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IN THE ECONOMIC CONTEXT: Concerning 'Efficiency'

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OUTLINE

The study of terminology related to 'effectiveness' may range in different areas where there are differences in ideas and usage. Comparisons are wanted, and here the concern is with topics more or less related to economics. It is suitable to deal with 'efficiency', which in that context is the term with proximity to 'effectiveness' and often replaces it. After attention to the general use of terms, a review is made of branches. These may have to do with such topics as production, consumption, cost and benefit, games, choice theory, welfare economics, and the market. The interest might often be straightforward but, especially when abuse of ideas is an issue, it can also involve controversy.

In dealing with any 'effectiveness', one should like to know what it is that has it, and also what it is for. It must be possessed by a specific thing, in relation to a specific outcome. At least, that is a position that has entertainment here.

The possibilities may be either complete effectiveness or complete ineffectiveness, a hit or miss, as in some cases. Otherwise the effectiveness could be to some intermediate extent. A medical treatment works to bring about cure of an illness, with success or failure as outcome, in some instances, and in others an intermediate result may be recognized. A further complication may have to do with side-effects, or joint-products, and these could be important. For instance, there could be prejudice in favour of keeping the patient alive.

Hence with 'effectiveness' there is always an agency in view, and an objective. This is a pattern for the term to be recognized. It should have to be mentioned since, as will be noticed, without having some such pattern in full view there can be an easy drift into obscurity.

Beside 'effectiveness' our undertaking has also to do with 'efficiency'. Everything said so far about 'effectiveness' could be said here also. In a way, therefore, these terms are synonyms, or at least closely related. If a difference between them should be found, it might be in respect to context factors which influence use. Which term is used may convey a message about context.

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Complexities with the terms which come into view in Symposium papers dealing with other areas are not present in economics.

At this point to gain settlement concerning proximity, if that should be needed, let us consult “Pitman's Book of Synonyms and Antonyms” (4th edition, 1949). Under ‘Efficient’ it says: See ‘Effective’. And under ‘Effective’ is a list that includes ‘Efficient’.

The terms may be virtual synonyms but can still have a difference. The magazine “Health and Efficiency” could not just as well have been called Health and Effectiveness, at least, not with the same tone, or expectation about contents. In reference to some mechanical engine, one would expect efficiency to have some straightforward definition, like miles/gal, or, if it be a steam-engine, conversion of heat into mechanical energy. But for effectiveness of the engine, one might have to regard it as a tool and ask what it is supposed to do. At least, not break down too often. Perhaps more likely, one might deal with effectiveness of an economic policy, or of a social programme. In any case, words can have a certain looseness, and the user some liberty.

But still, respect for common usage has importance. This has an appearance in the case where ‘rationality’ is taken to be nothing else than a maximization, or optimization; in other words, a sort of efficiency. However, referring again to Pitman's Synonyms and Antonyms, under ‘Rational’ it shows “Sane, intelligent, reasonable, intellectual”. That does not settle the matter, as would be gathered from history of a seminar on ‘Rationality’ that has been going on in Paris for years. There could be hazard of something like the same fate for ‘Efficiency’. Another case, to have more attention, is where “Pareto Optimum”, in a retreat from a dubious usage, becomes replaced by “Pareto efficient”, as if this made any difference.

Issues to do with efficiency and effectiveness may be recognized and well understood by many. However, like the power of reason, such appreciation should not be completely taken for granted. A new manager, brought in to modernize a certain hotel in a certain country, encountered unexpected obstacles to reforms he proposed for the sake of what he viewed as improved efficiency. No one understood what he was driving at. The only reason anyone understood for the way of doing anything was that it had always been done that way. It was similar with the preacher who insisted that piety did not lie in doing anything because it was reasonable.

The study “Effectiveness and Efficiency: a comparison

between settled agriculturists and semi-nomadic agro-pastoralists in Eastern Sudan" by Giorgio Ausenda deals with two social groups, in two aspects, first, "the 'effectiveness' of each group in achieving a life secure from natural unforeseens by the production and storage of a greater quantity of food", and second, "the 'efficiency' of each social group in maintaining the continuity of the group itself ." In either case, as required, there is a statement of the objective in view. The objectives are different but with fair acceptance the same single term might have been used. However, use of both terms enables an understanding about which objective is being talked about.

The two attributes for the two groups have magnitudes, based on data, and it is to be seen "whether and in what way they are related." Here is recognition of a scale of achievement for either of the objectives.

In some contexts the measurement may have undecided or arbitrary aspects, or present some kind of a problem. For instance, ranking treatments for an illness can become complicated by allowance for side-effects. Or an industrial process, once declared efficient, loses favour after an environmental impact study.

Any dealing with efficiency can raise the question of how it should be measured. There has to be an objective in view, and so a preference order on outcomes—which may or may not be represented numerically—deciding the extent to which the objective is achieved.

To be efficient without further qualification is to achieve the objective to the greatest extent possible, and to arrive at the *optimum,* and to proximity with the language of *choice* and *preference* and of *utility* and *welfare* found in economics. Our undertaking is to report on features in this area where efficiency is certainly important, if not a most central idea. This brings forward "welfare economics", the welfare and efficiency theory or doctrine about the market, highly peculiar to economics and a main focus of what we have to consider.

An interesting phase in the review of terminology is the consideration of its abuses. The "optimism" associated with economics, and with Leibniz, gives the term a claim for attention, but more straightforward matters will be dealt with first. Concern with some form of effectiveness or efficiency is widespread, in every kind of connection, and gives rise to questions about procedure, concept, method, standards, definitions and so forth.

This is brought out by deliveries at the Symposium, beside being no doubt a cause for having it in the first place.

A *cost-benefit analysis* deals with some project, a road, bridge, hospital or whatever, which provides a benefit and has a cost. There are two questions, first, is it possible to obtain greater benefit with no greater cost? If no, the project is *cost-effective*, providing maximum benefit obtainable with the given cost constraint. Second, is it possible to obtain as much benefit at a lower cost? If no, the project is *cost-efficient*, obtaining the given benefit at minimum cost.

The terms efficient and effective have a difference here, which might appear to deny they are synonyms. However, they are employed to distinguish two types of efficiency, or effectiveness, just as in the study of the two social groups to which a reference has been made.

Effectivity functions in game theory, introduced by H. Moulin and Bezalel Peleg (1982), show another area where 'effectiveness' has made an entry. Such a function describes, for each coalition S, the set of subsets within which S can force an outcome by means of some coordinated action of its members.[2] This use seems in line with expressions about a medical treatment, or some agency or institution, or person, being effective or ineffective, as concerns production of intended outcomes.

Typical economic efficiency concepts have to do with production. In a first notion of a production function f, it determines the output $q = f(x)$ obtained with any inputs $x \in C$.[3] Quite usually now it instead determines the maximum possible output for the given inputs, so $q \leq f(x)$ for any feasible input-output operation $(x, q) \in C \times \Omega$. The *efficiency* of the operation is then $e = q/f(x)$, that is, actual output as a fraction of the maximum possible. This quotient has also been called the *coefficient of resource utilization*. Necessarily $0 \leq e \leq 1$, with $e = 1$ in the not-so-likely case of a completely efficient operation.

With given production data $(x_r, q_r) \in C \times \Omega\, (r = 1, \ldots, m)$ econometricians have attempted to estimate a production function

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[2] For awareness of this subject, and information about it, I am indebted to Stefano Vannucci, University of Siena, from personal communications, and from his papers (1998a,b).

[3] In present notation, with Ω as the non-negative numbers, $B = \Omega_n$ is the *budget space* (non-negative row vectors), and $C = \Omega^n$ the *commodity space* (column vectors). Then any $p \in B$, $x \in C$ determine $px \in \Omega$ for the cost of the commodity bundle x at the prices p. Sometimes when dealing with a demand function, Ω should be the positive numbers. A scalar always multiplies a row vector on the left and a column vector on the right.

f from a non-linear regression of q upon x, to give a close fit for the relation $q_r = f(x_r)$. In this case the errors in the relation come out two sided, positive and negative. However, $q_r \leq f(x_r)$ or one-sided errors would be required of any production function f that represented as feasible the production operations that are actually observed, and given as data. One such function is

$$\widehat{f}(x) = \max\left\{\sum_r q_r t_r : \sum_r x_r t_r \leq x, \sum_r t_r = 1, t_r \geq 0\right\}.$$

It is on the classical model, monotonic and concave, and if f is any other then $\widehat{f}(x) \leq f(x)$ for all x. Hence $\widehat{f}$ is the smallest classical function that represents as feasible all the given production operations. This is the *frontier production function*[4]. It determines efficiencies

$$\widehat{e}_r = q_r / \widehat{f}(x_r).$$

The function is insensitive to data points for which $\widehat{e}_r < 1$ and these can be dropped without altering the function. While it has a certain usefulness[5], this exposes its limitation, to be dealt with again. First there can be comment on the modified function

$$\widetilde{f}(x) = \max\left\{\sum_r q_r t_r : \sum_r x_r t_r \leq x, t_r \geq 0\right\},$$

which is again classical, being monotone and concave, and in addition has the conical or constant returns property

$$\widetilde{f}(x\sigma) = \widetilde{f}(x)\sigma \ (\sigma \geq 0).$$

This enables a function representation of the production efficiencies resulting from Farrell's method[6], these now being

$$\widetilde{e}_r = q_r / \widetilde{f}(x_r),$$

and such that

$$\widetilde{e}_r \leq \widehat{e}_r.$$

These frontier, or envelope, functions are not anything like estimates of a true production function. This is evident from the efficiency distributions they determine for the observed production operations that serve as data, where all the operations to which they are sensitive come out with efficiency 1. For a true function, representing the maximum possible output for any given inputs, it

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[4] S. N. Afriat, "Efficiency estimation of production functions", *International Economic Review* 13, 3 (October 1972), 568-98.

[5] It makes a starting point for 'data envelope analysis'.

[6] M. J. Farrell, "The measurement of economic efficiency", *J. Roy. Stat. Soc.* CXX (Part 3, 1957), 253-281.

is more acceptable that no observed operations should come out with efficiency 1. A parallel case is illustrative.

Suppose we have speeds v_r of runners over 100 yards and their efficiencies are represented as given by $e_r = v_r/v^*$ where v^* is the greatest speed anyone could run 100 yards. A problem with this concept is that v^* is unknown; athletic records are continually broken, without a sign of finality. The interest now is in an approach to estimation of v^*, which should be greater than any observed v_r. A similar problem concerns heights to which water level rises during floods, in some city.[7] There is the need, in taking preventative measures, to have an estimate of the greatest height that could occur. The approach could also be applicable to the production function, this being understood to determine the unattained greatest possible output for given inputs.

The original approach was based on a model for the distribution of efficiency on the interval $[0, 1]$, with density falling to zero at the endpoints, and a single peak, having two parameters to specify location of the peak and concentration around it, determined simultaneously with v^*—or, in the further production application of the idea, simultaneously with the parameters of a production function on some model, originally the Cobb-Douglas—by the method of maximum likelihood. This method, which was a new approach to production function estimation, obtains a *stochastic frontier production function* f^* and a density ρ for the distribution of production efficiency, such that the likelihood of the efficiencies

$$e_r^* = q_r / f^*(x_r),$$

is at a maximum.

Should prices $p \in B$ for the input factors be given, so input $x \in C$ has associated with it a cost px, further efficiency concepts become available, corresponding to the cardinal concepts of cost-benefit analysis, cost-efficiency and cost-effectiveness. These are now formulated in application to a production operation (p, x, q), where output q is produced from inputs x with prices p. Beside the efficiency requirement $q = f(x)$ there are two others, H' and H'', involving the pair (p, x) and the function f. Thus for H',

[7] I am indebted to Ali Dogramaci, Bilkent University, Ankara, for drawing my attention to this similar case.

$$f(x) = \max\{f(y) : py \leq px\},$$

which is to say

$$py \leq px \;\Rightarrow\; f(y) \leq f(x),$$

which corresponds to the *cost-effectiveness* familiar in cost-benefit analysis, and asserts *x* produces as much as any other input bundle that costs no more at the prices. And for H'',

$$px = \min\{py : f(y) \geq f(x)\},$$

or equivalently,

$$f(y) \geq f(x) \Rightarrow\; py \geq px,$$

which corresponds to *cost-efficiency*, that any input bundle that produces as much as *x* costs at least as much.

While H' represents $output$*-maximization*, making *x* a bundle that has maximum output for the expenditure on input, H'' represents *cost-minimization*, making *x* have minimum cost for the output obtained. These are equally compelling basic economic principles. They are generally independent. However, properties of the function f can produce relations between them. For instance, if f is continuous monotonic increasing, as usual for a classical production function, then they are equivalent, defining a single condition H. Otherwise, H can be defined as the conjunction of the two efficiencies H' and H''.

In most matters practice falls short of ideal efficiency, or effectiveness. The condition H is therefore too stringent for realistic application. It can be relaxed by introduction of a parameter e, with $0 \leq e \leq 1$, the *level of cost-efficiency*, to make the condition $H'(e)$ defined by

$$f(x) \geq \max\{f(y) : py \leq epx\},$$

or equivalently,

$$py \leq epx \Rightarrow f(y) \leq f(x),$$

or the condition $H''(e)$ defined by

$$epx \leq \min\{py : f(y) \geq f(x)\},$$

equivalently,

$$f(y) \geq f(x) \Rightarrow py \geq epx.$$

While H' requires x to be an input bundle that provides exactly the greatest possible output attainable for its cost, for the more tolerant $H'(e)$ it is enough to obtain at least the output attainable with some fraction e of the cost.

Evidently, for

$$H(e) \equiv H'(e) \wedge H''(e),$$

we have

$$H \Leftrightarrow H(1),\ e \geq e'. \Rightarrow . H(e) \Rightarrow H(e'),$$

since we have this for either of the H' and H'' components, so $H(e)$ becomes progressively more stringent as e is increased, and coincides with the original H for $e = 1$.

Introducing the binary relation $R \subset C \times C$ by

$$xRy \equiv f(x) \geq f(y)$$

this is an order relation, reflexive and transitive,

$$xRx,\ xRyRz \Rightarrow xRz.$$

It is the order represented by the function f.[8] The partial efficiency conditions involving (p, x) with the function f now have statements where they depend on f just to the extent of this order,

$$H'(e) \equiv yRx \Rightarrow\ py \geq epx,$$

$$H''(e) \equiv py \leq epx \Rightarrow xRy.$$

The classical view of consumption is that it has a measurable utility, which consumers maximize, subject to a budget constraint. A utility function $\phi(x) \in \Omega(x \in C)$ shows the utility which results from consumption of, or making use of, any commodities, so it is like a production function, where the product is 'use'. But unlike the production function, the product in this case is not immediately observable.

An observation on the consumer has the form of a demand element $(p, x) \in B \times C$ for which $px > 0$, showing quantities x obtained at prices p. The cost therefore is px, and in choosing x the consumer should have the maximum utility attainable with this cost. In other words, the demand (p, x), when taken with the utility function ϕ, should be cost-effective,

$$py \leq px\ \Rightarrow\ \phi(y) \leq \phi(x).$$

A more stringent condition makes x the unique utility maximum subject to the cost constraint, that is,

$$py \leq px\ \wedge y \neq x \Rightarrow\ \phi(y) < \phi(x).$$

While utility is talked about it is, as said, not in principle directly observable. But it is entertained as the hypothetical

[8] With a binary relation *R*, beside the usual $(x, y) \in R$ because *R* is set, also the statements *xRy*, $x \in Ry$ or $y \in xR$ are available to assert (x, y) is an element of *R*, or that *x* has the relation *R* to *y*.

determinant of demand behaviour, which is in principle observable. Hence if utility is to be constructible, it should be on the basis of demand observations. Whether and how it is possible to do that is an old question that has continued to have attention.

Given any collection of demand elements, or *demand correspondence*, $D \subset B \times C$, it can be asked if there exists a utility which represents every element as efficient. If any, there would be many. Originally the utility was understood to be numerically measurable. Then remarks of Pareto about relevance to behaviour lead to emphasis on the "indifference map", or the preference order, free of numerical representation. Any numerical utility ϕ would have effect only the extent of the *preference order* R represented by it, for which

$$xRy \Leftrightarrow \phi(x) \geq \phi(y),$$

and many functions represent the same order, all those related by a monotonic transformation. Hence utility should be understood now as given not necessarily by a function but, more significantly, by an order. The utilities that are allowed are those, if any, that can represent given demand observations as showing efficient behaviour.

In consistency with this non-numerical approach Samuelson introduced the "revealed preference" method. According to this, if x was demanded when y was available at no greater cost, then this reveals the preference of x over y. In other words, if R is the preference system that governs demand, then, in regard to any demand observation (p, x),

$$(H')\, py \leq px \Rightarrow xRy,$$

or x is optimal in the cost set. In fact—though he did not put it that way—because he dealt not with a general demand correspondence but with a single-valued demand function, he made x the unique optimum and so had instead the stricter requirement

$$(H^*)\, py \leq px \wedge y \neq x \;\Rightarrow\; xRy \wedge \sim yRx.$$

From this comes Samuelson's well known "axiom", which was given its needed logical extension by Houthakker. This condition also has the consequence

$$(H'')\, yRx \Rightarrow py \geq px.$$

For the conjunction

$$H = H' \wedge H'',$$

therefore,

$$H^* \Rightarrow H.$$

The conditions H and H^* determine a relation between a demand element (p, x) and a utility order R, of *compatibility* and *strict compatibility*. While the efficiencies H' and H'' are in general independent, conditions on R produce relations between them, about to be stated.

There is *oversatiation* for R at a point *y* if a bundle *z* exists which is less in all amounts but also at least as good, that is,

$$z < y \ \wedge zRy.$$

The denial of such a possibility, or *insatiability*, requires

$$z < y \ \Rightarrow \ \sim zRy.^9$$

1 If *R* is complete and *xR* is closed, then $H'' \Rightarrow H'$.
2 If *R* is insatiable, then $H' \Rightarrow H''$.
3 If *R* is representable by a continuous semi-increasing function, then $H' \Leftrightarrow H''$.

It can be noted that 3 is a consequence of 1 and 2. With the representation by a continuous function, we have the completeness and closure which provides $H'' \Rightarrow H'$ by 1. If also the function is semi-increasing, the insatiability condition in 2 is obtained so that $H' \Rightarrow H''$.

Consequently, for the conjunction *H* required by compatibility, under such usual conditions we have both $H \Leftrightarrow H'$ and $H \Leftrightarrow H''$. Or in place of the traditional H' of demand analysis, for cost effectiveness or utility maximization, the H'' for cost-efficiency or cost minimization can serve just as well.

Important in Samuelson's approach is the idea—the "revealed preference" principle—that in any act of choice, that is, picking an element out of some set, if x is chosen while y is some other

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[9] For notation,

$$x \leq y \equiv x_i \leq y_i \text{ for all } i, \ x \lesssim y \equiv x \leq y \wedge x \neq y, \text{ and } \mathrm{x} < \mathrm{y} \equiv \mathrm{x}_i < \mathrm{y}_i \text{ for all i.}$$

A function $\phi(x)$ is *non-decreasing*, *semi-increasing*, *or increasing* according to the conditions

$$x \leq y \Rightarrow \phi(x) \leq \phi(y), \ x < y \Rightarrow \phi(x) < \phi(y), \ x \lesssim y \Rightarrow \phi(x) < \phi(y).$$

The three different conditions are increasingly restrictive for a continuous function. For a differentiable function ϕ with gradient *g* they require

$$g \geq o, \ g \gtrsim o, \ g > o.$$

For an example of the intermediate case, the Leontief type function

$$\phi(x) = \max \{t : at \leq x\}, \text{ for } a \in C$$

is semi-increasing but not increasing. Representation of utility by a semi-increasing function assures insatiability.

element in the set at the same time available, then we have the preference of x over y, or it is *revealed*. Treated as generally available, as it seems to have been in some hands, the unrestricted principle amounts to taking choice and preference to be synonyms, or to make any choice a result of *efficiency* in respect to some hypothetical objective, or preference system.

An unrestricted appeal to the revealed preference principle, whereby an efficiency is attributed to elections carried out by voting, leads to the well known "Voting Paradox". Attention to this topic in our dealings with efficiency, though it may not belong strictly to economics, is necessary, and prepares the way for further considerations about groups, in particular about the market economy.

Having an election by means of voting is a way a group of individuals, all of whom might have different ideas about what is good but are still committed to act together, go about making a choice, picking one element out of a set of possibilities, or candidates. The winner is not the *best* for the group, merely the elected one. Had there been some available prior definition of best candidate, there would have been no need to have an election in the first place. But still we have the Voting Paradox, where there is determination to see the winner as best, and surprise at the result.

Consider three electors A, B, C and three candidates x, y, z. The electors give orders to the candidates, expressing their preferences between them which will determine the votes they cast in elections:

A	x, y, z
B	y, z, x
C	z, x, y

For example, A prefers x to y, so in choice between x and y would choose x, and vote accordingly. Three elections are conducted, running the candidates against each other in pairs:

election candidates	A B C votes	election results
y, z	y y z	y defeats z
z, x	x z z	z defeats x
x, y	x y x	x defeats y
x, y, z	x y z	none

The added last line is for the inconclusive election in which all candidates run and get one vote each. In the others, the winner defeats the loser with a decisive 2 votes to 1 in each case. For instance, x defeats y, 2 to 1. From this it appears that x is definitely superior to y, for the group, the group being understood to have a preference system, represented by an order relation R, reflexive and transitive. That is, x is as good as y, or xRy, and y definitely not as good as x, or ~yRx, that is, xPy, P being the strict or antisymmetric part of R, necessarily irreflexive and transitive.

Thus we have the cyclical pattern where x defeats y, y defeats z, and z defeats x, 2 to 1 in each case.The scheme described is the basis for the well known Voting Paradox. What is a paradox and what else must have been going on in thought to see one here?

> *paradox* (Gr. doxa, opinion) A statement, view, etc., contrary to received opinion; an assertion seemingly absurd but really correct; a self-contradictory statement or phenomenon.
>
> Cassell's *Concise English Dictionary*

A list of the three branches may help the enquiry:

(A) A statement, view, etc., contrary to received opinion.
(B) An assertion seemingly absurd but really correct.
(C) A self-contradictory statement or phenomenon.

The study is useful for present purposes, beside that it serves an understanding of existing thought to know why a paradox has been found.

A promising approach to finding a paradox is to work with (A) from the dictionary statement and entertain a notion about received opinion. We do not have to look far for such a notion in prevailing ideas:

(I) Groups have preferences.

Encouragement comes from the welfare function and efficiency doctrines about a market economy.

(II) Preferences are revealed in choices, and with election choices where the winner has definitely more votes than the others, they are strict preferences.

That is the Revealed Preference principle, with a bit added.

(III) Strict preferences are antisymmetric and transitive.

Preferences of the same system belong to an order and so must be transitive as a matter of meaning, and strict preferences are antisymmetric by definition.

If one goes along with that, we do have a paradox. The elections between x, y and y, z reveal the strict preference of x over y and of y over z, or xPyPz. One should conclude then, from the transitivity of strict preferences, that x is strictly preferred to z, or xPz. However, the election between x and z shows the opposite, zPx, so there is a contradiction.

The scheme considered is imaginable and so it should be taken seriously. But when the received opinion is brought in a contradiction follows. A possibility is at odds with received opinion and so, according to (A), we have a paradox.

There is nothing to be done about the first side, and so there must be something wrong with the second, the received opinion. A resolution of the paradox is that groups might make choices, possibly by means of elections, but that does not mean they have preferences, and if they do not have preferences then no preferences can be taken to be revealed by any means.

What we have is seemingly absurd if one adheres to the received opinion and is really correct if one does not, so the (B) criteria are met after a fashion.With the adherence we found a preference that must be both present and not present, so giving the self-contradictory phenomenon required by (C). From all sides we have a paradox, if one believes that wherever there are choices they reveal preferences, and then adds the strictness of preference when one candidate has more votes than another. Without the strictness we would have group indifference between all the alternatives, which is consistent with the indecisiveness of the election when all three candidates run together. If that is an escape it takes away from the *main point*, which is simply that the group does not have preferences. Then the elected candidate is not the best candidate, since there is no criterion for the better and worse, but simply the elected candidate; it is the absence of such a criterion that would be cause for having an election in the first place.

A value of settling the well-known paradox is that it has sustaining connections with ideas, in particular about efficiency of the market, whose shortcomings are undramatized by paradox.

A most distinguished authority having to do with efficiency is the philosopher Leibniz who, in his *Théodisée* (1710), takes the Creator as a model. This is where he propounds the doctrine that the actual world is the “best of all possible worlds”, chosen by the Creator out of all the possible worlds which were present in his thoughts by the criterion of being the world in which the most

good could be obtained at the cost of the least evil. This is the doctrine known as “Optimism”; in its time it drew a great deal of attention, and is famous still. Voltaire's *Candide, ou l'Optimisme* (1759), with the well-known character of Dr Pangloss, was “written to refute the system of optimism, which it has done with brilliant success.” This and further information is in the Oxford English Dictionary. It was Leibniz who introduced ‘optimum’ as a technical term, on the model of a maximum, and it first came into a dictionary in 1752. We are told:

> The optimism of Leibniz was based on the following trilemma:- If this world be not the best possible, God must either
>
> 1. not have known how to make a better,
> 2. not have been able,
> 3. not have chosen.
>
> The first proposition contradicts his omniscience, the second his omnipotence, the third his benevolence.

As concerns efficiency doctrine related to the market economy, a clue about its beginnings is in historic simultaneity and other coincidences involving Leibniz’s Optimism. Though ridiculed by Voltaire and now without influence as such, it found a niche in economics where it has been able to survive with better protection. The arguments about the economy are not quite like that. Instead there is a page of calculus promising infinitesimal precision; it matters not about what, the results are quite the same. This is a parallel of the Maximum Doctrine that came into economics with François Quesnay and the physiocrats and flourishes still. It is impressive to find Quesnay’s Economic Principle “greatest satisfaction to be attained at the cost of the least labour-pain” perfectly represented in Leibniz’s doctrine *vis-à-vis* the Creator’s choice criterion.

The absurd double optimization, found again with the “greatest happiness of the greatest number” formula, is avoided in the Pareto Optimum. This is not an optimum in the sense intended by Leibniz, even though he abused it, which continues to the present as the understood proper usage. But calling it an optimum gives continuity with the old tradition. Under Pareto Optimisim, in regard to the good and evil of the world, there would be the greatest good attainable with the given evil, and the least evil suffered for the good. Begging the main question by a cost-benefit analysis, suitable to mortals who have to get on with the job but quite likely contrary to the law of Heaven, Creation would have been delayed by the need to make a choice between points in the

‘good-evil possibility set’, as we now would say. Leibniz omitted a criterion for that. Were there a marginal price to resolve the matter, with the return of good for evil diminishing to a point of equilibrium, the economic analysis of Creation could have gone further with a use of the new Calculus. There might have been discourse about the price, the author of it, and why it was not better, or worse.

There can be question about such doctrine that has early origins but still prevails and is represented in political speech, and the textbooks. A reading of the “Maximum Doctrine” of the physiocrats, meaningless taken literally, has been translated into a misreading of Adam Smith's doctrine of the “Invisible Hand”, and this by mathematical economists has been translated again, but not very well. In the latest version we have the Pareto optimum. When that is seen for what it is it is in no ordinary sense an optimum: it is just called that, while the power it has in economic thinking is as if it *were* that. Pareto fleetingly entertained the idea as analogous to a maximum and it has come to have exaggerated importance; it filled the vacuum created by shortage in the old doctrine.

Even if we are assured that Adam Smith did propose a maximality under government by the Invisible Hand—and it is possible that he did[10]—we still should not take it seriously. It could be a quaint residue of earlier thought—after all, Newton's mechanics is not vitiated by the importance he gave to number magic and alchemy (perhaps the contrary now, but we can put that aside). It does not matter what views the physiocrats or others had about automatic global economic optimization under various conditions that can be spelt out carefully at length, we still should not believe them for we cannot possibly know what they mean.

To the physiocrats the Maximum Doctrine was not a matter requiring proof—it was self-evident. There have been gestures to prove it since, out of a show of respect for the old words combined with obligation to modern science, but no one knows quite what it was that should be proved. Words have patterns, both with and apart from meaning, as recognized in songs.

Even if we put aside all the problems associated with choice and preference at the individual level, the transfer of the model for an individual to an arbitrary collection of individuals requires a pause. Such a transfer expresses something like the *volontée générale* of the eighteenth century, associated with a collection of

[10] Tom Settle, Guelph University, assured me that he did, and provided a copy of the relevant passage, which unfortunately I have lost.

individuals being so settled together in some way that it amounted to a unified organism representing an individual of a new order, with a will encompassing all the individual wills. Now we have the same idea, but it involves an arbitrary collection, an abstract set, since nothing is spelt out about the members and their relationship to each other that produces the result.

> I have tried to understand what it is that Adam Smith's "invisible hand" is supposed to be maximizing.
>
> Paul Samuelson
> "Maximum Principles in Analytical Economics"[11].

We have a concern with the Maximum Doctrine of Perfect Competition which is central to the dominant 'neoclassical' economics. It appears to have originated in the seventeenth century with François Quesnay and the Physiocrats and was imported from France into the United States by Dupont. For perfect competition there are the well known conditions, and the conclusion—which for the Physiocrats was nothing but self-evident—is that under these conditions the economy achieves an optimum. This is the "social maximum" alluded to by Kenneth J. Arrow in motivating his theory of *Social Choice and Individual Values*:

> If we continue the traditional identification of rationality with a maximization of some sort, then the problem of achieving a social maximum derived from individual desires is precisely the problem which has been central to the field of welfare economics.

Such a view of the rational seems not very traditional. There could just as well be a connection with the doctrine of free will, where man, being endowed with reason, has to choose between good and evil. Man knows good from evil, but the choice is still a problem. In welfare economics it is rather the other way round: the determination to choose the best, or maximum, is fully taken for granted; the problem, instead, is knowing the better from the worse. A fair connection might be found if the choice between good and evil were as simple as optimization, but apparently it is not and dispute is possible. Dr Pangloss was hanged (instead of being burnt—because it was raining) for speaking about the matter, and poor Candide was beaten just for listening.

What is the criterion in the Maximum Doctrine by which one can know the better and the worse and hence what is best, or the optimum? Nobody knows. Without a knowledge of this what we

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[11] Nobel Memorial Lecture, Stockholm, 11 December 1970. In *Les Prix Nobel en 1970*. Amsterdam and New York: Elsevier. Reprinted in *Science*, 10 September, 1971.

have been told must be quite empty. There could be cause for an abandonment of the whole idea, and surprise at how it is accepted without a tremor by the multitude of the faithful.

To be *free, and yet a good slave*—put that way it sounds ridiculous, though it should strike one the teaching is just like that. First there is the individual freedom in the self-regulated order, the market. Then as if here is not enough to the system, and in further praise of it, it is submitted that the overall result is *efficient*, as an obedient slave performing some precise duty to the utmost. It is a relief one never is told what the duty is. The social objective it is taken to exist and to govern because it is talked about. With more known about it there would be a better position to verify whether or not it is at a maximum. For some the loose end is put out of the way as 'the Aggregation Problem', but should we ever get to that problem we would not know what it is.

We are faced with a phenomenon appreciated in another case, the famous 'happiness' formula, known as a Marxist slogan though it has an earlier origin. P. P. Wiener attributes it to Frances Hutchison the teacher of Adam Smith. Its classic attribution is to the Utilitarians and Marxists must have borrowed it from them. According to I. Philips:

> John Bowring says in his Deontology [1834, p.100] that Jeremy Bentham recalled how on a visit to Oxford in 1768 he had first come across the phrase 'the greatest happiness of the greatest number', in Joseph Priestley's Essay on the first principles of Government, published in that year, 1768. "It was from that pamphlet [Bentham said] … that I drew the phrase, the words and import of which have been so widely diffused over the civilized world. At the sight of it, I cried out, as it were in an inward ecstasy like Archimedes on the discovery of the fundamental principle of hydrostatics, $Ευρηκα$."

We should try to find out what the stirring formula could possibly mean. Since "widely diffused" without any qualification, we may look for its import in a simple possible world, one where a cake is distributed over a number n and happiness is the size h of the slice anyone gets. Then the greatest h for the greatest n is wanted. To put all this mathematically, with the size of a slice measured by its angle in radians, so the whole cake is 2π, we have the constraint $hn \leq 2\pi$ and have to maximize h and n simultaneously. Let anyone try!

Economics students receive the notion that if no one can have more, unless someone has less, then we have an optimum. It is tagged with Pareto's name. It is just like with the cake, so

apparently you can distribute it around to everyone any way you please, it's always optimal. Good news for the party host as for the economics catechism. Since everyone wants more, this would have to be a case of "Multi-objective Optimization"—the title of a lecture I once saw announced. But there can be no such thing. If you have one objective to act on then you cannot at the same time also have another—you just have to make up your mind!

Impressive absurdities on the same model had occurred previously, for instance Quesnay's Economic Principle 'greatest satisfaction with the least labour-pain', and he must have drawn inspiration from Leibniz whose 'best of all possible worlds' provided the greatest good at the cost of the least evil. Of course we know reason has its intermittences; and F. A. Hayek has spoken well about authority in economics and the transmission of mistakes, how they are handed down with uncritical acceptance simply because of the prestige of their perpetrators.

Obviously if you choose to maximize one thing, then you cannot at the same time make a free choice of another. You may be lucky, for instance if (x, y) is subject to $x \leq 1$, $y \leq 1$ and you want to simultaneously maximize x and y, this is provided by $(1, 1)$. But we do not have a case like this in dealing with the 'happiness' formula, or the cake; for when n is made large h is forced to be small, and *vice versa*.

It may be wondered how anyone, whose respected output is supposed to be rational in an ordinary sense, can make such remarks, and how they can then have acceptance, even be awarded prizes. On submitting about wrong reasons to Chalongphob Sussangkarn, on a visit with a Thai trade delegation, he gave a healthy answer: "We have the right thing—never mind those reasons!"

Here is another remark bright with free market optimism, from Robert Heilbroner in *The Worldly Philosophers*:

> Edgeworth's pleasure machine assumption bore wonderful intellectual fruit … it could be shown—with all the irrefutability of the differential calculus—that in a world of perfect competition each pleasure machine would achieve the highest amount of pleasure that could be meted out by society.

Enjoyment of the wonderful fruit should in this case be spoilt by a suspicion of worms. What is all the irrefutability of the differential calculus? Is it irresistible authority of the Chain Rule; or final truth in the Infinitesimal, unphased by digital diversions; or the incomprehension and boredom of all those readers who give a

passing glance at the exhibition of machinery and then get on with the text? We should do that first, since the outer skin of this fruit is not without blemishes. We are faced once more with the Leibnizian double optimum expanded into *n* dimensions. That ought to be a relief since now there should really be no need to go back to that skipped-over calculus after all. However, belief that there is complete relief is feeble optimism, a dream of rationality. The particular calculus turns up in countless textbooks—at least now we may know where it started.

For a separate matter where there is an inconsistency something like in the 'happiness' formula, Mr James Baker the erstwhile U. S. Secretary of State toured the newly independent republics of Central Asia, speaking with their leaders and submitting what is expected of them: "democratic government and free-market economics". The principle of such government must include a certain independence. In requiring that, how can it also be laid down what they should decide? A people could well wish to maintain competition to brace up performance and exploit local capacities, without putting themselves at the mercy of a noisy global competition for which they are thoroughly ill-prepared; in other words, settle for living happily with their comparative disadvantage. Instead of doubling their population in thirty years or so, they might even choose to limit themselves—and pursue 'greatest happiness' for their steady number! After all, if one couple have three children its an appalling 50% expansion in one generation, draining away surplus for improvements, if any. Human rights, which get eager attention and have been listed at Helsinki, are perhaps good. Nonetheless, it is not in every way clear where the rights come from, and whether people in a chaotically crowded world have any rights at all. What about obligations, should they not come first?

The neoclassical outlook originated from the time of Newton and the euphoria over his mechanics, and Leibniz with his Optimism. The economy had then to be approached as a machine, not well understood because nobody around had made it or had the plan. Hence the models economists play with, and the cult of the optimum. "The best is enemy of the good" may be recalled at this point.

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