Social Networks and Efficient Evolutionary Selection in Common Interest Games: Three Simple Models

Stefano Vannucci

1. Introduction

In the last decade or so there has been a remarkable growth of interest in "informal" coordination and decision procedures. This is arguably due to a newly established large consensus on the pivotal role of institution building processes as combined with a growing appreciation of the limited effectiveness of a simple-minded "top-down" approach to institutional reforms. Moreover, the now widely held view that incomplete contracts are indeed a pervasive reality provides additional theoretical arguments for focussing on the role of trust and informal agreements or conventions in determining the actual outcomes of contractual arrangements. Be it as it may, there seems to be now a largely shared presumption that social networks arising from memberships in certain associations, organizations or communities might positively affect the efficiency of economic and political institutions. If true, that effect would obviously contribute a significant factor to the explanation of well-known – and sometimes striking – examples of differential performance concerning either private firms or public agencies operating in different areas under largely similar legal frameworks. Also, it would suggest that certain social networks might indeed embody a positive externality, thereby implying a clear policy prescription to the effect that, ceteris paribus, the relevant associations should be encouraged and supported in any suitable way by public agencies.

Thus, the present paper tentatively *probes the idea that some "informal" interaction structures and decision rules might indeed enhance the performance of "formal" institutions*. This is done by *treating the working of institutions as a coordination problem*, and by *modelling the role of certain social networks as effective coordination devices*. In order to accomplish this task, a few basic simplifying assumptions and modelling choices are made concerning social networks and the coordination problem to be solved. Indeed, the following strong restrictions and assumptions will be introduced below:

- ***"horizontal" social networks of civic associations are singled out for analysis*** : this restricted focus will help us to stick to a reasonably specific and especially plausible case (see e.g. Putnam(1993) for a most inspiring case study on the putative efficiency-enhancing impact of "civic" social networks on local political institutions) .

- ***social networks of civic associations are taken to result in a set of repeated pure coordination* (PC) *games*** i.e. games involving coordination efforts but no relevant conflicts of interest : these are in a sense the simplest interactions that can be safely expected to evolve towards efficient outcomes with no need whatsoever for explicit, formal decision rules. Therefore, "horizontal" social networks are modelled as certain labelled hypergraphs, having players as nodes, associations as hyperedges, and certain sets of (repeated, symmetric) PC games as labels of hyperedges.

- ***the coordination problem to be solved is represented by means of a recurrent common interest* (*CI*) *game of the Stag Hunt variety***, namely a game with a unique unanimously preferred outcome and at least *another – inefficient* but possibly less risky or *risk-dominant- symmetric strict Nash equilibrium.* The underlying intuition is of course that a smooth functioning of the relevant institution is most desirable for each player, but requires adoption of "cooperative" behaviour, which turns out to be ineffective and costly if some player "defects".

Under the foregoing assumptions, the general issue of our concern on the possible role of social networks in improving the performance of economic and political institutions reduces to a problem in reverse comparative statics, namely: *by what means players engaged in a set of repeated PC games could be plausibly more successful than their "socially disconnected" counterparts in coordinating on the efficient outcome of a recurrent Stag Hunt CI game ?*

Therefore, our basic issue is converted into a certain *equilibrium selection problem,* i.e. the "selection" of the *efficient* outcome in a recurrent Stag Hunt game (we recall here that a *recurrent* game is a game repeatedly played with varying opponents) . Moreover, we confine ourselves to simple *evolutionary* models, endorsing the now widely held view that evolutionary reasoning is by far the most promising approach to equilibrium selection. This paper presents *three* simple evolutionary selection models. Each of them provides a distinct mechanism that delivers a social-network-based "efficient selection" of sorts in a recurrent CI game (though – as we shall see below – the notions of "selection" and "evolutionary" are admittedly to be taken in a quite broad sense as far as our third model is concerned).

The first mechanism relies on *opponent-discriminating behaviour* within interactions: strategies can evolve that selectively cooperate with their copies while defecting against "others", thereby undermining (semi-neutral) evolutionary stability of inefficient strategies. Here, *thick social networks operate as reliable information transmission channels* that effectively help a large population of farsighted players to tell "cooperators" from "defectors".

The second mechanism is based upon the hypothesis that under certain conditions - involving a population of "busy" boundedly rational agents that play simultaneously the Stag Hunt and many similar and hardly-distinguishable (CI) games – *playing schemas* (i.e. families of *strategy-types,* as opposed to strategies) *are the relevant replicating units.* Whenever this is the case, it is easily shown that – in a *noisy* environment – "horizontal" social networks can favour the selection of the efficient fully "cooperating" state as uniquely stochastically stable for an underlying monotonic dynamics even if the inefficient equilibrium of the Stag Hunt is risk-dominant. Here, social networks resulting in a suitably large set of repeated PC games *provide a favourable environment for the evolution of "cooperative" playing schemas* within recurrent CI games. This effect works *by lowering the threshold frequency of "cooperating" players required for fixation of cooperation in the ultralong run* (i.e. the time horizon required for mutation-driven transitions between absorbing states).

The third mechanism deals with a scenario where boundedly rational players are repeatedly matched to play a 2×2 Stag Hunt game, and at each round – due to an inertia factor – have to stick to a fixed strategy of their choice. Here, players are endowed with *aspiration levels that are determined by their participation patterns* in the given "horizontal" social network (namely, a player's aspiration level is the projection of her expected payoff from efficient play of the repeated PC games that correspond to her memberships). We focus on the short run adjustment dynamics which obtains when players stick to the current strategy if their current payoff is at least as high as their aspiration level, and shift to the other one if not. This amounts to a non-monotonic non-deterministic selection dynamics, and the resulting short run evolution of population states is clearly dictated by the prevailing profile of aspiration levels. It can be shown that for (almost) any profile of aspiration levels the fully efficient "cooperating" state is *absorbing* (indeed the only absorbing *state*), while the fully "defecting" state is not. Hence, the given short-run dynamics – while not providing a full-fledged efficient selection result- embodies a definite *pro-efficient bias.*

The paper is organized as follows. Section 2 introduces the models and the results. Section 3 discusses some related literature. Section 4 concludes with some remarks on the significance and limitations of our results. The (simple) proofs are confined to an appendix.

2. Models and results

Let N be a non-empty set (the player set) endowed with a measure μ: $P(X) \to R_+$ (if N is finite, μ is the usual counting measure ; we also assume $\#N\geq 2$ in order to avoid trivialities). We say that (N,μ) is **large** if μ is non-atomic i.e. for any $S\subseteq N$ s.t. $\mu(S)\neq 0$ a $T\subseteq N$ exists s.t. $0\neq\mu(T) < \mu(S)$. Let us now consider a set Γ of two-person [infinitely repeated] pure *coordination* games with a *two-valued payoff matrix*. A **(Γ-labeled) social network** on N (on (N,Γ)) is a *hypergraph* (N,**E**) (a *labeled hypergraph* (N,**E**,L)) with set of nodes N, set of hyperedges $\mathbf{E}\subseteq\{S\subseteq N : \#S >1\}$, (and labeling $L:\mathbf{E}\to P(\Gamma)\setminus\{\varnothing\}$); thus, nodes are players while each hyperedge $E\in\mathbf{E}$ represents an *association* of players (*playing the games in* L(E)). The *type* of a player in (N,**E**) is a function τ: $\mathbf{E}\to\{0,1\}$. For any $i\in N$, the *type* τ_i of player i is fully determined by the set $\tau_i^{-1}(1)$ of memberships of i, namely the set **E**(i) of hyperedges $E\in\mathbf{E}$ s.t. $i\in E$. We denote by $\rho(\tau)$ the relative frequency of τ-type players in a social network (N,**E**) with population (N,μ) (i.e. $\rho(\tau)=(\mu(N))^{-1}\cdot[\mu(\{i\in N : \tau_i=\tau\})]$). A social network (N,**E**) is **thick w.r.t. μ** - or **μ-thick** – if (N,μ) is large and $\mu(\bigcup\mathbf{E}(i))\neq 0$ for any $i\in N$.

We shall consider the following general scenario. At any time $t\in\mathbf{Z}_+$, players in N play (repeated) pure coordination games as specified by the given labeled social network, and are (pairwise) randomly matched to play the symmetric 2×2 Stag-Hunt game $G^\circ=(\{I,II\},(\{s,t\},\{s,t\}),(u_I, u_{II}))$ with payoff matrix

$$\Pi(G^\circ)=\begin{bmatrix} a & c \\ b & d \end{bmatrix} \quad \text{where } a>b>d>c.$$

Hence, the efficient and inefficient strict Nash equilibrium payoffs of G° are $a=u_i(s,s)$ and $d=u_i(t,t)$ (i=I,II), respectively. The basic selection dilemma for G° : the "cooperating" strict equilibrium outcome *(a,a) is uniquely Pareto-efficient*, but the "defecting" strict equilibrium outcome *(d,d)* is *risk-dominant* or "less risky" whenever *(d-c)>(a-b)*.

As mentioned in the text above, we shall consider *three* distinct models that can provide a social-network-driven efficient selection result for a recurrent Stag Hunt.

First model. Semi-neutral evolutionary stability and "handshakes" between farsighted players: social networks as reliable information transmission channels.

In this model, the focus is on *strategies* for recurrent play of G° within a *large* population (N,μ) of players. Strategies for recurrent games are identified with the minimal (Mealy) automata that implement them. A (deterministic) *initial Mealy automaton* is a t-uple

A= (Q(A), q*(A), Y(A), δ(A), X(A), h(A)) where Q(A) is the (finite) set of states, q*(A)∈Q(A) is the initial state, Y(A) is the input set, δ(A): Q(A)×Y(A) → Q(A) is the state dynamics, X(A) is the output set, and h(A): Q(A)×Y(A) → X(A) is the output function. We shall consider automata for recurrent play of G° with limited observability (and no discounting), i.e. Mealy automata A with $Y(A)=\bigcup_{k=0}^{\infty}(S \cup \{0^*\})^k$, and Q(A)=X(A)=S×N, where 0* denotes "unobserved action" and S is the common strategy set of players of G°. A strategy for recurrent play of G° in discrete time $\mathbf{Z}_+$ is a function

$\sigma: \bigcup_{k=0}^{\infty}(S \cup \{0^*\})^k \to S$ mapping finite strings of elements of (S∪{0*}) into S.

A strategy σ for recurrent play of G° is obviously implementable by a suitable initial Mealy automaton A=A(σ) . Moreover, *minimal* Mealy automata having such a property can be canonically described by standard techniques. We denote by **A**(G°) the set of (minimal) initial Mealy automata implementing strategies for $(G^\circ_k)_{k=1}^{\infty}$, i.e. for recurrent play of G°. Each player i∈N of a social network (N, **E**) is endowed with a *recall function* $\rho_i: \bigcup_{k=0}^{\infty}(S \times N)^k \to \bigcup_{k=0}^{\infty}(S \cup \{0^*\})^k$ defined as follows: for any finite string $(x_1,..,x_k)\in(S\times N)^k$, $\rho_i(x_1,..,x_k)=(x_1^*,..,x_k^*)$ where $x_h^*=s$ if $x_h=(s,j)$ and $\mathbf{E}(i)\cap\mathbf{E}(j)\neq\emptyset$, and $x_h=0^*$ if $x_h=(s,j)$ with $\mathbf{E}(i)\cap\mathbf{E}(j)=\emptyset$, h=1,..,k . In words, player i "recalls" a previous (e.g. the initial) action of another player j if and only if i and j share the membership of at least one "association". Hence, the *recall function profile* $\rho=(\rho_i)_{i\in N}$ of recall functions is fully determined by the social network (N, **E**), i.e. ρ=ρ(N, **E**). By definition, *recall functions model the role of "horizontal"social networks as reliable information transmission channels.*

It is easily seen that- given (N,**E**)- any sequence of matchings between automata in **A**(G°) fully determines a sequence $(s_I^k, s_{II}^k)_{k=1}^{\infty}$ of plays (i.e. strategy profiles) of G° in the following way. Let A,B∈**A**(G°) the automata of players i,j – respectively- that are matched at time k, and σ=σ(A), σ'=σ(B) the strategies they induce for recurrent play of G°. Having played previously at times 1,..,k-1 both A and B have *output histories* (of length k-1) x and x', respectively. Then, $s^0_I=\sigma(\emptyset)$, $s^0_{II}=\sigma'(\emptyset)$, $s^1_I=\sigma(\rho_i(\sigma'(\emptyset),j))$, $s^1_{II}=\sigma(\rho_j(\sigma(\emptyset),i))$, and so on. The payoffs of i and j at time k after respective output histories x,x' are as follows :

$\pi_i(\sigma, \sigma';x,x',\rho_i,\rho_j)= u_I(A,B; x,x',\rho_i,\rho_j)=$

$= u_I(h(A)[(\sigma(\rho_i(x')),j),(x',\sigma'(\rho_j(x)))], h(B)[(\sigma'(\rho_j(x)),i), (x,\sigma(\rho_i(x')))])$, and

$\pi_j(\sigma,\sigma';x,x', \rho_i,\rho_j)= u_{II}(A,B ; x,x', \rho_i,\rho_j)$.

Clearly, at any time k and for any pair of players i,j∈N , the *expected* payoff

$\Pi^k_{ij}(A,B|P)$ of automaton $A \in \mathbf{A}(G^\circ)$ as chosen by player $i \in N$ - when playing with automaton $B \in \mathbf{A}(G^\circ)$ as chosen by player j - depends upon the (joint) probability distribution p^k_x of pairs of output histories of length k-1, that in turn is determined by the *current population of automata* P (i.e. $p^k_x = p^k_x(P)$). Hence,

$$\Pi^k_{ij}(A,B\ |P) = \int_{p^k_x} u_I(A,B;x,x';\rho_i,\rho_j)dp^k_x \quad .$$

Now, following Binmore,Samuelson(1992) among others, we shall regard choices of automata-strategies in $\mathbf{A}(G^\circ)$ as outcomes of a suitable *long-run* evolutionary process, and define a (pure) ***semi-neutral evolutionarily stable strategy (SNESS)*** of $(G^\circ_k)_{k=1}^{\infty}$ *-at recall function profile* $\rho = \rho(N, \mathbf{E})$- as an automaton $A \in \mathbf{A}(G^\circ)$ such that for any ("mutant") $B \in \mathbf{A}(G^\circ)$, for any $i,j \in N$, and for all $k > k^*$

either i) $\Pi^k_{ij}(A,A \mid P_{A,\varepsilon}) > \Pi^k_{ij}(B,A \mid P_{A,\varepsilon})$

or ii) $\Pi^k_{ij}(A,A \mid P_{A,\varepsilon}) = \Pi^k_{ij}(B,A \mid P_{A,\varepsilon})$ and $\Pi^k_{ij}(A,B \mid P_{A,\varepsilon}) > \Pi^k_{ij}(B,B \mid P_{A,\varepsilon})$

or else iii) $\Pi^k_{ij}(A,A \mid P_{A,\varepsilon}) = \Pi^k_{ij}(B,A \mid P_{A,\varepsilon})$, $\Pi^k_{ij}(A,B \mid P_{A,\varepsilon}) = \Pi^k_{ij}(B,B \mid P_{A,\varepsilon})$, and $\#Q(A) \leq \#Q(B)$

(where k* is an arbitrarily fixed positive integer, ε is a suitably "small" positive real number, and $P_{A,\varepsilon}$ is an arbitrarily fixed population of automata such that, for some injective function $f: N \rightarrow \mathbf{A}(G^\circ)$, $\mu(\{f(i) : f(i) \neq A\}) = \varepsilon$).

The following Proposition holds true :

Proposition 1. Let (N,μ) be a *large* population of players that are embedded in a *μ-thick* social network $(N, \mathbf{E})$. Then, an automaton A is a *semi-neutral evolutionarily stable strategy* of $(G^\circ_k)_{k=1}^{\infty}$ *at recall function profile $\rho = \rho(N, \mathbf{E})$* only if $(\sigma(A),\sigma(A))$ is a Pareto-efficient strategy profile of $(G^\circ_k)_{k=1}^{\infty}$ (i.e. a positive integer k* exists such that, for any $k > k^*$, $(\sigma(A),\sigma(A))$ induces the uniquely Pareto-efficient strict Nash equilibrium of G°).

Thus, the efficient selection result just established is independent of non-ordinal properties of payoff-parameters and rests upon the role of social networks as reliable information transmission channels that enable the players to enact a discriminating behaviour within interactions. The main subsidiary assumptions are : i) a *large* population, a requirement that is needed in order to make sense of (semi-neutral) evolutionary stability as a solution concept, and to provide thick social networks with a significant role; ii) *perfect farsightedness* of the players, which makes free their opponent-discriminating behaviour; iii) *virtual irrelevance of implementation-complexity costs of strategies* as compared to prospective payoff-benefits (a requirement embodied in semi-neutral evolutionary stability).

Second model. Stochastic stability and the power of successful episodes for busy boundedly rational players: social networks as a pro-efficient environment.

This model is focussed on *playing schemas* as opposed to strategies, and produces a *parameter-dependent* efficient selection. The population (N,μ) of players is *small*, i.e. N={1,..,n} is finite. Players are envisaged to play many games simultaneously, while being generally unable to perceive the exact nature of the games being played, or the identity of the opponents. Therefore, they are assumed to rely on a certain *shared* repertoire **T** of *strategy-types*, e.g. "cooperate", "defect" and so on (we also assume $\#\mathbf{T} \geq 2$ in order to avoid trivialities). Indeed, a strategy-type $\mathbf{t}\in\mathbf{T}$ may be regarded as a function that maps each member G of a given family of *symmetric* games into a subset **t**(G) of its strategy-set S(G) .[Asymmetric games can be "symmetrized" in the usual fashion, i.e. by taking role-conditional actions as strategies of the "symmetrized" game. The choice of a given strategy-type $\mathbf{t}\in\mathbf{T}$ for game G on the part of player i results in the mixed strategy σ(**t**(G)) corresponding to the *uniform distribution* on **t**(G).] A game G is said to be **T**-*nontrivial* if $\mathbf{t}(G)\neq S(G)$ for any $\mathbf{t}\in\mathbf{T}$. Moreover, the repertoire **T** of strategy-types is said to be *complete* for a family Γ' of symmetric games- or *Γ'-complete-* if $\cup_{\mathbf{t}\in\mathbf{T}}\mathbf{t}(G)=S(G)$ for any $G\in\Gamma'$. Two symmetric games G,G' are said to be **T**-*similar* whenever i)**T** is complete for G and G', and ii) for any $\mathbf{t},\mathbf{t'}\in\mathbf{T}$, $i\in\{1,2\}$, $s\in\mathbf{t}(G)$, $s'\in\mathbf{t'}(G)$, $t\in\mathbf{t}(G')$, $t'\in\mathbf{t'}(G')$: $(s, s)\geq_i(s', s')$ if and only if $(t, t)\geq_i(t', t')$.

A ***playing schema*** for a set Γ' of games under a Γ'-complete repertoire **T** of strategy-types is a function $P: \Gamma'\to\mathbf{T}$ such that P(G)=P(G') whenever G and G' are **T**-similar (a playing schema could be easily implemented by a simple *classifier system*: see e.g. Holland,Holyoak,Nisbett,Thagard(1986)).

Now, let (N,**E**,L) be a Γ-labelled social network, and $\Gamma^*=\Gamma\cup\{G°\}$ the set of games to be played by players in N. It is assumed that *each game* $G\in\Gamma^*$ is **T**-*nontrivial*. Also, we model the previously mentioned inability of players to discriminate between games in Γ* *by assuming all such games to be* **T**-*similar* (perhaps the players perceive Common Interest as the focal property of those games). But then, *playing schemas for* $\Gamma^*=\Gamma\cup\{G°\}$ *have a singleton-domain and reduce therefore to a single strategy-type* $\mathbf{t}\in\mathbf{T}$. Since all games in Γ* are- by hypothesis- 2×2-games, it follows that a set **P(Γ*)** of playing schemas for Γ* under repertoire **T** is bijective to a set $\mathbf{T^*}\subseteq\mathbf{T}$ such that $\#\mathbf{T^*}=2$ (i.e. $\mathbf{P}(\Gamma^*)=\{\mathbf{t},\mathbf{t'}\}\subseteq\mathbf{T}$, modulo bijections).

All this allows the frequency distribution – or population state - of playing schemas for Γ* to be represented by a single integer number $x\in X \equiv\{0,1,..,n\}$ denoting the number of players that are currently employing, say, the efficient "cooperating" playing schema **t** that is conducive to the unique Pareto-efficient payoff *a* of the recurrent Stag-Hunt game $G°\in\Gamma^*$.

In that setting, it is quite plausible that *the more games are being played where individual "cooperating" behaviour pays in terms of valuable resources the better "cooperating" playing schemas thrive*. Hence, it should be the case that a social network resulting in a set of (repeated) pure coordination games helps the evolution of the "cooperating" playing schema by establishing a more favourable environment for the latter. The present model probes this general idea under the "busy-boundedly-rational-players" –scenario outlined above, and using *stochastic stability* of population states –to be defined below- as a solution concept (see Foster,Young(1990), Kandori,Mailath,Rob(1993)).

Here, population states are to be regarded as (discrete-)time-varying realizations x_t of a random variable X. The evolution of the population state is affected both by a certain *monotonic selection dynamics* $f{:}X \rightarrow X$ (i.e. according to f the strategy-type or playing schema with the highest payoff either stays put or increases its frequency at the next period) and by *noise*(a small, positive and constant-across-states probability of *error* or *mutation)*. In order to analyze the role of a Γ^*-labeled social network (N,**E**,L) in that connection we take $\Gamma^* = \cup L(\mathbf{E}) \cup \{G^\circ\}$ (i.e. Γ^* consists of the repeated pure coordination games that provide the "labels" of the given social network and the recurrent stag-hunt game G° under consideration) . Moreover, we normalize by attaching a null payoff to the inefficient (pure) outcomes of the pure coordination games in $\cup L(\mathbf{E})$: therefore the payoff matrix of a game G in $\cup L(E)$ is

$$\begin{bmatrix} \alpha_G & 0 \\ 0 & 0 \end{bmatrix} \text{ where } \alpha_G > 0 .$$

Under the previous simplifying assumptions an (N, **E**, L)-*situated monotonic selection dynamics* on **T*** is a function $f : X \rightarrow X$ that satisfies the following condition :

$$f(x_t) \underset{\leq}{\overset{\geq}{=}} x_t \text{ if } [a\cdot(x_t-1) + c\cdot(n-x_t)]\cdot(n-1)^{-1} + \sum_{\tau} \rho(\tau) \sum_{G \in \{L(E):\tau(E)=1\}} \alpha_G \cdot (\# L^{-1}(G) - 1) \underset{<}{\overset{>}{=}}$$

$$\underset{<}{\overset{>}{=}} [\, b\cdot x_t + d\cdot(n-x_t-1)\,]\cdot(n-1)^{-1}$$

(where $\rho(\tau)$ denotes the relative frequency of τ-type players in (N,**E**)).

(If (N, **E**, L)=(N, ∅, ∅) we shall also say that f is trivially situated, or *unsituated)*. Indeed, the terms of the second set of inequalities above are to be interpreted as *expected* payoffs to playing schemas. Of course, such an interpretation makes sense if players are involved in a large number of interactions (games) at any time.

Now, consider a small probability of "mutation" ε that is –as mentioned above- constant across population states. Then, take the probability transition matrix $\mathbf{M}(f,\varepsilon)$ (between state-pairs) which is canonically induced by f and ε, and observe that $\varepsilon>0$ implies that the corresponding stochastic process is an irreducible aperiodic Markov chain on the finite state space X. Hence, by the elementary theory of Markov chains, it follows that the long-run behaviour of the process can be summarized by a *unique stationary probability distribution* $p(f,\varepsilon)$ which is invariant with respect to the initial population state. Also, it can be shown that – since X is finite– a (unique) limit distribution $p^*(f)=\lim_{\varepsilon\rightarrow 0} p(f,\varepsilon)$ exists (see e.g. Kandori,Mailath,Rob(1993), Vega-Redondo(1996)) . A population state $x\in X$ is a ***stochastically stable state*** of a (either situated or unsituated) monotonic selection dynamics f if it is in the support of $p^*(f)$, i.e. $(p^*(f))(x)>0$. A stochastically stable state under f can be shown to be necessarily included into an *absorbing set* of $\mathbf{M}(f,0)$, and is fully determined by the *basins of attractions* of the absorbing sets of $\mathbf{M}(f,0)$; (a set $A\subseteq X$ is an *absorbing set* of $\mathbf{M}(f,0)$ if i) for any $(x,x')\in A\times X$ $(\mathbf{M}(f,0))(x,x')>0$ implies $x'\in A$, and ii) for any $(x,x')\in A\times A$ a nonnegative integer k exists such that $(\mathbf{M}^k(f,0))(x,x')> 0$; the *basin of attraction* of an absorbing set A of **M**(f,0) is the set $D_f(A)=\{x\in X : (M^k(f,0))(x,x')>0$ for some nonnegative integer k and some $x'\in A\}$: see e.g. Kandori,Mailath,Rob(1996), Vega-Redondo(1996)).

Proposition 2. Let (N,μ) be a *finite* population of players, Γ a set of repeated 2-person 2-valued PC games, (N, **E**, L) a Γ-labeled social network, $\Gamma^*=\Gamma\cup\{G^\circ\}$, **T** a *shared* Γ*-complete set of strategy types such that all games in Γ* are **T**-*similar* for any i∈N, and f* an (N, **E**, L)-*situated monotonic* selection dynamics on the *shared* set **T*** of *playing schemas* for Γ* as defined above. Then, the efficient fully "cooperative" population state n∈X is the *unique stochastically stable state* of f* if and only if

$$2\cdot(n-1)\cdot\sum_{\tau}\rho(\tau)\cdot\sum_{G\in\{L(E):\tau(E)=1\}}\alpha_G\cdot(\#L^{-1}(G)-1)\;>\;n\cdot(d-c-a+b)+2\cdot(a-d).$$

Hence, in particular, n∈X can be the unique stochastically stable state of f* *even if the efficient equilibrium of Stag Hunt game* G° *is risk-dominated:* this is the case whenever

$(d-c)>(a-b)$ and

$$k^*>\left[(d-c-a+b)+(a-d)\cdot\frac{2}{n}\right]\cdot\frac{n}{2\cdot(n-1)}$$

(where $k^*=k^*(N,E,L)=\sum_{\tau}\rho(\tau)\cdot\sum_{G\in\{L(E):\tau(E)=1\}}\alpha_G\cdot(\#L^{-1}(G)-1)$).

Clearly, the efficient selection result embodied in the foregoing proposition is payoff-parameter-dependent and thus is in a sense weaker than the one encapsulated within Proposition 1 above. Indeed, Proposition 2 relies heavily on "bounded rationality" considerations as combined with the assumption that all the players be persistently *simultaneously* engaged in *many* similar games, and endowed with the *same* classification scheme of strategy-types across such games. The last requirement amounts, in fact, to assuming that *the players share a culture* that makes possible a common understanding of the relevant behaviour in their interactions. While this assumption is arguably both plausible and implicitly required by most game-theoretic constructs, it also puts a severe restriction on the scope of the present model. Moreover, it should also be remarked that Proposition 1 – and its method of proof - are confined to the hyper-stylized world of 2×2 games.

Third Model. Short run absorbing states and the establishment of demanding standards by aspiration-level-driven boundedly rational players: social networks as pro-efficiency-biased benchmark providers.

This model focuses on a non-monotonic and non-deterministic short run dynamics, wherein "boundedly rational players" are repeatedly matched to play a Stag Hunt CI game but have to stick to a fixed strategy of their choice at each round. The players are equipped with a *fixed* exogenous *aspiration level* that controls their choices. Thus, bounded rationality is essentially modelled here in terms of *short-run-inertia* and- consistently with that emphasis on short run behaviour- attractors duly replace stochastic stability as a solution concept. The suggested role of (Γ-labelled) social networks reduces to *determining the profile of aspiration levels (through projection of the players' expected payoff from the relevant repeated PC games)* .

The model to be presented below provides *no efficient selection result* as such but, rather, a much *weaker pro-efficient discrimination,* in that it makes *-for almost any profile of aspiration levels* -the efficient fully "cooperating" state a (possibly unique) *stationary or absorbing state*, and the fully "defecting" state *not absorbing* (and possibly *the only transient or aperiodic state* : a state is *transient* if it belongs to the basin of attraction of an absorbing set to which it does *not* belong). *However, other absorbing sets –namely cycles of period 2- do typically exist.* Thus, the efficient strict equilibrium outcome of G° is "selected" for only in the admittedly weak sense that *the former -as opposed to the inefficient strict equilibrium outcome- is typically retained among the possible solutions (attractors) of the relevant selection dynamics.* Moreover, at a certain profile of aspiration levels the monomorphic state that corresponds to universal adoption of the inefficient strategy may *belong to the basin of attraction of the efficient fully "cooperating" state, while the converse is never the case.*

Here, we consider again a finite population of boundedly rational players such that at every round each pair play a Stag Hunt game G°- without being able to change strategy within a given round (an *inertia*-hypothesis : see e.g. Kandori,Mailath,Rob(1993) for a similar assumption, and for an extensive discussion of the possible underlying motivations). As in the previous models, the recurrent Stag Hunt game G° is to be explicitly contrasted with a set Γ of repeated PC games – the "civic" games- that are efficiently played *and establish the relevant standards- i.e. the* ***projective*** *aspiration levels* $(\alpha_i=\alpha_i(N,\mathbf{E},L),\ i\in N)$ *that – by hypothesis- control the players' choices and amount to the (time-invariant) expected payoff that players enjoy by playing the PC games in* Γ *that correspond to their memberships (i.e. for any* $i\in N$ $\alpha_i=\sum_{G\in E(i)}\alpha_G$). If the possible individual payoffs of G° are $\{a,b,c,d\}$ with $a>b>c>d$, *five* relevant types of aspiration levels are to be distinguished w.r.t.G°: i) (*"exacting"*) $t^E(G°)=\{\alpha\in\mathbf{R} : \alpha\in(a,\infty)\}$, ii) (*"demanding"*) $t^D(G°)=\{\alpha\in\mathbf{R}: \alpha\in(b,a]\}$, iii) (*"mild"*) $t^M(G°)=\{\alpha\in\mathbf{R} : \alpha\in(c,b]\}$, iv) (*"permissive"*) $t^P(G°)=\{\alpha\in\mathbf{R}: \alpha\in(d,c]\}$, v) (*"flat"*) $t^F(G°)=\{\alpha\in\mathbf{R}: \alpha\in(-\infty,d]\}$.

Thus, we envisage here a *two-stage sequential process.* First, a profile $\alpha=\alpha(N,\mathbf{E},L)$ of *projective* aspiration levels for games in $\Gamma°$ is induced through the Γ-labelled social network of repeated PC games. Then the *projective-aspiration (PA)* selection dynamics $f=f(\alpha(N,\mathbf{E},L))$ operates on the set $X=\{0,1,..,n\}$ of population states as characterized by the frequency of the players that choose the efficient "cooperating" strategy s* of the Stag Hunt game G°. Namely, under the PA dynamics *players stick to their strategy if the payoff they achieve is not lower than their aspiration level, and shift to the other strategy, otherwise* (see e.g. Nowak,Sigmund(1993) for a discussion of the merits of this sort of behaviour – which they dub "Pavlov" - within a version of the repeated prisoner dilemma game). Clearly, the selection dynamics $f(\alpha(N,\mathbf{E},L))$ is best regarded as a short run dynamics, and is both non-deterministic and *non-monotonic* (see e.g. the case of an aspiration profile made up of one G°-flat type and (n-1) G°-demanding ones). Now, suppose that profile $\alpha(N,\mathbf{E},L)$ has *no G°*-exacting components (i.e. for any $i\in N$ the aspiration level of player i is not G°-exacting). Then, *the fully "cooperating" population state* n *is* an *absorbing state of* **M**(f(α(N,**E**,L))) (the degenerate probability transition matrix of f(α(N,**E**,L))).By contrast, *the fully "defecting" state 0 is not an absorbing state, unless* α(N,**E**,L) *is a* G°*-permissive-flat profile(* i.e. α_i(N,**E**,L) is either G°-permissive or G°-flat for any i∈N). Moreover, if the profile α(N,**E**,L) is G°*-demanding then state 0 is transient, and is in the basin of attraction of n.* These facts may be summarized as follows:

Proposition 3. Let (N,μ) be a *finite* population of players, Γ a set of repeated 2-person 2-valued PC games, (N,E,L) a Γ-labeled social network, α(N,**E**,L) its projective-aspiration profile, G° the Stag Hunt game, and f(α(N,**E**,L)) the *projective-aspiration* dynamics for G° as defined above. Then,

i) the fully "cooperating" state n is an *absorbing state* of **M**(f(α(N,**E**,L))) iff -for any i∈N- α_i(N,**E**,L)$\notin t^E$(G°) (i.e. it is *not G°-exacting);*

ii) the fully "defecting" state 0 is an *absorbing state* of **M**(f(α(N,**E**,L))) iff – for any i∈N - α_i(N,**E**,L)$\in t^P$(G°)$\cup t^F$(G°) (i.e. α_i(N,**E**,L) is either *G°-permissive or G°-flat);*

iii) when α(N,**E**,L) is a mixed *G°-demanding/G°-flat* profile, with an *odd* number of players having a G°-demanding aspiration level, the fully "cooperating" state n is the *unique absorbing state* of **M**(f(α(N,**E**,L))) . Moreover, at a monomorphic *G°-demanding profile the fully "cooperating" state n is absorbing with basin of attraction* {n,0} .

Thus, the present model only delivers a *comparatively weak – if definite- pro-efficient bias.* Hence, its relevance to the topic of efficient selection *proper* might be arguably regarded as rather tenuous. However, I think it proper to introduce this model here in that it indeed provides a *definite and social-network-induced pro-efficient discrimination* between the two strict equilibria of a Stag Hunt game under the following significant – if highly stylized- conditions : i) *absorbing sets* are used, that is an attractor-like solution concept that -by definition- operates over a much *shorter* time horizon than stochastic stability; ii) the underlying projective-aspiration dynamics is (generally speaking) *non-monotonic* and –by definition- exhibits a minimal amount of learning or, equivalently, a conspicuous path-dependency of outcomes (indeed, it can be regarded as a *short run dynamics).* Again, such a pro-efficient bias operates even if the efficient outcome is *risk-dominated.*.

To best appreciate the strict asymmetry between 0 and n established by this result, and the role of social networks in that connection, consider again the attractors of a projective-aspiration selection dynamics under alternative profiles of aspiration levels, when the Stag Hunt game G° has a *nonnegative* payoff matrix. Proposition 3 suggests that the strongest pro-efficient discrimination obtains when *every player has a G°-demanding standard.* This is, of course, an extreme hypothesis. But consider other monomorphic profiles, namely the *uniformly G°-permissive profile, and the uniformly G°-flat profile.* The uniformly G°-permissive profile is in a sense the most favourable to the inefficient equilibrium strategy, in that a player with a G°-permissive aspiration level will *invariably* stick to s, but not necessarily to s*. Nevertheless, at that profile *both* 0 and n are obviously absorbing states. Finally, consider the uniformly G°-flat profile that results- under our stipulations- from *lack of social networks* at all: namely, a profile of *zero aspiration levels.* Now, if each player has a *zero* aspiration level (*and* the Stag Hunt game G° has a *nonnegative* payoff matrix), then any player is of a "passive" type and thus – as it is easily checked – *any* state $x \in \{0,1,..,n\}$ is absorbing. An extreme version of path-dependency prevails here: no discrimination at all between states is achieved. Summing up, Proposition 3- while not conducive to a full-fledged efficient selection result – *does* tell us that the efficient fully "cooperating" state is in a sense "more likely" to obtain than the fully "defecting" state under the projective-aspiration dynamics.

3. Related literature

The main theme of the present paper is the *efficiency-enhancing potential of certain social networks* as modelled in terms of an *efficient equilibrium selection problem in common interest games.* Moreover, the latter problem is tackled using simple evolutionary game theoretic models and is therefore reduced to a *special case of the evolution of cooperation* when the component game has multiple equilibria . The amount of related work is therefore simply enormous. Hence, in what follows we have to confine ourselves to a short discussion of a few basic connections with the relevant literature.

i) Social networks and institutional performance.

The potential role of social networks – and especially "horizontal" social networks – in improving the efficiency of economic and political institutions has been widely recognized in the last decade or so (see e.g. Putnam(1993), Ostrom,Gardner,Walker(1994), Bowles,Gintis(1998) among others). Arguably, one of the main sources of this widespread interest in social networks is the growing consensus on the view that a) incomplete contracts - as opposed to complete ones- are the rule rather than the exception , and b) the range of enacted incomplete contracts – and the quality of their outcomes – may largely depend on the network of informal coordination and decision procedures into which they happen to be embedded.

More generally, one salient point frequently made in the recent literature is that the most relevant role of social networks must reside in their contributions to the effective solution of *current* coordination and decision problems, as opposed to -say- mere transmission of traditional norms (see e.g. Bowles,Gintis(1998)). The thrust of the present paper totally concurs with this suggestion. After all, traditional norms can be good or bad, and their ability to provide resources for solving efficiently current coordination problems (e.g. by abating asymmetric-information-related agency costs) certainly qualifies as a pivotal criterion in order to assess their quality including perhaps their prospective stability.

However, the mechanisms that produce the putative pro-efficient effect of "horizontal" social networks are apparently not yet well understood. Indeed, in his extensive study on the positive correlation between the "density" of networks of "civic" associations and the performance of local political institutions, Putnam(1993) offers a tentative list of possible effects of such networks that might explain their role in fostering reciprocity and coop :.ative norms, namely:

- **increasing the scope for repeated interactions and interdependence :**

- **producing templates for the relevant cooperative behaviour ;**

- **establishing a favourable environment for the evolution of cooperative behaviour ;**

- **providing a reliable information transmission system .**

Each one of the effects listed above does in fact embody some suggestive if vague intuitions on the nature of possibly relevant mechanisms. The bulk of the present paper has been in fact devoted to probing the strength and scope of those intuitions within some simple evolutionary game-theoretic models. In that connection, it is quite remarkable that *all the items in Putnam's list have a well-defined counterpart in some of the models presented in Section 2 above*. In fact, the existence of a tight relationship between social networks and repeated interactions is a basic assumption in all of our models, and the role of "horizontal" social networks as reliable information transmission channels is a key feature of the "handshake" model. Moreover, the "busy-boundedly-rational-players" model provides a mechanism through which both the pro-cooperative-environment and the cooperative-template effects might plausibly operate in a pro-efficient manner. Finally, the "projective-aspiration" model presented above suggests a *further* "short-run" mechanism (*not* included in Putnam's list), that relies on social networks as *benchmark providers*.

ii) ***Efficient equilibrium selection in common interest games.***
There is of course a large body of literature on efficient equilibrium selection in CI games (with no role at all for social networks). It may be useful to distinguish between *unconditional* efficient selection results, and *parameter-dependent ones* (i.e. those selection results that consist in specifying some sets of values of suitable parameters that support the efficient outcome). Indeed, many unconditional efficient selection results have been obtained by focussing on *repeated* CI games, where earlier interactions can be somehow used as effective communication devices that may even come for free if the players are farsighted : see Binmore,Samuelson(1992), from which the solution concept, the farsightedness assumption, and the "handshake" mechanism of our *first* model are borrowed; see also Aumann,Sorin(1989), Maskin,Fudenberg(1990), Anderlini, Sabourian(1995) for some interesting variants of this general signalling principle, and Tadelis(1995) for an efficient selection result on certain repeated *extensive* CI games that relies on a version of Von Neumann-Morgenstern stable sets – namely, "nondiscriminating optimistically stable standards of behaviour" - as a solution concept.

In the *non-repeated* case, many unconditional efficient selection results for CI games have been obtained by adapting the solution concept itself in a more or less radical manner and/or by embedding the relevant CI game in a larger two-stage game with a preliminary communication phase. Thus, Harsanyi,Selten(1988) plainly advocate priority of payoff-dominance over risk-dominance as an equilibrium selection criterion. Robson(1990) – inspired by some ideas previously advanced by Dawkins on genetically-based signalling (the hypothetical "green beard" effect: see e.g. Dawkins(1982))- obtains an early "secret handshake"-driven efficient selection result relying on unamended evolutionary stability, but *assuming* the (supposedly genetically-based) evolution of strategies endowed with distinctive *costless signals*, and the ability to detect them.. Sobel(1993) and Vega-Redondo(1996) obtain other efficient selection results for general CI games by adding a pre-play communication stage *and* introducing new non-equilibrium (static) notions of evolutionary stability that are meant to capture the role of drift. Matsui(1991) adds a pre-play communication phase *and* relies on "cyclically stable sets" i.e. essentially the absorbing sets of the best-reply dynamics. Kim,Sobel(1995) also add a pre-play communication stage and focus on a class of payoff-monotonic adjustment dynamics using (nonempty, minimal) absorbing sets as a solution concept.

Further unconditional efficient selection results for CI games in the *non-repeated* case have been provided by means of *group selection* arguments of sorts, i.e. models whereby *many distinct –but virtually identical – interaction loci exist, and players may move among them* (in a possibly constrained way). Thus, Mailath(1997) and Dieckmann(1997) obtain unconditional efficient selections in terms of asymptotically stable sets of the replicator dynamics and of stochastic stability, respectively, working under largely similar scenarios where several interaction loci exist, and players are able to *choose* their locus and to *observe* – with a positive probability – strategy profiles at every locus.

In that connection, our first model shows how – under farsightedness of players – social networks can be instrumental in extending "handshake"-driven unconditional efficient selection results to *recurrent* one-shot CI games *without* appending to them a communication phase, invoking group selection effects, or embarking on major reforms of the solution concept. Indeed, the selection result offered by our "handshake"-model is very much like the result of Robson(1990) as quoted above. In a sense, our result amounts to a particular "implementation" of such "handshake"-principle. Such "implementation"- relying on social networks as opposed to an undocumented and apparently rather implausible genetic adaptation- offers what is arguably a more sensible basis for an explanation of efficient interactive behaviour than a direct appeal to genetics.

Turning to parameter-dependent efficient selection results for CI games, we face another quite varied landscape. A few efficient selections have been produced relying on (some variety of) *non-random interactions*, including of course *group-selection processes*. Thus, Eshel, Cavalli Sforza(1982) pursue an extension of the notion of evolutionary stability in order to accommodate the case of non-random interactions resulting both from spatial constraints on the matching process (or "viscosity" of the population structure) and from the ability of players to discriminate between prospective partners and selectively choose or avoid encounters *after* matching (relying presumably on some *available signals*) . They show that the efficient outcome of a CI game is uniquely evolutionarily stable (in their extended sense) provided that : a) the combined effect of spatial constraints on interactions and localization of "mutations" is strong enough to produce a sufficiently high proportion of non-random encounters between identically behaved units, or b) the foregoing effect is weak or even negligible, but is compensated by the *ability of players to choose their partners* after matching, as combined with a *sufficiently large expected number of meetings*. In a similar vein, Myerson,Pollock, Swinkels(1991) rely on ecological considerations concerning the spatially constrained structure of interactions in order to motivate "fluid equilibria" as limits of suitably defined "viscous equilibria", thereby producing an efficient selection in CI games in terms of the latter equilibria for a *large enough value of the viscosity parameter*. With a standard-setting problem as their main motivation, Goyal,Janssen(1997) combine a *spatially constrained matching structure*, that generates a dimorphic locally homogeneous population, and certain *flexibility-cost parameters* (i.e. the cost of acquiring the ability to adopt either standard): the efficient equilibrium outcome of a CI game is selected as the *unique stationary state* of a suitable deterministic monotonic dynamics for *sufficiently low values of those cost parameters*. Boyd,Richerson(1990) obtain a parameter-dependent efficient selection in CI games by means of a *group-selection* model, namely by assuming the existence of several interaction-loci with associated subpopulations or "demes", and a persistent variation among them. The latter is sustained by the local intrademic "conforming" pressure – possibly enhanced by "conformist" cultural transmission - provided that such pressure is *strong enough* to overcome random migration effects. Then, efficiency of the outcomes of that group-selection process obtains if a *large enough fraction of the founders of newly formed "demes" come from the same old "deme"* (see Boyd,Richerson(1990)).

Several powerful parameter-dependent selection results have been recently produced by introducing various sorts of "trembles" with quite different motivations, typically *not* related to non-randomness of the matching structure (see Kim(1996) for a most useful critical survey). *Stochastic stability* has been the most prominent solution concept within that literature. The typical result established by means of such models is that *the efficient outcome of a 2×2 CI game is selected as uniquely stochastically stable for a payoff-monotonic dynamics only if it happens to be risk-dominant as well: otherwise the inefficient risk-dominant outcome prevails* (see Kandori,Mailath,Rob(1993), Young(1993)). This is apparently a quite robust –if definitely "long run" – result. Indeed, adding a spatially constrained (i.e. localized) interaction pattern to these models essentially results in speeding-up convergence to the (uniquely stochastically stable) risk-dominant outcome (see Ellison(1993), Vega-Redondo(1996))). Moreover, a similar selection result favouring the risk-dominant equilibrium has been replicated by means of models whose "trembles" and solutions are *not* phrased in evolutionary terms (including the incomplete-information model proposed by Carlsson,Van Damme(1993) – to be discussed below – that uses iterated strict dominance as a solution concept).

Two notable apparent exceptions are Binmore,Samuelson,Vaughan(1995) and Robson,Vega-Redondo(1996). In fact, both of them provide models wherein the population state corresponding to the *efficient outcome* of a 2×2 Stag Hunt game *can be uniquely stochastically stable even if it fails to be risk-dominant*. The Binmore,Samuelson,Vaughan(BSV) selection result is based upon a "muddling" model where even the selection component of the evolutionary dynamics is *randomized and non-monotonic* : while switching to more successful strategies is more likely than the opposite, as long as a strategy is used by some player there is a positive probability that it will attract more players, however poor its comparative performance in terms of current payoffs (see Binmore,Samuelson,Vaughan(1995) and Samuelson(1997)). A major feature of such a "muddling" adjustment dynamics is that all states except the monomorphic or boundary ones belong to the basins of attraction of *both* the monomorphic states themselves: mutations are only required to escape the latter. Moreover, a *single* mutation is needed for that, as opposed to the combination of simultaneous mutations that is typically required in order to escape the basin of attraction of a boundary (absorbing) state under a monotonic selection dynamics of the Kandori-Mailath-Rob (KMR) type. Incidentally, this also implies that, for small mutation rates, long run (or rather ultralong, in BSV-terminology) convergence to the unique stochastically stable state is typically much *faster* in a "muddling" model than under a KMR dynamics. In the BSV selection model the efficient outcome of a 2×2 CI game is uniquely stochastically stable if the population is *large enough* and *the ratio between the probabilities of switching away from the "efficient" and the "inefficient" equilibrium strategies, respectively, is sufficiently small* (see Samuelson(1997)). A similar pro-efficient selection result obtains if the underlying dynamics is a convex combination of the "muddling" dynamics described above and an entirely random i.e. payoff-independent imitation dynamics. When the weight of such a random imitation dynamics is large enough, the foregoing result may be interpreted in terms of "background payoffs". Namely, it can be said that the efficient outcome turns out to be selected whenever the "background payoff " is low enough to confer adequate prominence to the CI game under consideration. Conversely, if the selection dynamics consists of a convex combination of the "muddling" and the *best-reply* dynamics – with a suitably large weight put on the latter – then the *risk-dominant outcome reemerges as uniquely stochastically stable* (see again Binmore,Samuelson,Vaughan(1995), and Samuelson(1997)). Clearly enough, the BSV-efficient selection result is indeed driven *by reliance on a "muddling" selection dynamics as opposed to a monotonic one.*

By contrast, the model proposed by Robson,Vega-Redondo(1996) sticks to a monotonic selection dynamics of the KMR type. In that model the population is finite but allowed to grow indefinitely large, and the probability of mutation is taken as usual to be vanishing. However, it is assumed that a possibly large but *fixed finite number of rounds* is to be played by every player at each period with one strategy of her choice. Thus, whenever the population is *large enough,* at each period every player necessarily interacts with a *small sample* of the entire population of players. As a result, *matching-noise* (hence the prevailing, typically non-uniform matching structure) is allowed to gain prominence over mutation-noise, and a small-"deme"-effect of sorts is produced enabling the efficient outcome to emerge as uniquely stochastically stable given a large enough overall population. Thus, the Robson-Vega-Redondo selection model apparently shares some key general features of certain *group selection* models (see Robson,Vega-Redondo(1996), and Vega-Redondo(1996)).

The foregoing parameter-dependent efficient selection models can be fruitfully contrasted with the second social-network-based model for busy boundedly rational players as presented above. Indeed, it should be recalled here that such a model provides a parameter-dependent efficient selection result for 2×2 CI games that *is consistent with lack of risk-dominance of the efficient outcome, relies on a standard monotonic selection dynamics, and does not invoke locally constrained interactions or group selection processes.* This is not the place to dwell on the respective merits and drawbacks of these general properties of my second model, and of alternative assumptions. My point here is rather to emphasize again that, arguably, social networks as modelled above may have after all a distinct contribution to offer for a solution of the equilibrium selection problem in Stag Hunt games.

iii) Evolution of cooperation and common interest games.

The evolution of cooperation is of course a major topic in game theory, in economics, and in evolutionary biology. However, it should be emphasized that under this general heading at least two quite distinct issues have been typically referred to. More often than not, the evolution of cooperation has been formalized as the problem of "altruism" i.e. the emergence of "cooperative" behaviour in a repeated and/or recurrent Prisoner-Dilemma-like game (see e.g. Maynard Smith(1982), Axelrod(1984), Dawkins(1989)). Alternatively, the evolution of cooperation can be of course equated with the selection of efficient outcomes in certain coordination games (see e.g. Eshel,Cavalli Sforza (1982), Boyd,Richerson(1990), Maynard Smith,Szàthmary(1995)). While the models proposed in the current paper are only concerned with the latter –and essentially weaker- version of the evolution-of-cooperation-problem, it turns out that some of them bear similarities even to models produced in order to solve some version of the "altruism"-problem. Therefore, this subsection is devoted to a short discussion of some relevant work and ideas on the evolution of cooperation under its most comprehensive interpretation.

To begin with, it should be emphasized again the obvious fact that our "handshake"-model builds heavily on previous "handshake" ideas and models as mentioned and discussed above (i.e. Dawkins(1982,1989), Robson(1990), Binmore,Samuelson(1992)).

Kandori(1992) advances two further ideas that are somehow related to certain aspects of the "handshake"- model presented above. Indeed, Kandori explicitly states that "observability in the community is a substitute for having a long-term relationship with a fixed partner" (Kandori(1992), p.68). Moreover, he emphasizes the focal contribution of certain *reliable information transmission mechanisms (* including *social memberships)* in sustaining *all* feasible individually rational payoff vectors of certain component games as perfect equilibrium outcomes of the corresponding recurrent games with discounting. However, the focus of that work is on collective enforcement and on *extensions of folk theorems for repeated games to recurrent ones*, hence on *multiplicity* of equilibria as opposed to *selection* among them.

Simon(1990) proposes to explain *"altruism"* as a bounded-rationality-induced by-product of "docility"(i.e. a certain propensity towards "social learning"), hence in a way that bears a certain similarity to our "busy-boundedly-rational-players" model. However, on top of other significant differences in scope, detail and concern, Simon's emphasis on "docility" apparently evokes a special role for "conformist" cultural transmission, which is not implied at all by the general monotony requirement for the selection dynamics of our model. It should also be mentioned here that the basic scenario underlying the selection model proposed in Carlsson,Van Damme(1993) involves *boundedly rational players that play many different games without being able to distinguish them from each other.* Such a model – which, as mentioned above, is not phrased in evolutionary terms and uses iteratively strictly undominated strategy profiles as a solution concept – provides a sharp selection result *favouring the risk-dominant equilibrium outcome* in Stag Hunt games. This should be contrasted with the opposite result provided by our second model, that also relies on a somehow related scenario. Indeed, apart from other significant technical differences, this contrast between the respective predictions of Carlsson-Van Damme (CVD) and our model can be essentially traced to the fact that in the former model the class of indistinguishable 2×2 games is definitely *larger* than in the latter. This is so because the class of indistinguishable games in the CVD equilibrium selection model has to include both games where a strategy – or rather strategy-type - is strictly dominant *and* games where *the other* strategy has such a property. In a sense, CVD-players are "busier" or "less rational" than the busy boundedly players of our second social-network-based selection model.

Concerning our *projective-aspiration* "selection" model, it should be recalled here that it builds upon the so called "simpleton" (or "Pavlov") strategy as first introduced by Rapoport (see e.g. Rapoport,Chammah(1965)) for the repeated Prisoner Dilemma game and recently studied by Nowak and Sigmund as an alternative to tit-for-tat in the same game (see Nowak,Sigmund(1993)).

Finally, the literature on the evolution of cooperation provides some broad explanatory principles that have been repeatedly mentioned above in discussing various models of efficient evolutionary selection, namely a) *discriminatory behaviour within interactions,* b) *non-uniform patterns of interactions between players as engendered by non-randomness of interactions*, and –as a specially prominent case of b) – c) *group selection processes*. It is remarkable that – as we shall further discuss shortly below in the next concluding remarks – group selection has apparently no essential role to play in our social-network-based efficient selection models.

4. Concluding remarks

This last section is devoted to a short discussion of the significance, scope and limitations of the models and results proposed in the present paper.

To begin with, a general issue concerning the interpretation of our models and their possible relevance is to be addressed here. It is probably fair to say that, according to standard economic wisdom, the most obvious job for social networks would consist in supporting collusion in oligopolies and related practices that are as a rule prejudicial to overall economic efficiency. Now, collusion in oligopolistic markets can be possibly modelled in terms of *coordination* games for the relevant subpopulation of agents (e.g. producers), but definitely not via *common interest* games (except perhaps under far-fetched assumptions). Hence, our models do *not* apply to the most typical textbook case of collusive behaviour. However, if social networks do indeed favour efficient coordination in *any* common interest game then they may well be occasionally detrimental to the public interest. In general, when it comes to possible policy implications of our models it all depends on the relationship between the population of players that play the relevant Stag Hunt CI games and the constituency whose interests the policy under consideration is meant to promote. According to the selection models presented above it transpires that – as a rule - *if the relevant constituency reduces to the population of players then social networks are* (ceteris paribus) "*good*". If, on the contrary, *the population of players is a proper subset of the relevant constituency, then social networks are typically* "*bad*". In the latter case the most obvious policy implication of our results would be indeed *against* the relevant social networks, which should be discouraged and possibly dismantled by public agencies. As a matter of fact in the current paper the first –positive – interpretation of the role of "horizontal" social networks has been consistently emphasized. There are in my view at least two sound reasons for insisting on that somewhat single-handed attitude. Firstly, the denunciation of the damaging effects of collusive behaviour has been –and is – a pervasive theme in economists at least since Adam Smith's notorious observations on market collusion. By contrast, the relationship between certain social networks, the diffusion of trust, and an effective performance of economic and political institutions has been by far less prominent (to put it mildly) in modern economics and related disciplines. In that connection the single-handed emphasis of the current paper on the pro-efficient case reflects the intention to stress precisely the hardly obvious point that *certain social networks can possibly be instrumental in promoting efficiency from the point of view of an entire polity*. Secondly, contrasting collusive practices in an effective way definitely requires a well-designed, stable and smoothly functioning institutional setup. To the extent that "good" civic social networks do indeed support the smooth functioning of the relevant institutions, they may arguably also contribute to check and counteract the detrimental effects of "bad" social networks.

A first obvious limitation of the selection models discussed in Section 2 is that social networks are treated as entirely *exogenous*. No attempt whatsoever is made at modelling their origins or evolution (see e.g. Mailath(1997), Barberà,Maschler,Shalev(1998) for two interesting approaches to those issues). This stance can be readily justified on practical grounds, given the specific aims of the present analysis. However, it should also be remarked that by taking social networks as parameters our models implicitly assume that evolutionary processes involving civic structures are orders of magnitude *slower* than the evolutionary dynamics of the main interactions to be analyzed. This may or may not be a wise postulate. It is in any case far from being an indisputable assumption since the selection processes which constitute the focus of our models are themselves – except for the third one - long run (or perhaps ultralong run) evolutionary episodes.

The *time horizon* required by our models in order to deliver their efficient selections involves another crucial issue. Indeed, a general implication of the results established in this paper is that – if given time to operate - "horizontal" social networks of civic associations may really embody a significant *positive externality* and bring about considerable benefits for both members and non-members. This in turn suggests an obvious policy implication to the effect that – ceteris paribus – those "horizontal" social networks should be encouraged and promoted in any suitable way by governments and public agencies. However, the foregoing argument loses much of its practical appeal if the putative beneficial effect of such social networks takes too long to materialize. Now, this is certainly not a problem as far as the *projective-aspiration* model is concerned, since its underlying dynamics is definitely *short run*. However, the other selection models presented above rely on solution concepts that only make sense from a long run perspective. In particular, stochastic stability - under its most natural interpretation that involves transitions between absorbing states as driven by combinations of rare "mutations"- refers to a *very* long run (*ultralong* is in fact the label suggested by some authors as mentioned previously). It might be sensibly observed that the cultural nature of the "replicators" involved could arguably speed-up the entire selection process. In any case, it should be admitted that the long run character of the selection processes provided by both the "handshake" and the "busy-boundedly-rational-players" models as described above might well result in a major limitation to their practical significance.

In a more positive vein, a few significant aspects of the foregoing analysis should be pointed out. Firstly, as discussed at length in the previous section all the reasons typically mentioned in the informal literature in order to provide tentative explanations of the pro-efficient effects of some social networks have been given a precise counterpart in some of the mechanisms described in Section 2. In fact, the list of such possible explanatory mechanisms has been expanded by one item, adding the role of social networks as benchmark providers. This suggests that, indeed, further additions might well be available. Secondly, it should be noticed that the social-networks-driven efficient selection models provided in the present paper rely on typical pro-efficient evolutionary mechanisms in a definitely *selective* way. To see this, recall that in the previous section *three* basic ingredients of efficient evolutionary mechanisms for CI games have been distinguished: *signalling-supported intra-game discriminatory behaviour*, *non-uniformity of interaction patterns due to (some source of) non-randomness of interactions*, and *group selection*. Intra-game discriminatory behaviour is of course a basic component of our "handshake"-model. Non-randomness of interactions plays a comparatively minor if pervasive role in our models. In fact – while social networks in all our models correspond to non-random *repeated* interactions between players – the *recurrent* Stag Hunt games that are the main focus of our analysis *are invariably played according to either random or uniform matching rules*. Thus, *non-random matching effects are definitely ruled out as far as the latter (recurrent) interactions are concerned*. Rather, our models rely on *two* classes of interactions - non-uniformly and uniformly patterned, respectively - *and show the influence of the former on the latter*. Finally, *group selection processes do not play any role at all in our efficient selection models*.

It should also be emphasized again that our models provide some *qualifications* concerning the validity of the proposed explanations of putative social networks' pro-efficient effects, namely : a) a large population of farsighted players, a long-run horizon, a noiseless monotonic adjustment dynamics, and low-complexity costs for the first model; b) a possibly small population of "busy" boundedly rational players, an ultralong-run horizon, a noisy monotonic adjustment dynamics, and high-complexity costs for the second model; c) a finite population of boundedly rational players, a short-run horizon, and a non-monotonic – and non-deterministic -adjustment dynamics for the third model .

Since those qualifications point to quite different conditions, the results established in the present paper suggest that some of the intuitive reasons previously advanced in order to explain the pro-efficient effects of "civic" social networks might well apply to mutually exclusive sets of situations. On balance, it seems to me that such preliminary results on social-network-driven efficient evolutionary selection are promising enough to invite further theoretical and experimental inquiry.

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Appendix

Proof of Proposition 1. Suppose not (i.e. A is a Pareto-inefficient SNESS of $(G^\circ_k)_{k=1}^{\infty}$ within (N,E), which implies that the sequence of action-output profiles induced by (σ(A), σ(A)) comprises a subsequence of Pareto-inefficient strategy profiles of G°).

Then, consider a "mutant" $A' \in \mathbf{A}(G^\circ)$ having the following properties:

a) $\sigma(A')(\varnothing) \neq \sigma(A)(\varnothing)$;

b) $\sigma(A')(x)=s^*$ for any $x \neq \varnothing$ such that $(\rho_i(x))_1 = \sigma(A')(x)$ (where s* is the "efficient" strategy of G° and i denotes any player that plays A') ;

c) $\sigma(A')(x) = \sigma(A)(x)$ for any $x \neq \varnothing$ such that $(\rho_i(x))_1 \neq \sigma(A')(\varnothing)$ (where $i \in N$ is any player that plays A').

(In words, A' selects a first action-output that is *different* from the first action-output selected by A, *but subsequently replicates faithfully* the behaviour of automaton A *unless it recognizes its opponent to be a "mutant" that has selected the very same output as itself at the first period, in which case A' selects the efficient action s**).

Therefore, for any k>1

$$\Pi^k_{ij}(A',A' \mid P_{A,\varepsilon}) = \mu(N)^{-1} \cdot [\, \mu(\cup \mathbf{E}(i)) \cdot a + (\mu(N) - \mu(\cup \mathbf{E}(i))) \cdot \Pi^k_{ij}(A,A \mid P_{A,\varepsilon})\,]$$

where a is the unique Pareto-efficient payoff of G°.

Since (N,E) is μ-thick and $\Pi^k_{ij}(A,A \mid P_{A,\varepsilon}) < a$ for a suitable subsequence of positive integers, it follows that for any k in that subsequence

$$\Pi^k_{ij}(A',A' \mid P_{A,\varepsilon}) > \Pi^k_{ij}(A,A \mid P_{A,\varepsilon}) = \Pi^k_{ij}(A',A \mid P_{A,\varepsilon}) = \Pi^k_{ij}(A,A' \mid P_{A,\varepsilon})$$

(where ε denotes the probability density of "mutation" A').

As a result, A cannot be a SNESS of $(G^\circ_k)_{k=1}^{\infty}$ in (N,E) , a contradiction . □

Proof of Proposition 2. By definition of f , {n} is clearly an absorbing set of **M**(f,0), and for any state $x \in X$, $x \in D_f(\{n\})$ whenever the expected payoff at x of the efficient playing schema is greater than the expected payoff at x of the inefficient one, namely if

$$[\, a\cdot(x-1)+c\cdot(n-x)\,]\cdot(n-1)^{-1}+k^* > [b\cdot x+d\cdot(n-x-1)]\cdot(n-1)^{-1} .$$

Thus, the critical state x* i.e. the greatest lower bound of $D_f(\{n\})$ is

$$(*) \qquad x^* = \frac{n\cdot(d-c)+a-d-k^*\cdot(n-1)}{a-c-b+d}$$

Using the combinatorial techniques due to Freidlin,Wentzell(1984) as presented by Kandori,Mailath,Rob(1993) or Vega-Redondo(1996), it is easily checked that n is the unique stochastically stable state of f if $\#D_f(\{n\}) > (n+1)/2$ or, equivalently, if

$$(**) \qquad x^* < (n+1)/2 .$$

Then, the thesis follows immediately from some straightforward algebraic manipulations after substituting the RHS of (*) for x* in (**). □

Proof of Proposition 3. i) Let $\alpha(N,\mathbf{E},L)$ be a profile without G°-exacting types. Then, the state n is an absorbing set because at n each player $i\in N$ gains the maximum payoff $a \geq \alpha_i(N,\mathbf{E},L)$ - since by hypothesis $\alpha_i(N,\mathbf{E},L)\notin t^E(G°)$. Therefore, at state n each player sticks to the efficient "cooperating" strategy, and n persists, i.e. it is an absorbing state. Conversely, let $\alpha(N,\mathbf{E},L)$ be such that $\alpha_i(N,\mathbf{E},L)\in t^E(G°)$ for some $i\in N$. Then, at n player i shifts to the inefficient strategy s. Hence, $f(\alpha(N,\mathbf{E},L))(n)\neq n$.

ii) if $\alpha(N,\mathbf{E},L)$ is a mixed G°-permissive/G°-flat profile, then –by definition- for any $i\in N$ $\alpha_i(N,\mathbf{E},L)$ is a lower bound to $\{b,d\}$, the set of possible G° -payoffs achievable by means of the inefficient strategy s. Therefore, under $f(\alpha(N,\mathbf{E},L))$ each player sticks to s if this is her current strategy. Conversely, if - for some $i\in N$ - $\alpha_i(N,\mathbf{E},L)\notin t^P(G°)\cup t^F(G°)$ then at 0 player i shifts to s*(because her payoff is $d<\alpha_i(N,\mathbf{E},L)$).

iii) Let $x=f^t(\alpha(N,\mathbf{E},L))(x_0)$, $x\neq n$, (hence for some player $j\in N$, j chooses– at x- the inefficient strategy $t\neq s$) . Let $H\subseteq N$ – with $h=\#H$ - the set of players with a G°-demanding aspiration level, and $N\backslash H$ the set of players with a G°-flat aspiration level. Hence, for any $i\in H$ - $\pi_i(G°)< \alpha_i(N,\mathbf{E},L)\leq \max_{hk}\{a_{hk}(G°)\}$, where $\pi_i(G°)$ is the payoff accruing to i from her n-1 plays of G° at the given period. But then, at the next period t+1 $s^{t+1}_i\neq s^t_i$ if and only if $i\in H$. Two cases are to be distinguished, i.e. x=0 and $x\neq 0$. Indeed, if x=0, then $f(\alpha(N,\mathbf{E},L))(0)=h$. In particular, it follows that at a monomorphic G°-demanding profile, i.e. for h=n, $f(\alpha(N,\mathbf{E},L))(0)=n$, which is an absorbing state as observed above. Hence at such a monomorphic profile state 0 is transient and belongs to the basin of attraction of n. By contrast, if $x\neq 0$ then - by the foregoing argument – $y\in f(\alpha(N,\mathbf{E},L))(x)$ iff $y=h - h_1 +k_1$ where $h_1=\#\{i\in H : s^t_i =s\}$ and $k_1=\#\{i\in N\backslash H : s^t_i=s\}$, hence $x=h_1+k_1$. Therefore, if $x\neq 0$ then $x\in f(\alpha(N,\mathbf{E},L))(x)$ iff $h_1+k_1= h - h_1+k_1$ i.e. $h=2\cdot h_1$. Hence n is indeed the unique absorbing state at profile $\alpha(N,\mathbf{E},L)$ whenever h is odd. In particular, it follows that if h=n then $n\in f(\alpha(N,\mathbf{E},L)(x)$ iff $n=n - h_1$ i.e. iff $h_1 =0$ or equivalently $n=h=h_2$, a contradiction since $x\neq 0$. Thus, at a monomorphic G°-demanding profile the basin of attraction of n is $\{0,n\}$. □