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On investment criteria based on the internal rate of return

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1. Introduction*

An internal rate of return attached to a project can be unambiguously used in decision-making procedures only if such a rate is unique. This explains two trends in the literature: the first, developed in the late sixties and early seventies, aimed to render unique the internal rate of return by the truncation of productive projects; the second, particularly successful in recent years, sought larger and larger classes of projects to which a unique internal rate of return is attached(1).

The main purpose of this paper is to point out that uniqueness is definitely not the only property an internal rate of return must have in order to be fully available for decision-making. An <u>additional property</u> will be shown to be necessary in the light of the outcome of an investigation that we shall make into the economic meaning of the internal rate of return.

Moreover this property will be proved to be <u>more restrictive</u> than uniqueness and hence this final result will be obtained: full validity of standard criteria for accepting, or rejecting any single project, and for ranking projects, is limited to a <u>proper subset</u> of the set of projects whose internal rate of return is unique.

Researchers should take this into account and not concern

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* This is a fully revised version of <u>Quaderno nº 11</u>: the general framework has been changed and new results added. I am grateful to Proff. E. Zaghini and G. D'Ippolito of Rome University and to Prof. G. Patrizi of Siena University for their most useful comments on the first draft of this paper.

(1) Special reference is made to a considerable number of contributions published in the <u>Journal of Financial and Quantitative Analysis</u> during 1973--1980.

themselves solely with uniqueness.

A minor purpose of this paper, achieved as a by-product, is to generalize sligthtly Soper's well known sufficient condition for the uniqueness of the internal rate of return.

The following definitions will be adopted throughout this paper.

Definition 1. We define as a productive project a vector of expected net outputs $(a_0 , \dots , a_n)$ such that $a_0 < 0$, $a_j > 0$ for at least one $j = 1 , \dots , n$, and $a_n \neq 0$.

Definition 2. We define as an internal rate of return attached to project $(a_0 , \dots , a_n)$, an interest rate $r_1 > -1$ such that[2]:

$$\sum_{j=0}^{n} a_j (1+r_1)^{n-j} = 0 .$$

Both these definitions are the most general to have been proposed in the literature.

2. A sufficient condition for the uniqueness of the internal rate of return

In this section we are going to prove the following proposition.

(2) An internal rate of return is usually defined as an interest rate $r_1 > -1$ such that:

$$\sum_{j=0}^{n} a_j (1+r_1)^{-j} = 0 ,$$

i.e. such that the present value of the project is equal to zero. It is obvious that this definition is equivalent to Definition 2.

Proposition 1. A (given) internal rate of return r_1 attached to a (given) project $(a_0 , \dots , a_n)$ is unique for such a project if the following inequalities are satisfied:

$$\text{(I)} \quad \sum_{j=0}^{i} a_j (1+r_1)^{i-j} \leq 0 \qquad (i = 0 , \dots , n-1) .$$

Proof. According to Definition 2, in order to prove the proposition, we can prove that, if the following inequalities are satisfied:

$$\text{(II)} \quad \sum_{j=0}^{i} a_j x_1^{i-j} \leq 0 \qquad (i = 0 , \dots , n-1) ,$$

then polynomial

$$\text{(III)} \quad A(x) := \sum_{j=0}^{n} a_j x^{n-j}$$

has no positive root other than $x_1 := 1+r_1$. It is well known[3] that there exists a (n-1).th degree polynomial $Q(x)$ such that:

$$A(x) = (x-x_1) Q(x) .$$

Therefore the positive roots of polynomial (III) other than x_1 (if any) are the same as the positive roots of polynomial $Q(x)$. Moreover it is well known that the i.th coefficient of polynomial $Q(x)$ is as follows:

$$q_i := \sum_{j=0}^{i} a_j x_1^{i-j} \qquad (i = 0 , \dots , n-1) .$$

(3) See, for example, A. Kurosh, Course d'Algebre Superieure, Moscou: Editions Mir, 1971, pp. 148-152.

Therefore, under condition (II) all q_i ' s are non-positive, so that Descartes' rule of signs[4] guarantees that polynomial $Q(x)$ has no real and positive root. Q.E.D..

It is important to stress that condition (I) is not necessary for the uniqueness of r_1. To show this, let us consider the counter-example represented by the internal rate of return 0.7 attached to project (-100 , 270 , - 270 , 170). It can easily be verified that such an internal rate is unique, even if it does not satisfy condition (I) (in fact we have $a_0 (1+r_1) + a_1 = 170 > 0$ and $a_0 (1+r_1)^2 + a_1 (1+r_1) + a_2 = 19 > 0$).

3. On the nature of the internal rate of return

In this section we shall investigate the meaning of the internal rate of return. The outcome of this investigation will be applied in Section 5 to the economic interpretation of uniqueness condition (I) that we have already established.

It is well known that Definition 2 is an attempt to formalize the intuitive idea of a rate of growth of the funds invested in a project (or a rate of interest earned on such funds).

While some authors[5] believe that such a formalization is

(4) See, for example, A. Kurosh, Op. Cit., pp. 263-267.

(5) J. Hirshleifer, "On the Theory of Optimal Investment Decision", Journal of Political Economy, August 1958, vol. 66, pp. 329-352. See also M. Trovato, "Sulla Validità del Tasso Interno di Rendimento come Criterio di Selezione di Progetti di Investimento", Giornale degli Economisti e Annali di Economia, settembre-ottobre 1972, vol. 31, pp. 678-691, and P. Puccinelli, "Alcuni Aspetti Controversi della Teoria della Domanda di Investimenti", Note Economiche, gennaio-febbraio 1976, vol. 9, pp. 35-93. Hirshleifer's

fallacious, we shall show that an internal rate of return attached to a project actually means the rate of growth of invested funds, though in 'certain circumstances' it also takes on 'some additional meaning'.

Our analysis will be centred on Propositions 2 and 3 which will follow. Their contents are quite simple, but their formal statements are somewhat cumbersome. This is why we introduce them with the following example.

Let us consider the project:

$$\text{(IV)} \quad (-10 \, , \, 17 \, , \, -6 \, , \, -20 \, , \, 24) \; .$$

Moreover, let us imagine the four consecutive, single-period operations given by the following pairs of net outputs[6]:

$$\text{(V)} \quad \{(-10 \, , \, 12) \; , \; (5 \, , \, -6) \; , \; (0 \, , \, 0) \; , \; (-20 \, , \, 24)\} \; .$$

The first operation is imagined to start at time zero and the last is imagined to end at time four. In this way, the whole

argument is also touched upon by A.A. Alchian, "The Rate of Interest, Fisher's Rate of Return over Costs and Keynes' Internal Rate of Return", American Economic Review, December 1955, vol. 45, pp. 938-943; by E. Solomon, "The Arithmetic of Capital Budgeting Decisions", Journal of Business, April 1956, vol. 29, pp. 124-129 and by P.H. Karmel, "The Marginal Efficiency of Capital", Economic Record, December 1959, vol. 35, pp. 429-434. Nevertheless these authors' main interest is in explaining the differences between internal rate of return rules and net present value rules.

(6) We use the term operations (instead of projects) for pairs (V) because some of these pairs do not fit into Definition 1.

of set (V) is <u>imagined</u> to start when project (IV) starts and to end when project (IV) ends. The operations contained in set (V) have three main features:

(i) the two net outputs which constitute each operation are either null or differ in sign. According to this feature, operations (V) can be classified into three groups. We shall call <u>investment operations</u> those operations whose first net output is negative, <u>financing operations</u> those operations whose first net output is positive, and <u>null operations</u> those operations whose first net output is null (null operations are one-period intervals located between one investment, or financing operation and the next).

(ii) All (non null) operations are performed at the <u>uniform</u> interest rate 0.2 (i.e. 20%).

(iii) Operations (V) are <u>equivalent</u> to project (IV) in the simple sense that an investor who undertook all these (<u>imaginary</u>) operations would obtain the same sequence of (total) net outputs as an investor who undertakes (<u>real</u>) project (IV), as shown here after:

10	12			
	5	-6		
		0	0	
			-20	24
10	17	-6	-20	24

This equivalence <u>ideally</u> allows us to <u>decompose</u> project (IV) into operations (V), i. e. to regard project (IV) as the set of these operations.

First of all, we want to show that no set of operations other

than set (V) can exist which still presents features (i) to (iii) described above.

More precisely, and more generally, we want to prove the following proposition.

<u>Proposition 2</u>. Given a project $(a_0 , \dots , a_n)$ and given a rate of interest r_1 (7), suppose there exist a set

$$B := \left\{ (b^i_{i-1} , b^i_i) \ : \ i = 1 , \dots , n \right\}$$

of n consecutive single-period operations (8), whose i.th is (b^i_{i-1} , b^i_i) starting at time i-1 and ending at time i, such that:

$$\text{(VI)}\quad b^i_i = -(1+r_1)\, b^i_{i-1} \qquad (i = 1 , \dots , n)$$

$$\text{(VII)}\quad b^1_0 = a_0$$

$$\text{(VIII)}\quad b^i_i + b^{i+1}_i = a_i \qquad (i = 1 , \dots , n-1)$$

$$\text{(IX)}\quad b^n_n = a_n \ .$$

Then set B is <u>unique</u> and we have:

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(7) In accordance with Definition 2, and following common practise, we define as an interest rate, a real number greater than -1 (interest rates smaller than, or equal to -1 are considered economically meaningless).

(8) An <u>operation</u> can be formally defined as an arbitrary vector of dated cash flows.

$$(X)\quad b_i^{i+1} = \sum_{j=0}^{i} a_j (1+r_1)^{i-j} \qquad (i = 0 , \ldots , n-1)$$

$$(XI)\quad b_{i+1}^{i+1} = -(1+r_1)\, b_i^{i+1} \qquad (i = 0 , \ldots , n-1) .$$

Proof. The proof is quite simple. All that we have to do is to consider constraints (VI) to (VIII) (excluding constraint (IX)) as a system of equations whose parameters are $a_0 , \ldots , a_n , r_1$ and whose unknowns are b_j^i $(i=1,\ldots,n \ ; \ j=i-1,i)$. Then we realize easily that these unknowns are uniquely determined in terms of the parameters, according to (X) and (XI). Q.E.D..

Proposition 2 states that if a set B satisfying constraints (VI) to (IX) exists, then it is unique, but it says nothing about when such a set exists. This question is answered instead by the following proposition.

Proposition 3. Given a project $(a_0 , \ldots , a_n)$ and given a rate of interest r_1 , there exists a set B satisfying equalities (VI) to (IX) if and only if r_1 is an internal rate of return attached to $(a_0 , \ldots , a_n)$.

Proof. The last equality (XI) is:

$$b_n^n = -\sum_{j=0}^{n-1} a_j (1+r_1)^{n-j-1} .$$

Therefore solution (X) to (XI), to constraints (VI) to (VIII), satisfies also constraint (IX) if and only if

$$\sum_{j=0}^{n} a_j (1+r_1)^{n-j} = 0 ,$$

i.e. if and only if r_1 is an internal rate of return attached to $(a_0 , \ldots , a_n)$. Q.E.D..

Going back to our numerical example, we are now aware that rate 0.2 which is uniformly applied to operations (V) , is an internal rate of return attached to project (IV). In fact we have:

$$-10(1+0.2)^4 + 17(1+0.2)^3 - 6(1+0.2)^2 - 20(1+0.2) + 24 = 0 .$$

Propositions 2 and 3 provide an insight into the very core of impenetrable, though familiar, Definition 2. Infact these propositions 'reveal' that an internal rate of return attached to a project is an interest rate uniformly applied to some consecutive, single period (investment, financing or null) operations into which such a project can be uniquely decomposed.

We can easily refine this crucial result as follows.

First of all, given a project and an internal rate of return attached to it, at least one operation in set B is an investment operation since $b_0^1 = a_0 < 0$ (see constraint (VII) and Definition 1).

On the contrary, it might well occur that no operation in set B is a financing operation. This is shown by the set B attached to project (-50 , 45 , -89 , 110) and to its internal rate of return 0.1. This set B is the following: {(-50 , 55) , (-10 , 11) ,(-100 , 110)}.

When no operation contained in set B is a financing operations, the internal rate of return is a pure lending rate earned (by the investor) on the funds invested in the project. Such invested funds are numbers $-b^i_{i-1}$ $(i=1,\dots,n)$.

On the other hand, when some operations are financing operations, the internal rate of return acquires a <u>mixed</u> (but still clear) economic meaning. In fact it must be regarded both as the <u>lending rate</u> earned (by the investor) on the funds invested in the project, and as the <u>borrowing rate</u> paid (still by the investor) on the <u>funds financed</u> by the project. Invested funds are numbers $-b^i_{i-1}$ $(1 \le i \le n$; $b^i_{i-1} < 0)$, while financed funds are b^i_{i-1} $(1 \le i \le n$; $b^i_{i-1} > 0)$.

In terms of the two numerical examples previously considered in this section, the internal rate 0.2 attached to project (-10 , 17 , -6 , -20 , 24) is a mixed rate (invested funds being 10 at time zero and 20 at time 3, and financed funds being 5 at time 1)[9],

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(9) Also the internal rate 0.7 attached to project (-100 , 270 , -270 , 170) (see the numerical example given in Section 2) is mixed. In fact we have:

-100 ,	170		
	100 ,	-170	
		-100 ,	170
-100 ,	270 ,	-270 ,	170

while the internal rate 0.1 attached to project (-50 , 45 , -89 , 110) is a pure rate (invested funds being 50 at time zero, 10 at time 1 and 100 at time 2).

We want to stress the importance of Proposition 2 stating the uniqueness of set B. This uniqueness ensures that an internal rate of return is <u>either</u> pure <u>or</u> mixed: in fact constraints (VI) to (IX) cannot be satisfied by two sets of operations, say B and B', such that B contains solely investment operations while B' contains at least one financing operation[10].

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(10) We want to stress also that uniqueness of B (stated by Proposition 2) is to be intended for a <u>given</u> project and an internal rate which is <u>given</u> as well. Therefore, if a project A admits two internal rates, say r_1 and r_2, there will be a <u>unique</u> set B_1 associated with project A and internal rate r_1 (i.e. with pair (A , r_1)), and there will be another <u>unique</u> set B_2 associated with project A and internal rate r_2 (i.e. with pair (A , r_2)). For example, let us consider project (-1, 5, -6) which admits internal rates 1 (100 %) and 2 (200 %). Unique set B_1 associated with pair $[(-1 , 5 , -6) , 1]$ is as follows:

-1 ,	2	
	3 ,	-6
-1 ,	5 ,	-6

Unique set B_2 associated with pair $[(-1 , 5 , -6) , 2]$ is as follows:

-1 ,	3	
	2 ,	-6
-1 ,	5 ,	-6

Incidentally, one can note that both internal rates 1 and 2 are mixed. In fact, the analysis we shall develop in Sections 5 and 6 will show that if two (or more) internal rates exist, then all of them are mixed.

4. Hirshleifer's interpretation

The results achieved in Section 3 are not entirely new, but the form in which they have been mentioned in previous contributions is inadequate[11].

On the contrary, the well known interpretation of the internal rate of return given by Hirshleifer[12] seems to be rather conflicting with ours. Hirshleifer's analysis can be summarized as follows.

First of all, " (...) the internal rate of return seems to be based upon the idea of finding (...) the rate of growth of capital funds invested in a project (...)"[13]. But this idea "(...) involves a ratio and cannot be uniquely defined unless one can uniquely value initial and terminal positions. Thus the investment option characterized by the annual cash-flow sequence (-1 , 0 , 0 , 8) clearly involves a growth rate of 100 per cent (compounding annually), because it really reduces to a two-period option with intermediate compounding (...) Consider, however, a more general investment option characterized by the sequence (-1 , 2 , 1) (...) How can a rate of

(11) Special reference is made here to M.J. Bailey, "Formal Criteria for Investment Decision", Journal of Political Economy, October 1959, vol. 67, pp. 476-488; J.F. Wright, "Notes on the Marginal Efficiency of Capital", Oxford Economic Paper, June 1963, vol. 15, pp. 124-129; J.F. Wright, "Some Further Comments on the Ambiguity and Usefulness of Marginal Efficiency as an Investment Criterion", Oxford Economic Paper, March 1965, vol. 17, pp. 81-89; C. Filippini e L. Filippini, "Nota Critica al Teorema del Troncamento", Ricerche Economiche, gennaio-dicembre 1974, vol. 28, pp. 3-18.

(12) See footnote 5.

(13) J.H. Hirshleifer, Op. Cit., p. 346.

growth for the initial capital outlay be determined?"[14].

Hirshleifer answers this question as follows.

Let $A := (a_0 , \dots , a_n)$ be a project and let r_1 be an internal rate of return attached to it. Let us consider:

$$b_n := \sum_{j=1}^{n} a_j (1+r_1)^{n-j} .$$

The internal rate r_1 is, by definition, such that:

$$-a_0 (1+r_1)^n = b^n .$$

Therefore, "initial and terminal positions" (so as to use Hirshleifer's terms) are, respectively, $-a_0$ and b_n, while r_1 is the rate at which $-a_0$ must grow in order to become b_n after n periods.

In other words, Hirshleifer believes that " (...) mathematical manipulations involved in the calculation of r_1 implicitly assume that all intermediate (...) cash flows are reinvested (or borrowed if cash flows are negative) at the rate r_1 itself"[15].

This belief leads Hirshleifer to rather pessimistic conclusions since " (...) this mathematical manipulation (...) is unreasonable in its economic implications. There will not normally be other investment opportunities arising for investment of intermediate cash proceeds at the rate r_1 , nor is it generally true that intermediate cash inflows (if required) must be obtained by borrowing at the rate

(14) J. Hirshleifer, Op. Cit., p. 347.

(15) J. Hirshleifer, Op. Cit., p. 350-351.

r_1 . The rate r_1 , arising from a mathematical manipulation, will only by rare coincidence represent relevant economic alternatives"[16].

Indeed, our analysis shows that an internal rate of return is the rate of growth of the funds invested in the relative project independently of any external opportunities. The point is that, in certain circumstances, it may also be the rate of growth of the funds financed by the project.

The consequences of this double meaning on standard decision--making procedures based on the internal rate of return, will be examined in Section 6.

5. The economic significance of the uniqueness condition established in Section 2

The economic interpretation of the uniqueness condition established in Section 2 is straightforward in the light of the investigation we have made of the economic meaning of the internal rate of return.

We only need to recall that set B, which satisfies equalities (VI) to (IX), is uniquely determined according to (X) and (XI), and then to compare (X) with condition (I).

Then we soon discover that under condition (I) all the operations contained in set B are either investment operations or null operations, while outside condition (I) set B contains at least one financing operation.

Therefore condition (I), economically interpreted, shows itself to be necessary and sufficient for an internal rate of return to

(16) J. Hirshleifer, Op. Cit., p. 350.

be a pure lending rate earned on the funds invested in the relative project.

6. Decision-making criteria based on the internal rate of return

In this section we shall discuss the consequences of the analysis we have developed so far, for the use of standard decision-making criteria based on the internal rate of return.

In Section 3 we have shown that an internal rate of return attached to a project may be either a pure rate of interest earned on invested funds, or a mixed rate of interest both earned on invested funds and paid on financed funds.

Suppose the second case applies to an internal rate of return r_1 attached to a project A , and suppose also that $r_1 > i$, where i is the market rate of interest[17]. As far as r_1 means the rate of interest earned on the funds invested in A , project A should be accepted (because these funds can be borrowed from the market at a lower rate). But, as far as r_1 means the rate of interest paid on the funds financed by A, project A should be rejected (because these funds cannot be lent to the market at a higher rate).

Since project A (i.e. set B) is indivisible, no meaningful decision can be taken on the simple basis of standard criteria.

Symmetrical reasoning applies when $r_1 < i$.

Let us now suppose that an internal rate of return r_a attached

(17) In the present context we do not worry about uniqueness or non-uniqueness of r_1.

to a project A and an internal rate of return r_b attached to a project B , still have the double meaning of the internal rate r_1 considered above. Suppose also that $r_a > r_b$.

As far as r_a and r_b mean the rates of interest earned on the funds invested in the respective projects, project A is preferable to project B . But the reverse choice should be taken when r_a and r_b are regarded as financing rates (because the lower financing rate is preferable).

Symmetrical reasoning applies when $r_a < r_b$.

Note that in case r_a is pure and r_b is mixed, meaningful ranking is also not possible. In fact the double nature of r_b cannot be ignored.

These arguments show that an internal rate of return (apart from uniqueness) can be <u>meaningfully</u> used for decision-making <u>only if</u> it is a <u>pure</u> lending rate earned on the funds invested in the relative project.

On the other hand, we have already said that an internal rate of return (apart from its economic meaning) can be <u>unambiguously</u> used for decision-making <u>only if</u> it is <u>unique</u>.

Therefore, an internal rate of return is available for <u>fully legitimate</u> (both meaningful and unambiguous) use in decision-making <u>if and only if</u> it is <u>both</u> pure <u>and</u> unique.

We now recall that condition (I) has been shown to be necessary and sufficient for an internal rate to be pure (Section 5) while the same condition has been proved to be sufficient, but not necessary, for an internal rate to be unique (Section 2). This implies that pureness is <u>more restrictive</u> than uniqueness. In other terms, if an internal rate of return is pure, then it is unique, but the converse is not true.

From the last two paragraphs the following conclusion may be drawn: full validity of standard criteria based on the internal rate of return is limited to a <u>proper subset</u> of the set of projects whose internal rate is unique. This is the subset of those projects that yield an internal rate which satisfies condition (I)[18].

Indeed, this result is that announced in the introduction to this paper.

7. Soper's sufficient condition for the uniqueness of the internal rate of return

In this section we shall show that Soper's well known sufficient condition for the uniqueness of the internal rate of return[19] is

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(18) Full validity is not to be intended in the sense that decision-making based on the internal rate of return is <u>consistent</u> with decision-making based on the net present value. This is true for the acceptance/rejection case (in fact it can be easily verified that condition (I) implies positive present values for $i < r_1$ and negative present values for $i > r_1$), but ceases to be true as far as ranking rules are concerned. To show this, let us consider the projects(-100 , 20 , 0 , 144) and (-100 , -80 , 230 , 12). They have in common the internal rate 0.2 which satisfies condition (I), no matter what the project refered to. According to decision-making based on the internal rate, the two projects are indifferent. Nevertheless, according to decision-making based on the net present value, the investor would be indifferent about the two projects only if it were $i = 0.1$ or $i = 0.2$; project (-100 , 20 , 0 ,144) would be preferred to project (-100 , -80 , 230 , 12) if it were $i < 0.1$, while the reverse choice applies for $i \in (0.1 , 0.2)$.

(19) C.S. Soper, "The Marginal Efficiency of Capital: a Further Note", <u>The Economic Journal</u>, March 1959, vol. 69, pp. 174-177.

generalized by the uniqueness condition we have proved in Section 2.

Put into our notation, Soper's condition (which is proved by the author in a rather cumbersome way) can be rewritten as follows:

$$\text{(XII)} \quad \sum_{j=0}^{i} a_j \, (1+r_1)^{-j} < 0 \qquad (i = 0 \,, \, \dots \,, \, n-1) \quad .$$

Multiplying both sides of the i.th inequality (XII) by $(1+r_1)^i$, we obtain:

$$\text{(XIII)} \quad \sum_{j=0}^{i} a_j \, (1+r_1)^{i-j} < 0 \qquad (i = 0 \,, \, \dots \,, \, n-1) \quad .$$

In the light of the investigation made in Section 3, Soper's condition (XIII), economically interpreted, merely states that all the operations contained in set B must be investment operations. Condition (I) is more general since null operations are also allowed.

Summary

The main purpose of this paper is to point out that uniqueness is not the only property an internal rate of return must have in order to be fully available for decision-making procedures, and to show that full validity of these procedures is limited to a proper subset of the set of projects whose internal rate of return is unique. A minor purpose is to generalize Soper's well known sufficient condition for the uniqueness of the internal rate of return.

Sommario

Lo scopo principale di questo lavoro è di mostrare che l'unicità non è la sola proprietà che un tasso interno di rendimento deve possedere al fine di poter essere legittimamente usato nelle procedure decisionali, e che queste procedure sono pienamente valide solo se applicate su un sottoinsieme proprio dell'insieme dei progetti il cui tasso interno di rendimento è unico. Un secondo scopo è di generalizzare la nota condizione di Soper, sufficiente per l'unicità del tasso interno di rendimento.

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