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1. Introduction.

A frequently voiced opinion on the relevance of the results of the Cambridge controversy in capital theory is that, although reswitching of techniques and reverse capital deepening are definite possibilities and the range of values of technical coefficients for which these phenomena can happen is not of measure zero (Schefold, 1976), still they are so unlikely, as not to endanger the traditional neoclassical approach to income distribution([1]).

The present paper intends to contribute to the discussion of this opinion, through 1) a reconsideration of the first attempt to assess the numerical probability that reswitching and reverse capital deepening may happen - an attempt, due to the late prof. D'Ippolito (1987), whose results were seen by its author and by others (e.g. Metcalfe and Steedman, 1995) as giving some support to that opinion - , and 2) through a discussion of the reasoning which appears to be implicit in the importance attributed to the supposed low likelihood of reverse capital deepening.

Parts I and II of the present paper question D'Ippolito's results. Part I starts by summarising D'Ippolito's procedure, what may be of interest to readers unable to read the Italian original; it then argues that D'Ippolito's very low values for the probability of 'perverse' switches are due to a logical slip. D'Ippolito obtains probabilities that a switch of techniques be 'perverse', which, for example, as the value of the average rate of profits increases from 5% to 30%, increase from approximately 2% to 8%; sufficiently low

[1] . Representative examples are Hicks (1965, p. 156; 1973, p. 44); Eltis (1973, Ch. 5); Stiglitz (1974); Malinvaud (1986); Laing (1991). For contrary views cf. e.g. Garegnani (1990, pp. 71-2); Ciccone (1996).

probabilities, he apparently concludes, as to allow economists to consider reverse capital deepening so improbable as not to endanger traditional reasonings. The correctness of this inference is questioned in Part III of the present paper; but it may be nonetheless of interest that, once the logical slip in D'Ippolito's procedure is corrected, these probabilities rise to, respectively, 36% and 45%. Another modification of the estimation procedure, perhaps closer to the spirit of what D'Ippolito may have had in mind, results in lower probabilities, but still significantly higher than D'Ippolito's: 7.5% and 10.7% respectively. In order to assess the meaningfulness of these exercises, a quite different approach is then illustrated in Part II, which I think has intuitive appeal because based in an immediate way on the shape of the w(r) curves. This method requires the estimation of complicated integrals, but numerical approximation methods make it possible easily to surmount this difficulty. It results in still different probabilities, e.g. 8.4% and 13.5% for r=5% and r=30% respectively. Table I reports the probabilities calculated with the different procedures for values of the rate of profits from 1% to 3000%.

In Part III, after noticing the unavoidable arbitrariness of these exercises, it is argued that, if one nonetheless believes these exercises to yield some useful information, then the message is that 'perverse' switches are a very relevant possibility. Then the argument, implicit in the importance attributed to the supposed low likelihood of reverse capital deepening, is discussed and found wanting for other reasons too.

PART I

2.

In the model studied by Samuelson (1962), Hicks (1965), Garegnani (1970) and D'Ippolito (1987), a single consumption good can be produced via different techniques, each one requiring a different circulating capital good and labour, with the capital good in turn produced by itself and labour. Thus the transition from one technique to the other is not studied; the exercises consist of comparisons of long-period positions in which the transitional technologies no longer appear. The production processes last one year. There are constant returns to scale; α_i, β_i are the technical coefficients, respectively, of capital good of type *i* and of direct labour in the production of the consumption good according to technique *i*; a_i, b_i are the technical coefficients respectively of the capital good and of direct labour in the production of capital good of type i. For simplicity each capital good will be measured here in such units that its production needs one unit of direct labour, so b_i=1, for all *i*. The consumption good is the numéraire. Wages are paid at the end of the production period. The price-of-production equations for any technique *i* are, with *w* the rate of wages in terms of the consumption good and p_i the price of the capital good:

[1] $1 = \alpha_i p_i(1+r)+ßw$
[2] $p_i = a_i p_i(1+r)+w$.

In the sequel, the subscript *i* will be omitted when unnecessary.

The technical coefficients are non-negative. These equations establish a functional dependence of *w* on *r*:

[3] $w = [1-(1+r)a]/[ß+(1+r)(\alpha-aß)]$

such that, as long as $a>0$ and that direct or indirect labour is necessary to produce one unit of net product (consisting of the consumption good), this function crosses the non-negative orthant with negative slope and positive intercepts on both axes, determined by:

[4] $R = (1-a)/a$
[5] $W = (1-a)/[ß+(\alpha-aß)]$,

I shall call *w(r) curve* this portion of the function *w(r)* defined by [3].

It is known that two *w(r)* curves can cross each other in the non-negative orthant at most twice. If they do cross twice, the switch point at the higher level of r gives rise to reverse capital deepening.

Given a *w(r)* curve, it is known that, having selected a point *(r,w)* on this curve, as long as the economy is stationary (which will be assumed here) the value of capital per unit of labour is given by the absolute value of the slope of the straight line connecting this point with the point *(0,W)*. This is because in a stationary economy the net product per unit of labour consists solely of the consumption good and equals *W*; at long-period prices it must be true that the net product distributes itself between wages and profits (or interest) i.e., with *k* the value of capital per unit of labour, $W=w+rk$, which can be re-written as

[6] $k=(W-w)/r$.

Therefore if at r^* there is switch between two techniques, and if we call technique 1 the one dominant for *r* slightly less than r^* and technique 2 the one dominant for *r* slightly greater than r^*, the switch gives rise to reverse capital deepening, i.e. $k_2(r^*)>k_1(r^*)$, if and only if $W_2>W_1$, cf. Fig. 1.

(insert Fig. 1 about here)

For this model, D'Ippolito tries to determine the 'a priori' probability $P_{me}(r)$ that, if two techniques have a switchpoint at a rate of profits equal to *r*, this switchpoint be associated with reverse capital deepening or, as he unscientifically puts it, be 'perverse'([2]). He too calls technique 2 the one which becomes dominant to the right of the switchpoint, i.e. by assumption

$$-dw_2(r)/dr \equiv -w_2'(r) < -dw_1(r)/dr \equiv -w_1'(r)$$

[2] . Ptolemaic astronomers might have, analogously, called 'perverse' the epicycles which went counter their geocentric vision. Interestingly, it does not seem that they had recourse to such terminology. Economics apparently raises more emotional responses than astronomy does.

where r is the rate of profits at the switchpoint. He proceeds as follows. Because $w_1(r)=w_2(r)$ the previous inequality can be re-written

$$[7] \quad -w_2'(r)/w_2(r) < -w_1'(r)/w_1(r).$$

Since

$$[8] \quad w'(r) \equiv dw(r)/dr =$$
$$= \{-a[\beta+(1+r)(\alpha-a\beta)]-[1-(1+r)a](\alpha-a\beta)\}/[\beta+(1+r)(\alpha-a\beta)]^2,$$

and

$$[9] \quad p=1/[\beta+(1+r)(\alpha-a\beta),$$

equation [7] simplifies to

$$[10] \quad \alpha_2 p_2/(1-(1+r)a_2) \leq \alpha_1 p_1/(1-(1+r)a_1).$$

In order for the switch to be a 'perverse' one, the (value of) capital per unit of labour at the switchpoint must be greater for technique 2 than for technique 1. D'Ippolito notices that, as techniques switch, capital per unit of labour k, and capital per unit of net output K, vary in the same direction([3]), so this condition can be written $K_2(r)>K_1(r)$, or, since $K=\alpha p/(1-a)$ (because $\alpha/(1-a)$ units of the capital good are employed in a stationary economy producing 1 unit of the consumption good as net product):

[3] . The vertical intercept of the w(r) curve measures the value of net output per unit of labour, i.e., in our case, the physical production of consumption good per unit of labour (because the economy is assumed to be stationary). Let y_1, y_2 be these net outputs per unit of labour for technique 1 and 2, and assume $y_1<y_2$. Then labour employment *per unit of output*, L=1/y, is smaller with technique 2. Put net output equal to 1; since net output must equal net income i.e. y=1=wL+rK, if L is smaller in technique 2, then K must be greater. But the value of capital per unit of labour will also be greater at a switchpoint if the vertical intercept is greater, cf. equation [6].

[11] $\alpha_2 p_2/(1-a_2) \leq \alpha_1 p_1/(1-a_1)$.

D'Ippolito puts

[12] $v \equiv \alpha_1 p_1/(\alpha_2 p_2)$, $\rho \equiv 1+r$

and re-writes inequalities [10], [11] as:

[13] $a_2 - 1/\rho \leq (1/v)(a_1 - 1/\rho)$.
[14] $a_2 - 1 > (1/v)(a_1 - 1)$.

Let us then represent the points (a_1,a_2) on the non-negative orthant of a plane. Assume a given r and a given v. Equation [13] implies $a_2 \leq a_1/v - 1/(v\rho) + 1/\rho$, i.e. a_2 must be below the straight line with slope $1/v$ and passing through the point $(1/\rho,1/\rho)$. Equation [14] implies $a_2 > a_1/v - 1/v + 1$, i.e. a_2 must be above the straight line with slope $1/v$ and passing through the point $(1,1)$. On the other hand neither a_1 nor a_2 can be greater than $1/\rho$ if w is to be non-negative, because the maximum rate of profits is $R=1/a - 1$. Hence the set of couples (a_1,a_2) which satisfy both [13] and [14] is the set of the points, internal to the square OCBQ (of side length equal to $1/\rho$), which are both to the right of the line AB and to the left of the line A'E in Fig. 2. This set is not empty only if $v<1$, because if $v>1$ the line S is to the right of the line E([4]). It is the shaded area F in Fig. 2a.

(insert Fig. 2 about here)

Having reached this pleasant graphical result, D'Ippolito proceeds in a way which appears marred by a logical slip.

He argues (D'Ippolito, 1987, p. 17) that, for given r and v, the probability that the switch be 'perverse' is given by the ratio between the surface of the area F of points satisfying both

[4] . The economic meaning of v<1 is that the capital per unit of product in the sole consumption good industry must be greater with technique 2 than with technique 1.

constraints, and the surface $1/\rho^2$ of the whole square OCBQ, i.e. is given by ρ^2 times $F(r,v)$ (if with $F(r,v)$ one indicates the surface of F).

This is difficult to accept. Let us concede for the sake of argument to D'Ippolito the right to limit the inquiry to the coefficients a_1, a_2, and to assume, as he clearly does, that all points (a_1,a_2) compatible with the given r might occur with equal probability([5]). But D'Ippolito forgets that this probability can only be equal *before* one separates the cases he is interested in from all possible occurrences of couples of techniques. Since he has *assumed* that the two techniques have a switchpoint at the given rate of profits r, and since he has *decided* to call technique 2 the one which becomes dominant to the right of r, then (a_1,a_2) satisfies [13] by assumption; the points above line AB are therefore out of the question. So for given r and v, by assumption (a_1,a_2) is in the triangle ABC of Fig. 2a if $v<1$, or in the trapeze OCBH of Fig. 2b if $v>1$. Then the correct ratio - I will call it $Z(r,v)$ - between area of 'perverse' cases and area of possible cases, for a given r and a given $v<1$, is the ratio between $F(r,v)$, and the surface - I shall call it $D(r,v)$ - of the triangle ABC (the shaded area in Fig. 3).

(insert Fig. 3 about here)

Under the assumption, which D'Ippolito explicitly makes (*ibid.*, p. 18), that all values of v are equiprobable([6]), one may therefore

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[5] . It might on the contrary be argued that very low values of the coefficient a, implying very high values of the maximum rate of profits R, are less and less plausible the more they approach zero. It might also be argued that a cannot be very close to $1/(1+r)$

[6] . He appears here to mean all values of v between 0 and 1, as made clear by the limits of integration in footnote 15, p. 18 of his article; if one were to interpret him literally, then since v can vary from 0 to $+\infty$, the probability that v fall in any finite interval would be zero, i.e. the probability would be all concentrated at the value $v=+\infty$. The a priori symmetry of the possibilities $v<1$ and $v>1$ suggests instead to consider the two cases as equally probable for a random picking out of two techniques (giving or not rise to a switch: cf. Section 3), i.e. to consider the probability that $v'<v<v''<1$ with v' and v'' assigned, equal to the probability that $1/v''<1/v<1/v'$. D'Ippolito appears to concur in this view (cf. below in the text).

proceed to calculate the average probability $Z^*(r)$ that at a given r a switch be 'perverse' under the assumption that v is random but <1, by integrating $Z(r,v)$ with respect to v from 0 to 1. $Z^*(r)$ is not the average probability $P_{me}(r)$ that at a given r a switch be 'perverse', because it is determined under the assumption that $v<1$ so it leaves out the possibility that $v>1$. But D'Ippolito says that the case $v>1$ "would cover the remaining 50% of cases" (*ibid.*, p. 16), so he appears to authorise us to assume that the probability that $v<1$ is 50%. Then $P_{me}(r)$ is simply one half of $Z^*(r)$.

D'Ippolito, on the contrary, having said that the probability of a 'perverse' switch, for a given r and a given $v<1$ is given by $\rho^2 F$, simply goes on to integrate this probability over v from 0 to 1 in order to obtain his $P_{me}(r)$, without mentioning any more the fact that there is also the case $v>1$. I have been unable to find a way to make his several statements consistent([7]). His probabilities are reported in column 9 of Table I, under the heading 'D'Ippolito original'. They converge to 25% as r tends to +∞.

The calculation of the probability that a switch be 'perverse' with the correction I find necessary, i.e. as $Z^*(r)/2$, yields quite different values from the ones calculated by D'Ippolito; they are listed in column 10 of Table I under the heading 'D'Ippolito corrected'. They converge to 50% as r tends to +∞.

Appendix A shows how to determine the surfaces, whose ratios determine the probability of a perverse switch for given (r,v) according to D'Ippolito, and according to my correction.

3.

It cannot be excluded that D'Ippolito thought that he had the right to neglect to consider the cases v>1 because in some way he was

[7] . I am unable to accept Ciccone's attempt (1996, pp. 51-54) to justify D'Ippolito's procedure. Ciccone writes (p. 52, my translation): "Because of the symmetry between the conditions $v<1$ and $v>1$, the $P_{me}(r)$ calculated for values of v included between 0 and 1 comes out to be in fact equal to the average probability obtainable for values of v included between 1 and +∞"; but this is false, because if one follows D'Ippolito in calling technique 2 the one dominant to the right of r, then the second average probability is simply zero.

already taking the existence of those cases into account, by dividing *F(r,v)* by the whole surface of the square OBCQ instead of by the sole surface *D(r,v)*([8]).

But then a more consistent estimation procedure would appear to be the following one.

Let us drop the assumption that all values of $v<1$ are equiprobable, by noticing that $v<1$ and $v>1$ are symmetrical and hence equiprobable only if no constraint on the coefficients is added so that all points in OCBQ might be picked by the random technique selection process; while here there ***is*** a constraint, and this is that at the switchpoint it is technique 2 which becomes dominant to the right of *r*. This constraint makes only the points in ABC eligible, i.e. a fraction of the area of OCBQ the smaller, the smaller is *v*. If one then considers all values of a_1, a_2 as equally probable([9]), one may conclude that the values of *v* are not all equiprobable, but are the more probable, the greater the portion of OCBQ which makes the switch possible. The natural assumption then is to assume that the probability of each value of *v*, or of each value of $1/v$ if $v>1$([10]), is proportional to the ratio between *D(r,v)* and the area of OCBQ.

In other words, let us replace *v* in *Z(r,v)* with a variable *y*, $0 \le y \le 2$, defined as $y=v$ if $v \le 1$ and $y=2-(1/v)$ if $v>1$. The density function of *y*, *p(y)*, is assumed linear, going from 0 to 1 as *y* goes from 0 to 2, corresponding to the ratio $D(r,y)/(1/\rho^2)$ between the surface of ABC or of OCBH, and the surface of OCBQ. Then the average probability that a switch be perverse $P_{me}(r)$ is the definite integral of *Z(r,y)p(y)* over *y* from 0 to 2, divided by 2; but since $Z(r,y)=0$ for $y>1$, it suffices to calculate the definite integral of *Z(r,v)p(v)* over *v* from 0 to 1, and then divide by 2. Since for $v<1$ it is

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[8] . It may be noticed that if one associated to each $v<1$ the corresponding $1/v$, the eligible portions of OCBQ would sum to exactly the area of OCBQ.

[9] . To consider all points in OCBQ equally probable is clearly arbitrary in that they would not be equally probable if one decided e.g. that it is all admissible couples (α_1,α_2) that are equally probable. This arbitrariness is ineliminable from exercises of this kind.

[10] . Cf. footnote 6 above on the need to replace *v* with $1/v$ when $v>1$ in order to avoid having a zero probability of all finite values of *v*.

$D(r,v)=v/(2\rho^2)$ (cf. Appendix A) then $p(v)= v/2$, so $P_{me}(r)$ is one half of the definite integral of $vZ(r,v)$ over v from 0 to 1.

The resulting probabilities are listed in the last column of Table I, under the heading 'D'Ippolito reinterpreted'. I do not claim that this 'reinterpretation' has strong textual support.

Part II

4.

Now I explore a different method of estimating $P_{me}(r)$, which is not based on the coefficient v.

This method starts by assuming an initially given switchpoint C=(r^*,w^*) in the (r,w) plane in which one draws the $w(r)$ curves of the different techniques. For brevity in the sequel I drop the asterisks.

The form of a $w(r)$ curve in this model depends on α,β,a. There are therefore three degrees of freedom. If we establish that the curve must pass through C and must have a given $R \geq r$ (and $>a$ \r if $w>0$) and a given $W \geq w$ (and $>w$ if $r>0$), the three degrees of freedom are eliminated and the $w(r)$ curve, i.e. the technique, is completely determined.

The curve will not always correspond to an economically acceptable technique, though. I take from D'Ippolito (1987, p. 34) the following reasoning. For a given point C and a given a, the values of α and β must satisfy equations [3] and [5] where w, r and a are given, what can be re-written as

[15] $w(1+r)\alpha + w[1-(1+r)a]\beta = 1-(1+r)a$

[16] $W\alpha + W(1-a)\beta = 1-a$.

For a given W, this is a system of 2 linear equations in α, β. The solutions are

[17] $$\beta = \frac{w(1+r)(1-a)-W[1-(1+r)a]}{wWr}$$

$$[18] \quad \alpha = \frac{(W-w)[1-(1+r)a](1-a)}{wWr}$$

As long as $wWr>0$, for $ß$ to be non-negative the numerator on the right-hand side of [9] must be non-negative i.e.

$w(1+r)(1-a) \geq W[1-(1+r)a]$

which, since $(1-a)/a=R$ implies $[1-(1+r)a]/(1-a)=(R-r)/R$, can be re-written as

$$[11] \quad W \leq w(1+r)\frac{R}{R-r} \equiv W_{max}$$

The right-hand side of this inequality defines a new variable W_{max} whose meaning will be clarified presently.

Now $w/(R-r)$ is the absolute slope of the straight line connecting C with R (the point where the $w(r)$ curve touches the horizontal axis), so $wR/(R-r)$ is the value of w where this straight line crosses the vertical axis, see Fig. 4.

(insert Fig. 4 about here)

Therefore W cannot exceed a value $W_{max}(r,w,R)$ determined by the value of this point multiplied by $(1+r)$.

This one, $W \leq W_{max}$, is the sole constraint besides $W>w$ (assuming $r>0$): $\alpha \geq 0$ does not pose a constraint because $W-w \geq 0$ and $[1-(1+r)a](1-a)=(R-r)/R \geq 0$; as to R, it can be chosen arbitrarily close to r by increasing a; and W can be chosen arbitrarily close to w through the sufficiently high values of α and β determined by equations [17] and [18].

The $w(r)$ curve will be concave (downwards) if W is below the point $W_{max}/(1+r) = wR/(R-r)$ where the straight line through R and C crosses the vertical axis; it will be convex if W is in between this point and the point of ordinate W_{max}.

For given points C (with $r>0$) and R, therefore, a given W satisfying the constraint $w<W\leq W_{max}$ uniquely determines the $w(r)$ curve. This means that all the possible $w(r)$ curves passing through given points C=(r,w) and R can be generated by letting W vary in the interval $(w, W_{max}]$.

It will be assumed in the sequel that each value of W in this interval has the same probability. This does not appear to be a more arbitrary assumption than the analogous ones in D'Ippolito's analysis.

With C always fixed let us now suppose it to be known with certainty that no available technique, of those whose curve passes through C, has an associated R greater than a certain finite value R_{sup}.

I find such an assumption (which does not prevent one from fixing a very high R_{sup}, nor from admitting that technical progress increases R_{sup}) more reasonable than the assumption (implicit in D'Ippolito) that R can take any value, however great: what would imply that one can get as near as one likes to producing with unassisted labour. Anyway this assumption is not necessary to the method proposed here, it can be seen as only an intermediate step to assuming $R_{sup}=+\infty$.

Let us suppose that two (admissible) couplets (R,W) are repeatedly randomly selected, thus randomly selecting two $w(r)$ curves through C and hence two techniques. The probability that the two values of R coincide is zero, so let us assume that they differ, and let us call technique 1 the one associated with the lower R, and technique 2 the other one. Hence $R_1<R_2$ by assumption. (Notice that technique 2 is no longer defined as the one which is dominant to the right of the switchpoint, but as the one with the higher R.)

The probability that $W_1=W_2$ is analogously zero. A necessary condition for there to be a second intersection of the two $w(r)$ curves either to the left or to the right of C, is that $W_1<W_2$, see Figs. 5a, 5b.

(insert Fig. 5 about here)

This condition is not quite also sufficient, because it is possible that the two $w(r)$ curves be tangent in C (see Fig. 5c); but if W_1 and W_2 are, as we are assuming, continuous variables, the probability of this case is zero: once r, w, R_2, W_2 and $R_1<R_2$ are assigned, the condition that the two $w(r)$ curves be tangent in C uniquely determines W_1, as will be shown later. Thus this case can be neglected. Therefore except for this negligible fluke, $W_1<W_2$ is a necessary and sufficient condition for the two $w(r)$ curves to reswitch.

The probability that $W_1<W_2$, to be indicated with the symbol μ^*, is the probability that there be a second switchpoint, conditional on the curves passing through the assigned point C. Since this second switchpoint might be to the right of C, in which case the switch in C does not give rise to reverse capital deepening (i.e. is not 'perverse'), we must also determine the probability - to be indicated with the symbol P - that, in case the two curves reswitch, the second switch be to the left of C; then the probability that the switch in C be 'perverse' will be given by the product μ^*P and will be indicated with the symbol $\pi(r,R_{sup})$ - it will be shown that it depends only on the values of r and R_{sup}.

5.

Let us now determine the probability μ^* that, having assigned the switchpoint C=(r,w) with $r,w>0$, having randomly selected two $w(r)$ curves passing through C, and having called 'technique 2' the one with the greater R, it is the case that $W_1<W_2$.

Let R_1 and R_2 be initially given, and let us consider the admissible intervals for W_1 and W_2:

$$[20] \quad w < W_1 \leq w(1+r)\frac{R_1}{R_1-r} \equiv W_{1max}$$

$$[21] \quad w < W_2 \leq w(1+r)\frac{R_2}{R_2-r} \equiv W_{2max}$$

Obviously $(W_{2max}-r)<(W_{1max}-r)$ because W_{max} decreases as R increases with C fixed. So it is possible that $W_1>W_{2max}$ but it is excluded that $W_2>W_{1max}$.

If we consider all values of W_i as equiprobable within its admissible interval, then the probability, that $W_1<W_2$ conditional on $W_1\leq W_{2max}$, is 1/2; while the probability that $W_1<W_2$ conditional on $W_1>W_{2max}$ is zero; so the unconditional probability that $W_1<W_2$ must be 1/2 times the ratio, to be indicated as Q, between $(W_{2max}-w)$ and $(W_{1max}-w)$. This ratio is given by:

$$[22] \quad Q \equiv (W_{2max}-w)/(W_{1max}-w) =$$

$$= \{[(1+r)wR_2/(R_2-r)]-w\}/\{[(1+r)wR_1/(R_1-r)]-w\} =$$

$$= \frac{(R_1-r)/(R_2-r)}{(R_1+1)/(R_2+1)}.$$

Q comes out not to depend on w; the probability, that $W_1<W_2$ for an assigned quadruple (w,r,R_1,R_2), remains the same if only w is varied (i.e if the point C is moved vertically). So from now on we forget about w.

Let us now suppose that only r and R_2 are assigned, and let us determine the average probability $\mu(r,R_2)$ (not yet μ*!) that $W_1<W_2$ as R_1 is made to vary from r to R_2.

In order to have an intuitive grasp, let us start by noticing that Q tends to zero as R_1 tends to r, and tends to 1 as R_1 tends to R_2. For each assigned R_1 the probability that $W_1<W_2$ is Q/2, so it varies from 0 to 1/2 as R_1 is made to vary from r to R_2. If at the denominator of Q, instead of $(R_1+1)/(R_2+1)$, there were 1, then Q would indicate the proportion of the distance between r and R_2 travelled by R_1, so it would increase linearly from 0 to 1, its mean

would be 1/2, and so the average probability μ would be 1/4 if one considers all values of R_1 between r and R_2 as equiprobable. That the denominator of Q is on the contrary always less than 1 except when $R_1=R_2$ means that μ must be greater than 1/4; furthermore since Q/2 cannot be greater than 1/2 (because for given r and R_2, Q is a strictly increasing function of R_1 as shown by its derivative, and it tends to 1 as R_1 tends to R_2), μ cannot be greater than 1/2.

Formally,

$$[23]\quad \mu = \frac{1}{2(R_2-r)}\int_r^{R_2} \frac{(R_1-r)(R_2+1)}{R_1+1)(R_2-r)} dR_1 = 1/2 \cdot (R_2+1)[R_2-r\cdot \ln(R_2+1)+$$

$$+r\cdot \ln(1+r)+\ln(1+r)-r]/(r-R_2)^2.$$

It will be useful to notice that this is an increasing function of R_2, because its derivative is

$$[24]\quad \partial\mu/\partial R_2 = -1/2 \cdot (1+r)\{(r+R_2)[\ln(R_2+1)- \\ -\ln(r+1)]+2[r-\ln(1+r)]-2[R_2-\ln(1+R_2)]\}/(-R_2+r)^3$$

always positive for $0<r<R_2$, because, since the denominator is negative, the sign of $\partial\mu/\partial R_2$ depends on the sign of

$$[25]\quad (r+R_2)[\ln(R_2+1)-\ln(r+1)]+2[r-\ln(1+r)]- \\ -2[R_2-\ln(1+R_2)];$$

this expression reduces to zero if $R_2=r$, where its first and second derivatives with respect to R_2 are zero, while the third derivative, which is $(2r-R_2+1)/(R_2+1)^3$, reduces to $1/(1+r)^2>0$ if $R_2=r$. Therefore when $R_2=r$, expression [25] is zero but is an increasing function of R_2, so to the right of $R_2=r$ expression [24] is positive. And since the second derivative of expression [25] with respect to R_2 is $(R_2-r)/(R_2+1)^2$ which is always positive for $R_2>r$, the first derivative is always an increasing function of R_2 and so it too is positive to the right of $R_2=r$, and therefore expression [25] too is increasing and therefore positive to the right of $R_2=r$, what completes the demonstration.

The limit of μ for R_2 tending to r from the right is 1/4, for R_2 tending to $+\infty$ is 1/2, for r tending to zero is

$$\lim_{r\to 0_+} \mu = \frac{R_2+1}{2R_2^2} \cdot (R_2-\ln(R_2+1)).$$

Having determined $\mu(r,R_2)$ and having found how it varies with R_2, the average probability $\mu^*(r,R_{sup})$ that $W_1<W_2$ when R_2 is also free to vary on an interval $(r,R_{sup}]$ is simply the definite integral of $\mu(r,R_2)$ over R_2 from r to R_{sup}.

Through e.g. Maple one can obtain the exact analytical expression for $\mu^*(r,R_{sup})$ as just defined (it is very long and for this reason omitted here). But for the purpose of arriving at numerical estimates of μ^* also for very high values of R_{sup}, and in the limit for R_{sup} tending to $+\infty$, it seems preferable to replace the assumption, of a uniformly distributed probability of all values of R between r and R_{sup}, with the assumption of a uniform probability distribution of all values of the coefficient $a=1/(1+R)$ in the corresponding interval, because, as we let R_{sup} tend to $+\infty$, the first assumption would result in a probability tending to zero of all values of R in any finite interval, i.e. in terms of the coefficient a we would be assigning, in the limit, probability zero to all nonzero values of this coefficient, what is absurd. There is of course some arbitrariness in assuming a uniform probability distribution of all admissible values of a, but some such arbitrariness appears inevitable in this kind of exercises.

Replacing then R_1 and R_2 with $(1-a_1)/a_1$ and $(1-a_2)/a_2$ in the equations from [20] to [23] we have:

[26] $W_{1max}=w(1+r)(1-a_1)/(1-a_1-ra_1)$

[27] $W_{2max}=w(1+r)(1-a_2)/(1-a_2-ra_2)$

[28] $Q/2 \equiv (W_{2max}-w)/[2(W_{1max}-w)] =$

$= \{w(1+r)(1-a_2)/(1-a_2-ra_2)\}/2\{w(1+r)(1-a_1)/(1-a_1-ra_1)\}=$

$$= \frac{1}{2} \cdot \frac{(1-a_1-ra_1)(1-a_2)}{(1-a_2-ra_2)(1-a_1)}.$$

Q/2 as here defined is the probability that $W_1<W_2$, i.e. that there is a second switchpoint, once r, a_1 and a_2 are assigned.

Now μ is re-defined as the average probability that $W_1<W_2$ as a_1 varies from a_2 to $1/(1+r)$, equal to the definite integration of $Q/2$ over a_1 from a_2 to $1/(1+r)$:

$$[29] \quad \mu = \frac{1/2}{[1/(1+r)-a_2]} \int_{a_2}^{1/(1+r)} \frac{(1-a_1-ra_1)(1-a_2)}{(1-a_2-ra_2)(1-a_1)} \, da_1 \, .$$

And $\mu^*(r,R_{sup})$ is given by the following definite integral:

$$[30] \quad \mu^* = \frac{1}{1/(1+r)-1/(1+R_{sup})} \int_{1/(1+R_{sup})}^{1/(1+r)} \mu \, da_2$$

The analytical solution of these integrals is very complex and in the end unnecessary. The function at the bottom of these integrations is $Q/2$ as defined by equation [28], which is a smooth, well-behaved function, so one can legitimately approximate the calculation of these integrals with the method of rectangles. I have used MapleV Release3 (Student Edition) to approximate μ and μ^* through the area of 40 rectangles of equal basis and of height equal to the value of the function in their middle point. MapleV determines the analytical expressions for this approximation, and substituting into it the assigned values of r and of R_{sup} one obtains the probabilities that there be a second switch point. The numerical values of these probabilities are in Table II in the Appendix. (The MapleV file for these and the other calculations in this paper will be supplied upon request.)

6.

We must now determine *P*, the probability that, assuming there is another switchpoint between the two randomly selected *w(r)* curves passing through a given point C, this second switchpoint is to the left of C. (*P* too will come out to be independent of *w*.)

Let us initially take as given not only the point C but also the $w_2(r)$ curve i.e. R_2 and W_2. For each R_1 such that $r<R_1<R_2$, there is a unique value of W_1 such that the two *w(r)* curves are tangent in C. Let $W_1\hat{}(R_1)$ indicate this value of W_1. $W_1\hat{}$ is determined by considering W_1 as a variable in equation [5] in the system of equations [3]-[4]-[5] applied to technique 1, and adding, first, the equations [3], [4], [5] applied to technique 2 with W_2 given, and second, the condition that in C the slopes of the two w(r) curves must be the same i.e.

$$[31] \quad \frac{-\alpha_1}{(-\alpha_1 - \alpha_1\cdot r - ß_1 + a_1\cdot ß_1 + a_1\cdot ß_1\cdot r)^2} = \frac{-\alpha_2}{(-\alpha_2 - \alpha_2\cdot r - ß_2 + a_2\cdot ß_2 + a_2\cdot ß_2\cdot r)^2}.$$

The uniqueness of $W_1\hat{}(R_1)$ derives from the fact that to each triplet of points (C, R, W) there corresponds a unique *w(r)* curve and that, for given C and *R*, the convexity of the *w(r)* curve monotonically increases with *W*, passing from initially concave to straight to convex: so also the absolute value of the slope in C of the *w(r)* curve monotonically increases with *W*; therefore there will be a unique value of W_1 making the slope of w_1(r) in C equal to the assigned slope of the $w_2(r)$ curve. Indeed let us demonstrate that for given *r* and *a* the absolute value of the slope of a *w(r)* curve is an increasing function of *W*.

In:

[32] $dw/dr=-\alpha/[ß+(1+r)(\alpha-aß)]$

let us replace α and β with the expressions determining them in equations [17] and [18]; simplifying we obtain:

[33] $-dw/dr=\{w(Wa-wa-W+w)\}/\{Wr(a-1+ar)\}$

whose derivative is:

[34] $d^2r/dr^2 = w^2(1-a) / W^2r[1-a(1+r)] >0$

because the numerator is positive ($0<a<1$ if $R>0$), and the denominator is positive because $0<a(1+r)<1$, owing to $a=1/(1+R)$ and therefore $a(1+r)=(1+r)/(1+R)<1$.

(Using a here instead of R has the same motivation as in equations [26] and ff.)

W_1^ is a function of $a1$, $a2$, r, w, W_2:

[35] $W_1\text{^} = W_2\cdot w(-a_1+a_1\cdot a_2+ra_1a_2+1-a_2-ra_2)/$
$/(-rW_2a_2+wa_1a_2-wa_2+wra_1a_2+a_1rW_2-wa_1+w-wra_1)$

Since $w(r)$ is a hyperbola, as R_1 increases continuously in the interval (r,R_2) also W_1^ increases continuously and goes through all values in the interval (w,W_2). Thus, for a given R_1, if $W_1<W_1$^ then the slope in C of $w_1(r)$ is less (in absolute value) than the slope of $w_2(r)$, so the second switchpoint is to the right of C; if $W_1>W_1$^, the second switschpoint is to the left of C. The probability that the switch in C be 'perverse' depends therefore on the probability that $W_1>W_1$^. We may assume that this probability is to 1 like the length of the interval $(W_1\text{^},W_2)$ is to the length of the interval (w,W_2), and therefore that it is equal to $(W_2-W_1\text{^})/(W_2-w)$.

(For W_2 and/or $_w$ tending to $+\infty$ it might seem that the same problem arises, which earlier induced me to replace R with a; but this problem will not arise because W_2 and w will disappear through simplification from the formulas.)

For R_1 tending to R_2 this ratio tends to zero, and it tends to 1 for R_1 tending to r. But how it varies within the interval (r,R_2) is a complicated thing and for the calculation of the average value of $(W_2-W_1\hat{})/(W_2-w)$, when both W_2, R_1 (or rather a_1), and R_2 (or rather a_2) are considered random variables with a uniform probability distribution within the respective admissible intervals, MapleV has been again indispensable.

7.

For given values of r, w, R_2 (or rather a_2), the values of W_2 can vary in the interval (w,W_{max}). It will be assumed that all values of W_2 in this interval are equiprobable. The probability, that for given r, w, R_2 and R_1 one finds that $W_1>W_1\hat{}$, is given by the definite integral:

$$[36] \qquad \frac{1}{W_{2max}-w} \cdot \int_{w}^{W_{2max}} [(W_2-W_1\hat{})/(W_2-w)]dW_2$$

where w is given, $W_{2max}=w(1+r)R_2/(R_2-r)=w(1+r)(1-a_2)/(1-a_2-ra_2)$, and $W_1\hat{}$ is given by equation [22] and is therefore a function of r, w, a_1, a_2 and W_2. This probability is a function of r, w, a_1 and a_2.

It is useful to reach this same probability in another way. Let x be a scalar, variable between 0 and 1, and, in the expression $(W_2-W_1\hat{})/(W_2-w)$ let us replace $W_1\hat{}$ with its value given by equation [27], and let us replace W_2 with its expression in terms of w, W_{max} and x, i.e. with:

$$[37] \qquad W_2 = w + x[w(1+r)(1-a_2)/(1-a_2-ra_2) - w].$$

(The expression inside the square brackets on the right hand side of [37] is $W_{max}-w$; so as x varies from 0 to 1, W_2 varies from w to W_{max}.) If we perform these two substitutions in the expression

$(W_2-W_1\hat{})/(W_2-w)$, w is eliminated and we obtain an expression for $(W_2-W_1\hat{})/(W_2-w)$ to be indicated as PA:

$$[38] \quad PA \equiv (W_2-W_1\hat{})/(W_2-w) = r(a_2-a_1-a_2^2-ra_2^2+xra_2- -xra_1+ra_1\cdot a_2+a_1\cdot a_2)/(2ra_2-2a_1a_2-2ra_1a_2-2ra_2^2- -r^2a_2^2+a_2^2a_1+2ra_2^2a_1+r^2a_2^2a_1+xr^2a_2-xr^2a_1+a_1+2a_2-1-a_2^2).$$

PA indicates the probability that, if two $w(r)$ curves cross in C, this switchpoint is 'perverse', when r, a_1, a_2 and W_2 (i.e. x) are assigned. (As announced, W_2 and w have disappeared.) PA is the basic function in what follows.

Since x varies between 0 and 1, the integral [36] becomes:

$$[39] \quad PX \equiv \int_0^1 PA \, dx$$

PX is the average probability that the switch in C is 'perverse' if there is another switchpoint, when r, $a1$ and a_2 are given while W_2 is random between w and W_{2max}. PX is independent of w, like PA; it is a function of r, a_1 and a_2. Again it can be calculated by approximation.

Now let us consider only r and a_2 as assigned and let us suppose all values of a_1 in the interval $(a_2,1/(1+r))$ to be equiprobable; by integrating PX with respect to a_1 on the interval $(a_2,1/(1+r))$ and dividing by $[1/(1+r)]-a_2$, we now determine the average probability that the switch in C is 'perverse' if there is another switchpoint, when only r and a_2 are assigned. Let this probability be indicated as $PA1$:

$$[40] \quad PA1 \equiv \frac{1}{[1/(1+r)]-a_2} \int_{a_2}^{1/(1+r)} PX \, da_1$$

Lastly, let $a_{2inf} \equiv 1/(1+R_{sup})$ be the minimum possible value of a_2 (and of a_1), i.e. the one corresponding to the assigned R_{sup}. We can then integrate *PA1* with respect to a_2 on the interval $(a_{2inf},1/(1+r))$ and divide by the length of this interval, and in this way we obtain the average probability *P* that the switch in C be the 'perverse' one if there is another switchpoint, with only *r* and R_{sup} given:

$$[41] \quad P(r,R_{sup}) \equiv \frac{1}{1/(1+r)-a_{2inf}} \int_{a_{2inf}}^{1/(1+r)} PX\, da_2 \ , \ a_{2inf}=(1+R_{sup})^{-1}.$$

MapleV Release 3 (Student version) again determines without difficulty the values of these integrals by approximating them with the method of rectangles (again I have used 40 rectangles). The basic function *PA* is very 'regular' so the approximations are certainly very good. The approximating function tends to 1 as r tends to R_{sup}.

Table III shows the values of *P* for the same values of *r* and R_{sup} as Table II does for μ^*.

8.

We have now what we need: Table I, columns 2 to 8 (under the heading 'My method based on w(r) curves'), shows the values of $\pi(r,R_{sup})=\mu^*P$, which indicate the average probability that the switch in C be 'perverse', as a function of *r* and R_{sup} for selected values of these variables. This probability is higher the higher *r* for a given R_{sup}; it is on the contrary lower the higher R_{sup} for a given *r*. This shows that admitting no limit to R_{sup} tends to underestimate this probability relative to the - more plausible, I would argue - cases in which the possible techniques can be presumed never to have an R_{sup} above a certain finite value. Anyway the limits to which the probability tends for R_{sup} tending to $+\infty$ are also shown: the probability that a switch be 'perverse' is about 10% for r=8%, about 13% for r=25%. Definitely not negligible.

Part III. Some concluding remarks.

9.

In conclusion, D'Ippolito's results were deeply misleading. His probabilities are significantly lower, for the plausible values of the rate of profits, than the probabilities determined in the three ways proposed here. In particular the calculation which follows his approach and statements most closely but corrects his logical slip (column 10) arrives at probabilities enormously higher than his. The other two calculations reach results less far from D'Ippolito's, but they nonetheless arrive at much higher probabilities than D'Ippolito's especially for low values of the rate of profits, e.g. for r=5% they estimate probabilities around 8% against the 2.2% of D'Ippolito.

What also emerges is a significant dependence of the results on the assumptions about the distribution of the probabilities of the technical coefficients. It is unclear how one might decrease the arbitrariness of these assumptions. There is a danger that, by changing them, one may obtain nearly any result. There is therefore room for further reflection on how to evaluate the likelihood of reswitching and reverse capital deepening([11]). Still, if one believes the kind of exercises attempted here to yield some useful information, then the message appears to be that the Samuelson-Hicks-Garegnani model supplies no basis at all for believing that the

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[11] . It might for example be argued that what is important is simply the "potential generality" of those phenomena (Garegnani, 1990, p. 72), i.e., if I understand correctly, the impossibility of confining their occurrence to very peculiar situations. Such a potential generality would seem no doubt to be there, given the ease with which, given a *w(r)* curve and an assigned value of *r*, one may draw a second *w(r)* curve which produces a 'perverse' switch with the first one at the given *r* (a greater ease, it seems to me, than in producing e.g. instances of Giffen goods).

likelihood of 'perverse' switches can be considered negligible - rather the opposite([12]).

10.

Since there is a danger that exercises like the ones attempted in this paper may be inconclusive owing to an ineliminable margin of arbitrariness in the assumptions, are there other considerations which may be of help in assessing the relevance of reverse capital deepening? In this concluding paragraph I advance some remarks on this issue (for reasons of space I shall be very brief and often refer to other writings).

The answers, coming from the defenders of the supply-and-demand (or neoclassical) approach, to the thesis that reswitching and reverse capital deepening destroy the foundations of their approach have been of two kinds.

The first one, coming mostly from general equilibrium specialists, has been that general equilibrium theory in its modern versions has no need for capital aggregation and that therefore it survives reswitching unscathed. The results of the present inquiry have little bearing on this position; but it is slowly being perceived by the profession that this line of defence of the neoclassical approach is very weak: the 'modern versions' of general

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[12] . It seems a fair guess that the method based on the *w(r)* curves proposed in Part II should be also applicable to the two-sector model studied by Mainwaring and Steedman (1995), and I would not be surprised if it yielded significantly higher probabilities than the low ones estimated by the authors (for the Samuelson-Hicks-Garegnani model, the probabilities estimated in this way that two w(r) curves cross twice - which is the probability Metcalfe and Steedman try to estimate for their model - is given in Table II and is quite high). Anyway, as noticed by Ciccone (1996, p. 45, fn. 8), a basic difference between the Samuelson-Hicks-Garegnani model and the Mainwaring-Steedman model is that the latter assumes both goods to be common to both techniques. This is very restrictive, since in real economies different methods of production of a commodity usually require different and specific intermediate goods or machines. The probability of reswitching is thereby in all likelihood underestimated by the Mainwaring-Steedman model, because their assumptions allow reswitching to occur only if both *w(r)* curves are concave, or both are convex, what is not required in the Samuelson-Hicks-Garegnani model. The same criticism applies to D'Ippolito (1989).

equilibrium theory are unable to support the neoclassical explanation of distribution and employment precisely because they try to do without the traditional notion of capital. An equilibrium determined on the basis of a given vector of initial endowments of the several capital goods is deprived of any persistence: its data are quickly altered by disequilibrium actions, so it is unable to indicate the situation the economy tends to([13]); furthermore, even the implausible assumption of instantaneous adjustment is unable to justify the full-employment nature of these equilibria, because, once one drops the traditional notion of capital and, with it, the belief in the negative elasticity of aggregate investment with respect to the rate of interest, it is impossible to presume that the savings-investment market is capable of reaching an equilibrium at the full-employment level of savings (Petri, 1997; 1998).

The second line of defence has tried to argue that reswitching and reverse capital deepening are rare phenomena, and therefore as negligible as the possibilities of unstable and multiple equilibria which have long been known to derive from income effects([14]). This line of defence is, I would argue, considerably weakened by the results of the present paper. But it seems possible to add that it was a weak line of defence to start with.

This line of defence implicitly admits that the notion of a downward-sloping long-period 'demand' curve for capital the single factor (an amount of value) has a very important role in the neoclassical approach: it implicitly admits that, without that notion, also the dependence of aggregate investment on the rate of interest would become highly doubtful - and with it the tendency to the full employment of labour in the long run, and the 'neoclassical

[13] . "In a real economy, however, trading, as well as production and consumption, goes on out of equilibrium. It follows that, in the course of convergence to equilibrium (assuming that occurs), endowments change. In turn this changes the set of equilibria. Put more succinctly, the set of equilibria is path dependent [....] [This path dependence] makes the calculation of equilibria corresponding to the initial state of the system essentially irrelevant." (Fisher, 1983, p. 14)

[14] . Cf. in particular Hicks (1965, p. 154) who comments as follows on the possibility of reswitching: "Not a very satisfactory situation, but one that has parallels in other parts of economic theory!".

synthesis', and the current revival of pre-Keynesian growth theory. It thereby also implicitly admits that neo-Walrasian general equilibrium theory, owing to its difficulty with proving stability and its need to assume the instantaneity of adjustments, cannot replace the reliance of the neoclassical approach to distribution and employment on the traditional mechanisms of long-period substitution between labour and 'capital'. What it tries to argue is that reverse capital deepening, because of its low likelihood, does not really undermine these mechanisms. The (again, largely implicit) argument appears to be the following. In any economy there is ample scope for technical choice, and the range of techniques available is vast, so along the outer envelope of the w(r) curves there must be numerous switchpoints; if only a small proportion of these give rise to reverse capital deepening, then the 'demand curve' for capital the value factor is still essentially downward-sloping: the upward-sloping sections are in all likelihood few and short and apart from each other, so they are rendered almost unnoticeable by the prevalence of 'non-perverse' switches.

If this is the implicit defensive argument in the minds of the upholders of the negligibility of reverse capital deepening, then it has a number of weaknesses, even apart from the little support for the presumption of a low likelihood of 'perverse' switches.

First, the absence of reswitching of techniques is not enough to ensure the absence of 'perverse' behaviour of the value of capital per unit of labour. Fig. 6 provides one example. In this example (which is compatible with the assumptions of the Samuelson-Hicks-Garegnani model as well as, of course, with more complex models) all the techniques have concave *w(r)* curves, so the value of capital per unit of labour increases along any given *w(r)* curve; then, as shown by Fig. 6, the absence of reswitching guarantees that the change of the value of capital at the switch points is not 'perverse', but this is insufficient to make the value of capital per unit of labour a decreasing function of the rate of interest.

(insert Fig. 6 about here)

Another possible reason for 'perverse' behaviour of the value of capital per unit of labour without reswitching of alternative techniques is the possibility that (for reasons, in fact, strictly analogous to those which may cause the reswitching of alternative techniques) as the rate of interest decreases the relative prices of consumption goods may move in a 'perverse' direction (consumption goods employing more capital per unit of labour than other consumption goods may rise in price relative to the second ones), what may cause the *substitution* effect in consumer choices - and not only the income effects to be discussed below - to work against the neoclassical presumptions.

Second, the neoclassical approach requires that the demand curve for capital be not only non-increasing but also quite elastic; e.g. a downward-sloping but highly inelastic demand curve for capital would imply a highly inelastic investment schedule, what would be as fatal as an increasing one to the plausibility of the thesis that investment adapts to savings rather than the other way round([15]); now, any upward-sloping section will diminish the overall elasticity of the "demand-for-capital" curve; so reverse capital deepening, even if restricted to a small set of ranges of values of the rate of profits, diminishes anyway the plausibility of the theory. We have remembered above another possible reason for upward-sloping sections: the possible concavity of the *w(r)* curves, which, even when not provoking the type of phenomena illustrated in Fig. 6, may still contribute to decreasing the elasticity of the "demand-for-capital" curve.

Third, the fact that there are many known alternative techniques is no proof that there are many switchpoints on the envelope of the *w(r)* curves; a single technique might be dominant for ample intervals of values of the rate of profits, and this may again imply a rigid investment schedule.

[15] . The great flexibility of production in response to changes in demand means that unless investment adapts quickly to savings, it will be aggregate output to change and to alter savings, as Keynes and Kalecki noticed. On the connection between demand for capital and investment schedule cf. Petri (1997).

Fourth, the analogy with the presumed irrelevance of income effects is more damaging than helpful to the neoclassical theorist. The destructive implications for the neoclassical approach of the possible instabilities and multiplicities of equilibria caused by income effects are much more serious than it is generally admitted. Income effects may cause the supply curves of factors to be 'backward bending', and it is generally admitted that, especially for labour, this may well be often the case. Now, even conceding the legitimacy of the notion of a downward-sloping demand curve for labour([16]), a downward-sloping supply curve of labour may cause not only multiple equilibria, but also, what is at least as damaging to the theory, a quasi-coincidence of the demand curve and the supply curve over some interval, as in the case illustrated in Fig. 7a; if one admits, as one should, that the forces making for a change of the price in a market are in all likelihood the weaker, the smaller the excess demand on that market, one must then also admit that in the situation of Fig. 7a the real wage is to all practical effects indeterminate between the values w_1 and w_2. This possibility of what we may call 'practically indeterminate equilibria' has been seldom discussed but is indubitable and it means that, even leaving aside the debates about capital, in some perfectly possible situations neoclassical theory is unable to determine income distribution even when, strictly speaking, the equilibrium is unique and stable.

(insert Fig. 7 about here)

The likelihood of multiple or practically indeterminate equilibria on factor markets is the lower, the greater the elasticity of the demand curves for factors; but on this elasticity too income

[16] . This curve must describe the locus of equilibrium real wages corresponding to a parametrically shifting employment of labour, when the endowments of other factors are given (and a 'Clower constraint' makes the income of consumers equal to the income of employed factors only); its determination becomes impossible the moment one admits that in deriving it one can take as given neither the endowment of 'capital' conceived as a single factor (owing to the Cambridge criticism), nor the vector of endowments of the several capital goods (because these would be altered by disequilibrium actions), cf. Petri (1997), and paragraph 10 below.

effects may have highly damaging consequences. For example, let us imagine an economy where the only factors of production are labour and land; and where wages go to purchase a basket of consumption goods which requires for its production a higher-than-average labour/land ratio([17]). Now the real wage decreases owing to labour unemployment; the share of wages in total income decreases; if the composition of demand from each type of income remains unaltered, the demand for labour (assuming a given total employment of land) decreases. This income effect goes therefore against the consumer substitution effect and the technological substitution effect, and even when it is unable to overpower them it may nonetheless significantly decrease the elasticity of the demand for labour, making it more nearly vertical([18]). So the likelihood is increased, of multiple or practically indeterminate equilibria owing to backward-bending factor supply curves; furthermore, it is possible that the only full-employment equilibrium be characterised by an implausible income distribution which gives the entire or nearly the entire income to only one factor (cf. Fig. 7b), a prediction which deprives the theory of plausibility.

Now, since in the last two centuries history has produced a great variety of historical situations, it seems highly unlikely that none of the 'perverse' cases just illustrated should have ever occurred. Then the following consideration appears relevant:

> However small the evaluated probability of the instances in which the principle of substitution does not operate, obviously prices and incomes would take shape, and would therefore have an explanation, also in those circumstances. One would thus be implicitly admitting the existence of a

[17] . So income from land rent is employed in the purchase of consumption goods which require for their production a lower-than-average labour/land ratio.

[18] . Applying the same analysis to labour and 'capital' (conceding for the sake of argument the legitimacy of the latter concept), the absence of any significant substitution between labour and 'capital' would also imply an insensitivity of investment to the rate of interest, and thus would render inoperative the mechanism which should ensure the tendency of investment to adapt to savings, with the consequence pointed out in footnote 12 above.

> theory of distribution, alternative to the neoclassical one, and without any basis for excluding that this alternative theory, differently from the neoclassical one, may apply to the generality of cases. (Ciccone, 1996, p. 42, our transl.)

(In the light of the above considerations, the puzzling thing is rather the extent of the unquestioned acceptance of the neoclassical approach to distribution in the last 100 years.)

The similar argument applying to reverse capital deepening can only make the situation even worse for neoclassical theory. We have had now two centuries of capitalism, with all its technical changes; so we have had many 'random' extractions of sets of alternative techniques; then even as low probabilities as D'Ippolito calculates would not make it unlikely that at least in some countries and some historical periods, the number of 'perverse' switchpoints on the outer envelope of the w(r) curves may have been significant, and the demand curve for 'capital' in value terms([19]) may accordingly have had significant upward-sloping sections - what should have resulted in instabilities or other 'perverse' phenomena if distribution were determined by the interplay of supply and demand as postulated by the neoclassical approach. I am not aware that phenomena of this nature have ever been noticed. The conclusion must be, it would seem, the implausibility of a theory, which predicts that at least sometimes one should have observed phenomena of which there is no trace([20]).

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[19] . And conceding for the sake of argument a non-backward-bending supply curve of labour. It should not be forgotten that a backward-bending supply curve of labour may imply that, when the rate of interest decreases and the real wage increases, even if the capital-labour ratio increases the demand for capital (assuming the full employment of labour)

[20] . "Thus, after following in the footsteps of traditional theory and attempting an analysis of distribution in terms of 'demand' and 'supply', we are forced to the conclusion that a change, however small, in the 'supply' or 'demand' conditions of labour or capital (saving) may result in drastic changes of r and w. That analysis would even force us to admit that r may fall to zero or rise to its maximum, and hence w rise to its maximum or to fall to zero, without bringing to equality the quantities supplied and demanded of the two factors. Now, no such instability of an economy's wage and interest rates has ever been observed. The natural conclusion is that, in order to explain distribution, we must rely on forces other than 'supply' and 'demand'." (Garegnani, 1970, pp. 427-8)

Fifth, and most importantly, even if in a certain economy it could be proven that the available techniques do not give rise to reverse capital deepening, would that be enough to make the neoclassical approach to distribution defensible for that economy? Besides the point made by Ciccone, the following crucial difficulty for that approach must not be forgotten. A general equilibrium would still have to be determined, toward which the spontaneous working of supply and demand would be pushing the economy. How can one specify for this equilibrium the data relative to the endowment of capital? An unhappy dilemma appears to be inescapable for the neoclassical theorist, a dilemma which haunted e.g. Hicks (Petri, 1991). A given vector of endowments of the several capital goods (as in Arrow-Debreu or in temporary equilibria) would be changed by any time-consuming disequilibrium adjustment, so the position toward which the disequilibrium processes push the economy would be indeterminable. A given amount of 'capital' conceived as a single factor of variable 'form' (as in traditional neoclassical long-period equilibria) would avoid this problem but would have to be measured as an amount of value([21]) and would not therefore be ascertainable independently of prices i.e. of the variables the equilibrium should determine. There appears therefore to be no acceptable way to specify the endowment of capital, hence no acceptable notion of general equilibrium, hence no way to give a foundation to the neoclassical theory of distribution.

In the light of the above, the importance of reswitching and reverse capital deepening appears to lie, above all, in their confirming the radical difference between produced means of production, on the one hand, and labour or land on the other. The extent of this difference has been made less easy to perceive by the neoclassical approach also on other counts. For example, the neoclassical concentration on full-employment situations tends to

[21] . This problem appears to have escaped e.g. Laing (1991). Champernowne's chain index is of no help in this respect because it is unable to give a reliable measure of the quantity of capital which is to change 'form' without changing in 'quantity' in disequilibrium. Changes of technique do *not* involve the rate of interest stopping at the value at which two techniques are equiprofitable and the change in 'form' happening at the corresponding prices with no change in the value of the capital goods in the meanwhile!

induce economists to underestimate the importance of the flexibility of output in response to variations of demand, a flexibility which implies that the production of capital goods too can be significantly influenced by the demand for them, and can therefore be generally increased without any previous decision to abstain from consumption([22]) - another aspect of reality hardly reconcilable with the neoclassical view of the nature and origins of capital.

[22] . On this issue cf. Kurz, 1992; Garegnani and Palumbo, 1998; and the literature there cited.

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APPENDIX A

I take from D'Ippolito (1987, p. 32) the following determination of the surface *F(r,v)*. Refer to Fig. 2a which is reproduced here as Fig. 8.

(insert Fig. 8 about here)

F(r,v) is the difference between the areas of the triangles ABC and A'B'C. Since:

OC=BC=$1/\rho$

OM=ME=1

CM=r/ρ

AC/CB=A'C/CB'=A'M/ME=A'M=v

A'C=A'M-CM=$v-r/\rho$

one obtains the areas of the two triangles as, respectively,

$ABC=(AC\cdot CB)/2=CB^2 v/2=v/(2\rho)$

$A'B'C=(A'C\cdot CB')/2=A'C^2/(2v)=(v-r/\rho)^2/(2v)$ as long as $v\geq r/\rho$, otherwise A'B'C=0.

Hence the area of F is

$$F = \frac{1}{2\rho^2}\left[v - \frac{1}{\rho}(\rho v-r)^2\right] \text{ if } v\geq r/\rho; \quad F = v/(2\rho) \text{ if } v<r/\rho.$$

Hence $Z(r,v) = F(r,v)/D(r,v) = F(r,v)/ABC = 1 - \dfrac{(v - r/\rho)^2}{\rho^2}$ if $v\geq r/\rho$, otherwise $Z(r,v)=1$. Therefore

$$Z^*(r) = \int_0^{r/\rho} dv + \int_{r/\rho}^{1} [1-(v-r/\rho)^2/\rho^2]dv.$$

Table I

Probabilities that a switch be 'perverse'.

r	$\pi(r,R_{sup})$ (My method based on w(r) curves) R_{sup} = 0.5	1	2	5	10	100	->∞	D'Ippolito original	D'Ippolito corrected	D'Ippolito reinterpreted
0.01	.0692	.0573	.0494	.0441	.0421	.0401	.0399	.0048	.3414	.0653
0.02	.0909	.0778	.0688	.0623	.0597	.0573	.0571	.0092	.3490	.0680
0.03	.1040	.0906	.0812	.0742	.0715	.0689	.0686	.0134	.3562	.0706
0.04	.1130	.0997	.0903	.0830	.0802	.0775	.0772	.0174	.3630	.0730
0.05	.1199	.1068	.0973	.0900	.0871	.0843	.0840	.0212	.3694	.0753
0.06	.1254	.1124	.1030	.0956	.0928	.0900	.0896	.0248	.3755	.0774
0.08	.1339	.1211	.1118	.1045	.1016	.0988	.0985	.0317	.3866	.0814
0.10	.1403	.1276	.1184	.1112	.1084	.1056	.1052	.0349	.3965	.0850
0.12	.1455	.1327	.1237	.1165	.1137	.1109	.1106	.0439	.4054	.0883
0.14	.1501	.1370	.1280	.1209	.1181	.1153	.1150	.0494	.4134	.0912
0.16	.1540	.1406	.1316	.1246	.1219	.1191	.1188	.0546	.4206	.0939
0.18	.1577	.1439	.1348	.1278	.1250	.1223	.1220	.0595	.4271	.0963
0.20	.1612	.1467	.1376	.1305	.1278	.1251	.1248	.0642	.4330	.0985
0.25	.1695	.1529	.1433	.1363	.1336	.1309	.1306	.0747	.4454	.1032
0.30	.1777	.1580	.1479	.1408	.1380	.1353	.1350	.0809	.4551	.1069
0.40	.1976	.1671	.1553	.1476	.1448	.1421	.1418	.0998	.4690	.1123
0.50		.1753	.1612	.1529	.1499	.1470	.1467	.1127	.4780	.1159
0.60		.1837	.1663	.1571	.1539	.1510	.1506	.1235	.4841	.1183
0.80		.2031	.1753	.1639	.1603	.1570	.1566	.1405	.4912	.1212
1.00			.1837	.1695	.1653	.1616	.1612	.1534	.4948	.1227
1.20			.1919	.1742	.1695	.1653	.1649	.1636	.4968	.1236
1.60			.2105	.1824	.1763	.1713	.1708	.1786	.4986	.1244
2.00				.1895	.1819	.1759	.1753	.1891	.4993	.1247
3.00				.2053	.1928	.1844	.1836	.2054	.4998	.1249
4.00				.2216	.2015	.1905	.1895	.2149	.4999	.1249
5.00					.2090	.1952	.1940	.2210	.4999	.125
10						.2093	.2072	.2345	.5	.125
20						.2223	.2189	.2420	.5	.125
30						.2294	.2248	.2446	.5	.125

Table II

μ*, i.e. probability, according to my method based on the w(r) curves, that two w(r) curves intersect twice.

	R_{sup}						
r	0.5	1	2	5	10	100	->∞
0.01	.4227	.4374	.4466	.4530	.4553	.4575	.4578
0.02	.3934	.4114	.4231	.4314	.4345	.4374	.4377
0.03	.3741	.3938	.4068	.4162	.4197	.4231	.4235
0.04	.3597	.3804	.3943	.4044	.4082	.4119	.4123
0.05	.3484	.3697	.3841	.3947	.3987	.4026	.4030
0.06	.3391	.3607	.3755	.3865	.3907	.3947	.3952
0.08	.3246	.3465	.3617	.3732	.3776	.3818	.3823
0.10	.3135	.3354	.3509	.3627	.3672	.3716	.3721
0.12	.3047	.3265	.3421	.3540	.3586	.3631	.3636
0.14	.2976	.3192	.3347	.3468	.3514	.3559	.3565
0.16	.2915	.3129	.3284	.3405	.3452	.3498	.3503
0.18	.2864	.3076	.3230	.3351	.3398	.3444	.3449
0.20	.2819	.3028	.3182	.3303	.3350	.3396	.3401
0.25	.2730	.2933	.3084	.3204	.3251	.3297	.3303
0.30	.2662	.2859	.3008	.3127	.3174	.3219	.3225
0.40	.2566	.2753	.2897	.3013	.3059	.3105	.3110
0.50		.2680	.2819	.2933	.2978	.3023	.3028
0.60		.2626	.2761	.2872	.2917	.2961	.2966
0.80		.2550	.2680	.2787	.2830	.2874	.2879
1.00			.2626	.2730	.2772	.2814	.2819
1.20			.2586	.2688	.2730	.2771	.2776
1.60			.2534	.2633	.2673	.2713	.2718
2.00				.2596	.2636	.2675	.2680
3.00				.2545	.2583	.2621	.2625
4.00				.2517	.2554	.2592	.2596
5.00					.2537	.2574	.2578
10						.2536	.2540
20						.2516	.2521
30						.2510	.2514

Table III

P, i.e. probability, according to my method based on the w(r) curves, that, if two techniques switch twice, the switch at the given value of r is associated with reverse capital deepening.

				R_{sup}			
	0.5	1	2	5	10	100	->∞
r							
0.01	.1636	.1311	.1112	.0974	.0924	.0877	.0871
0.02	.2311	.1890	.1627	.1443	.1375	.1311	.1304
0.03	.2779	.2300	.1997	.1782	.1703	.1628	.1619
0.04	.3141	.2621	.2289	.2052	.1965	.1881	.1872
0.05	.3442	.2888	.2533	.2279	.2184	.2095	.2084
0.06	.3698	.3116	.2742	.2474	.2374	.2279	.2268
0.08	.4125	.3495	.3091	.2800	.2691	.2588	.2576
0.10	.4475	.3803	.3375	.3066	.2951	.2841	.2828
0.12	.4776	.4065	.3615	.3291	.3171	.3055	.3042
0.14	.5042	.4293	.3824	.3487	.3361	.3241	.3227
0.16	.5284	.4495	.4008	.3659	.3530	.3406	.3391
0.18	.5507	.4678	.4173	.3814	.3680	.3552	.3538
0.20	.5718	.4845	.4323	.3953	.3816	.3685	.3670
0.25	.6207	.5212	.4647	.4253	.4108	.3969	.3953
0.30	.6676	.5527	.4918	.4502	.4349	.4204	.4187
0.40	.7701	.6067	.5360	.4899	.4733	.4576	.4558
0.50		.6542	.5718	.5212	.5032	.4864	.4845
0.60		.6994	.6023	.5469	.5277	.5098	.5078
0.80		.7966	.6542	.5881	.5663	.5464	.5441
1.00			.6994	.6207	.5962	.5742	.5718
1.20			.7418	.6480	.6207	.5967	.5940
1.60			.8309	.6928	.6595	.6314	.6283
2.00				.7298	.6899	.6577	.6542
3.00				.8069	.7465	.7037	.6994
4.00				.8805	.7888	.7350	.7298
5.00					.8238	.7584	.7524
10						.8255	.8159
20						.8837	.8685
30						.9138	.8940

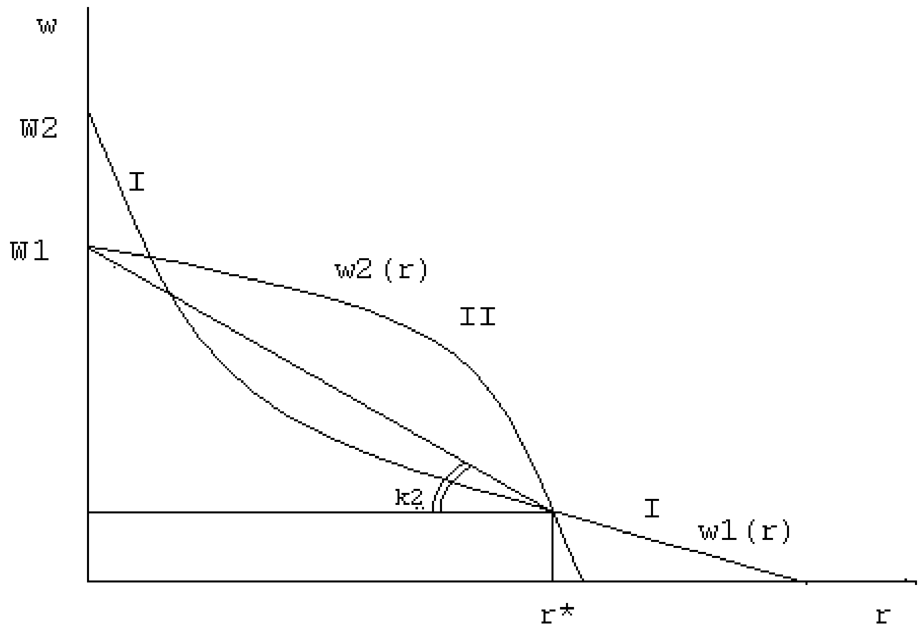
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Fig. 1

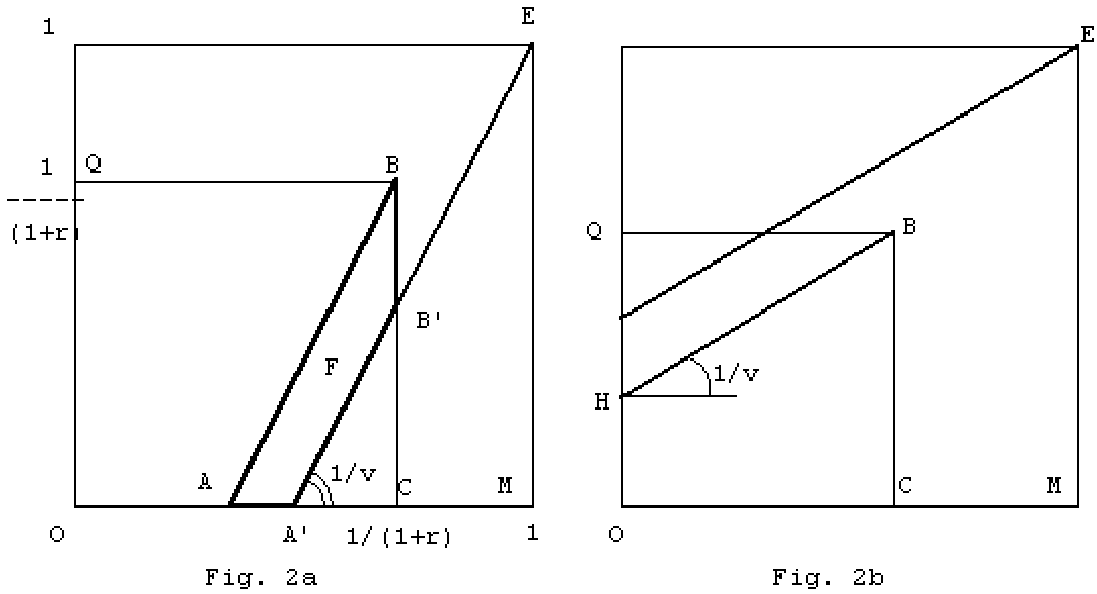
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Fig. 2a
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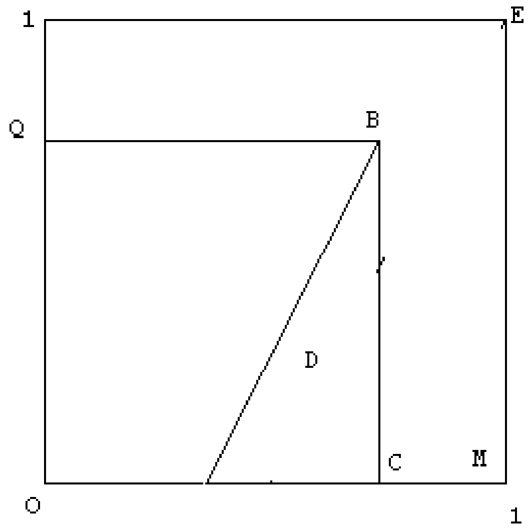
Fig. 2b



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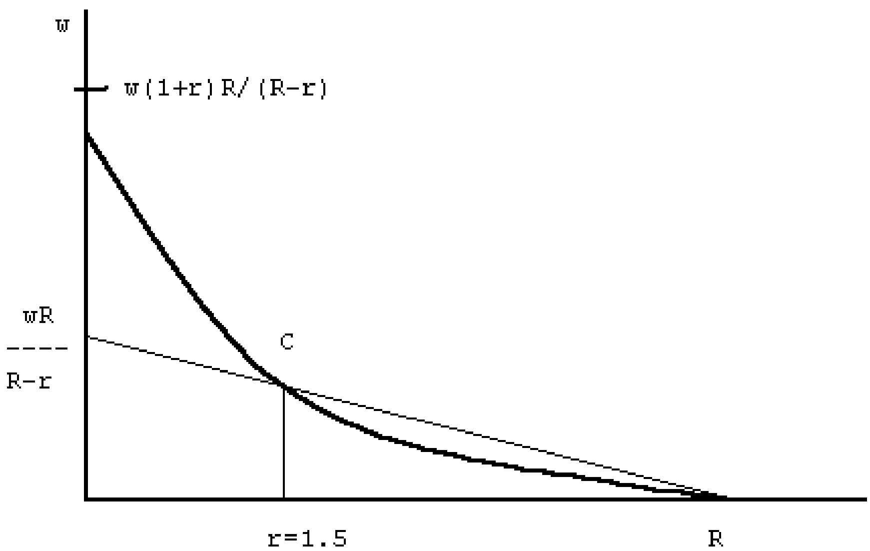

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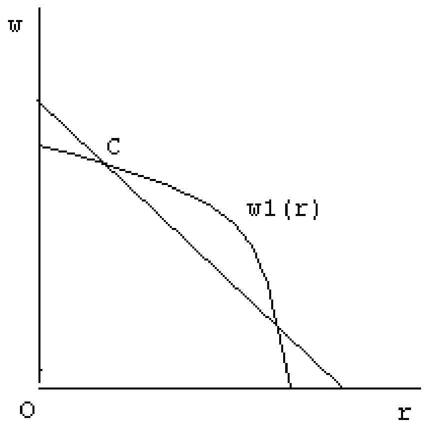

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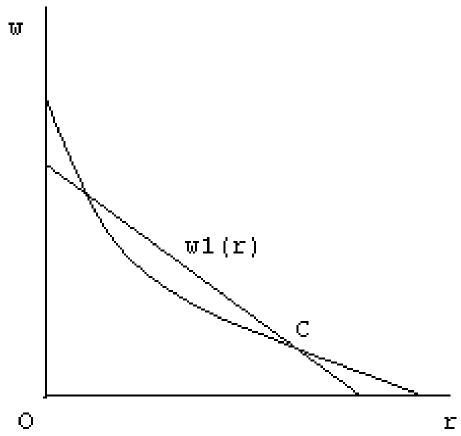
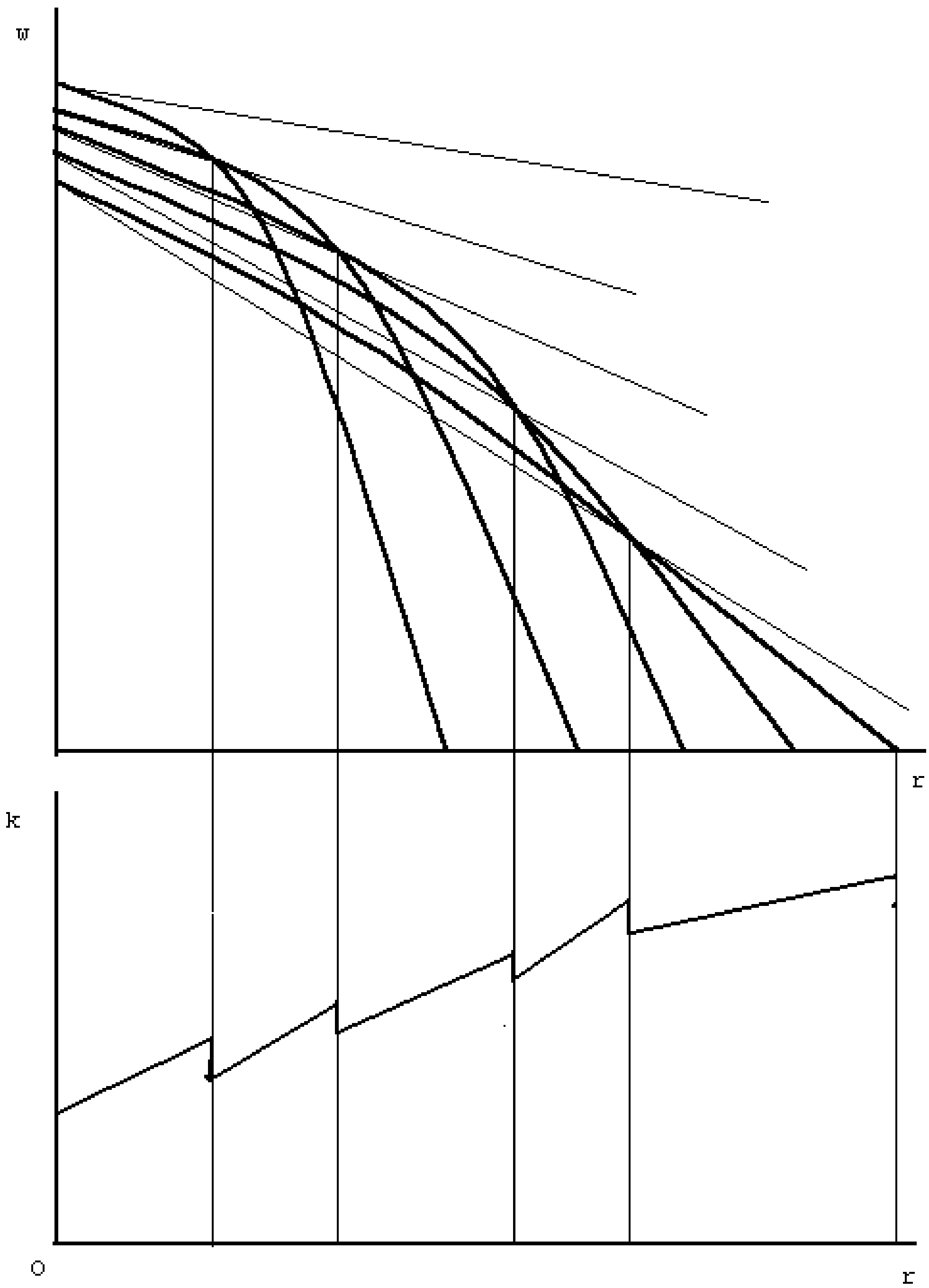


Fig. 5b



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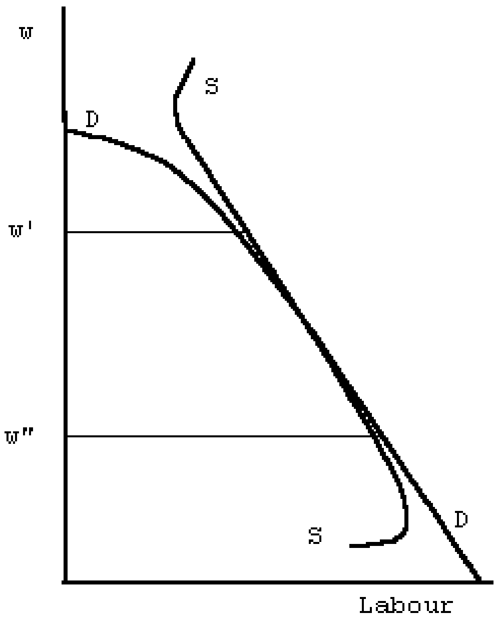

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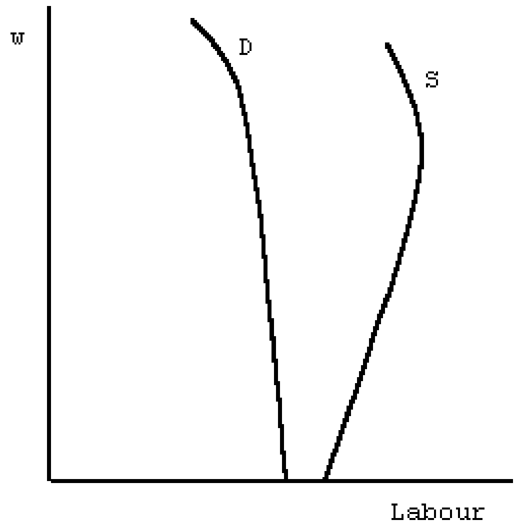

Fig. 7b

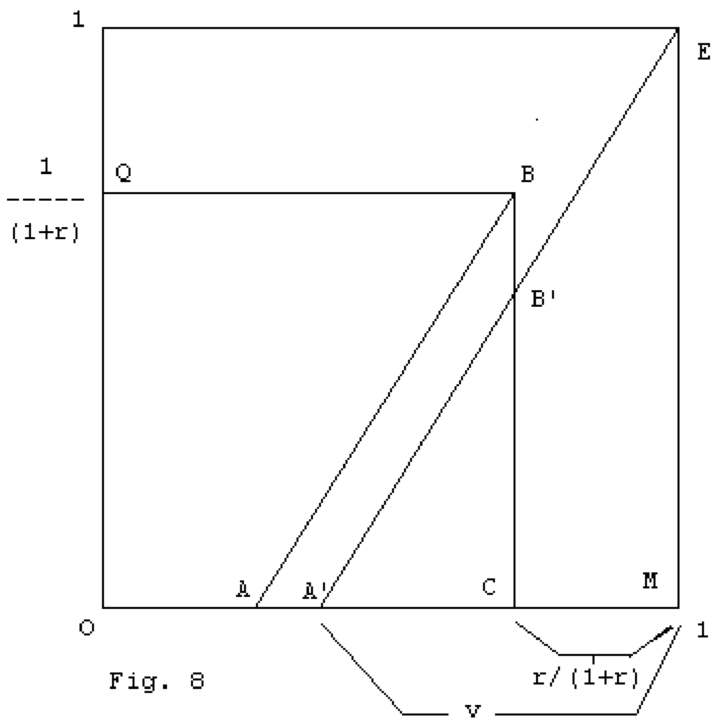

Fig. 8