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Some Comments on Time-Inconsistency in a Game with Absentmindedness

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1. Introduction

In a recent stimulating paper Piccione-Rubinstein (1997) (PR henceforth) analyzed the choice of an "absentminded driver" (AMD henceforth), a decision maker whose peculiar lack of memory entails, in the decision problem faced, a type of imperfect recall that may lead to time inconsistent behavior on her part. The game considered by PR is as in Fig 1 below.

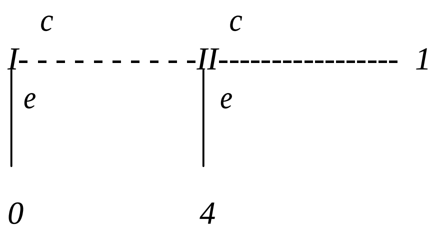

Fig 1

The payoffs associated to terminal histories of the (individual) extensive form game reveal that her goal is to obtain *4,* the highest reward. There is absentmindedness since the driver, upon reaching one of them, is unable to distinguish between node *I* and *II*. Namely she has a single information set, indicated by the dotted line connecting nodes *I* and *II*. The actions available at each intersection are either *c {continue}* or *e {exit}*; hence, AMD's goal is achieved if and only if the sequence of actions *ce* (in this precise order) is chosen.

In this simplest decision problem Piccione-Rubinstein detect the possibility of time-inconsistent behavior when both pure and (randomized) behavioral strategies are considered. Namely, the plan (on how to choose in the game) formed by AMD before she starts playing may be revised, in spite of unchanged preferences, upon reaching the information set.

A possible way out to inconsistency was suggested by PR, and a number of other authors[1], to be found in the *multiself* approach. Namely a decision maker's behavior could turn out to be time-consistent when the individual is modeled as a collection of different selves, each acting at different decision nodes (histories), and the actions chosen characterize an equilibrium profile of the game played by the various selves.

This work takes a different stance in that it argues that consistent behavior could be recovered also within a purely *oneself* approach, by showing that inconsistency may arise when the AMD's reasoning would not fully exploit awareness of absentmindedness[2]. In this sense, inconsistency could be interpreted as a form of bounded rationality. However, no claim is made that this approach is necessarily superior to other proposals; indeed, we very much share the view, made explicit by PR and Lipman (1997), that there may be more than one plausible way to model a decision maker mental process in this class of games.

Since behavioral strategies represent the general case, in Section 2 we deliver the issue by referring to them. In Section 3 we further discuss the matter and conclude the paper.

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[1] See, among others, Aumann-Hart-Perry (1997) - Battigalli (1997) - Gilboa (1997) – Lipman (1997)

[2] I was made aware by Piccione and Rubinstein that a similar approach was independently undertaken by Segal (1995) in a still (to my knowledge) unpublished paper.

2. Time Consistency in Behavioral Strategy

We shall argue on the possibility of AMD time-consistent behavior by considering a generalized version of the game in Fig 1, where the number of decision nodes is an arbitrary positive integer *T*[3]. Figure 2 below shows this game.

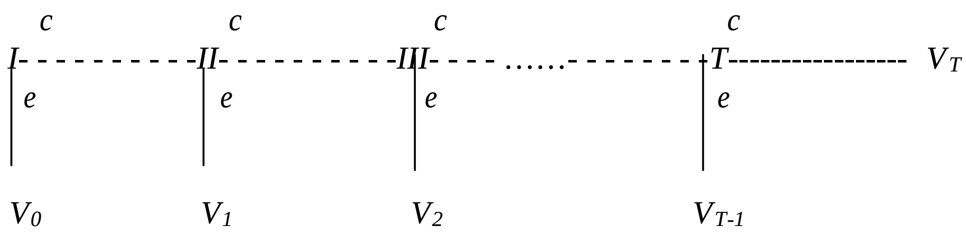

Fig 2

However, before analyzing the more general form of the problem it may be useful to start recalling briefly, and somewhat informally, the PR argument within the context of the original game of Figure 1. In what follows *p* will be the ex-ante (planned before the game starts) probability of choosing action *c {continue}, 1-p* the probability of action *e {exit}* at each node (in the behavioral strategy), *q* the ex-post (at the information set) probability of choosing *c, 1-q* the probability of choosing *e* and α the subjective probabilistic belief to be at node *I* conditional to having reached the information set.

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[3] In so doing we proceed according to an observation of PR who underline that a more realistic version of the problem could be seen as describing an individual who has to exit at the *17th* intersection of an highway (Piccione-Rubinstein 1997, pag 7). To be as general as possible we considered a version of the problem both payoff-structure and length independent.

i) Ex-ante Before the game starts AMD's expected payoff is $p(4\text{-}3p)$ so that the optimal strategy is $p^*=2/3$.

ii) Ex-post Conditional to being at the information set AMD's expected utility is $\alpha [q(4\text{-}3q)]+(1\text{-}\alpha)[4\text{-}3q]$. Hence, the optimal strategy is $q^*=max\ [0,(7\alpha\text{-}3)/6\alpha]$, different from p^* unless $\alpha=1$. However, in this case too there would still be inconsistency since, under the presumption of being at node *I* with probability one, AMD will clearly choose $q^*=1$.

We now discuss how consistency could be recovered once AMD's reasoning is formalized in a way that would fully incorporate the (explicit and implicit) model assumptions that we spell out below.

A1) (History-independent information partition) *At all non terminal histories AMD's information partition is the same (in particular as in Fig.2).*

In extensive form games and decision problems this is the standard assumption concerning an agent's information processing skills. It is however worth underlying it since taken together with the next one (A2) will entail some important implications. Notice further that forms of absentmindedness could evidently be conceived even if (A1) would not hold. For example, if ex-ante AMD would imagine that she will be able to recognize precisely every junction, so that the problem initially perceived is as in Fig. 3 below

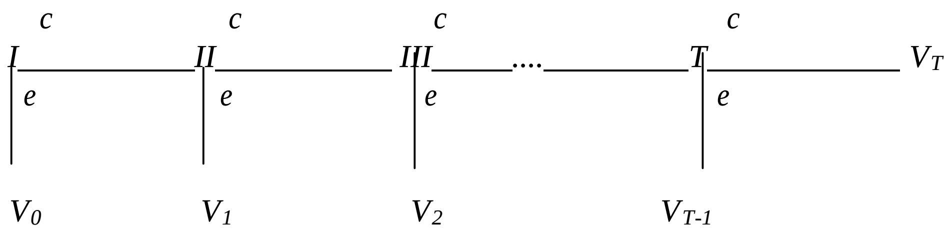

Fig 3

but ex-post (upon reaching any decision history) she would perceive a game as that of Fig. 2, then AMD would not be absentminded at all non terminal histories. She might, in this case, be considered to have a lower degree of rationality, with respect to when Fig. 2 is viewed as the game at all histories, because of her failure to initially foresee that she will be absentminded later in the game.

Being standard, in PR assumption (A1) is left implicit; what is made explicit instead is the following one. Note that the role played by (A2) is fundamental: indeed, without it dynamic inconsistency may not even be an issue.

A2) (Awareness[4] of Absentmindedness*) At all non terminal histories AMD is aware of her absentmindedness, and takes this knowledge into account when calculating her expected payoff.*

Few comments are in order here. Absentmindedness is introduced as AMD inability to distinguish between highway junctions, at all decision histories in the problem. Consequently, for both (A1) and (A2) to hold it must necessarily be true the further

[4] The notion of (un)awareness is introduced here only on intuitive grounds. For a formal treatment of the concept see Modica-Rustichini (1994,1999) and Dekel-Lipman-Rustichini (1998).

implication that absentmindedness should also regard any *element* that may provide AMD with information concerning the node she is at. The *element* we shall particularly be interested in is her mental process at decision nodes.

A possible informal argument supporting the above assertion could be the following. Consider the problem in Fig. 2 which, suppose, illustrates the game structure imagined by AMD at all non terminal histories. Without loss of generality, assume now that at node *IV* (say) AMD could recall (for example) a consideration that she has made at some previous node, different from the start of the game, and so either at node *I, II* or *III*. Not knowing that she is at *IV* she could clearly not infer that the consideration was made at one of the first three junctions. Nonetheless, she could certainly deduce not to be at node *I* and so her information partition at history *IV* can not be as in Fig 2, with histories *I* and *IV* included in the same information set.

The following two assumptions are not in PR and will be crucial for the result

A3) (Payoff Symmetry) *At all histories in the information set AMD thinks that her perceived expected payoff is identical to that of any other history in the set. Formally, if* $\Pi(q;T\,|\,i)$ *is the (ex-post) conditional, to being at node i, with* $i=I,..,T-1$, *perceived expected payoff at the next node of the information set then* $\Pi(q;T)=\Pi(q;T\,|\,i)$ *where* $\Pi(q;T)$ *is the ex-post (conditional) to being at the information set perceived expected payoff.*

Though (A3) might be given independently of the first two assumptions, its introduction could be motivated by means of the following informal argument, much akin to the one given above for (A2). Assume that upon reaching (again say) junction *IV* the individual's perceived expected payoff is v and she believes that this is so at all nodes in the information set except (say) junction V where it would be different from v; then, node V could not be part of the information set.

A4) (Belief Consistency) *Conditional to having reached the information set, the probabilistic belief of being at a particular node must be consistent with the behavioral strategy chosen at the information set (ex-post) and not at the start of the game (ex-ante)*[5]*. Formally, with reference to the generalized game of Fig. 2, if* α_i *(with i=I,..,T and* $\alpha_I=\alpha$*), is AMD belief to be at the ith node then* $\alpha_i=\alpha q^{i-1}$ *where q is the behavioral (probabilistic) strategy chosen at the information set. This would clearly imply that*

$$\alpha+\alpha q+\alpha q^2+\ldots+\alpha q^{T-1}=1$$

and so that $\alpha=(1-q)/(1-q^T)$*.*

In order to motivate (A4) it is simpler to consider the game in Fig. 1; in it, to calculate her conditional expected payoff AMD could reason as follows:

"If I am at node *I*, my belief to be at it is α and the *ex-post* behavioral strategy is q, then my belief $(1-\alpha)$ to be at node *II* should necessarily satisfy the equality $(1-\alpha)=\alpha q$, since transition from *I* to *II* can only occur *via* the ex-post behavioral probabilistic strategy, namely with the probability q that I would choose now. If I am at node *II* instead, as I have no elements to believe that at history *I* my reasoning has been different from the

[5] We underline so since in PR belief consistency is defined with respect to the probabilistic behavioral strategy p^* chosen ex-ante. Namely, they require $1-\alpha=\alpha(2/3)$. Our view on adopting a different hypothesis is articulated in what follows.

present one and so I assume that it must have been the same. This implies that I must have passed from *I* to *II* with the very same probability q that I would choose now."

Having specified the main assumptions of the model we are now ready to enter the analysis that, as we said, will be conducted with reference to the generalized game of Fig. 2 where conclusions will indeed be independent of the game-length and payoff structure.

iii) Ex-ante The ex-ante expected payoff $\Pi(p;T)$ is clearly

$$\Pi(p;T) = (1-p)\Sigma_i p^i V_i + p^T V_T \quad i=0,..,T-1 \quad (1)$$

so that the optimal $p=p^*$ is obtained by maximizing (1).

iv) Ex-post Let, as in (A3), $\Pi(q;T)$ be the ex-post (conditional to being at the information set) expected payoff of the driver. Since α_i , with $i=I,..,T$, is the probabilistic belief of being at node i and $\Pi^*(q;T\,|\,i)$ the conditional (to being at node i) expected payoff it must be that

$$\Pi(q;T) = \Sigma_i\ \alpha_i\ \Pi^*(q;T\,|\,i) \qquad i=I,..,T \qquad (2)$$

For a better appreciation of (A3) and (A4), before going further it may be useful to briefly see first what they entail in the PR case of $T=2$. Above we have seen that in this game PR define

$$\Pi^*(q;2\,|\,I) = q(4\text{-}3q) \text{ and } \Pi^*(q;2\,|\,II)=(4\text{-}3q) \qquad (3)$$

and so p=p*, maximizing

$$\Pi(p;2) = p(4\text{-}3p) \qquad (4)$$

is in general different from q=q* maximizing $\Pi(q;2)$, which would entail time-inconsistency.

If we instead now incorporate (A3) we obtain

$$\Pi(q;2) = \alpha q \Pi(q;2) + (1\text{-}\alpha)\Pi^*(q;2\,|\,II) \qquad (5)$$

while (A4) entails

$$(1\text{-}\alpha)=\alpha q \qquad (6)$$

so that

$$\Pi(q;2)=q(4\text{-}3q)$$

which has the same functional form of $\Pi(p;2)$, hence the conclusion $p^*=2/3=q^*$.

Let us go back now to the generalized problem in Fig. 2. Payoff symmetry entails $\Pi(q;T)= \Pi(q,T\,|\,i)$, for $i=I,..,T\text{-}1$, so that (2) becomes

$$\Pi(q;T)= \alpha[q\,\Pi(q;T)+(1-q)V_0]+\alpha q[q\,\Pi(q;T)+(1-q)V_1]+..+\alpha q^{T-1}[(1-q)V_{T-1}+qV_T] \qquad (7)$$

from which it follows that

$$\Pi(q;T)=\alpha q\Pi(q;T)[(1-q^{T-1})/(1-q)]+\alpha(1-q)\Sigma_i q^i V_i+\alpha q^T V_T = (1-q)\Sigma_i q^i V_i + q^T V_T\,;\ i=0,..,T-1 \qquad (8)$$

namely that $\Pi(p;T)$ and $\Pi(q;T)$ (as functions respectively of p and q) have the same functional form, so that $p^*=q^*$ and inconsistency would not arise.

3. Conclusions

In Piccione-Rubinstein the absentminded driver is dynamically inconsistent because some "elements of inconsistency" are built into her reasoning. In particular in the paper we argued that two assumptions, *belief consistency* and *payoff symmetry*, play a decisive role for time-coherent decisions to be made. The former specifies that, upon reaching the information set, the individual's belief to be at a history should be consistent with the ex-post, rather than ex-ante, behavioral probabilistic strategy; the latter instead captures an agent who perceives the same expected payoff at all histories in the information set.

We view payoff symmetry as a necessary consequence of the fundamental epistemic assumptions that the individual is always aware of her own absentmindedness and that her information partition is history-independent. More explicitly, at a junction the driver can not imagine herself to perceive at a (possible) future junction a payoff different from the one she is presently perceiving. This last possibility may be plausible if the individual were to anticipate the reasons why in the future she will perceive a different expected payoff; but this might mean that she has information that could make her capable to discriminate among decision nodes.

Belief consistency captures the idea that upon reaching an information set the conditional probability of being at a history should only depend upon the behavioral strategy chosen ex-post, namely should not be determined by the ex-ante behavioral strategy. This too is motivated by the individual's awareness of her own absentmindedness. In the generalized version of the game, with an unspecified number of nodes, it is not without difficulty to think of an agent who imagines to possibly reach the "*17th* junction" with transition probability, from one node to the next, given by the ex-ante behavioral strategy. Indeed, for this to be the case the individual should be convinced of never have changed her behavioral strategy and so, it could also be argued, reconsideration of it at the information set would be pointless.

Once these two assumptions are incorporated into the model it is shown that time-consistency emerges.

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