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Stability of Risk Premia in Italian Stock Market

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Abstract

In this work we estimate the Arbitrage Pricing Theory on the Italian Stock Market using the Reduced-Rank Regression technique recently proposed by Bekker, Dobbelstein e Wansbeek (1996). Due to its computational simplicity, this technique allows extensive empirical analysis of the properties of the estimator employed. Specifically, in this work we carry out a first exploration of the stability of the risk premia estimates in relation to the stocks' sample composition.

Key Words: Arbitrage Pricing Theory, Reduced-Rank Regression.
JEL classification: G11, G12

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1 Introduction

In this work we estimate the Arbitrage Pricing Theory on the Italian Stock Market using the Reduced-Rank Regression technique recently proposed by Bekker, Dobbelstein e Wansbeek (1996). Due to its computational simplicity, this technique allows extensive empirical analysis of the property of the estimator employed. Specifically, in this work we carry out a first exploration of the properties of the estimates obtained through this technique in relation to the composition of the sample of stocks used to estimate the model.

2 The Linear Factor Model

The Arbitrage Pricing Theory (APT, Ross, 1976) assumes the existence of a linear return generation process, known to market participants:

$$\tilde{Z}_t = a + B\tilde{f}_t + \tilde{\varepsilon}_t \, , \tag{1}$$

with

$$\begin{aligned}
E(\tilde{\varepsilon}_t) &= 0, \, \forall t, \\
E(\tilde{f}_t) &= 0, \, \forall t, \\
E(\tilde{f}_t\tilde{f}_s^{'}) &= I, \, t = s, \, E(\tilde{f}_t\tilde{f}_s^{'}) = 0, \, t \neq s, \\
E(\tilde{\varepsilon}_t\tilde{f}_s^{'}) &= 0, \, \forall t, s, \\
E(\tilde{\varepsilon}_t\tilde{\varepsilon}_s^{'}) &= \Sigma, \, t = s, E(\tilde{\varepsilon}_t\tilde{\varepsilon}_s^{'}) = 0, \, t \neq s,
\end{aligned}$$

where $\tilde{Z}_t' = (\tilde{Z}_{1t}, \tilde{Z}_{2t}, \ldots, \tilde{Z}_{Nt})$ is the random vector $(1 \times N)$ of returns on the N assets in period $t = 1, \ldots, T$; a is the vector of expected returns on the N assets; $\tilde{f}_t' = (\tilde{f}_{1t}, \tilde{f}_{2t}, \ldots, \tilde{f}_{Kt})$ describes common factors affecting assets returns; B is a matrix ($N \times K$, of rank K) in which the ith row is the vector of factor loadings of the ith asset; $\tilde{\varepsilon}_t' = (\tilde{\varepsilon}_{1t}, \tilde{\varepsilon}_{2t}, \ldots, \tilde{\varepsilon}_{Nt})$ represents an additional random component specific to each asset. Under the usual assumption that residuals are not correlated, ($\Sigma = D$, a diagonal matrix), and that there exist no asymptotic arbitrage possibilities, from the fundamental APT theorem there exist $K + 1$ constants, $\lambda_0, \lambda_1, \lambda_2, \ldots, \lambda_K$ such that, approximately:

$$a = \lambda_0 i_N + B\lambda\,, \tag{2}$$

where i_N is a vector $(N \times 1)$ of ones, while $\lambda' = (\lambda_1, \lambda_2, \ldots, \lambda_K)$. The constant λ_0 may be interpreted as the risk free rate, the elements of λ as the risk premia for the exposure to the corresponding factors. Ingersoll (1984) demonstrates that (2) obtains also if residuals are correlated if the norm of the residuals correlation matrix is bounded for every N.

3 Reduced Rank Regression Estimators

Empirical tests of the APT model have been carried out through different approaches (Connor, 1995). A commonly used procedure is that of obtaining, in a first pass, estimates of factor loadings through factor analysis on the sample of asset returns. In a second pass, estimates of the risk premia are obtained through a cross-section regression where the factor loadings estimated in the first pass are used as independent variables to explain the assets' average return (Roll e Ross (1980), Chen (1983)). From an econometric point of view, this approach presents some problems, mainly arising from the interaction of the error in variables, as some estimated variables, the factor loadings, are used as independent variables to obtain further estimates, the risk premia[1]. Furthermore, risk premia obtained through this procedure are not unique, due to the factor rotation issue, and they do not allow an economic interpretation. An alternative approach is that of exogenously specifying factors on the basis of economic theory considerations (Chen, Roll e Ross, 1986). Based on such factors, a test of the theory may once again be carried out through a two step procedure, as described above.

McElroy and Burmeister (1988) showed that, by combining (1) and (2), estimation of the Arbitrage Pricing Theory model may be achieved in a single pass considering the model a system of nonlinear equations, Non Linear Seemingly Unrelated Regression (NLSUR), and minimizing an appropriate objective function. By iterating this procedure, estimates asymptotically equivalent to maximum likelihood estimates are obtained. This procedure has been widely used as a valid alternative to the two pass approach in recent empirical works. However, the estimation procedure proposed by

[1]To reduce the error in variables problem in the second pass, a commonly used technique is to group securities into portfolios.

McElroy and Burmeister is computationally intensive, as the optimization of the objective function resulting from NLSUR system of equations, usually of large dimension, must be solved numerically.

In a recent paper, Bekker, Dobbelstein and Wansbeek (1996) show that the APT estimation problem may be recast as a Reduced Rank Regression (RRR) estimation, which is exactly equivalent to the Non Linear Seemingly Unrelated Regression approach proposed by McElroy and Burmeister. Bekker, Dobbelstein and Wansbeek (1996) essentially show how the McElroy and Burmeister model may be conveniently solved: risk premia estimates may be obtained by solving an eigenvalue problem of small dimensionality, while factor loadings estimates are simply obtained through an ordinary least squares regression on rescaled data.

Defining the vector of returns in excess of the risk free rate, $\tilde{Y}_t = \tilde{Z}_t - i_N\lambda_0$ and the matrices $Y' = (\tilde{Y}_1, \tilde{Y}_2, \ldots, \tilde{Y}_T)$, $(N \times T)$, $f' = (\tilde{f}_1, \tilde{f}_2, \ldots, \tilde{f}_T)$, $(K \times T)$, $U' = (\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \ldots, \tilde{\varepsilon}_T)$, $(N \times T)$, e $X = f + i_T\lambda'$, the APT model may be rewritten as:

$$Y = XB' + U\,. \tag{3}$$

Bekker, Dobbelstein and Wansbeek (1996) showed that the model (3) may be interpreted as a RRR model. Defining $A \equiv (B\lambda,\, B) = B\,(\lambda,\, I_K)$, where I_K is an identity matrix, $F \equiv (i_T, f)$, we have,

$$XB' = F \begin{bmatrix} \lambda' \\ I_K \end{bmatrix} B' = FA'\,,$$

and the system of equations (3) may be redefined as:

$$Y = FA' + U \ ,$$

subject to the restriction $A\mu = 0$, where

$$\mu = \begin{bmatrix} 1 \\ -\lambda \end{bmatrix} .$$

This is a RRR model as the matrix of regression coefficients, A, is not of full rank. So, the estimation problem may be set up as:

$$\min_{B,\lambda} \ tr(U'WU)$$

where W $(N \times N)$ is an appropriate weighting matrix. If we set

$$P = F'YWY'F(F'F)^{-1}$$

the risk premia λ are estimated by choosing the eigenvector, μ, corresponding to the smallest eigenvalue of P. By normalizing the first element of this eigenvector to 1, the remaining elements yield the estimate of λ. The estimator of the factor loadings matrix, $\hat{B}$, is then given by $Y'X(XX')^{-1}$, where the matrix X is obtained scaling common factors by the risk premia estimates, $\hat{\lambda}$. Asymptotically efficient estimates are obtained by choosing a weighting matrix W proportional to the estimator of Σ^{-1}, and a suggested choice is $W = T(Y'(I_T - F(F'F)^{-1}F')Y)^{-1}$. Bekker, Dobbelstein and Wansbeek also provide an original and elegant expression for the asymptotic covariance matrix of the RRR estimator applied to the APT model:

$$\sqrt{T}\begin{bmatrix} vec\left(\hat{B}-B\right) \\ \hat{\lambda}-\lambda \end{bmatrix} \rightarrow N\left(0,\, plim\, Q\right),$$

with

$$\begin{aligned} Q &= \begin{bmatrix} (X'X)^{-1}\otimes\Sigma & 0 \\ 0 & 0 \end{bmatrix} + \frac{1}{\phi}\begin{bmatrix} \left(X'X\right)^{-1}X'i_T\otimes B \\ -I_K \end{bmatrix} \\ &\times \left(B'WB\right)^{-1}B'W\Sigma WB\left(B'WB\right)^{-1} \\ &\times \left[i_T'X\left(X'X\right)^{-1}\otimes B',\, -I_K\right], \end{aligned}$$

where $\phi = T - i_T'X(X'X)^{-1}X'i_T$.

4 Estimation and Stability of Risk Premia

Bekker, Dobbelstein and Wansbeek (1996) stress that the RRR methodology allows extensive research on the statistical properties of the model, as it eliminates the computational problems associated with the McElroy and Burmeister approach and therefore allows easy replication of the estimation for different samples.

In the concluding section of their paper, Bekker, Dobbelstein and Wansbeek, taking advantage of this computational ease, investigate the stability of the risk premia estimated on the Dutch stock market using exogenously specified factors in the spirit of Chen, Roll and Ross (1986). Specifically, they consider the stability of the estimated $\hat{\lambda}$ coefficients in relation to the composition of the sample, by allowing the number of assets considered in the estimation procedure to increase from four to forty. They find that, for all the factors considered, the estimates of the λ coefficients are very sensitive to the number of assets in the sample, and, even worst, the sign of the estimated risk premia keeps oscillating as new assets are added to the sample without reaching a stable value. As the risk premium represents the expected return in excess of the risk free rate on a portfolio that replicates a factor, this excess return should not be influenced by the number or choice of assets in the sample, except for the fact that the APT model holds only asymptotically, and not for a finite sample of assets. Such instability in the estimated coefficients may be considered therefore as indirect evidence against the model.

Bekker, Dobbelstein and Wansbeek possibly attribute these empirical findings to the small size of the Dutch stock market, which provides for the analysis too small a number of securities to allow the identification of stable risk premia in the APT model. The authors also investigate whether the instability they found may be caused by the choice of the weighting matrix, W, used in the estimation procedure. They repeat the estimation procedure setting $W = I$, and in this case find a lower variability of the estimates, although the sign of the estimated risk premia keeps changing as the sample size increases. Furthermore, a weighting matrix not proportional to the inverse of the data covariance matrix results in less efficient estimates.

4.1 Estimates Stability on the Italian Stock Market

In this work, starting from the results obtained by Bekker, Dobbelstein and Wansbeek, we analyze the stability of the λ coefficients estimated in the Italian Stock Market. With respect to Bekker, Dobbelstein and Wansbeek, we employ a larger sample of stocks and we attempt to understand better the reason for the instability of the risk premia.

Our data are analogous to those in Roma and Schlitzer (1996). We consider stocks continuously listed in the period January 1989-June 1995 (78 observations). Following the approach of Chen, Roll and Ross (1986), the innovations in six macroeconomic variables are used as factors: industrial production, ERR3, inflation rate, UI, term premium, UTS, oil price, OIL, real exchange rate, ER, stock market return, RMIB, and a residual market factor, MK.

Table 1 reports the estimates of the risk premia obtained through the McElroy and Burmeister Iterated Nonlinear Seemingly Unrelated Regression (ITNLSUR) method used by Roma and Schlitzer, and those obtained through Reduced-Rank Regression, without iterating with respect to the weighting matrix W. Point estimates obtained through the two methods on the same sample considered by Roma and Schlitzer are very close and always have the same sign.

Subsequently, the instability of the risk premia estimated on the Italian Stock Market was investigated. To this end, the λ coefficients were repetitively estimated increasing each time the number of stocks in the sample until the full sample of sixty stocks was considered. Figure 1, which summarizes the result of this estimation experiment, shows that in the Italian market as well estimated risk premia are markedly sample specific and unstable, although their instability decreases after the sample size exceeds forty stocks. This is consistent with the idea that the instability of the estimated risk premia may be due to an insufficiently large sample of stocks. However, sixty stocks seem to be enough, in the Italian market, to ensure some "equilibrium" in the estimates obtained, which appear more stable and show less pronounced sign swings.

Following Bekker, Dobbelstein and Wansbeek, we also estimated the risk premia on the same sequence of stocks samples setting $W = I$. The result is shown in Figure 2, where a substantial reduction in the estimates variability, compared to Figure 1, is apparent. By choosing the identity matrix as the weighting matrix W, the risk premia instability decreases dramatically even for samples containing a low number of stocks. This fact suggest further considerations. The choice of an identity weighting matrix, besides corresponding to the least squares estimation method, implies the hypothesis of no collinearity in the stock returns data matrix. Since the choice which guarantees the minimum variance of the estimated λ coefficients is to set W proportional to Σ^{-1}, setting $W = I$ is equivalent to the assumption that the returns covariance matrix is an identity matrix of order N and that, therefore, the columns of Y are mutually orthogonal. A possible cause of the instability may then be the pronounced collinearity of the columns of the Y matrix, which generates in turns an unstable weighting matrix.

We recall that if a linear return generation mechanism is assumed with arbitrary correlation between the residuals, a sufficient condition insuring a vanishing mean squared pricing error for N large is that the norm of the residuals correlation matrix is bounded for every N, as shown by Ingersoll (1984). This result provides the intuition that pricing errors also depend on the correlation among asset returns in the sample: assets with highly correlated residuals are subject to larger pricing errors in the APT model. Connor (1984) pinpointed that the economy "regularity condition" insuring both the existence of perfectly diversified portfolios and a finite limit for the sequence of risk premia is given by:

$$\lim_{N\to\infty} \left\| \left(B_N' B_N \right)^{-1} \right\| = \lim_{N\to\infty} \left\| Q_N^{-1} \right\| = 0$$

where B_N is the matrix of factor loadings for the N assets. This limit condition corresponds to the absence of collinearity among factor loadings and guarantees the absence of redundant assets.

In order to carry out a more detailed analysis of the instability of the λ coefficients, we may consider the behavior of $\left\| Q_N^{-1} \right\|$ in relation to the collinearity in the stock returns matrix. We computed $\left\| Q_N^{-1} \right\|$ for every sample size considered.

Table 2 shows the values of $\left\| Q_N^{-1} \right\|$ for the last thirty samples of the sixty stocks considered by Roma and Schlitzer. As it is apparent from the Table, for the small sample sizes for which the instability of the estimated coefficients is most pronounced the norm of this matrix presents larger values, while for sample sizes larger than forty stocks its value is closer to zero. To understand whether the instability is related to the collinearity in the returns data we may use the condition number of the stock returns matrix, defined as:

$$K(Y) = \left(\frac{\sqrt{\lambda_{\max}}}{\sqrt{\lambda_{\min}}} \right)$$

where $\lambda_{\max}$ and $\lambda_{\min}$ indicate, respectively, the largest and the smallest eigenvalue of Y. If the matrix Y is standardized in such a way that each column has unit length, then the condition number is equal to one when the columns of Y are mutually orthogonal and becomes larger as the orthogonality is lost, that is the more the columns are collinear. Condition numbers larger than twenty usually indicate the presence of collinearity (Johnston, 1984). As reported in Table 2, the condition number for the entire sample of sixty stocks is 3.44.

Based on the above theoretical considerations, we investigated whether a more appropriate strategy for the construction of the samples used in the estimation yields better results in terms of coefficients stability. We adopted the following two strategies:

a) we increased the sample size by adding stocks from different industries maintaining the industry composition of the sample approximately constant as it grows larger [2];

b) we formed different samples by grouping stocks according to their average transaction volume over the period.

Through the first classification we tried to eliminate that component of the coefficients instability arising from a random selection of stocks which may lead to stocks samples which do not reflect the actual composition of the market, with the risk that for small samples the weight of stocks of a particular industry may be overwhelming. Stocks belonging to the same industry are usually influenced in a similar way by macroeconomic conditions, and their returns are highly correlated. The second classification instead aims at "filling" the samples by grouping stocks with more similar turnover levels, so that the effect of this characteristic may be isolated. Previous empirical analyses, in fact, (see Murgia, 1989) showed that the more liquid stocks have larger factor loadings on macroeconomic variables than less liquid stocks.

According to strategy (a), we used 39 stocks from the Insurance, Banking, Chemical, Mechanical and Textile industries (Table 3a). We increased the sample size by "cycling" over the stock list in Table 3a, each time peaking a stock in the next industry group. As Figure 3 reveals, under strategy (a) the behavior of the estimated risk premia in relation to the sample size is considerably more stable, while the data collinearity is low (as evidenced by

[2] We grouped stocks into industries according to the classification of the Consiglio di Borsa.

the condition number, Table 4). The low collinearity in the Y returns matrix keeps the weighting matrix stable, and the resulting estimates are therefore more similar, in terms of stability, to those obtained setting $W = I$. Table 3 also reports the risk premia point estimates for the sample of 39 stocks, which are all statistically significant.

To implement strategy (b) we considered ninety stocks (we excluded the seven least traded stocks) which we ranked according to their average turnover in the period April 1994 - June 1995. We then subdivided the data into two groups of forty-five stocks and we proceeded to analyze the risk premia stability in relation to the sample size separately for the two groups. Figures 4 and 5 show the behavior of the estimated risk premia in relation to the sample size, for the high turnover and for the low turnover stocks, respectively. As it is apparent from Figure 4, more liquid stocks generate high peaks of variation in the estimated λ coefficients. A possible explanation for this instability is the higher correlation of the returns of the larger and more liquid stocks, with respect to the more idiosyncratic behavior of the returns on the less liquid stocks. Indeed, the low turnover stocks produce more stable risk premia (Figure 5). We must also report, however, that for the two volume sorted groups of stocks the estimated risk premia are often not significant.

In conclusion, at a first empirical analysis, it seems that in the Italian Stock Market for sufficiently large samples of stocks the problem of the instability of the APT estimated risk premia may be less serious than what Bekker, Dobbelstein and Wansbeek (1996) found for the Dutch stock market. Furthermore, an appropriate choice of the sample of stocks, to insure a sufficient cross section dispersion of the estimated factor loadings, contributes to decrease the instability.

Table 1a

Risk Premia Estimation through the RRR and ITNLSUR Approach. January 1989-June 1995

The Table shows the risk premia estimated through the RRR and ITNLSUR* approach on the Roma and Schlitzer (1996) randomly selected sample of sixty stocks, in which all industries are represented.

	UTS	UI	ERR3	OIL	ER
RRR	0.0664	-0.0249	0.3506	-0.3679	0.1223
	(8.31)	(-7.93)	(13.11)	(-5.50)	(6.71)
ITNLSUR	0.0665	-0.023	0.2264	-0.3435	0.1015
	(4.56)	(-7.27)	(8.68)	(-5.24)	(4.82)

$*$ The ITNLSUR λ coefficients are taken from Roma and Schlitzer (1996), Table 5 Panel B.

Table 1b

Risk Premia Estimation through the RRR and ITNLSUR Approach. January 1989-June 1995

The Table shows the risk premia estimated through the RRR and ITNLSUR* approach on the Roma and Schlitzer (1996) randomly selected sample of sixty stocks, in which all industries are represented.The RMIB factor is the return on the Milan Stock Exchange MIB index.

	UTS	UI	ERR3	OIL	ER	RMIB
RRR	0.0797	-0.0267	0.3713	-0.4479	0.0989	0.1922
	(8.16)	(-7.42)	(11.96)	(-5.93)	(4.94)	(19.72)
ITNLSUR	0.0759	-0.0237	0.2453	-0.4173	0.0902	0.1370
	(7.41)	(-6.81)	(8.15)	(-5.64)	(3.89)	(3.36)

$*$ The ITNLSUR λ coefficients are taken from Roma and Schlitzer (1996), Table 5 Panel C.

Table 1c

Risk Premia Estimation through the RRR and ITNLSUR Approach. January 1989-June 1995

The Table shows the risk premia estimated through the RRR and ITNLSUR* approach on the Roma and Schlitzer (1996) randomly selected sample of sixty stocks, in which all industries are represented. The MK factor is defined as the residuals of an OLS regression of the return of the Milan Stock Exchange MIB index on the remaining macroeconomic factors.

	UTS	UI	ERR3	OIL	ER	MK
RRR	0.0797	-0.0267	0.371	-0.4479	0.0989	-0.3566
	(8.16)	(-7.42)	(11.96)	(-5.93)	(4.94)	(-7.13)
ITNLSUR	0.0758	-0.0237	0.2451	-0.4170	0.0903	-0.3673
	(7.40)	(-6.81)	(8.15)	(-5.65)	(3.90)	(-7.06)

$*$ The ITNLSUR λ coefficients are taken from Roma and Schlitzer (1996), Table 5 Panel D.

Table 2

The Table shows the value of the norm of the matrix Q_N^{-1} for the last 30 samples formed out of the Roma and Schlitzer (1996) stocks. In the table, "Stocks" denotes the number of stocks in each sample. The table also shows the condition number K(Y).

Stocks	$\left\Vert Q_N^{-1} \right\Vert$	K(Y)
30	51.02	11.43
31	13.77	11.22
32	2207.19	10.67
33	4980.08	10.26
34	2188.32	9.60
35	580.83	9.30
36	491.42	9.00
37	485.54	8.71
38	646.23	8.50
39	240.44	7.29
40	78.59	7.10
41	47.63	6.86
42	18.26	6.39
43	24.18	6.16
44	174.87	5.68
45	2.83	1.00
46	2.27	5.43
47	1.91	1.36
48	1.91	1.40
49	1.78	5.33
50	2.66	1.92
51	3.51	5.00
52	5.13	2.18
53	4.69	2.52
54	3.70	4.74
55	3.69	2.86
56	2.58	4.40
57	2.58	4.21
58	1.68	3.85
59	1.66	3.56
60	1.50	3.44

Table 3

The table reports the risk premia estimated on the sample of 39 industry sorted stocks, giving approximately equal weight to the Insurance, Banking, Chemical, Mechanical and Textile stocks (Student's t in parenthesis).

	λ	t-stat
UTS	0.10	(3.90)
UI	-0.02	(3.15)
ERR3	0.37	(4.14)
OIL	-0.97	(-3.49)
ER	-0.21	(-2.21)
RMIB	0.14	(6.21)

Table 3a

The Table reports the stocks used to form the samples with approximately equal weight on each industry.

	Insurance	20)	Caffaro Ord.
1)	Latina Ass. Ord.	21)	Snia Bpd
2)	La Fondiaria Ass. SPA	22)	Snia Risp.
3)	Alleanza Ass.	23)	Pirelli SPA
4)	Generali Ass.		**Mechanic. and Automob.**
5)	Milano Ass. Ord.	24)	Westinghouse
6)	Toro Ass. Priv.	25)	Fiat Priv.
7)	Toro Ass. Ord.	26)	Fiat
8)	L'Abeille	27)	Olivetti Priv.
9)	Fondiaria	28)	Olivetti Ord.
10)	Sai	29)	Olivetti Risp.nc
	Banking	30)	Sasib
11)	B.ca Commerciale Ital.	31)	Rejna
12)	Credito Italiano		**Textiles**
13)	Mediobanca	32)	Cantoni Itc.
	Cement and Chemicals	33)	Cucirini Cantoni C.
14)	Boero Bartolomeo	34)	Zucchi SPA
15)	Italgas	35)	Marzotto Risp. cv
16)	Perlier	36)	Marzotto
17)	Saffa Risp.	37)	Linificio 500
18)	Saffa Ord.	38)	Linificio 500 Risp. nc
19)	Caffaro Risp. cv	39)	Manif. Rotondi

Table 4

The Table shows the value of the norm of the matrix Q_N^{-1} for the samples formed out of the 39 industry sorted stocks, giving approximately equal weight to the Insurance, Banking, Chemical, Mechanical and Textile stocks. In the table, "Stocks" denotes the number of stocks in each sample. The table also shows the condition number K(Y).

Stocks	$\|Q_N^{-1}\|$	K(Y)
6	487.96	8.91
7	207.64	8.46
8	324.01	8.18
9	83.10	7.60
10	31.03	7.46
11	30.57	6.97
12	20.27	6.47
13	4.58	5.73
14	4.52	5.86
15	4.36	5.87
16	3.47	5.41
17	5.75	5.24
18	5.59	4.74
19	5.89	4.54
20	4.42	4.36
21	4.91	4.20
22	5.42	3.91
23	4.95	3.66
24	4.63	3.40
25	3.29	3.22
26	3.11	3.10
27	2.59	2.91
28	3.00	2.82
29	4.82	2.58
30	4.46	1.00
31	2.76	1.24
32	3.04	1.33
33	3.00	1.52
34	3.23	2.37
35	3.57	2.07
36	3.48	1.70
37	7.12	1.76
38	7.30	1.87
39	7.23	2.30

Table 5

The Table shows the value of the norm of the matrix Q_N^{-1} for the last 26 samples formed out of more liquid stocks. In the table, "Stocks" denotes the number of stocks in each sample.

Stocks	$\|Q_N^{-1}\|$
19	20.37
20	35.61
21	64.49
22	85.66
23	521.35
24	138.83
25	89.89
26	118.33
27	191.11
28	63528.1
29	26.55
30	237.61
31	200.67
32	77.9
33	202.25
34	340.83
35	154.2
36	191.03
37	22.8
38	23.33
39	12.53
40	12.77
41	28.96
42	27.4
43	35.74
44	34.76
45	34.53

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Figure 1

Estimated Risk Premia and Sample Size

The Figure shows the behavior of the estimated risk premia for increasing sample sizes. We consider the list of 60 stocks reported by Roma and Schlitzer (1996), Table 6. The W weighting matrix is proportional to the estimated Σ^{-1} matrix.

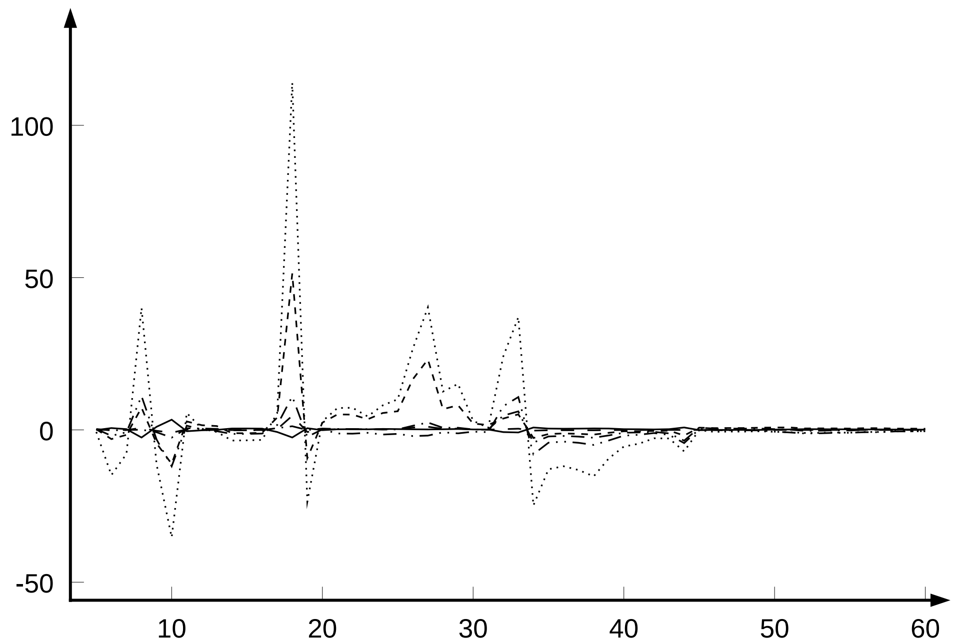

Figure 2

Estimated Risk Premia and Sample Size

The Figure shows the behavior of the estimated risk premia for increasing sample sizes. We consider the list of 60 stocks reported by Roma and Schlitzer (1996), Table 6. As weighting matrix, we set $W = I$.

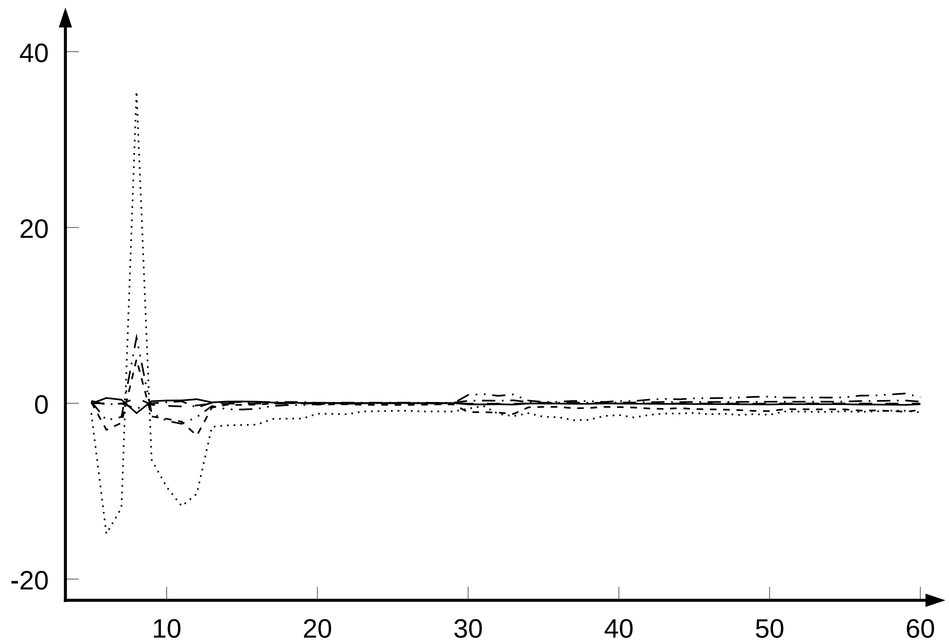

Figure 3

Estimated Risk Premia and Sample Size

The Figure shows the behavior of the estimated risk premia for increasing sample sizes. We consider samples formed from 39 industry sorted stocks, giving approximately equal weight to the Insurance, Banking, Chemical, Mechanical and Textile industries.

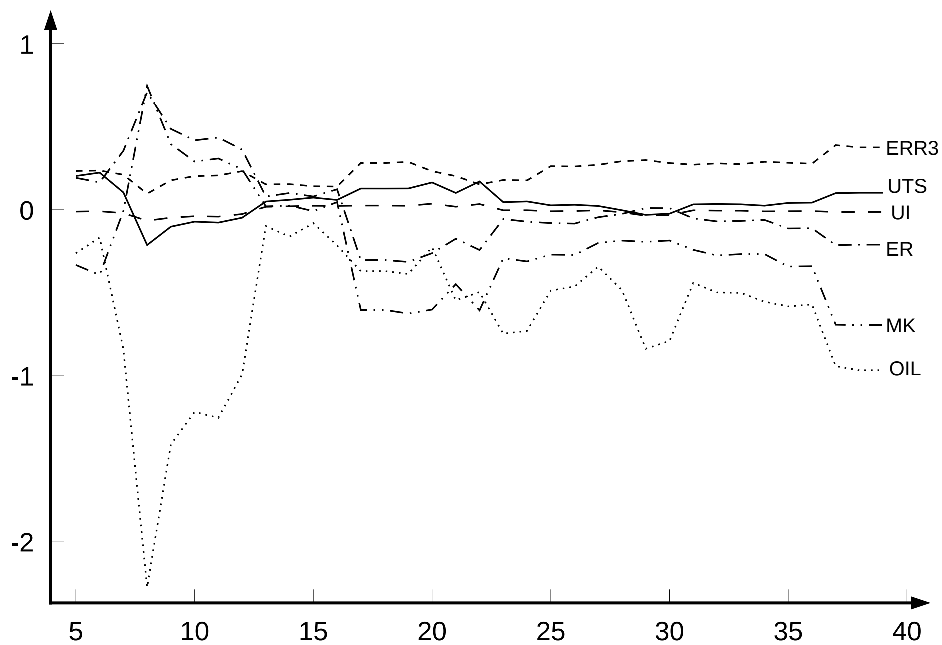

Figure 4

Estimated Risk Premia and Sample Size

The Figure shows the behavior of the estimated risk premia for increasing sample sizes. The 45 stocks with the highest turnover are considered.

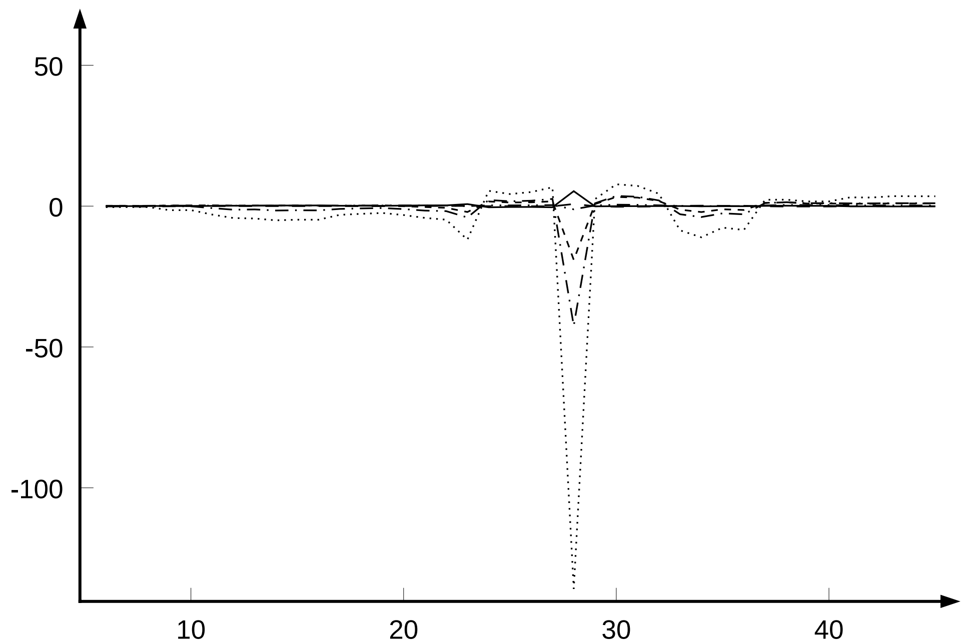

Figure 5

Estimated Risk Premia and Sample Size

The Figure shows the behavior of the estimated risk premia for increasing sample sizes. The 45 stocks with the lowest turnover are considered.