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Quasi - Option Values: Empirical Measures

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1. INTRODUCTION

Two main aspects characterize each development decision involving natural environment: irreversibility and uncertainty. Renewable and exhaustible assets embedded in any economic activity of both consumption and production of commodities are capable of providing a flow of services indefinitely into the future, given at least a sustainable rate of exploitation. Nevertheless environmental resources and goods such as biodiversity, rain forest, watersheds, and topsoil can be degraded permanently by human exploitation.[1] The process of environmental degradation may lead to severe ecological disruption (global warming, greenhouse effect, deforestation in Amazonia), that is economic development may induce consequences that are impossible to reverse. Moreover, the permanent degradation of environmental assets might prevent the possibility to enjoy their potential use values, which are actually unknown, such as the discovery of medicinal drugs that can be used to treat and cure illnesses.[2]

Human beings have always exploited and managed natural environment adroitly. Since the most fertile land has already been cultivated, human beings are converting wild land such as rain forest, tropical wetland, and primitive forest for livestock, agriculture, and other development. As a consequence four biological species perish a day, wild areas disappear at an increasing rate[3], and climate changes occur. Decisions of investing involve large wild land and entire ecosystems and, to a large extent, man-driven changes cannot be reversed in any short period of time. As a result, irreversibility of environmental modifications has to be explicitly considered and evaluated as an output of economic activity. Irreversibility breaks the temporal symmetry between the past and the future, this means that restoration to an original natural state can be technically impossible or extremely expensive. The intuitive concept of irreversibility as a technological or physical constraint can be generalized to include irreversibility as a restoration cost. Uncertainty is another key characteristic of decision processes involving environmental assets and the distinction between different modalities of uncertainty is crucial for the analysis of environmental risk associated with any economic activity. Uncertainty means that consequences of development decisions cannot be fully determined ex-ante and all the uncontrolled variables of the decision process are random variables, which only depend on the possible state of nature that will occur in the future. Moreover, economic agents have imperfect knowledge and might improve their beliefs by learning as time passes (independent learning). Risk alludes to the possibility that an unfavorable event occurs. Global warming is a clear example of uncertain event. There is uncertainty about the link between pollution, gaseous emissions and global mean temperature; moreover there is uncertainty about the relationship between global mean temperature and climate. Floods, storms, droughts are risky events that global warming could induce.[4]

In their seminal article Arrow and Fisher [1] introduce the notion of quasi-option value. They argue that whenever uncertainty is assumed, "even where it is not appropriate to postulate risk aversion in evaluating an activity, something of the feel of risk aversion is produced by a restriction on reversibility of decision" [1, p.318]. Even under risk neutrality, they are able to identify "a quasi-option value having an effect in the same direction as risk aversion, namely, a reduction in net benefits from development" [1, p. 315]. In their two-period model, the decision-maker has to choose between preservation and development. Assuming that the decision-maker replaces the uncertain variables with their expected values, Arrow and Fisher show that "the expected value of benefits under uncertainty is seen to be less then the value of benefits under certainty" [1, p.317]. The value of this difference is called quasi-option value and is related to the possibility of acquiring and exploiting the potential information in the second period.

In the same year, but independently of Arrow and Fisher, Henry [15, 16] publishes two seminal articles that further clarify what Arrow and Fisher have called quasi-option value, although from a slightly different perspective. Henry [16] proves that coexistence of uncertainty and irreversibility prevents the use of the certainty equivalence methods to solve intertemporal decision problems, even if the payoff function is quadratic.[5] Henry shows that replacing the initial random problem by an associated riskless problem, i.e. an equivalent certainty case, the decision-maker may obtain a non-optimal solution, even if she/he is risk neutral. In fact, 'if the solution of the associated problem doesn't imply an immediate irreversible decision, then the solution of the initial problem doesn't either; but it may happen that the solution of the associated problems implies an immediate irreversible decision, whereas the solution of the initial problem does not' [16, p. 1008]. Henry indicates that if the future is uncertainty, some indivisible actions are more irreversible than others, the decision process is sequential, and uncertainty can be reduced by gaining better information, the 'existence of a (positive) option value is a very general property, true whatever the number of periods, the ways of coming to fuller information, the intertemporal shape of the utility function and the lesser or greater the uncertainty of the benefits and costs' [15, p.94]. Henry explains that only the reversible act allows one to exploit additional information in the future. This asymmetry between reversible and irreversible acts encourages a more conservative choice.[6]

In the 1990s, on the basis of the recent developments in decision theory (e.g., Vercelli [26]) with regard to the distinction between soft uncertainty (Knightian risk) and hard uncertainty (Knightian uncertainty), Basili and Vercelli [4] and Basili [2] show the case in which the space of future environmental states of the world is misspecified. In this case the decision-maker has ambiguous beliefs and faces hard uncertainty. Basili and Vercelli [4] suggest a generalized notion of quasi-h-option value (*intertemporal h-option value*), which extends the notion of quasi-option value taking into account hard uncertainty. In fact, hard uncertainty is a more common condition when unanticipated outcomes such as the virus Ebola, the melting of the polar ice or the use of genetically modified organisms in food are involved. They also enlist some properties of quasi-h-option value, regarding its sign and the relationship with the characteristic of prospective learning. Basili [2] indicates that quasi-h-option value is independent of risk attitude; indeed it is a sort of positive hard uncertainty premium that derives from asymmetry between reversible and irreversible actions and from independent learning process. Quasi-h-option value represents a further correction factor that has to be introduced in the evaluation of total economic value of natural assets.

Some different methods of measuring quasi-option value have been suggested in the context of empirical decision problems. The most notable of them is due to Pindyck [22]. Pindyck considers an irreversible action as a financial call option, that gives up the possibility of waiting for new information to arrive that might affect the desirability or timing of the expenditure. The analogy between an irreversible action and a financial call option on common stock allows him to extend standard pricing methods and to estimate quasi-option value.[7]

To the best of my knowledge, a measure of quasi-h-option value has not yet been suggested in the context of empirical decision problems. In this paper I propose an approach that allows the possibility of obtaining two quasi-option values for each investment project. I consider the greater between them as an empirical measure of quasi-h-option value.

The plan of the paper is as follows. Section 2 describes the relationship between quasi-option value and the option value derived by Pindyck. In section 3, hard uncertainty attitude and quasi-h-option value are defined. A general empirical measure of quasi-h-option value is set in section 4. Section 5 concludes.

2. AN EMPIRICAL MEASURE OF QUASI-OPTION VALUE

Henry [16] shows how large irreversibility effect involved in logging a forest might be, by using a decision model with T periods. For the sake of simplicity I collapse all the future in one period and consider a two-period process, i.e. the present and the future.[8] In the two-period decision process, the decision-maker faces either reversible or irreversible actions, i.e. preserving or logging a rain forest. In the first period the decision-maker is uncertain about the state of the world Ω that will occur. At the end of the first period the decision-maker obtains additional information about the future state of nature that will occur, that is he has a finer partition F_2 with respect to the trivial partition on Ω. The decision-maker can choose at the beginning of both periods, but her/his second period choice depends on the previous choice. Development is possible in both periods, that is it is not a 'now or never' opportunity, but a 'now or next period' one. When the decision-maker faces dynamic choice situations[9] and asymmetric actions, she/he has to take into account the potential learning derived from additional information, that only depends on the passage of time.

Henry [15, 16] derives the concept of quasi-option value by comparing the revenue of the random problem with the revenue of the associated riskless problem. He compares the maximum possible conditional[10] expected value of future returns of a feasible action with the maximum possible value of future returns of the same action. In fact, "to the initial random problem we will associate a riskless problem where, for every period and every sequence of decisions affecting this period, the random returns from this period given the sequence of decisions are replaced by their expected value calculated at time 0, i.e., at time the decision-maker solves his problem in order to decide what must be done immediately" [16, p. 1007].

Let $S=2^{\Omega}$ be the set of all the mutually exclusive and exhaustive subsets (events) of $\Omega=\{A,B,C\}$ and let $p:S\rightarrow R$ be a probability (linear and additive) on all subsets or equiprobable events of Ω, such that $p(A)=p(B)=p(C)$ and $p(A)+p(B)+p(C)=1$. Consider a decision-maker having two different investment opportunities each of them depending on her/his behavior with respect to potential additional news.

In the first scenario[11], the decision-maker does not take into account the potential information she/he can gather at the end of the first period, when independent learning occurs. This behavior could be represented by a linear function. In this function d_1 and d_2, expressing the amount or level of the irreversible act chosen in the first and the second period subjected to the constraints $d_1\geq 0$, $d_2\geq 0$

and $d_1+d_2\leq 1$. The decision-maker faces the following production function $V^*=id_1+\max E(i(d_1+d_2))$, assumed $i_A=1$, $i_B=0$, $i_C=-1$ and $i=(p_Ai_A+p_Ci_C)$.

In the second scenario, the decision-maker takes into account learning, that is the F_2 finer partition, with respect to the trivial partition on Ω, occurs among $\{A,\{B,C\}\}$, $\{\{A,B\},C\}$, $\{\{A,C\},B\}$, respectively called F_{21}, F_{22}, F_{23}, at the end of the first period. Let q_1, q_2, q_3 be the probabilities of F_{21}, F_{22}, F_{23}, such that $q_1=q_2=q_3$ and $q_1+q_2+q_3=1$ (equiprobable information structures). The decision-maker faces the following function $V^{**}=id_1+E(\max(i(d_1+d_2)))$, subjected to the constraints $d_1\geq 0$, $d_2\geq 0$ and $d_1+d_2\leq 1$.

If the decision-maker disregards the prospect of additional information, she/he will replace uncertain outcomes with their certainty equivalents, that is she/he will substitute the random problem with the associated riskless one and she/he will evaluate the function V^*. On the other hand, if the decision-maker takes into account acquisition of potential additional news, she/he will consider the random problem and she/he will evaluate the function V^{**}.

In the first condition, the decision-maker solves $V^*=id_1+\max E(i(d_1+d_2))$, subject to $d_1\geq 0$, $d_2\geq 0$ and $d_1+d_2\leq 1$. The optimal solutions[12] are $d_1^*=0$ and $d_1^*=1$, that is the corner solutions prevail. If $i<0$, then $d_1^*=0$ and $V^*=id_2^*$; if $i>0$, then $d_1^*=1$ and $V^*=i[d_1^*+d_1^*]=2id_1^*=2i$.

In the second condition, the decision-maker solves $V^{**}=id_1+(1-q_2)[p_Ci_Cd_1+p_Ai_A]+q_2i(d_1+d_2)$, subject to $d_1\geq 0$, $d_2\geq 0$ and $d_1+d_2\leq 1$. The corner solutions prevail and the optimal solutions[13] are $d_1^{**}=0$ and $d_1^{**}=1$.

The corner solutions always prevail, that is either full complete irreversible or complete reversible act holds, and Bayes' rule holds. The expectation of a given random variable is always equal to the conditional expectation of the random variable with respect to a finer partition on the set of the states (iterative law of expectation), that is $E(.)=E(E(.|F_2.)$. As a result, it is possible to consider the difference between the two outcomes. Consider the difference $V^{**}(d_1^{**})-V^*(d_1^*)$.

- When full development is chosen in the first period, the difference between the expected value of the two options is $V^{**}(1) - V^{*}(1)=i+q_2i+(1-q_2)i-2i=0$ and the decision-maker is indifferent with respect to them.
- When preservation is chosen in the first period, the difference between the expected value of the two options is $V^{**}(0)-V^{*}(0)=[(1-q_2)p_Ai_A+q_2i]d_2-id_2=[(1-q_2)p_Ai_A+q_2p_Ai_A+q_2p_Ci_C]d_2-p_Ci_C+p_Ai_A)d_2= =(p_Ai_A+q_2p_Ci_C-p_Ci_C-p_Ai_A)d_2=[-(1-q_2)p_Ci_C]d_2\geq 0$, that is the second option dominates the first one.

It is possible to obtain the quasi-option value as the difference between the outcomes of the two alternatives. The two options are indifferent with respect to the solution $d_1^{**}= d_1^{*}=1$, because learning has no value. The second option dominates the first one, if the solution $d_1^{**}= d_1^{*}=0$ is chosen at the beginning and the quasi-option value, which measures this dominance, is always positive (strictly positive for $d_2>0$) and induces to make a more conservative choice. Quasi-option value (QOV) is equal to the maximum difference between the expected revenue of the random problem and the riskless one, that is $QOV=\max [V^{**}(0) - V^{*}(0), 0]$. Quasi-option value has an effect similar to soft uncertainty aversion, even if the decision-maker is risk neutral and her/his marginal utility is constant. The concept of quasi-option value represents the conditional value of information, conditional to the reversible action. The existence of the quasi-option value does not allow the possibility to reduce a dynamic process to a timeless one.

Irreversibility and uncertainty are supposed to profoundly affect the decision to invest and intertemporal flexibility encompasses the concept that “good current action may be those which permit good later responses to later observations” [20, p. 42]. Strictly speaking, the option value is a concept whose existence and magnitude rest on intertemporal flexibility preference. If an investment decision is now or in the future opportunity, the investment expenditure can be delayed and postponed. An investment decision entails sunk cost, that is the initial cost of investment can only be partially recovered. The degree of investment reversibility depends on possibility of investment dismissing (disinvestment). Disinvestment depends on the resale price of the specific capital that involves the existence of a secondary market for plants, equipment, and machinery industry-specific. An investment opportunity is equivalent to a call option, where the investment expenditure is the exercise price and the project value, that is the expected payoff from investing, is a share of the underlying asset. The analogy between an irreversible investment decision and a financial perpetual call option[14] on common stock allows the extension of standard pricing methods to estimate quasi-option value. Dixit and Pindyck [12] emphasize the option-like characteristics of investment decisions by noting stochastic fluctuations in the investment expenditure and the expected payoff may induce the decision-maker to revise the optimal timing of investment. Moreover, they consider that "waiting allows a separate optimization in each of the contingencies of a price rise and a price fall, whereas immediate action must be based on only the average of the two" [12, p.98]. In a two-period model, applying dynamic programming and splitting the whole sequence of decisions into immediate decision and the future ones, Dixit and Pindyck [12] derive the expected net present value of all cash-flows, by the Bellman equation that maximizes the immediate profit and the continuation value. As Henry [16] does, Dixit and Pindyck solve the random problem by applying the principles of optimality in stochastic dynamic programming and the riskless problem by certainty equivalence method. Dixit and Pindyck obtain "the value of the extra freedom, namely the option to postpone the decision" [12, p.97] or quasi-option value, as the difference between the expected net present values of random and associated riskless problems. The option to postpone is always positive because "the maximum is a convex function, so by Jensen's inequality the average of the separate maxima is greater than the maximum of corresponding averages" [12, p. 98], as the model, which it is introduced to represent the Henry's approach, does.

Pindyck [22] observes that dynamic programming and contingent claims analysis give the identical solution (rule that maximizes the market value of the investment opportunity), if the decision-maker is risk neutral.[15] The only difference is that in the contingent claims solution the risk-free interest rate replaces the discount rate. As a consequence, Pindyck determines the value of a project and the decision to invest, that is the value of the option to invest in the project, by constructing a dynamic portfolio of assets or an asset, whose holdings are adjusted continuously as the underlying asset price changes. The price of this dynamic portfolio (replicating portfolio) has to be perfectly correlated with the stochastic value of the investment project. It is supposed that the markets are either complete or at least sufficiently complete[16] (spanning assumption) so that "the firm's decisions do not affect the opportunity set available to investors" [12, p. 147]. The optimal investment rule and the value of the investment opportunity can be derived as functions of the output price (underlying asset price), by using option-pricing methods. Standard contingent claims analysis can be considered a general method of measuring quasi-option value in the context of empirical decision problems if capital markets are at least sufficiently complete. In some applications of contingent claims analysis, Dixit and Pindyck [12] show the option value of an undeveloped oil reserve and the value of the option of retrofitting a plant to reduce polluting emissions.

3. HARD UNCERTAINTY ATTITUDE AND QUASI-H-OPTION VALUE

Models explaining quasi-option value assume that environmental states of the world have an additive probability of occurring, that is the decision-maker's description of states of the world is exhaustive. The decision-maker has (explicitly or implicitly) a unique and fully reliable probability distribution over events; moreover she/he possesses an expected utility function linear in probabilities.[17]

Consider a decision problem in which states of the world included in the model do not exhaust the actual ones. A description of the world is considered as a misspecified model whenever that omitted states are not explicitly included in the model. If the decision-maker does not know how many states are omitted, she/he can represent her/his beliefs by means of a capacity or a non-additive measure μ on the set of events. The decision-maker faces hard uncertainty if she/he has a misspecified description of the states of the world, she/he is unable to assign a reliable probability distribution to states of the world because they are ambiguous, she/he has more than one probability distribution, none of which is not fully reliable, over states of the world, she/he has ignorance of the world in which she/he has to act and attaches an interval of probabilities to each event.

Let $\Omega=\{w_1,...,w_n\}$ be a non empty set of states of the world and let $S=2^\Omega$ be the set of all events. A function $\mu:S\to R_+$ is a capacity or a non-additive signed measure if it assigns a value 0 to the event $\varnothing$ and value 1 to the universal event Ω, that is the measure is normalized, and for all $s_1,s_2\in S$ such that $s_1\supset s_2$, $\mu(s_1)\geq\mu(s_2)$, that is the measure is monotone. A capacity is convex or supermodular (concave or submodular) if for all $s_1,s_2\in S$ such that $s_1\cup s_2$, $s_1\cap s_2\in S$, $\mu(s_1\cup s_2)+\mu(s_1\cap s_2)\geq(\leq)\mu(s_1)+\mu(s_2)$. It is superadditive (subadditive) if $\mu(s_1\cup s_2)\geq(\leq)\mu(s_1)+\mu(s_2)$ for all $s_1,s_2\in S$ such that $s_1\cup s_2\in S$, $s_1\cap s_2=\varnothing$. Given a real-valued function $f:\Omega\to R$, f is a measurable function if for every $t\in R$, $\{w|f(w)\geq t\}$ and $\{w|f(w)>t\}$are elements of S. Since μ is non-additive, the integration of a real-valued function f with respect to μ is impossible in the Lebesgue sense. The proper integral for a non-additive measures is the Choquet integral, originally defined by Choquet [9] and discussed in Schmeidler [23], and it requires that states of the world are ranked from the most to the least favorable ones with respect to their consequences. The Choquet integral of f with respect to a capacity μ is defined as

$$\int fd\mu=\int_0^\infty \mu(\{w|f(w)\geq t\})dt+\int_{-\infty}^0 [\mu(\{w|f(w)\geq t\})-\mu(\Omega)]dt$$

The Choquet integral with respect to a capacity is a generalization of the Lebesgue integral, moreover capacities are a generalization of additive probabilities. As a result, the Choquet integral is a generalization of mathematical expectation with respect to a capacity.

The decision-maker expresses hard uncertainty aversion (preference) if she/he assigns larger probabilities to states when they are unfavorable (favorable), than when they are favorable (unfavorable), that is if her/his non-additive measure is convex (concave). Hence, the convexity (concavity) of the capacity, that implies superadditivity (subadditivity) of the Choquet integral, captures the decision-maker's attitude toward hard uncertainty.

The concept of quasi-option value is defined under soft uncertainty; nevertheless the decision-maker can experience a lack of information and faces hard uncertainty. If uncertainty is hard and the use of environmental assets entails irreversible effects, an individual has to consider that natural assets get an additional value, which has been called quasi-h-option value (intertemporal h-option value). Assuming a set of comonotonic feasible actions, that is a set of actions that induce the same ordering of favorable state and the same permutation, Basili [2] shows the characteristics of quasi-h-option value. Differently from Arrow and Fisher, who assume events have an additive probability to occur and can thus derive the expected value associated to each possible action, in the Basili's model events have a non-additive probability of occurring. If the decision-maker ignores uncertainty, she/he replays random variables by their Choquet expected values. As a result, 'benefits under uncertainty are at most equal to benefits under certainty and…the method based on Choquet certainty equivalents also fails to solve decision problem characterized by hard uncertainty and irreversibility' [2, p. 424]. Basili points out the existence of a positive quasi-h-option value, whenever the decision-maker is either optimistic or pessimistic. In environmental economics, the quasi-h-option value is particularly relevant because the uncertainty faced is typically hard, and irreversibility is a very serious matter. For instance in the case of genetically modified food, quasi-h-option price might represent the ecological hazards of genetically engineered crops and reliability of potential hazards.

4. AN EMPIRICAL MEASURE OF QUASI-H-OPTION VALUE

Pindyck [22] considers that the decision-maker faces various forms of risk, such as uncertainty over the future product prices, operating costs, future interest rates, cost and timing of the investment itself. Uncertainty is represented by a finite set of states of the world, one of which will be revealed as true and option pricing will determine the optimal exercise rule of an investment. In this framework, Pindyck assumes that, given competitive markets, no arbitrage conditions and asset prices that follow a particular diffusion process, there is a unique probability distribution on the measurable space (Ω,S) such that market value of any asset is the expectation of its payments. In this way the asset, which spans the stochastic changes in the project worth, may be considered a random variable $\beta:\Omega\rightarrow R$ of its payments and its unique market value equals $\int_{\Omega} \beta \partial \wp$. As a consequence, there is only one opportunity value of an investment project (e.g., Dixit and Pindyck [12]).

In finance it has been proved that if an agent has a non-additive measure on the measurable space, the valuation of an asset will not be the Lebesgue integral of the asset payments (linear pricing rule) but it will be obtained by the Choquet integral of the asset payments (non-linear pricing rule). If a non-linear pricing rule holds, there might be an interval of prices within which the agent neither buys nor sells short the asset, that is they show either *inertia* or *partial inertia* (e.g., Dow and Werlang [13], Simonsen and Werlang[25], Basili [3]).

Because of Knightian uncertainty, in this paper it is suggested of considering the bid and ask prices of the perfectly correlated asset as respectively the worst and the best expectation of an optimistic decision-maker.[18] As a result of hard uncertainty attitude on an investment valuation, there will exist two investment opportunity values. Let the decision-maker be ambiguity seeking (optimist) and let she/he face an investment opportunity. Because of optimism, the decision-maker has a concave capacity ν: $S\rightarrow[0,1]$ on the measurable space (Ω,S), such that normalization and monotonicity with respect to set inclusion hold. Moreover, the capacity ν is monotonely sequentially continuous and compatible with a probability p, that is for all $s_1,s_2 \in S$, $p(s_1) \le p(s_2)$ implies $\nu(s_1) \le \nu(s_2)$ and for all $s \in S$, $s_n \uparrow s$ implies $\nu(s_n) \uparrow \nu(s)$ and $s_n \downarrow s$ implies $\nu(s_n) \downarrow \nu(s)$.

The optimistic decision-maker considers the ask price of the asset β as the maximum expected value consistent with her/his beliefs. There exists a unique ν (e.g., Chateauneuf [6]) such that

$$\int_\Omega \beta \partial \nu = \int_\Omega \beta \partial p \text{ with } p \in P = core(\nu)$$

The Choquet integral of β with respect to ν is equal to the maximum of a family of Lebesgue integrals with respect to the family of probability distributions P. Considering ν as an unanimity game, the c*ore*(ν) can be considered as the set P of additive probabilities on S, such that for all $s_1 \in S$ $\nu(s_1) \ge p(s_1)$. This threshold value is the highest investment value or the maximum price that the decision-maker can ask for the asset β (*upper bound*).

The optimistic decision-maker will wish to buy β at the lowest possible price, compatible with her/his beliefs and by the asymmetry of the Choquet integral (e.g., Denneberg [11])

$$\int_\Omega -\beta \partial \nu = -\int_\Omega \beta \partial \nu^*$$

There exists a unique capacity ν^* on (Ω, S) called the conjugate or dual capacity for ν (e.g., Chateauneuf [6]), such that

$$\int_\Omega \beta \partial \nu^* = \int_\Omega \beta \partial p^* \text{ with } p^* \in P = core(\nu)$$

The conjugate capacity ν^* is defined by $\nu^*(s_i) = \nu(\Omega) - \nu(s_i^C)$ for all $s_i \in S$, where s_i^C is the complement of s_i and it may be considered to what extent the decision-maker believes the negation of s_i. The Choquet integral of β with respect to ν^* reveals the worst expectation of the optimistic decision-maker. This threshold value can be considered as the lowest investment value or the minimum price at which the decision-maker buys the asset β (*lower bound*).

Roughly speaking, the decision-maker assumes that the true probability distribution of the asset β payments is located in the set P, even if she/he has complete ignorance about its location. As a consequence, the selling price of the asset β is supposed to be the supremum of the family of mathematical expectation with respect to every probability distribution in P, whereas the buying price is supposed to be the infimum of the same family of mathematical expectation. These two asset values crucially depends on hard uncertainty and the lesser the faith in likelihood[19] of events is, the longer the interval is.

Once both ν and ν* are guessed and the two probability distribution p and p* have been derived, it is possible to apply Pindyck's method to assess the investment opportunity value. Because of hard uncertainty there will exist two opportunity values F(V) and F(V*), the former valued with respect to the maximum probability distribution in P, the latter valued with respect to the minimum probability distribution in P. Like in Dixit and Pindyck, the two values of the decision-maker's option to invest, given by both contingent claims and dynamic programming, must satisfy some boundary conditions [12, p.141] and one of them, indeed the *value-matching condition*, permits one to evaluate the critical value of investment project either V or V*, that is the value at which it is optimal to invest. Under Knightian uncertainty it is suggested considering F(V*) as a measure of quasi-h-option value induced by the investment project. In this case, the value-matching condition is V*=F(V*)+D, setting the value of the project equal to the full cost (direct cost D plus the quasi-h-option value) of making the investment. As a consequence, the investment rule corrects the simple net present value rule by including the greater option value, that is maximum opportunity cost F(V*), compatible with the decision-maker beliefs.

Including the maximum opportunity value compatible with the decision-maker's beliefs increases the critical value of an investment project and might induce postponement of the investment decision.

5. CONCLUDING REMARKS

This paper indicates some useful outcomes. It shows that the value of an investment opportunity defined by Pindyck completely overlaps the concept of quasi-option value defined by Henry. It has been shown that the option-like characteristics of investment decisions and the contingent claims analysis used by Pindyck are able to give an empirical measure of quasi-option value. These outcomes have a large relevance because they should induce modification of the optimal rule of investment by considering the opportunity cost of killing an option to invest when soft uncertainty, irreversibility and independence learning are involved.

Since hard uncertainty is a common condition for environmental investment decisions, this paper suggests an empirical measure of the quasi-h-option value. This empirical measure exploits the dual relationship that exists between capacities. Generalizing the option approach to hard uncertainty condition and evaluating the dynamic replicating asset by the Choquet integral of its payments, it is possible to obtain two investment opportunity values. These option values are derived by considering the subadditive measure and its conjugate that the decision-maker attaches to ambiguous events. The subadditive measure and its conjugate can be guessed by considering the bid and ask prices of the replicating asset. The empirical measure of quasi-h-option value is obtained by considering the minimum expected value of the investment, compatible with the decision-maker's beliefs. As a consequence under Knightian uncertainty, the critical value of an investment project increases. Considering quasi-h-option value can induce a more conservative policy whenever investment decisions involve depletion of natural resources and severe damage of ecosystems. As an example, the possibility of obtaining new information about the effects (likely diseases) of genetically modified food on vital organs and immune system of rats could negatively affect the production of genetically modified foods.

APPENDIX A

When the decision-maker solves $V^*=id_1+\max E(i(d_1+d_2))$, subject to $d_1\geq 0$, $d_2\geq 0$ and $d_1+d_2\leq 1$, he maximizes $L=(p_Ai_A+p_Ci_C)[2d_1+d_2]-\lambda(1-d_1-d_2)+\mu d_1+\upsilon d_2$.

By the First Order Conditions, assumed $i=(p_Ai_A+p_Ci_C)$, $2i-\lambda+\mu=0$ and $i-\lambda+\upsilon=0$, that is $\upsilon=\lambda-i$ and $\mu=\lambda-2i=\upsilon-i$. Moreover the First Order Conditions imply that: $\mu\geq 0$, $d_1\geq 0$ and $\mu d_1=0$; $\upsilon\geq 0$, $d_2\geq 0$ and $\upsilon d_2=0$ and then $(\mu,\upsilon)\neq(0,0)$, that is no interior solution exists, then $(d_1^*,d_2^*)\in(0,1]\times(0,1]$.

It is possible to distinguish some cases:

$d_1^*\in(0,1]$, $\mu=0$ and $\lambda=2i$, impossible if $i<0$

$d_2^*\in(0,1]$, $\upsilon=0$, $\lambda=i$, impossible if $i<0$ and $\mu=-i$ impossible if $i>0$

$d_2^*=0$, $i<0$, then $d_1^*=0$ and $d_1^*\in(0,1]$, $\mu=0$, $i>0$, $\lambda=2i$ by the constraint $d_1^*+d_2^*=1 \Rightarrow d_1^*=1$

APPENDIX B

When the decision-maker regards prospective learning, he solves

$$\begin{aligned}V^{**}=&q_1\{id_1+[p_A\max_{d_2}i_A(d_1+d_2)]+[(p_B+p_C)\max_{d_2}(p_B|(p_B+p_C))i_B(d_1+d_2)+(p_C|(p_B+p_C))i_C(d_1+d_2)\}+q_2\{id_1+\\&+p_B\max_{d_2}i_B(d_1+d_2)]+[(p_A+p_C)\max_{d_2}(p_A|(p_A+p_C))i_A(d_1+d_2)+(p_C|(p_A+p_C))i_C(d_1+d_2)\}+q_3\{id_1+\\&+p_C\max_{d_2}i_C(d_1+d_2)]+[(p_A+p_B)\max_{d_2}(p_A|(p_A+p_B))i_A(d_1+d_2)+(p_B|(p_A+p_B))i_B(d_1+d_2)\}=\\&= q_1\{id_1+p_Ai_A(d_1+d_2)+p_Ci_C(d_1)\}+q_2\{id_1+(p_Ai_A+p_Ci_C)(d_1+d_2)\}+q_3\{id_1+p_Ci_C(d_1)+p_Ai_A(d_1+d_2)\}=\\&=(q_1+q_2+q_3)id_1+(q_1+q_3)(p_Ai_A+p_Ci_Cd_1)+q_2i(d_1+d_2)= id_1+(q_1+q_3)(p_Ai_A+p_Ci_Cd_1)+q_2i(d_1+d_2)\end{aligned}$$

Then the decision-maker calculates $\max L=id_1+(q_1+q_3)(p_Ai_A+p_Ci_Cd_1)+q_2i(d_1+d_2)-\lambda(1-d_1-d_2)+\mu d_1+\nu d_2$

By the First Order Conditions:

$\delta L/\delta d_1=i+(1-q_2)p_Ci_C+q_2i-\lambda+\mu=0$; $\delta L/\delta d_2=q_2i-\lambda+\nu=0$; moreover $d_1\geq 0$, $\mu\geq 0$, $\mu d_1=0$; $d_2\geq 0$, $\nu\geq 0$, $\nu d_2=0$.

As a result, $\nu=q_2i-\lambda$ and $\mu=-i-(1-q_2)p_Ci_C-q_2i+\lambda=-i-(1-q_2)p_Ci_C+\nu$

If $\nu=0$, then $\mu=-i-(1-q_2)p_C i_C=-i-p_C i_C+q_2 p_C i_C<0$, but $\mu<0$ is impossible and $(\mu,\nu)\neq(0,0)$, there are no interior solutions and $(d_1^{**},d_2^{**})\in(0,1]\times(0,1]$.

If $d_1^{**}\in(0,1]$, $\mu=0$ and $\lambda=i+(1-q_2)p_C i_C+q_2 i$, but this solution is impossible if $i<0$

If $d_1^{**}\in(0,1]$ and $\mu=0$, $\lambda=i+(1-q_2)p_C i_C+q_2 i$, this solution is possible if $i>0$. By the constraint $d_1^{**}+d_2^{**}=1$, then $d_1^{**}=1$

If $d_2^{**}\in(0,1]$, $\nu=0$ $\lambda=q_2 i$, this solution is impossible if $i>0$, then $\mu=-i-(1-q_2)p_C i_C$, but this solution is impossible if $i<0$; if $d_2^{**}=0$, $\nu>0$ then $i<0$ and $d_1^{**}=0$

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[1] A period between one hundred and five hundred years is expected to be necessary for the formation of one centimeter of topsoil, under natural conditions of vegetation cover.

[2] Seventy per cent of the three thousand officinal and medical plants classified by the National Cancer Institute (NCI) as active anti-cancer substances come from tropical forest.

[3] It is reported that 11.300.000 ha of forest are logged a year.

[4] It is reported that damage induced by floods, storms, hurricanes, droughts were about ninety billions of U.S. dollars in 1998.

[5] If 'the criterion function is quadratic, the planning problem for the case of uncertainty can be reduced to the problem for the case of certainty simply replacing, in computation of the optimal first period action, the certainty future values of variables by their unconditional expectations. In this sense, the unconditional expected values of these variables may be regarded as a set of sufficient statistics for the entire joint probability distribution, or alternatively as a set of certainty equivalence' [24, p.74]. Malinvaud [19] generalizes the applicability of certainty equivalent method to risky situations in which payoff function is not quadratic but functions involved are twice differentiable. However, this approach is inapplicable with irreversibility, which introduces discontinuity in the derivatives of either functions or payoff.

[6] Fisher and Hanemann [14] provide a very simple model to represent the quasi-option value. In a two-period model with independent learning, they point out that quasi-option value is always positive, by convexity of maximum operator, and it does not depend on the decision-maker's risk aversion.

[7] A different method is based on a specific contingent valuation survey, in which the decision-maker has to answer to a particular question, indeed 'what would you be willing to pay for information about future benefits of preservation and development, information that would be available before you had to decide whether to preserve or develop, assuming you do not foreclose the option to preserve in the future by choosing to develop now?' [14, p. 675]. Implementation of this method might be very difficult because the contingent valuation survey has to elicit the decision-maker preferences for information.

[8] In appendix, Hanry [16] assumes that $T=2$.

[9] "A situation is said to involve dynamic choice if it involves decisions that are made after the resolution of some uncertainty. This could occur for a couple of reasons. One is simply that the individual may not have to commit to a decision until after some uncertainty is resolved. Another reason might be that the available set of choice depends upon

the outcome of uncertainty. In any event, a dynamic choice situation will include at least some choices that the individual can (or must) postpone until after nature has made at least some of her moves" [17, p.1623].

[10] Conditional refers to the state of the information available at time when the action is made.

[11] For short $p_j = p(j)$ for every $j \in 2^{\Omega}$.

[12] See APPENDIX A.

[13] See APPENDIX B.

[14] A disinvestment opportunity (partial reversibility) is equivalent to a put option and the act to disinvest is equivalent to exercise such an option.

[15] Risk neutrality means that the discount rate is equal to the risk-free rate (e.g., Cox and Ross [10]).

[16] If spanning assumption does not hold, it is possible to value the investment project and the decision to invest by dynamic programming with an exogenous discount rate.

[17] When the decision-maker "possesses a unique, well-defined classical probability distribution over events, possesses a von Neumann-Morgenstern utility function over outcomes, and ranks subjectively uncertain acts according to the expected utility of their induced probability distributions over outcomes... [she/he] is a probabilistically sophisticated expected utility maximizer" [18, p.747].

[18] Cherubini [8] uses continuous fuzzy measures, called g_{λ}-measures, and introduces a fuzzified version of both the Merton model [21] and the Black and Scholes model [5] to parameterize the upper and lower Choquet integrals by bid and ask prices.

[19] The faith in likelihood of events represents the weight of evidence and it can be measured by $[1-\nu(s_1)-\nu(s_1^C)]$ for all $s_1 \in S$. Obviously, if there is a unique and additive probability measure on S, the capacity will be equal to its dual, the upper and lower Choquet integrals will collapse down to the standard Lebesgue integral and a unique investment opportunity value will be obtained.