

**Measuring the degree of poverty in a dynamic and  
comparative context: A multi-dimensional approach using  
fuzzy set theory**

**by**

**G. Betti and V.K. Verma**

**Working Paper n. 22, 1998**

# Measuring the degree of poverty in a dynamic and comparative context: A multi-dimensional approach using fuzzy set theory<sup>1</sup>

*Gianni BETTI*  
*Dipartimento di Metodi Quantitativi*  
*Università di Siena*

*Vijay K. VERMA*  
*Director, International Social Research*  
*ORC International*

## 1. Introduction

The traditional approach to poverty measurement identifies the extent of poverty by the proportion of persons whose net monetary household income, 'equivalised' in some way to reflect economies of scale of the size and composition of the person's household, is below the 'poverty line', i.e. a certain percentage of the mean or median of the overall income distribution. While not without a certain degree of descriptive value, such a measure has a number of limitations. Firstly, it is evident that partitioning the population into the simple dichotomy of 'the poor' versus the 'non poor' is an over-simplification. As pointed out by Cheli and Lemmi (1995), poverty is not a simple attribute that characterises an individual in terms of its presence or absence; the relative hardship or well-being of a person is clearly a matter of degree. Secondly, it is insufficient to define poverty in a single dimension, merely in terms of net monetary income; in reality deprivation is multi-dimensional. Thirdly, in the dynamic context, mobility is measured simply in terms of movements across some designated poverty line, rather than to reflect the actual magnitude of the changes affecting individuals at all points of the distribution.

---

<sup>1</sup> This paper has been produced in the framework of the research project "Lavoro e disoccupazione: questione di misura e di analisi", co-financed by MURST 1997-98, and has been presented at the Sixth Islamic Countries Conference on Statistical Sciences, Lahore (Pakistan) 27-31 August 1999.

Consequently, the degree of mobility of persons near to the chosen poverty line tends to be over-estimated, while that of persons far from that line grossly under-estimated. Furthermore in the research on poverty dynamics spells usually replace households or individuals as units of analysis and this leads to a concentration on the duration of poverty and loses sight of its severity. Finally, the conventional measures are purely relative, not taking into account the actual levels of deprivation involved.

This paper reports some ongoing research involving the development of an alternative statistical approach aimed at overcoming some of these shortcomings.

## **2. Beyond the income poverty line**

The objective here is to develop a systematic approach to the extension of the conventional dichotomous classification of the population into ‘the poor’ and ‘the non-poor’ based on monetary income alone. The following steps are involved.

### **2.1 The conventional poverty line**

We begin with the dichotomy classification based on the conventional poverty line. The net monetary annual income of each household is equivalised to take into account economies of scale resulting from differences in household size and composition. Then units (households or persons) are ranked according to their equivalised income, and those below a certain percentage of some measure such as the mean or median of the overall distribution (the poverty line) are deemed to be ‘the poor’. The remainder are considered ‘the non-poor’. The proportion classified as the poor is termed as the ‘head-count ratio’.

Several choices are involved in the actual application of the procedure: the data source on income and other characteristics; the definition of income (what components are included and excluded, the reference period, units of measurement, etc.); the equivalence scale used to convert this to equalised income; the units of analysis (e.g. households versus persons); the population within which the income distribution is studied (e.g. subnational regions, countries, groups of countries); the

statistical measures used for defining the poverty line (e.g. 50% of the mean, 60% of the median, etc.). The specific choices in our numerical results are noted in Section 3.1. These choices of course affects the actual estimates obtained, but are not a critical to our main concern, which is to extend the conventional approach.

## **2.2 Poverty as a matter of degree**

The first extension is to replace the simple poor/non-poor dichotomy by a measure of the degree of, or propensity to, income poverty as a function of the individual's position in the income distribution. This propensity is defined to be in the range 1 (the poorest) to 0 (the richest). The poor therefore are a fuzzy set (as originally proposed by Cerioli and Zani, 1990), comprising in principle the whole population, but each individual only to a degree. Choices need to be made concerning the functional form of this distribution (the 'membership function'), and how it relates to the conventional measure.

## **2.3 Supplementary indicators of the level of living**

In addition to the level of monetary income, the level of living of households and persons can be described by a host of indicators, both quantitative and qualitative including subjective variables, such as housing conditions, possession of durable goods, the general financial situation, perception of hardship, expectations, norms and values, etc. Each of these indicators may be quantified assuming some appropriate functional form as a measure of hardship or degree of *supplementary poverty*. Since most such variables are ordinal (such as yes-no dichotomies or ordered scales), assumptions are required to convert them into metric (numerical) indices.

## **2.4 Composite supplementary poverty index**

Next, it is desirable to combine the diverse indicators into a single (or, at the most, a few) composite index, which supplements the basic income poverty index. This requires the choice of an appropriate system of weighting the individual indices.

## **2.5 Multi-dimensional poverty: overall index combining income and supplementary poverty**

Our basic proposal is to develop income and supplementary poverty indices separately as described above, and then to combine them into an overall index summarising poverty/hardship in its multiple dimensions. This procedure helps in retaining a clear relationship with the conventional measures. Again, a choice is required of the weighting or scaling procedure for combining the income and non-income components. In Section 3.5 we propose two forms of this combination: the manifest poverty index (indicating the presence of both income and supplementary poverty); and the more inclusive latent poverty index (indicating the presence of either form of poverty).

## **2.6 Poverty dynamics: persistent versus transient poverty**

To what extent do individuals and households move in and out of poverty from one period to another? The conventional measure traces this as a count of movements across some chosen poverty line. In the extended framework we study this in terms of changes in the degree of (propensity to) poverty over time. We can distinguish between persistent poverty (present throughout the time interval); and the more inclusive any-time poverty (present at one or more periods comprising the interval). Transient poverty is the difference of the two (present at some but not at all the periods).

## **2.7 Absolute versus relative poverty**

The common measures used in poverty analysis are defined entirely in terms of the distribution of income or other resources within the population, independently of their actual level, and are in this sense purely relative. This applies to most of our analysis as well. Absolute measures require the introduction of the notion of some basic or minimum needs. However, even purely relative measures, but based on the common distribution pooled over several populations, acquire a degree of ‘absoluteness’ as concerns any individual population in the pool – in so far as the common distribution can be seen as an external standard. Furthermore, supplementary (non-income) indicators may provide less relativistic

measures than those based purely on income distribution – in so far as they reflect common standards across different populations. We aim to explore these aspects in the future.

### **3. The model**

This section provides details of the data base and the statistical models we have used in generating the numerical results presented here. Some of these choices are tentative, and alternative and better models can certainly be developed. Our emphasis in this paper is on the general approach proposed, rather than on specific details of the models chosen.

#### **3.1 The data and the conventional poverty indicator ( $p_i$ )**

The empirical illustrations provided here are based on first two waves of the European Community Household Panel (ECHP), which is a multi-country comparative household panel survey conducted annually by following the same sample of households and persons in Member States of the European Union. The survey covers a wide range of topics on living conditions such as income, employment and unemployment, health, housing, financial situation, degree of satisfaction with work and other aspects of life, and so on. Its comparability across countries, measurement of income in detail, multi-subject coverage, and longitudinal or panel design permits us to construct the various measures described below<sup>2</sup>. Based on these data, we have constructed the conventional poverty measures as follows.

Income. The income concept in ECHP is that of net total household income over a full calendar year (1993 and 1994 for the data from the first two waves analysed here). Amounts specified in national currencies have been converted to common units using the ‘purchasing power parities’ for the periods concerned published by Eurostat.

---

<sup>2</sup> The authors are thankful to Eurostat for the preparation of the ECHP Users’ Data Base and making it available for bona fide social research.

Equivalisation. The household incomes are equivalised using the modified-OECD scale, which assigns a weight of 1.0 to the first adult in a household, 0.5 to each subsequent member aged 14+, and 0.3 to each child under 14.

Units of analysis. The units of analysis are persons aged 16+, with the equivalised household income assigned to each person. Similarly, a wide range of household-level supplementary variables are assigned to each household member.

Population. The sample for analysis consists of persons interviewed at both waves with all necessary information obtained, so as to permit longitudinal analysis. Sample cases have been appropriately weighted to provide valid national estimates. Income distribution measures are computed within each country separately. Our main concern, however, is methodological, and it is sufficient to show most of the results aggregated over the whole EU. For this purpose, country estimates are pooled in proportion to the size of the national population aged 16+ residing in private households (see Table 1).

Statistical measure. Within each country, persons aged 16+ are ranked according to equivalised income. The conventional poverty line has been taken as 50% of the mean of that distribution, and persons with income below that line are defined as the poor ( $p_i = 1$ ), and the remaining as the non-poor ( $p_i = 0$ ).

Tab 1: Sample information.

	Original sample size 1994	Original sample size 1995	Matched sample	Full information sample	Population 16+ years	Mean Equivalent income (ECU / year)
Denmark DK	5,903	5,504	5,110	5,103	1.5	14,127
The Netherlands NL	9,407	9,151	8,482	8,353	4.4	13,044
Luxembourg LU	2,046	1,968	1,905	1,897	0.1	22,897
Belgium BE	8,121	7,732	7,086	6,937	2.9	14,021
France FR	14,333	13,306	12,674	12,584	16.1	13,654
Germany GE	8,516	7,958	7,777	7,665	23.8	13,527
Italy IT	17,729	17,780	16,628	15,790	16.9	10,246
Spain ES	17,907	16,276	15,234	15,027	11.1	9,020
United Kingdom UK	10,517	8,391	8,013	7,985	16.3	14,049
Greece GR	12,492	12,271	11,229	11,073	3.0	8,323
Ireland IE	9,904	8,531	7,942	7,892	0.9	12,193
Portugal PT	11,621	11,858	10,955	10,733	2.9	7,667
European Union EU	128,494	120,722	113,030	111,039	100.0	

### 3.2 Income-poverty index ( $q_i$ )

The income poverty index ( $q_i$ ) associated to each individual  $i$  is related to its rank and share in the equivalised income distribution. It is a relative measure. The model we propose is as follow:

$$q_i = V_i^\alpha \quad (1)$$

with  $V_i = \sum_{j=i+1}^n v_j$ ,  $i = 1$  to  $n$ ;  $V_n = 0$ , where  $v_j = \frac{y_j}{\sum_{i=1}^n y_i}$  is the share of total

equivalised income ( $y_j$ ) received by individual of rank  $j$  in the ascending income distribution.  $V_i$  varies from  $V_1 \cong 1$  for the poorest, to  $V_n = 0$  for the richest individual. It is the share of the total equivalised income received by all individuals less poor than the person concerned. As in Cheli (1995), we have determined parameter  $\alpha$  such that the population mean of the index  $q$ , i.e.  $\bar{q}$ , is equal to the proportion in poverty ( $\bar{p}$ ), according to the conventional approach<sup>3</sup>.

Tab 2:  $q_i$  versus conventional measure  $p_i$  and new measures  $x_i$  and  $\mu_i$

	Wave1			Wave2			Wave1			Wave2		
$q_i$	$p_i = 1$	$p_i = 0$	all	$p_i = 1$	$p_i = 0$	all	$x_i > q_i$	$q_i > x_i$	$\mu_i$	$x_i > q_i$	$q_i > x_i$	$\mu_i$
> 1							0.3	0.0	0.3	0.4	0.0	0.4
1 - .9	23.7	0.0	3.9	23.7	0.0	3.8	0.2	3.8	4.0	0.3	3.7	4.0
.9 - .8	16.2	0.0	2.6	15.8	0.0	2.5	0.3	2.6	2.9	0.4	2.4	2.8
.8 - .7	15.2	0.0	2.5	15.4	0.0	2.5	0.5	2.4	2.9	0.5	2.3	2.9
.7 - .6	15.5	0.0	2.5	15.4	0.0	2.5	0.9	2.4>	3.2	1.0	2.3	3.3
.6 - .5	16.8	0.0	2.7	16.9	0.0	2.7	1.6	2.5	4.1	1.6	2.4	4.0
.5 - .4	12.6	1.1	3.0	12.8	1.2	3.0	2.4	2.5	5.0	2.5	2.6	5.1
.4 - .3	0.0	4.2	3.5	0.0	4.1	3.4	4.5	2.8	7.3	4.4	2.7	7.1
.3 - .2	0.0	5.2	4.4	0.0	5.2	4.4	9.3	2.8	12.1	8.6	2.9	11.6
.2 - .1	0.0	7.8	6.6	0.0	7.7	6.4	18.0	3.0	21.1	17.5	3.1	20.6
.1 - 0	0.0	81.6	68.3	0.0	81.9	68.7	29.2	7.9	37.1	29.4	8.9	38.3
Total	100.0	100.0	100.0	100.0	100.0	100.0	67.3	32.7	100.0	66.6	33.4	100.0
Mean	0.73	0.05	0.16	0.73	0.05	0.16						

<sup>3</sup> Large values of  $\alpha$  (in the range 6-13 in our data) are required to meet this condition. Note that with  $\alpha=1$ ,  $\bar{q} = (1+G)/2$ , (where  $G$  is the Gini coefficient of the income distribution), which is typically 3-5 times larger than  $\bar{p}$ . Large values of  $\alpha$  help to concentrate the distribution of  $q_i$  at the lower end.



The first part of Table 2 shows the distribution of the population according to (equal sized ranges of)  $q_i$  values, separately for the conventional poor ( $p_i = 1$ ) and the non-poor ( $p_i = 0$ ). The two measures are of course related. It is seen that a vast majority (over 80%) of the conventionally non-poor also have low  $q_i$  values in the range 0-0.1, while the conventional poor are fairly evenly distributed in the range  $q_i = 1.0-0.5$ .

### 3.3 Indices in terms of supplementary variables ( $s_{k,i}$ )

Table 3 shows the wide range of supplementary variables available for our analysis using the ECHP. These indicators concern amenities in the household, ability to afford durable goods, problems with accommodation, and subjective variables on perception of hardship<sup>4</sup>.

The merit of including also monthly net income here is to show that had we not separated out the income variable ( $q_i$ ), it would have been simply 'lost' with a small weight among the diverse supplementary variables.

Most of the supplementary variables (see Table 3) are ordinal, with mostly 2 but sometimes more categories. To treat them as metric, we assign values to the categories as proposed by Cerioli and Zani (1990). For each variable  $k$ , with ordered categories 1 (most deprived) to, say,  $M$  (least deprived), individuals in category  $m$  are given the score:

$$v_{(m)} = \frac{M - m}{M - 1} \quad (2)$$

so that  $v_{(1)} = 1$  and  $v_{(M)} = 0$ . From (2) we define the value  $V_{k,i}$  for each individual in category  $m$  as:

$$V_{k,i|i \in m} = \sum_{l=m+1}^M v_{(l)} f_{(l)} ; V_{k,i|i \in 1} = 1 ; V_{k,i|i \in M} = 0 \quad (3)$$

---

<sup>4</sup> In relation to non-possession of durable goods, only those wanting but unable to afford them are considered deprived.

where  $f_{(l)}$  is the relative frequency, in terms of individuals, in class  $l$ . Following the same form as for  $q_i$  above, we can define the degree of deprivation in terms of supplementary variable  $k$  for individual  $i$  as:

$$s_{k,i} = V_{k,i}^\alpha \tag{4}$$

using the same value  $\alpha$  as determined in (1). This parameter makes no difference in the case of a dichotomy (where the possible values of  $V$  are confined to 0 and 1), but with many categories (especially for continuous variables), large values of  $\alpha$  make the distribution of  $s$  concentrated at the lower end – which is a desirable property of a poverty index.

### 3.4 Supplementary poverty index ( $s_k$ )

In order to put together indices for various supplementary variables, we take their weighted sum:

$$s_i = \frac{\sum_{k=1}^K w_k \cdot s_{k,i}}{\sum_{k=1}^K w_k} \tag{5}$$

The weights  $w_k$  are determined by the following statistical considerations. Alternative models are possible. Furthermore, account may also be taken of substantive considerations in particular situations.

[a]. Firstly, the weight is determined by the variable's power to “discriminate” among individuals in the population, that is, by its dispersion. We take this as proportional to the coefficient of variation  $w_k^a \propto cv_k$ . Note that for small proportions, the weight varies inversely to the square-root of the proportion. Thus deprivations which affect only a small proportion of the population, and hence are likely to be considered more critical, get large weights; while those affecting large proportions, hence likely to be regarded less critical, get small weights.

[b]. From a non-redundant point of view, it is necessary to limit the influence of those characteristics that are highly correlated with the others. The weight of

variable  $k$  is taken as the inverse of an average measure of its correlation with all the other variables:

$$w_k^b \propto \left( \frac{1}{1 + \sum_{k'=1}^K \rho_{k,k'} \mid \rho_{k,k'} < \rho_H} \right) x \left( \frac{1}{\sum_{k'=1}^K \rho_{k,k'} \mid \rho_{k,k'} \geq \rho_H} \right) \quad (6)$$

where  $\rho_{k,k'} = \text{corr}(s_{k,i}, s_{k',i})$  is the correlation between the two indicators.

In the first term in the right side of (6), the sum is taken over all indicators whose correlation with the variable  $k$  is less than a certain value  $\rho_h$  (determined, for instance, by dividing the ordered set of correlation values at the point of the largest gap.). The sum in the second term always includes the case  $k' = k$ , when the correlation coefficient is 1.0. The motivation for this model is that (i)  $w_k^b$  is not affected by the introduction of variables entirely uncorrelated with  $k$ ; (ii) only marginally affected by small correlations; but (iii) is reduced in proportion to the number of highly correlated variables present. The final weight is taken as the product of the two factors:  $w_k \propto w_k^a \cdot w_k^b$ . Note that the scale of the weights can be arbitrary. In Table 3, these have been scaled to average 1.0, i.e.,  $\sum w_k = \sum w_k^a = \sum w_k^b = K$ , the number of variables.

### 3.5 Latent and manifest poverty indices ( $\mu_i, \nu_i$ )

A more problematic aspect is the putting together of the income and supplementary poverty indices,  $q_i$  and  $s_i$ . The main difficulty arises from the fact that, as defined above, the scale of  $s_i$  is essentially arbitrary (determined by the numerical scores assigned to ordinal categories). Tentatively, we propose to scale  $s_i$  to give the same average as  $q_i$ , i.e. define the supplementary poverty index as

$$x_i = \left( \frac{\bar{q}}{\bar{s}} \right) \cdot s_i \quad (7)$$

Tab 3: Supplementary variables.

	Wave 1				Wave 2			
	$S_k$	$W_{ka}$	$W_{kb}$	$W_k$	$S_k$	$W_{ka}$	$W_{kb}$	$W_k$
<i>Not having amenities</i>								
Toilet	0.02	2.59	1.07	2.68	0.02	2.59	1.08	2.66
Bath	0.02	2.31	0.99	2.21	0.02	2.67	0.95	2.45
Hot water	0.04	2.22	1.03	2.25	0.05	2.29	1.01	2.18
Kitchen	0.06	2.13	1.48	2.82	0.05	2.05	1.47	2.65
Garden	0.17	0.77	1.23	0.86	0.16	0.72	1.25	0.81
Central heating	0.24	0.58	0.92	0.51	0.22	0.58	0.92	0.49
<i>Unable to afford durable goods</i>								
Colour TV	0.02	2.51	1.26	2.96	0.01	2.68	1.28	3.15
Telephone	0.05	1.53	1.10	1.59	0.04	1.51	1.15	1.63
Car	0.08	1.14	1.03	1.14	0.07	1.05	1.03	1.03
Video recorder	0.10	0.85	0.96	0.75	0.09	0.85	0.95	0.73
Micro wave	0.12	0.82	0.94	0.72	0.11	0.80	0.92	0.68
Dish washer	0.17	0.65	0.95	0.58	0.16	0.61	0.94	0.53
Second home	0.39	0.35	1.24	0.42	0.39	0.33	1.25	0.38
<i>Problems with accommodation</i>								
Roof	0.07	1.00	1.13	1.02	0.06	1.00	1.12	1.01
Light	0.10	0.86	1.27	0.98	0.09	0.83	1.23	0.91
Rot	0.10	0.86	1.02	0.79	0.09	0.85	1.00	0.76
Heating	0.12	0.84	1.00	0.83	0.10	0.97	0.95	0.96
Damp	0.15	0.70	0.98	0.63	0.12	0.70	0.98	0.62
Pollution	0.17	0.62	1.31	0.74	0.17	0.57	1.24	0.64
Space	0.20	0.55	1.18	0.59	0.18	0.54	1.15	0.56
Vandalism	0.20	0.61	1.36	0.77	0.18	0.58	1.30	0.69
Noise	0.27	0.47	1.24	0.53	0.25	0.45	1.18	0.48
House	0.36	0.25	0.80	0.19	0.55	0.22	0.79	0.16
<i>Cannot afford</i>								
Eat meat etc.	0.07	1.15	0.88	0.95	0.06	1.15	0.90	0.97
New clothes	0.14	0.74	0.68	0.47	0.13	0.69	0.66	0.43
Entertain friends	0.16	0.65	0.66	0.40	0.14	0.63	0.68	0.39
Adequate heating	0.17	0.98	0.79	0.82	0.16	0.98	0.84	0.85
Annual holidays	0.31	0.43	0.48	0.20	0.31	0.40	0.53	0.20
New furnitures	0.39	0.35	0.47	0.15	0.40	0.31	0.51	0.14
<i>Financial problems</i>								
Mortgage	0.02	2.13	1.22	2.39	0.01	2.31	1.32	2.82
Higher purchase	0.03	1.65	1.04	1.60	0.02	1.76	1.12	1.85
Rent	0.03	1.81	1.06	1.84	0.02	1.79	1.12	1.89
Utility bills	0.06	1.29	0.89	1.08	0.05	1.32	0.96	1.21
Debt burden	0.15	0.96	1.16	1.09	0.13	0.65	1.13	0.69
Change in situation	0.26	0.11	1.20	0.12	0.24	0.09	1.11	0.09
Financial difficulty	0.29	0.13	0.39	0.05	0.28	0.12	0.42	0.04
Can save	0.60	0.22	0.72	0.14	0.58	0.20	0.71	0.13
Current tot net monthly income	0.16	0.18	0.87	0.14	0.16	0.16	0.86	0.12
All	0.07	1.00	1.00	1.00	0.06	1.00	1.00	1.00

Now by combining the income and supplementary indices, we have the following set of poverty indicators:

[0]	Conventional	0,1	$p_i$
[1]	Income poverty	0-1	$q_i$
[2]	Supplementary poverty		$x_i$
[3]	Latent poverty	Either of [1] or [2]	$\mu_i$
[4]	Manifest poverty	Both of [1] and [2]	$v_i$

[1] and [2] are fuzzy sets; [3] is their union

$$\mu_i = \max(q_i, x_i) \quad (8)$$

and [4] their intersection

$$v_i = \min(q_i, x_i) \quad (9)$$

The second part of Table 2 shows the distribution of the population according to (equal sized ranges of)  $\mu_i$  values. For each column, also shown is the breakdown according to the relative size of the income and supplementary indices,  $q_i$  and  $x_i$  for the individual concerned. A small minority are highly poor in terms of the supplementary index. Apart from that, high values of  $\mu_i$  are characterised by higher value of income compared with supplementary poverty ( $q_i > x_i$ ), and low  $\mu_i$  by its converse ( $x_i > q_i$ ) – implying that there is less inequality in terms of the supplementary index compared to that in terms of monetary income.

### 3.6 The dynamic context

Denote with  $\mu_i^t$  and  $\mu_i^{t+1}$  individual degrees of poverty of unit  $i$  at two successive points of time  $t$  and  $t+1$ . These are two fuzzy sets (Dubois and Prade, 1980, Zadeh, 1965), and hence we can define measures of persistence or otherwise of poverty at the individual level as follows. Persistent poverty, i.e. present at both times, as the intersection of the two sets:

$$\mu_i^{(P)} = \min(\mu_i^t, \mu_i^{t+1}) \quad (10)$$

Any-time poverty, that is at one or both of the times, as the union of the two fuzzy sets:

$$\mu_i^{(S)} = \max(\mu_i^t, \mu_i^{t+1}) \quad (11)$$

Transient poverty, i.e. at one but not both times, is the difference of the above, while non-poverty is the complement of (11).

The method can be extended to any number T of periods (years). The indices of persistent poverty and any-time poverty are defined as follow:

$$\mu_i^{(P)} = \min(\mu_i^1, \mu_i^2, \dots, \mu_i^t, \dots, \mu_i^T) \quad (12)$$

$$\mu_i^{(S)} = \max(\mu_i^1, \dots, \mu_i^t, \dots, \mu_i^T) \quad (13)$$

More indices can be calculated in the case of T periods. Let  $\mu_i^{[j]}$  be the ranked value for individual i so that  $\mu_i^{[1]} \leq \mu_i^{[2]} \leq \dots \leq \mu_i^{[T]}$ . Then  $\mu_i^{[1]} = \min(\mu_i^1, \mu_i^2, \dots, \mu_i^t, \dots, \mu_i^T) = \mu_i^{(P)}$  is the propensity of being always poor throughout the period T, i.e. the index of persistent poverty;  $\mu_i^{[T]}$  is the index of any-time poverty during T. in general,  $\mu_i^{[j]}$  can be seen as the propensity of being poor for at least  $(T + 1 - j)$  of the T periods. Since we can identify propensity to persist in poverty to varying degrees, it will be very instructive to compare  $\bar{\mu}^{[1]}$ ,  $\bar{\mu}^{[2]}$ , ...,  $\bar{\mu}^{[T]}$  against the average  $\bar{\mu}$  over T periods.

#### 4. The main results

Main results are reported in table 4; this composed of six panels. Panel one briefly reports aggregate statistics for the European Community, while panels 2 to 6 report statistics, country by country, on conventional measure  $p_i$ , fuzzy income  $q_i$ , supplementary variables  $x_i$ , income or supplementary  $\mu_i$ , income and supplementary  $v_i$ . Each panel is sorted by ascending traditional poverty measure. Columns 2 to 4 report each measure for wave 1, wave 2 and the arithmetic mean of the years; in columns 5 to 8 are reported persistent and anytime poverty, as well as their ratio on the average value of poverty in the two waves. Finally the last

column reports the ratio of the country measure, the mean of the two waves, to the European average from panel 1.

Tab 4: Comparison of conventional and new dynamic measures.

EU	wave1	wave2	m (mean)	persistent	any time	per/m	any/m	
$p$	0.16	0.16	0.16	0.10	0.22	0.62	1.38	
$q$	0.16	0.16	0.16	0.11	0.22	0.66	1.34	
$x$	0.16	0.16	0.16	0.12	0.21	0.73	1.27	
$\mu$	0.26	0.25	0.25	0.19	0.32	0.74	1.26	
$\nu$	0.07	0.07	0.07	0.04	0.09	0.63	1.37	
conventional, $p$								
	wave1	wave 2	mean	persistent	any time	persistent/ mean	any time / Mean	country/ EU
DK	0.07	0.08	0.07	0.03	0.11	0.46	1.54	0.44
NL	0.08	0.08	0.08	0.04	0.12	0.53	1.47	0.52
LU	0.14	0.13	0.14	0.09	0.19	0.63	1.37	0.84
BE	0.12	0.14	0.13	0.07	0.20	0.51	1.49	0.81
FR	0.15	0.14	0.15	0.09	0.20	0.62	1.38	0.90
GE	0.14	0.14	0.14	0.09	0.19	0.64	1.36	0.86
IT	0.17	0.17	0.17	0.10	0.23	0.60	1.40	1.03
ES	0.19	0.18	0.18	0.11	0.25	0.63	1.37	1.13
UK	0.20	0.19	0.20	0.12	0.27	0.62	1.38	1.23
GR	0.22	0.21	0.21	0.14	0.29	0.65	1.35	1.31
IE	0.19	0.24	0.22	0.15	0.29	0.67	1.33	1.34
PT	0.25	0.24	0.25	0.18	0.31	0.74	1.26	1.54
fuzzy, income, $q$								
	wave1	wave 2	mean	persistent	any time	persistent/ mean	any time / mean	country/ EU
DK	0.07	0.08	0.07	0.04	0.11	0.51	1.49	0.44
NL	0.08	0.08	0.08	0.05	0.12	0.56	1.44	0.52
LU	0.14	0.13	0.14	0.09	0.18	0.69	1.32	0.84
BE	0.12	0.14	0.13	0.07	0.19	0.57	1.43	0.81
FR	0.15	0.14	0.15	0.09	0.20	0.65	1.35	0.90
GE	0.14	0.14	0.14	0.09	0.19	0.67	1.33	0.86
IT	0.17	0.17	0.17	0.11	0.23	0.64	1.36	1.03
ES	0.19	0.18	0.18	0.12	0.24	0.67	1.33	1.13
UK	0.20	0.19	0.20	0.13	0.26	0.67	1.33	1.23
GR	0.22	0.21	0.21	0.15	0.28	0.70	1.30	1.31
IE	0.19	0.24	0.22	0.16	0.28	0.73	1.27	1.34
PT	0.25	0.24	0.25	0.19	0.31	0.77	1.23	1.54
supplementary not bounded, $x$								
	wave1	wave 2	mean	persistent	any time	persistent/ mean	any time / mean	country/ EU
DK	0.07	0.08	0.07	0.05	0.10	0.66	1.34	0.44
NL	0.08	0.08	0.08	0.06	0.11	0.70	1.30	0.52
LU	0.14	0.13	0.14	0.09	0.18	0.70	1.30	0.84
BE	0.12	0.14	0.13	0.09	0.17	0.70	1.30	0.81
FR	0.15	0.14	0.15	0.11	0.18	0.77	1.23	0.90
GE	0.14	0.14	0.14	0.10	0.18	0.69	1.31	0.86
IT	0.17	0.17	0.17	0.12	0.21	0.72	1.28	1.03
ES	0.19	0.18	0.18	0.14	0.23	0.74	1.26	1.13
UK	0.20	0.19	0.20	0.15	0.25	0.74	1.26	1.23
GR	0.22	0.21	0.21	0.16	0.26	0.75	1.25	1.31
IE	0.19	0.24	0.22	0.15	0.28	0.69	1.31	1.34
PT	0.25	0.24	0.25	0.20	0.30	0.80	1.20	1.54

Tab. 4: Continued.

income or supplementary, $\mu$								
	wave1	wave 2	mean	persistent	any time	persistent/ mean	any time / mean	country/ EU
DK	0.12	0.13	0.13	0.08	0.17	0.63	1.37	0.49
NL	0.14	0.14	0.14	0.09	0.19	0.66	1.34	0.56
LU	0.22	0.21	0.22	0.16	0.27	0.73	1.27	0.85
BE	0.20	0.23	0.22	0.15	0.29	0.68	1.32	0.86
FR	0.23	0.22	0.23	0.17	0.29	0.75	1.25	0.90
GE	0.22	0.23	0.23	0.17	0.29	0.73	1.27	0.90
IT	0.26	0.26	0.26	0.19	0.33	0.73	1.27	1.02
ES	0.29	0.27	0.28	0.21	0.36	0.75	1.25	1.12
UK	0.31	0.30	0.30	0.23	0.37	0.76	1.24	1.18
GR	0.33	0.32	0.33	0.25	0.40	0.77	1.23	1.28
IE	0.29	0.34	0.32	0.24	0.39	0.76	1.24	1.25
PT	0.38	0.36	0.37	0.30	0.44	0.82	1.18	1.46
income and supplementary, $\nu$								
	wave1	wave 2	mean	persistent	any time	persistent/ mean	any time / mean	country/ EU
DK	0.02	0.02	0.02	0.01	0.03	0.45	1.55	0.27
NL	0.03	0.03	0.03	0.02	0.04	0.57	1.43	0.39
LU	0.06	0.05	0.05	0.03	0.07	0.63	1.37	0.79
BE	0.04	0.05	0.05	0.03	0.07	0.58	1.42	0.66
FR	0.06	0.06	0.06	0.04	0.08	0.67	1.33	0.89
GE	0.05	0.05	0.05	0.03	0.07	0.59	1.41	0.71
IT	0.07	0.07	0.07	0.05	0.10	0.63	1.37	1.06
ES	0.09	0.08	0.08	0.05	0.11	0.65	1.35	1.18
UK	0.10	0.09	0.10	0.06	0.13	0.66	1.34	1.38
GR	0.10	0.10	0.10	0.07	0.13	0.67	1.33	1.44
IE	0.10	0.13	0.11	0.08	0.15	0.68	1.32	1.64
PT	0.13	0.12	0.12	0.09	0.16	0.74	1.26	1.80

We now compare conventional poverty with fuzzy income indicator; at a macro level the indices show little variability, but at a micro level we can observe that the former presents much less movements (for individuals around the poverty line) than the new measure (structure is high variable).

The poverty index from supplementary variables (panel 4) is more persistent than the previous two based on income; this is probably due to the inclusion of amenities, durable goods, problems with accommodation and financial variables in the aggregated index  $x$ . Moreover, there is small response variability in the measurement of income.

Finally it is very interesting to note how income and supplementary poverty overlap much less for richer countries than for poorer; this is evident comparing the wide range of the ratios of manifest poverty to the European average (last column, panel 6), with the range of the ratios of latent poverty to the European average (panel 5).



## **5. Further research**

We feel that the above approach is a useful one in general structure, though more careful work – both theoretical and empirical – is necessary to improve the specific models and weighting systems etc proposed. This applies in particular to the definition and aggregation of non-income indicators, and especially to the next step of combining the supplementary index with that based on income.

## REFERENCES

- Cerioli A., Zani S. (1990), A Fuzzy Approach to the Measurement of Poverty, in Dagum C., Zenga M. (eds.), *Income and Wealth Distribution, Inequality and Poverty*, (proc. Pavia, Italy), Studies in Contemporary Economics, Springer Verlag, Berlin, pp. 272-284.
- Cheli B. (1995), Totally Fuzzy and Relative Measures in Dynamics Context, *Metron* **53** (3/4), pp. 183-205.
- Cheli B., Lemmi A. (1995), A Totally Fuzzy and Relative Approach to the Multidimensional Analysis of Poverty, *Economic Notes*, **24**, pp. 115-134.
- Dubois D., Prade H. (1980), *Fuzzy Sets and Systems*, Academic Press, Boston, New York, London.
- Zadeh L.A. (1965), Fuzzy Sets, *Information and Control* **8**, pp.338-353.

## RESUME

The conventional approach to the measurement of poverty involves a rather simplistic division of the population into the 'poor' and the 'non-poor' in terms of the person's position in relation to some arbitrarily chosen point in the income distribution. Such measures have a number of serious limitations. We propose measures of *degrees of poverty*, thus avoiding the need to choose an essential arbitrary poverty line. Weighting procedures are proposed which make it possible to incorporate, in addition to the level of monetary income, *multidimensional aspects of deprivation* into the definition. In the dynamic, micro-level context, households and persons do not simply move across some arbitrary poverty line, but experience changes in the degree of deprivation over time. Some basic results of the *fuzzy set theory* are used to distinguish states such as *latent* versus *manifest* poverty, *persistent* versus *transient* or *any-time* poverty, and to some extent, *relative* versus *absolute* measures of poverty. We provide empirical illustrations of the procedure for countries of the European Union based on the *European Community Household Panel*.