Classical Growth Cycle

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Introduction*

The aim of this paper is to insert tentatively monetary conditions in a model of fluctuating growth, such as the one expounded by Goodwin in 1965.

Let me start by recalling that a theory aiming at a description and interpretation of economic cycles need to use a non-linear model: This is so because it is a well-known property of linear models that they can give rise to persistent, built-in oscillations only in very special cases. In other words some of the parameters of the model have to take on particular values. In all other cases either we have explosive oscillations or we have dampening ones (1). In the latter case we need therefore external shocks which let the "game" go on. This is not totally unsatisfactory as it is pretty clear that exogenous shocks do play a part in the explanation of persistent cycles, but it is also true that they can always be "spread" over it in order to give, as a result, different shapes to the cycle. The former case i.e. explosive oscillations, is not satisfactory since we are not really looking for explosive movements (2), but just for alternating movements.

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* I am indebted to M. Desai, R. Goodwin, L. Izzo, M. Krüger, J. Steindl and A. Vercelli for helpful discussions. Financial help from the University of Siena is gratefully acknowledged.

(1) We abstract from monotonic paths, of course.

(2) This argument may need qualifications: see Vercelli, A. (1982), "Is Instability Enough to Discredit a Model?", Economic Notes, vol. XI, p. 173.

The advantage of non-linear models is precisely that we get oscillatory movements in spite of any restraints on the values of the parameters. Moreover, in general, in order to make the model dynamic we do not need lags (which of course can be introduced), which are always of a disputable sort. This however does not imply that non-linear models are free from any limits. In general they show a dependence on initial conditions which may be a disappointing one (3). In the following I will also argue that this dependence can be weakened a bit in the modified models I will set up. Let me now turn to the presentation of the first of two models where I try to introduce monetary considerations into the investment function of the prey-predator model elaborated by Goodwin.

The reason for the choice of this model amongst many non linear models lies in the fact that this approach underlines one of the most interesting and persistent features of capitalist economies, i.e. the relationship between distribution of income and cyclical growth.

If one looks even cursory at the historical development of the capitalist system, one is struck by the following circumstance: to show a period of growth it needs labour and/or raw materials to be cheap and in large supply. Goodwin's model captures exactly the essential influence of one of these two factors.

(3) There is an exception however when one is able to show that the model has a unique limit cycle.

Part I

A Model with an Exogenous Rate of Money Supply Growth

1.1. The model I will present and comment on is derived from the paper A Growth Cycle which Goodwin presented at The First World Congress of the Econometric Society in Rome in 1965 (1) and was subsequently elaborated by Izzo (2). The latter is the first attempt to introduce monetary factors in Goodwin's model which underwent various modifications - none of which preserved the original features. The peculiarity of Izzo's presentation is that his model, due to various simplifications, does exhibit the same oscillatory behaviour as Goodwin's original model. This is because the influence of monetary factors on the working of the model is kept to a minimum.

However, I think, it is profitable to scrutinize Izzo's model closely because it allows us to draw some reflections on the interplay between monetary and real factors in business cycle theory. I just want to outline some of the consequences of introducing even in a rough way, monetary considerations into Goodwin's model.

(1) Cfr. Goodwin, R.M. (1967), "A Growth Cycle" in Feistein, C. H. (ed) Socialism, Capitalism and Economic Growth, Cambridge University Press.
(2) Cfr. Izzo, L. (1971), Saggi di analisi e teoria Monetaria, Milano, F. Angeli.

1.2. Let me quickly go through Izzo's model. (I will maintain his notation for convenience).

$$(1)\quad b = X/K$$
$$(2)\quad X/L = a_0 e^{\alpha t}$$
$$(3)\quad I = X-(w/p)L+nK(\vartheta - \mu \dot{X}/X)$$
$$(5)\quad I = \dot{K}$$
$$(6)\quad N = N_0 e^{\lambda t}$$
$$(7)\quad \dot{w}/w = -\varepsilon + \gamma\,(1-L/N)^{-1}$$
$$(8d)\quad \dot{p}/p = [\beta/(1+\beta)](\dot{w}/w - \alpha)$$
$$(11a)\quad S = S(X)$$
$$(10a)\quad M^d = f(p,X,\dot{p}/p,\Pi,K)$$
$$(12b)\quad p(I-S)+V(M^d-M) = 0$$

where X = product

L = employment

I = ex ante investment

$\dot{K} = \partial K/\partial t$

K = stock of capital

w = money wage

N = supply of labour

p = price level

θ = growth rate of money supply.

S = savings

Π = rate of interest

M^d = demand for money

M = stock of money

Eq. (1) maintains that there is fixed proportion between capital and production.

In Eq. (2) we have the usual assumption about technical progress, i.e. it is of the Harrod neutral type.

Eq. (5) says that investment plans are always fulfilled.

Eq. (7) is a non-linear Phillip's curve cast in money terms.

As for the determination of the rate of inflation a completely cost determined formula is used. The implication of this particular formulation is that the rate of price increase is lower than the difference between the growth rate of money wages and the rate of productivity growth. Consequently when $\dot{w}/w > \alpha$ the profits per unit of output (and capital as well) lower and increase for $\dot{w}/w < \alpha$.

Eq. (11a) and (10a) need no comment being, the first one an usual Keynesian savings function, and the second one a demand for money depending positively on p,X,K and negatively on $\dot{p}/p,\Pi$.

Eq. (12b) is a Walras' Law according to Stein's formulation(3). As for the monetary side we can visualize the system as consisting of a single bank which buys bonds issued by firms (4). Monetary base is made of loans to the bank by the Central Bank. We assume that the multiplier is the reciprocal of the reserve requirement ratio, so that money supply is

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(3) Cfr. J. Stein (1969), "Neoclassical and Keynes-Wicksell Monetary Growth Models", Journal of Money, Credit and Banking, Vol. I, p. 153.

(4) Bonds can be thought of as a variety of "call loans".

directly under the control of the monetary authorities (5). Let us now come to eq. (3).

It has a new term which is not in the OGM (6) where investments depend on profits. Now we admit the possibility that, owing to the explicit existence of banks, firms can invest more (or less) than the amount of profits depending on the absolute value of the difference between the growth rate of money supply and the growth rate of income. This difference can be taken as an index of permissive monetary policy, thus being a signal to firms that they will not be frustrated by lack of credit if they feel like investing more than current profits. By simple manipulations one can get the following pair of Lotka-Volterra differential equations:

$$\dot{y}_1/y_1 = h_1 - \beta_1 y_2$$

$$\dot{y}_2/y_2 = -\beta_2 + \sigma_2(1 - y_1)^{-1}$$

where

y_1 = the ratio between employment and labour supply

y_2 = the share of wages in the product

The model can be solved for the real as distinct from monetary side and it can be shown that the behaviour of the model is exactly the same as Goodwin's original model.

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(5) For a more explicit discussion of this, cfr. Izzo, L., op. cit., pages 187-189.

(6) Original Goodwin Model.

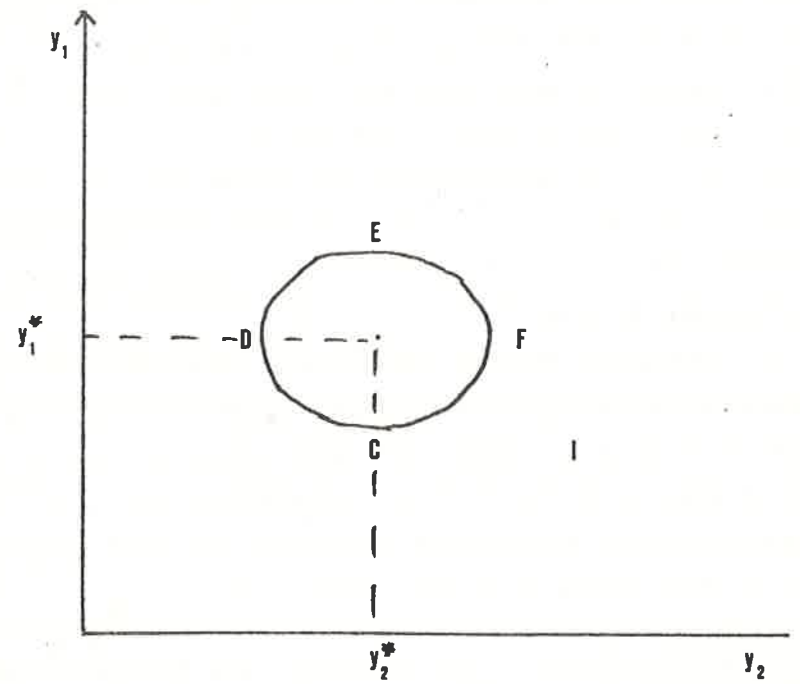
The solution is

$$y^*_2 = h_1 / \beta_1 \quad ; \quad y^*_1 = (\beta_2 - \sigma_2)/\beta_2$$

$$h_1 \equiv [b + n\Theta - (1+n\mu)(\alpha + \lambda)]/(1+n\mu); \quad \beta_1 \equiv b/(1+n\mu)$$

$$\sigma_2 \equiv z/(1+\beta) \qquad \beta_2 \equiv (\varepsilon + \alpha)/(1+\beta)$$

so we have the following diagram:



1.3. It can be shown also that the steady state solution (it occurs if appropriate initial conditions are met) is characterized by a "natural rate of growth" (7), zero rate of inflation and lapse from full employment. Moreover the wage share associated with the steady state solution depends on the particular growth rate of money supply which is in the hands of the Central Bank. Moreover the growth rate of money supply must lie in an open interval (See pp. 8 - 9). The dynamics of the economy when it is off the steady state can be briefly described as follows. In C the growth rate of income is the average one, the employment ratio is below its average value. Therefore money wages grow at a lower rate than productivity, letting the rate of profit rise. Starting from C consequently the growth rate of income rises too to get its maximum value at D where money wages growth equals the growth rate of productivity so that the rate of inflation is zero.

Starting from D the rising employment ratio forces money wages to grow faster than productivity. Consequently prices rise at a lower rate than wages do causing the rate of accumulation to fall to its average level in E. Then the system starts entering a recession and then finally comes to C again where it starts recovering.

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(7) By "natural rate of growth" I mean a growth rate equal to $\alpha + \lambda$, i.e. rate of productivity growth + growth rate of labour force.

1.4. We now look at the monetary side beginning from the case where the economy is on the steady state path.

The main result of the model is that the distribution of income, i.e. the value of y_2^*, is undetermined unless the rate of money growth is fixed. In particular the higher the rate of money growth the higher the share of wages in the product. Since the rate of income growth is exogeuously given then a higher Θ has to be associated with a higher wage share so that accumulation is financed by debt.

It can be shown that if

(i) at $t = t_0$ the rate of interest is equal to the rate of profit

(ii) at $t = t_0$ $I = S$ when $\dot{X}/X = \alpha + \lambda$

(iii) $[\partial M^d/\partial X](X/M^d) \equiv \eta$

then the rate of money growth is steady state has to be $\eta(\alpha + \lambda)$ so that the rate of interest can be continuously equal to the rate of profit, which is an equilibrium condition (8).

If $\eta = 1$ then Θ must be $\alpha + \lambda$.
Indeed the growth rate of real balances per unit of product is equal to $\Theta - \dot{p}/p - \dot{X}/X$.

Being $\dot{p}/p = 0$, Θ has to equal $\alpha + \lambda$ so that real balances can grow at the same rate as the product. It is now important to comment on the fact that Θ has to lie in an open interval:

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(8) Remember that on the steady state path $\dot{p}/p = 0$.

this is required in order to let y^*_2 be positive and less than 1 (9). Izzo shows that

$$(1+\eta\mu)(\alpha+\lambda)/\eta-b/\eta < \Theta < \left[(1+\eta\mu)/\eta\right](\alpha+\lambda) \qquad (26c)$$

Let us recall the meaning of η and μ in equation (3).

$$A \equiv I/K = \left[X-(w/p)L\right]/K-\eta(\Theta-\mu\dot{X}/X)$$

Now $\eta=\partial A/\partial(\Theta-\mu\dot{X}/X)$ and tells us how the rate of accumulation reacts to a given change in monetary policy (for a given value in μ). The larger (smaller) η, the greater (smaller) the effect on the rate of accumulation of a given discrepancy between Θ and $\mu\dot{X}/X$ (10). On the other hand the larger μ, the larger the difference between Θ and $\dot{X}/X$ has to be in order to produce a given effect on the rate of accumulation (11). If we want that even on the steady state path monetary

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(9) In other words we are not interested in solutions where either profits or wage shares are zero.

(10) It has been suggested that η should equal 1; however this is not necessary since the "excessive" liquidity can be hoarded. Of course this will depend inter alia on the level of the rate of interest.

(11) Now the question arises under which conditions the two limits in (26c) do not go to zero or to ∞. If $\eta \to 0$ then $1/\eta + \mu \to \infty$ and both limits "vanish". This is obvious since $\eta = 0$ implies an investment function where monetary factors do not play any role, and therefore η can take on any value. If $\eta \to \infty$ then $1/\eta + \mu \to \mu$ and the condition now becomes $\mu(\alpha+\lambda) < \Theta < \mu(\alpha+\lambda)$ which cannot be fulfilled. In other words when monetary policy is greatly effective the interval shrinks. In this limiting case the condition can be fulfil-

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policy plays a part in the model we require that $\eta \neq \mu$.

On the steady state path $\dot{X}/X = \alpha + \lambda$ and Θ equals $\eta(\alpha+\lambda)$. Now if $\eta = \mu$ then the second term in the investment equation is zero; in general however we can maintain that even on a steady state path monetary policy may play a role.

1.5. Before investigating what will happen to the monetary side when the system is off the steady state path, let me make a general remark on the relationship in this model between parameters and amplitude of fluctuations. It has been maintained by various writers (12) that in this model the amplitude of fluctuations depends on initial conditions. By the latter expression I mean the values of endogenous variables at $t=t_0$. I feel however that the argument should be viewed from a different perspective: given the initial conditions by historical events, what could have happened to the amplitude of fluctuations if the parameters had taken on different values at $t=t_0$, especially the parameters which can be under the control of monetary authorities?

Something can be said on this question which, I think, is perhaps more interesting than the usual question. Indeed performing a linear approximation of the system around equilibrium values and exploiting the well-known fact that,

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...(11) led if we admit the possibility of y_2 being equal to 1. Then Θ has to be $\mu(\alpha+\lambda)$ and accumulation is entirely financed by debt.

(12) Cfr. e.g. Medio A. (1979), Teoria nonlineare del ciclo economico, Bologna, Il Mulino, p. 40.

in this case, fluctuations are ellipse like (13), one can show that when the centre of fluctuations moves north-west (southeast) the amplitude of oscillations decreases (increases) (14). In particular it is interesting to note that as higher (lower) θ is associated, ceteris paribus, with a larger (smaller) amplitude of fluctuations.

Let us suppose now that when profits are lowest (in F) we have the accumulation of capital "sustained" by the Central Bank. This is tantamount to assume that θ attains the highest possible level consistent with the satisfaction of (26c) i.e. (slightly less than) $(1+n\mu)(\alpha+\lambda)/n$. For simplicity let us suppose that μ =1. In this case eq.(3) can be written as

$$I/K = b(1-y_2)_F+n(\alpha+\lambda)+\alpha+\lambda-n\dot{X}/X$$

In general it will be true that $(\alpha+\lambda)(1+n) > n\dot{X}/X$, so that capital accumulation is higher than in OGM. On the other hand when we are in D where the profits are greatest, it is plausible to assume that we have a higher rate of accumulation than in OGM, at least for small fluctuations. Take now the opposite case and assume that the Central Bank chooses θ to be slightly less than $(1+n)(\alpha+\lambda)/n-b/n$ which is the lowest limit which satisfies (26c). In this case eq.

(13) Cfr. Volterra V. (1931), Leçons sur la théorie mathématique de la lutte pur la vie, Paris, Gauthier Villars, p. 21.

(14) For the SW and NE displacements of the centre, the analysis is inconclusive. This is exactly as in the OGM.

(3) at point D becomes:

$$\dot{K}/K = b(1-y_2)_D+(1+n)(\alpha+\lambda)-b-n\dot{X}/X$$

Since $b > \alpha+\lambda$ we conclude that in general the rate of accumulation is lower that in OGM. On the other hand when we are in F it is plausible in the small fluctuations case that investments are lower than in OGM.

1.6. Now when we come to try to describe the actual behaviour of the rate of interest over the cycle we run into difficulties since many paths are feasible, given the unspecified form of the monetary subsystem. Therefore the pattern of the rate of interest behaviour Izzo choosed to give as an example is well-accomodated by the structure of the model which is flexible enough. As an example look at the following table.

	r	$\dot{p}/p$	Π
C	6	-2	6
D	9	0	9
E	6	+2	6
F	3	0	3

where by r we mean the rate of profit

Here we also have $r = \Pi$ throughout, but this is not necessary for the argument.

As a consequence of this case we have that the real rate of interest is higher (lower) than the rate of profit at the point C(E) and they are equal to each other in D and F. This behaviour (as any other) need not disturb the model since as we saw there is no feed-back from the monetary conditions to the real sector.

Let us now ask whether this is the end of the story as in Izzo's paper or are there some other considerations to be made which may lead to a more precise statement.

In this model the introduction of a monetary sector is justified essentially by saying that money is a complementary way of financing growth, in a strictly "Keynesian" fashion (see equation 3). The exercise we did with alternatives θ on a steady state path shows exactly that the path associated with a higher θ has the same rate of income growth in that the associated lower share of profits is compensated for by credit abundance.

Now Izzo shows correctly that on the steady state path the real rate of interest cannot differ from the rate of profit i.e. the rate of interest and the rate of profit are to be equal, the rate of inflation being zero. This, as we saw, implied that the rate of monetary growth takes on a definite value.

1.7. What about the other trajectories? It seems to me that the following considerations do fit into the logic of the model. The basic point is that we should not allow for price incentives which run against the quantity incentives embodied in equation (3). Now this argument seems to imply that when

the difference between θ and $\dot{X}/X$ is maximum (i.e. at point F) then the monetary rate of interest should be higher than or at least equal to the rate of profits in order to have willing savers (15). Remember that here the rate of interest is viewed with reference to the savers outside firms and it is not an element of the investment function i.e. such that investment are higher the lower the rate of interest, which is the cost of financing (16). In other words if the monetary rate of interest were very low in F, the quantity mechanism could possibly be impaired.

In D the opposite holds true, i.e. the monetary rate of interest should be lower than the rate of profit, since the difference between θ and $\dot{X}/X$ is minimum.

The following table differs from Izzo's one and reflects considerations expounded above.

	r	$\dot{p}/p$	Π
C	6	-2	4
D	9	0	6
E	6	+2	8
F	3	0	6

(15) In other words is is a cumbersome way of introducing a saving function which reacts to the rate of interest without destroying the qualitative feature of the original model.

(16) We will come to this point subsequently.

It is easy to show that we can provide more generally a set of hypothesis that will lead to a behaviour of π along the argument sketched above. Let us suppose quite simply, but reasonably, that

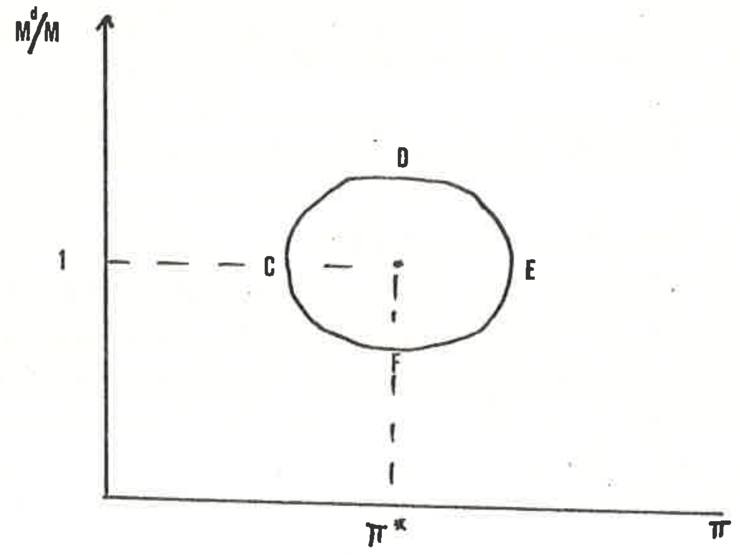
$$\dot{\pi} = lg(M^d/M^*)$$

$$\pi^* - \pi = \frac{\partial}{\partial t} lg(M^d/M)$$

In other words when M^d exceeds (falls short of) M then π is rising (falling) whereas if the growth rate of demand for and supply of money are equal to each other, the nominal rate of interest equals its average value (17). Let us suppose that in C(E) M^d=M and $\theta = \alpha + \lambda$. The latter are made as a benchmark hypotheses. In order to see which restraints we have to impose on the demand for money in order to get the above results, let us postulate a particular form of that equation, namely $M^d = TpX\, e(-\delta\pi)$ which shows a unitary elasticity with respect to p and X. This particular form is chosen only because it allows for a conveniently simple mathematical formulation of the model (18).

(17) Which is $\alpha + \lambda$.
(18) It has been already employed by Phillips A.W. (1961), "A Simple Model of Employment, Money and Prices in a Growing Economy", Economica, vol. XXVIII, p. 360.

The diagram is as follows:



In C $\dot{\Pi}$ =0 since M^d=M and it's a minimum for Π. Indeed $\Pi^* > \Pi$ so that $\frac{\partial}{\partial t}$ lg(M^d/M) = $\ddot{\Pi}$ should be positive.
Now $\frac{\partial}{\partial t} lg(M^d/M) = \dot{p}/p + \dot{X}/X - \delta\dot{\Pi} - (\alpha + \lambda) > 0$.
Now $\dot{X}/X = \alpha + \lambda$, ergo $\dot{p}/p$ should be positive.

This is impossible. Therefore it means that either θ has to be lower than $\alpha + \lambda$, if fluctuations are small, or that the elasticity of M^d with respect to X has to be higher than 1, (the higher the larger the fluctuations).
In D $\dot{\Pi} > 0$ and $\Pi^* = \Pi$.
Now $\dot{p}/p + \dot{X}/X - \delta\dot{\Pi} - (\alpha + \lambda)$ and since $\dot{p}/p = 0$

$$\dot{X}/X - (\alpha + \lambda)/S = \dot{\Pi}$$

This of course is possible since $\dot{X}/X > \alpha+\lambda$ but requires an appropriate δ which is higher the larger the amplitude of fluctuations.

In E $\dot{\Pi}=0$ and Π is at its maximum:

$$p/p+\dot{X}/X-\delta\,\dot{\Pi}-(\alpha+\lambda)<0$$

which is impossible, since $\dot{p}/p>0$.

Therefore either θ has to be higher than $\alpha+\lambda$ if fluctuations are small, or the elasticity of M^d with respect to X has to be lower than 1 and the lower the larger the fluctuations.

Finally in F $\dot{\Pi}<0$.

Now

$$\dot{X}/X-\delta\,\dot{\Pi}-(\alpha+\lambda)=0.$$

Even this is feasible since $\dot{X}/X<\alpha+\lambda$ but again it requires an appropriate δ, which is higher the larger the amplitude of fluctuations.

1.8. Two possible implications emerge from this case study: first that over the cycle θ may not be equal to $\alpha+\lambda$, secondly that alternatively the elasticity of M^d with respect to X should vary over the cycle, in particular falling

(rising) whenever $\dot{X}/X >(<) \alpha+\lambda$. Now another feature in $M^d=TpX\exp(-\delta\Pi)$ is that the elasticity of the demand for money with respect to the nominal rate of interest vary. Indeed $[\partial M^d/\partial\Pi]\Pi/M^d=-\delta\Pi$ therefore the higher the rate of interest the higher the elasticity. This is in contrast with the usual Keynesian assumptions, and indeed implies, as Phillips remarked a very peculiar LM curve, though it is a valid approximation for the usual LM curve everywhere except at very low and very high levels of Π (19). Now it is clear that the exercise is not to be taken too seriously; it only serves as a memorandum of the difficulties of building a real and monetary model of the trade cycle. It helps showing how, though indirectly, the monetary system affects the real behaviour of the economic system. As an example another option could have been chosen with regard to Π. We could have required the real rate of interest and the profit rate to be equal to each other throughout the cycle. This could also serve as an illustration of the restraints on the monetary system in order (not) to impair the real mechanism at work.

A further element which can be inserted and discussed is the dependence of n on Π. In other words it is plausible that when the monetary rate of interest is high (e.g. at point F) n is high meaning that hoarding is not a convenient way of holding wealth.

However the effect on the rate of accumulation will not be qualitatively different from the one sketched above.

(19) Cfr. Phillips A.W., "A Simple Model of Employment, Money and Prices in a Growing Economy", op.cit., p. 365.

Part II
A Model with an Exogenous Rate of Interest

2.1. A similar exercise can also be worked out assuming a different hypothesis on the behaviour of the Central Bank. Let us suppose that the latter aims at controling the money rate of interest rather than the money supply, as was the case for many countries during the 50's and 60's. In this case we leave the model unaltered except for the investment function.

We simply postulate that investments depend on profits and also on the difference between the rate of profit, r, and the money rate of interest. The higher the latter the lower the investment rate. Suppose that (3) is replaced by

$$(3) \quad I = X - (w/p)L + n_K (r - \bar{\Pi})$$

where $\bar{\Pi}$ is the level at which the Central Bank pegs the money rate of interest.
Therefore, by simple calculations we get the usual Lotka-Volterra equations

$$\dot{y}_1 / y_1 = b(1+n) - b(1+n)y_2 - n\bar{\Pi} - (\alpha+\lambda)$$

$$\dot{y}_2 / y_2 = -\beta_2 + \sigma_2 (1-y_1)^{-1}$$

Stationary values are $y_1^* = (\beta_2 - \sigma_2)/\beta_2$ (unchanged) and

$$y_2^* = 1 - [-n\bar{\Pi} + \alpha + \lambda] / b(1+n)$$

We need conditions to be verified in order to have $0 < y_2^* < 1$ (1)

So even in this case the distribution of income is affected by the value of monetary variables, i.e. by the level of Π. The share of wages in the product is higher, the lower Π i.e. the share of profits is higher the higher Π. This makes sense since the higher the share of wages the lower Π has to be to finance the given growth of income. It is easy to show that even in this example $\dot{X}/X = \alpha + \lambda$ on the steady state.

It is also clear that on the steady state path where the rate of profits cannot be different from the rate of interest (inflation being zero), monetary policy is not allowed to exert any influence on I/K. This traces a difference with the previous model where even on the steady state the accumulation can be financed by debt. If therefore the economic system happens to be on the steady state path then, we know, the rate of profit has to be equal to the money rate of interest (since the rate of inflation is zero). Let us call r^* the value r takes on when on the steady path.

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(1) For plausible values of the parameters the condition is satisfied.

$$r^* = b\,(1 - y_2^*)$$

$$= \alpha + \lambda / [1+n] + \bar{\Pi}\, n/1+n$$

The two are equal to each other if $\bar{\Pi} = \alpha+\lambda$, i.e. if the money rate of interest takes on the "natural" rate income growth. The monetary side of the model in this case reduces to an equation describing the rule the Central Bank has to follow in order to have $\bar{\Pi}$ equal to the rate of profit. If we start from a situation where $\bar{\Pi} = r$ and $M^d = M$, then it will be sufficient to furnish the economy a quantity of money which is able to cope with the demand for it which in turn depends on money income and prices only. Therefore let us suppose the same demand for money equations as before.

$$\frac{\dot{M}}{M} = \frac{\dot{M}'}{M'} = \dot{p}/p + \dot{X}/X - \delta\,\bar{\Pi}$$

If δ is quite low then $\dot{M}/M$ tends to $\alpha+\lambda$, otherwise a large δ means that in order to keep $M = M^d$ we have to slow down $\dot{M}/M$, because the demand is quite sensitive to variations in Π and there will permanently be a value Π lower than r^*.

2.2. Now the situation looks different if the economic system is off the steady state path. In order to analyse this case better, we postulate first that the real rate of interest

equals the rate of profit and secondly that I = S. Both conditions hold in C. The latter generally holds true for any cycle in some particular points of it (2). It is only for convenience that we impose it to be satisfied ad C. As for the former assumption it implies that in C the nominal rate of interest will be lower than the rate of profit, since $\dot{p}/p$ is negative.

We can find the value of $\bar{\Pi}$ chosen by the monetary authorities such that the rate of profit and the real rate of interest are equalised.

From $r^* = \bar{\Pi} - \dot{p}/p$ we get

$$\bar{\Pi} = \alpha + \lambda + (1+n)(\dot{p}/p) \quad (3)$$

As a consequence of the first assumption the rate of accumulation in C will be greater than in OGM.

It is also apparent that in D where the rate of profit achieves its maximum value, the rate of accumulation will be higher than in OGM, whereas it is also true that I>S. In E where the rate of profit is back to its average value

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(2) We know that $I = S \Rightarrow \dot{X}/X = (\partial S/\partial X)\, b$. For reasonable values of the marginal propensity to consume and of the coefficient of capital, there will be a rate of income growth which shows the above condition satisfied. Saying that it is going to happen in C (where $\dot{X}/X = \alpha+\lambda$) we are assuming the income rate of growth to be equal to the "natural rate of growth".

(3) The value for Π must not be inconsistent with the condition stated at p. 19. which is to be fulfilled anyway for the model to be economically meaningful. Therefore our argument is to be restricted to a subset of all cycles i.e. those for which $-(\alpha+\lambda)/n < \alpha+\lambda+(1+n)\,\dot{p}/p < b(1+n)/n - (\alpha+\lambda)n$ i.e. $-(\alpha+\lambda)/n < \dot{p}/p < b/n - (\alpha+\lambda)/n$ at point C.

we also have that the money rate of interest is lower than r^* .
Therefore the rate of accumulation will be higher than the OGM, whereas $I>S$.

Finally in F where the rate of profit is at its lowest level, the rate of accumulation could either be lower (and $I<S$) or higher (or even equal to) than that in OGM, depending on the amplitude of the cycle. Therefore if the monetary authorities for any given cycle will peg the money rate of interest to a level which is consistent with the equality (in C) of the real rate of interest and the rate of profit, then the above result will follow. It can be said in this case that the model shows a larger amplitude of oscillations than to OGM. It remains to be said how the monetary authorities can implement this program.

By Walras law excess demand (supply) in the market for goods has to be compensated for by excess supply (demand) in the money market. Consequently if we start in C where $M^d=M$ with a given $\bar{\Pi}$, then we need to have an excess supply of money in D and E. It is sufficient that from C to E the rate of growth of money supply be higher than the rate of growth of the demand for money which is given by.

$$\dot{M}^d/M^d = \dot{p}/p + \dot{X}/X - \delta\dot{\Pi}$$

if we stick to the particular form of the demand for money we already used i.e. $M^d = Tpx \exp(-\delta\Pi)$

Let us now compare the above results with those

obtained by assuming a different objective for the Central Bank. Assume that the monetary authorities wish the equality between the real rate of interest and the rate of profit to be achieved in D (4). Since there the rate of inflation is zero, then the rate of profit equals the money rate of interest too.

Consequently in D the rate of accumulation will be exactly as in the OGM; we assume again that $I = S$, which implay again that the income rate of growth equals the "natural rate". In E we have that the rate of profit is lower than the money rate of interest and the rate of accumulation will be lower than in the OGM. We also maintain that $I<S$.

In F again the money rate of interest will exceed the rate of profit and therefore the rate of accumulation will be lower than in OGM, the consequences is that $I<S$.

Finally in C we have exactly the same as in E and $I<S$ as well. Now in this case we can think of the monetary authorities as a stabilising force in fluctuations.
Even here we know that by Walras Law if we have $I = S$ in D then we need an excess demand for money in E so that is sufficient that from D to E the rate of growth of money supply be lower than the rate of growth of demand for money.

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(4) The reason for that need not concern us. It is sufficient to say that it could be chosen for external reasons (e.g. balance of payments difficulties, etc.)

There will be other cases according to the objective chosen by the Central Bank: the two above meant simply to be illustrative. The final step we will not attempt here is to integrate the monetary and the real aspects of the cycle i.e. by introducing an investment function depending inter alia on an endogenous rate of interest.

However this would be a major modification of the original model and therefore a new way of looking at cycles is probably called for.

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