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A Characterisation of Observability and Verifiability in Contracts

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Abstract - The notions of observability and verifiability are much wide spread in contract theory. Intuitively, both refer to the ability, respectively of the parties to the contract and the court of correctly detecting the contract contingencies. Despite their importance, the notions do not seem to have been given so far a formal characterisation. In this paper we aim at doing so by modelling parties and court as information processors, who may have subjective state spaces different from the objective one. The two concepts are modelled differently. Though both ask for correct detection of contract contingencies verifiability is modelled so as to incorporate also correct belief motivation. This to capture the idea that a court should not only emit the right verdict but also provide possibly correct motivation of it. The main results of the paper state that verifiability can be the case if and only if the court's subjective state space is included in the objective state space and the court information processing skills are at least of the same level as that of the parties.

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Key Words: Observability, Verifiability, Contracts

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1. Introduction

The issues of observability and verifiability are fundamental in contract theory. Indeed, they refer to how parties to a contract, and the possible arbitrator (court henceforth) that may have to enforce it, would process the available information emanated by the actual *state of affairs* (state henceforth). Even though both concepts refer, in some sense, to the capacity of *correctly evincing* the state on the part of the relevant agents typically (though not always), in the literature, observability is associated to parties and verifiability to the court. Despite their wide spread use and conceptual importance the two notions do not appear so far to have been given a characterisation that would go beyond an appeal to some intuitive meaning. This seems to be an important missing point in the theory for both because a formal characterisation should convey the ideas more crisply and (so) because it would help reasoning on the needed conditions for observability and verifiability to occur. The point is even more cogent in view of the fact that alternative notions may, in principle, be contemplated.

As it was mentioned above the fundamental intuition, whether explicitly stated or implicitly hinted at, appears to be that a contract is either observed and/or verified when state signals are correctly elaborated by the agents. In words, agents to a contract are in some sense depicted as rational (correct) information processors; from this general point of view

parties and court appear to be similar. A meaningful point however, often left unmentioned, that may potentially distinguish parties from the court, is that the latter would typically not only be asked to draw correct conclusions on the true state but also to provide a, possibly, *consistent* (internally and with respect to empirical evidence) argument motivating the verdict. In words, we notice that while observability appears conceivable in a manner that, in case of conflict, may exempt parties to partially, or even completely, provide a correct justification of their beliefs (positions), this would not be the case for the court. In essence, the capacity of verifying a contract seems to be calling upon an agent, the court, with a degree of information processing ability no lower than that of the parties. This distinction will lie at the heart of our characterisation.

Clearly, in reality, courts may have a role in resource allocation (Tirole, 1999) to the extent that factors like possible lack of competence, time constraint, difficulty in gathering information etc. may have an impact on the verdict. In this paper however, we shall only endeavour to provide a plausible characterisation of courts complying with the benchmark requisites mentioned above.

The framework we shall be using has been widely adopted in economic theory to deliver a number of fundamental theoretical issues crucially dependent upon agents' epistemics: among many others, two notable ones are consensus of opinions (Aumann, 1976) and conditions justifying Nash equilibrium in normal form games (Aumann-Brandenburger, 1995). We shall also briefly investigate how our proposal of verifiability could be linked to the notion of awareness, recently formalised and discussed by Modica-Rustichini (1994,1999) and further axiomatised by Dekel-Lipman-Rustichini (1998).

In what follows IP will stand for "information processor". Suppose Ω is the set of states (state space henceforth), with $\omega \in \Omega$ being the generic state, and let $P(\omega) \subseteq \Omega$ be the possibility set[1] associated to ω, namely the set of states that IP would think as being the possible ones when ω obtains. The sets $P(\omega)$ then represent a simple way to model IP abilities at the true state ω. The collection of $P(\omega)$, normally referred to as IP's *information structure* (Geanakoplos, 1989), is then taken to formalise her overall information processing skills. When this collection is a partition[2] of Ω the individual is typically considered to be a rational-consistent IP. Perfect information processing would then mean that $P(\omega)=\{\omega\}$, namely the finest possible partition of the state space.

Why rationality can be formalised by partitions can be better appreciated upon introducing the knowledge operator[3]. A partition of Ω obtains when for all $A \subseteq \Omega$, $KA \subseteq A$ and

[1] Technically, a possibility correspondence is simply a mapping $P: \Omega \rightarrow 2^{\Omega} - \{\varnothing\}$, where 2^{Ω} is the power set of Ω, namely the set of all possible subsets of Ω. The empty set can not be a possibility set since IP is supposed to think that something always happens. Subsets of Ω are called events.

[2] Namely if for all pairs $\omega, \omega' \in \Omega$ it is either $P(\omega)=P(\omega')$ or $P(\omega) \cap P(\omega') = \varnothing$ and $\cup_{\omega \in \Omega} P(\omega) = \Omega$.

[3] A knowledge operator K, a derived object in this case, is a mapping $K: 2^{\Omega} \rightarrow 2^{\Omega}$ defined, for all $A \subseteq \Omega$, as $KA = \{\omega \in \Omega \mid P(\omega) \subseteq A\}$. In words, knowledge of A is represented by the set of states in which IP believes (feels sure) that event A obtained. Notice that according to the definition, knowledge could be wrong in the common sense that personal beliefs may not correspond to reality. Notice further that since $KA \subseteq \Omega$ is in turn an event, expressions of the kind KKA are well defined in the framework.

$\neg KA \subseteq K\neg KA$[4]; in this case it would also follow that $KA \subseteq KKA$. In axiomatic approaches, the first property is often called the axiom of *truth* and simply says that whenever IP believes in *A* then it must be true. The second property, the axiom of *negative introspection*, in case of ignorance of *A* implies knowledge of one's ignorance. Analogously the third property called axiom of *positive introspection* entails, in case of knowledge of *A*, knowledge of one's knowledge.

Not surprisingly, since the court that we aim at modelling is (in some sense) a *proper* IP, we shall ask her skills to refer to some of the above properties. The fundamental request that the correct conclusion has to be drawn is meaningfully captured by correct beliefs while personal introspection, both positive and negative according to the situation, would seem to capture well the idea of a correct (self-public) justification for the decision.

As an example consider the following simple one. Two parties, *a* and *b*, signed a contract specifying that "if tomorrow it rains during day time *a* pays 1£ to *b* while if it snows *b* pays 1£ to *a*". Suppose that indeed it does rain so that *a* has to pay *b* but she refuses to. The court we have in mind should be able to gather, and for the purpose correctly process, the information needed to draw appropriate conclusion. Imagine that the only information the court gathers is "there were many people in the streets, there was some sun light, almost everybody was wet, streets were wet, there were thunders". The court should then rightly conclude that it rained, and so that *a* should pay *b* without, perhaps, considering the fact that "there was some sun light" as the decisive piece of evidence. We would moreover want the court to be able to draw correct conclusions even when a contingency not specified in the contract would occur. Indeed, if it was sunny, rather than rainy or snowy, we think that

[4] Notation $\neg$ stands for event complementation; i.e. $\neg A = \Omega - A$.

appropriate verification should again call for the right conclusion and possibly correct justification, so that no money would be transacted.

As far as observability is concerned, we understand its fundamental intuition as being that of self-evidence of contract contingencies. Formally, event A is self-evident for IP if $A \subseteq KA$, namely whenever it occurs IP believes in it. In a sense, the state would drive beliefs. This also implies $KA \subseteq KKA$ but not necessarily negative introspection unless $\neg A \subseteq K\neg A$[5] namely also the complementary event is self-evident.

As far as a contract is concerned however, the sense in which we understand observability does not seem to demand self-evidence over contingencies that are not in the contract space. In the above example if the weather is humid, but neither rainy nor snowy, this may not be self-evident to parties *a* and *b*; nonetheless, it would not prevent parties' consensus on contract contingencies when rain or snow were to obtain. More explicitly, observability does not appear to ask for correct information processing *off* the space of contingencies listed in the contract. Alternatively, we see the decision concerning dismissal of a possible case all in the hands of the court. Consider again the above example; party *b* (say), in case of absence of rain but presence of very high humidity may go to the court and make the case that "since it was very humid it rained" and so *a* should pay. The fundamental issue here appears to be not so much which contract contingency has obtained but, preliminarily to

[5] Or equivalently $A \subseteq KA$ and $\neg A \subseteq \neg KA$. Indeed, if $A \subseteq KA$ and $\neg A \subseteq K\neg A$ then $A = KA$ and $\neg A = K\neg A$ which implies $\neg A = \neg KA = K\neg A$ and so $\neg KA = K\neg KA$. Notice that the above conclusion is also valid in non-standard frameworks, namely where *true* (objective) and *subjective* state spaces do not necessarily coincide, that we shall consider in this paper.

that, whether the contract should at all be implemented. We see this as being a necessary part of verifiability but not of observability.

In view of the above considerations it will not come as a surprise the main characterisation of the paper stating that a necessary condition for a contract to be verifiable is that, at the time of verification, the court's information processing abilities should be such that it need to conceive, and rightly detect, all the true contingencies specified in the contract.

The paper has the following structure. In Section 2 we introduce the fundamentals of the model; more specifically, we formalise the information processing abilities of the actors to a contract, the notion of a contract and discuss the interpretation of the relevant state spaces that will appear in the analysis. In Section 3 we introduce observability and verifiability of a single event, of contracts and briefly discuss the relation between verifiability and unawareness as defined in Modica-Rustichini (MR henceforth) (1994). Section 4 presents the main characterisation results while Section 5 concludes the paper.

2. Agents, Contracts and Relevant Spaces

In this section we shall be modelling parties to a contract and court as information processors; prior to doing so however we need to introduce the relevant state spaces.

2.1 Relevant Spaces

There are N parties to a contract. To save on notation, N will indicate both the set and the number of parties; then $i \in N$ is the generic party, with $i=1,..,N$. The $N+1$th agent will indicate the court.

Definition 1 *(Contract Space) The contract space C, with generic element $c \in C$, is the set of states identified by the parties to the contract.*

A comment is in order here. Typically, in a contract, parties do not associate clauses to single states; they rather base their agreement on contingencies (events), sets of states with a common relevant characteristic[6]. For example, two parties may stipulate an agreement founded on whether or not *tomorrow it will rain* in a certain location. The events *tomorrow rains* or *tomorrow does not rain* in the specified location are defined by the set of states that include those two contingencies in the definition.

The above definition is flexible enough so as to encompass situations where some of the contingencies are indescribable ex-ante (Anderlini-Felli, 1994). Suppose, as in Hart-Moore (1999), that a seller agrees to provide a buyer with a "widget" the desired characteristics of which can not be completely described ex-ante, i.e. at the time the contract is signed, but only ex-post upon observing the state. Then, a (very weak) specification in the contract could be "*whatever the state* the seller will provide a widget that the buyer specifies, at a price to be agreed upon". Of course, if ex-post they do not agree on the price trade may not take place; nonetheless, even if contingencies are not describable ex-ante they may be so ex-post, either by parties or by the court or both[7]. In passing we notice that in the above agreement a court could not go beyond establishing what the state is; since the clause leaves discretion to both the seller and the buyer no trade or transaction is enforceable.

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[6] In words, a state could be thought in terms of a (possibly infinite) list of propositions (sentences), each of which describing a particular characteristic of the state itself. The same interpretation will hold true for states in all the spaces that will be introduced.

[7] Obviously, a situation in which there is indescribability both ex-ante and ex-post would not

Definition 2 *(True State Space) The true state space T, with generic element $t \in T$, is the set of states that may truly occur.*

The set *T* then represents all objectively possible states of world. In what follows the specification *ex-ante* will refer to "before state realisation".

Definition 3 *(Ex-Ante Subjective State Spaces) S^i, with generic element $s^i \in S^i$, is the set of states that party $i \in N$ subjectively imagines as being possible.*

The interpretation of S^i is analogous to that of *T* but with the following crucial difference: unlike states in *T* those in S^i may not truly occur. Consider, as an example, the following agreement between individual *a* and *b*, stipulated (say) on 31 July 1999: "If on 1 August 1999 the noon temperature in Rome is below –50° Celsius then *a* gives 100$ to *b* otherwise *b* gives 100$ to *a*". Even though it is perfectly conceivable that in Rome could be so cold in the middle of the summer, it did not happen and assume could not happen (given the actual situation in our solar system). In the agreement individual *b* is the one who is more likely to think the event as being truly possible and be worried about it. In this sense, her subjective view is wrong.

The definitions of *C* and S^i are given independently but our intuition suggests that typically is $C \subseteq \cup_{i \in N} S^i$. It may however be that parties' interaction, to decide what contingencies should be specified in the contract, would enhance learning so that, at a very general level, $\cup_{i \in N} S^i$ and *C* could in principle be conceived to be in any relationship.

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be of interest.

Moreover, without much loss of generality, to simplify matters henceforth we shall assume that $S^i=S$ for all $i\in N$, namely all parties have the same state space. Since S^i could be seen as, in some sense, representing party's i "view of the world", the hypothesis of common spaces might be justified by imagining that parties have similar experiences and information processing skills.

Finally, the following will also hold.

Assumption 1 $C=S$

The above assumption is less demanding than it could perhaps appear at first. Indeed, suppose $C\subset S$ and consider a contract defined on the space $C'=S-C$, with $c'\in C'$ being the generic state, as follows: "*at every state c' no agreement among parties is specified*". Then a contract defined on S such that, at all contingencies in C it coincides with a contract defined on C and at all states in C' with the above contract defined on C', will clearly have the same welfare implications of the contract defined on C.

Finally, notice that

Definition 4 *(Ex-Ante Court's State Space)* Z, *with generic element* $z\in Z$, *is the set of states that the court imagines as being possible.*

Simply, Z is the subjective space of the court.

Definition 5 *(General State Space)* $\Omega=T\cup S\cup Z$ *is the general state space, with generic element* $\omega\in\Omega$.

In words, Ω is the aggregate of true and conceived (by the agents to the contract) states in the framework. We are now ready to formalise the agents' information processing skills; after having done so below we shall then introduce a consistency assumption concerning agents' information processing which, in a sense, may be seen to *endogenise* the above subjective spaces.

2.2 Agents Information Processing Abilities

Parties Each $i \in N$ is endowed with a possibility correspondence $P^i: T \to 2^S - \{\varnothing\}$. Moreover, for all events $A \subseteq \Omega$, from P^i we derive the knowledge operator as

$$K^i A = \{ t \in T \mid P^i(t) \subseteq A \}$$

Notice that $K^i A \subseteq T$. If $\sim K^i A$ is the set of true states in which A is not known then, unlike the standard definition of set complementation, in this case $\sim K^i A = T - K^i A \subseteq T$ and not $\sim K^i A = \Omega - K^i A$ as it keeps being instead for non epistemic events. This will clearly hold true henceforth for all knowledge operators appearing in the paper.

Court The court has possibility correspondence $P^{N+1}: T \to 2^Z - \{\varnothing\}$ and knowledge operator analogously defined as

$$K^{N+1} A = \{ t \in T \mid P^{N+1}(t) \subseteq A \}$$

It is intuitive to interpret the sets $\cup_{t \in T} P^{N+1}(t)$ and $\cup_{t \in T} P^i(t)$, for all $i \in N$, as *ex-post* state spaces, since they represent the aggregate of states imagined as being possible by the

agents after information is processed, namely after the true state has obtained. Again, to save on notation and simplify matters, without much loss of generality we now introduce the following consistency assumption.

Assumption 2 *For all $i \in N$ is $S=\cup_{t\in T}P^i(t)$ and $Z=\cup_{t\in T}P^{N+1}(t)$.*

The reasons why the above condition need not always hold should be clear. For example, party *i* may widen her views ex-post so that $S^i \subset \cup_{t\in T}P^i(t)$ or perhaps narrow them entailing the opposite situation.

2.3 Contracts

Definition 6 *(Contract) A contract is a partition Π of C, with $\Pi(c)$ being the partition element containing state $c \in C$. Then $\Pi(c)$ is a contract contingency and, moreover, $\Pi(C)$ the set of all contracts (partitions) defined on space C.*

In words, since a contract could be seen as a collection of clauses, to each of which a contingency is associated, for the purpose of the paper a contract could simply be thought of as a collection of disjoint and exhaustive subsets of *C*, each defining a contract contingency (event).

3. Observability and Verifiability of Contracts

We are now ready to introduce the two main notions of the paper. We start formalising the concepts as far as a single event is concerned.

3.1 *Verifiability and Observability of an Event*

In the definition below the knowledge operator K refers to a generic information processor (IP) with subjective state space S; the true state space will still be T so that $\Omega=S\cup T$.

Definition 7 *(Event Verifiability) Event $A\subseteq\Omega$ is locally verifiable at state $t\in T$ if $t\in VA=(A\cap KA\cap KKA)\cup(\neg A\cap\sim KA\cap K\sim KA)\subseteq T$, where $V(\cdot)$ is the verifiability operator. Event A is globally verifiable if $VA=T$. Finally, the non-verifiability operator is defined as $\sim V(\cdot)=T-V(\cdot)\subseteq T$.*

The above formalises the intuition of verifiability according to the sense anticipated in the introduction. In broad terms, IP verifies an event when she always correctly believes whether it has occurred or not, and *properly* motivates the causes of her conviction (belief). This minimal request will also render the court able to properly decide whether a case based on a contract should or not be dismissed. Notice that by verifiability from now on we shall intend the global notion.

Definition 8 *(Event Observability) Event $A\subseteq\Omega$ is observable if $A\subseteq KA$ and $\neg A\subseteq\sim KA$.*

As previously anticipated, we model observability by evoking self-evidence of the relevant event and asking for IP not to believe in it when it does not occur. Since, by definition, is $A\cup\neg A=\Omega$; and $KA\cup\sim KA=T$ it follows that if at least one event $A\subseteq\Omega$ is observable then $T=\Omega$ and $S\subseteq T$, namely that the contingencies imagined as being possible by IP are always truly possible. This is an important implication and, not surprisingly, we shall see that the two definitions above capture the intuitive idea that when observability and

verifiability of a contract hold, both the parties and the court imagine truly possible states. They must be *proper* information processors with the important distinction however that observability does not require motivating beliefs at all true states. To better appreciate this last consideration take the following example.

Example 1 Let $T=\{a,b,c\}$, $P(a)=\{a\}$, $P(b)=\{b\}$, $P(c)=\{a,b\}$ so that $S=\{a,b\}$ and $\Omega=T$. Take $A=\{a\}$. Then $KA=\{a\}$, $\sim KA=\{b,c\}$ hence $A\subseteq KA$ and $\neg A=\{b,c\}\subseteq\sim KA$ implying A to be observable. However, $KKA=\{a\}$ and $K\sim KA=\{b\}$ and it is easy to see that $VA=\{a,b\}$ which implies that A is locally verifiable, in particular at $t\in\{a,b\}$ but not globally; therefore, for our purposes, non verifiable. What happens here is that when $t=c$ the individual rightly thinks that A is not the case but not because she realises that c obtained rather because she can not decide between a or b. In a sense, at $t=c$ she has "limited rationality", or "bounded information processing skills", either because c is too difficult to detect when it occurs and/or perhaps because she has some particular bias towards a and b or else. Whatever the reason, though A is observable, IP seems somehow to be lacking full comprehension of its nature since at c she thinks it as being possible.

The understanding of an event hinted at above calls for, in a natural way, some notion of awareness of that event. However, before discussing in more detail the relation between the verifiability and the awareness operator as defined in MR (1994), it is worth stating the following important property of $V(\cdot)$.

Proposition 1 *i) If $KT=\varnothing$ then for all $A\subseteq\Omega$ is $VA=\varnothing$ ii) If for all $A\subseteq\Omega$ is $VA=\varnothing$ then $KKT=\varnothing$*

Proof If $KT=\varnothing$ then $P(t)$ is not included in T, for all $t\in T$. This means that $P(t)$ can neither be included in KA nor in $\sim KA$ so that $KKA=\varnothing=K\sim KA$ which implies $VA=\varnothing$ for all $A\subseteq\Omega$. In passing notice also that $KT=\varnothing$ clearly entails $KKT=\varnothing$.

If instead, for all $A\subseteq\Omega$, is $VA=\varnothing$ then $VT=\varnothing$ and so $T\cap KT\cap KKT=\varnothing$ which entails $KT\cap KKT=KKT=\varnothing$.

The above proposition identifies minimal (rather intuitive) conditions for verifiability. Necessity says that at least one state t must exist where all contingencies believed by IP could truly obtain. It is worth underlying that $KKT=\varnothing$, alone, will not suffice to have $VA=\varnothing$, for all A, unless $KT=\varnothing$. Indeed, suppose $KKT=\varnothing$, $KT\neq\varnothing$ and $A=\varnothing$; then $V\varnothing=\neg\varnothing\cap\sim K\varnothing\cap K\sim K\varnothing=\Omega\cap T\cap KT=KT$.

3.2 Relation Between Verifiability and Awareness

As it was mentioned above, the idea of verifiability is intuitively linked to the notion of awareness. This is why in this paragraph we find it of some interest to see how $\sim VA$ relates to MR (1994) formalisation of unawareness. In a framework where subjective and objective state spaces coincide, though the considerations we shall make below hold in our more general framework too, MR define the unawareness operator $UA\subseteq T$ as

$$UA=\sim KA\cap\sim K\sim KA=(A\cap\sim KA\cap\sim K\sim KA)\cup(\neg A\cap\sim KA\cap\sim K\sim KA)$$

namely ignorance of one's ignorance, with the intention of capturing the epistemics of an individual who does not even have A in her mind. This is explicit in the right term of the equality, where it is clear that unawareness is independent of whether A obtains or not; i.e., a

state of unawareness is not related to empirical evidence. Hence, if at state t the individual is unaware of A, namely $t \in UA$ then either $t \in (A \cap \sim KA \cap \sim K \sim KA)$ or $t \in (\neg A \cap \sim KA \cap \sim K \sim KA)$; it is easy to see that in both cases $t \in \sim VA$ and so $UA \subseteq \sim VA$. This entails that the notion of non verifiability is less strict than unawareness; alternatively, if IP verifies event A then she is aware of it but not necessarily viceversa.

3.3 Verifiability and Observability of Contracts

We begin with the definition of verifiability.

Definition 9 *(Contract Verifiability) Contract $\Pi \in \Pi(C)$ is verifiable if there exists a court, with subjective state space Z, such that $V^{N+1}\Pi(c)=T$ for all $c \in C$.*

In words, we define a contract to be verifiable when a court could be found that would be able to verify all events of the partition contract. The way the definition is stated should suggest that verifiability hinges on the relationship between C, Z and T. Indeed, as we shall see in the next section, should parties somehow anticipate that a court will be able to verify a contract this will mean that, necessarily, C and Z have to be in a certain relationship.

In principle, given C, parties could consider any possible partition contract of it. Hence, the fundamental question that we pose in the next section, the answer of which will give rise to the main characterisation results, would be to determine the conditions for any contract based on C to be verifiable.

Definition 10 *(Contract Observability) Contract $\Pi \in \Pi(C)$ is observable if there exist N parties such that $\Pi(c) \subseteq K^i \Pi(c)$ and $\neg\Pi(c) \subseteq \sim K^i \Pi(c)$ for all $c \in C$ and $i \in N$*[8].

The definition simply generalises that given for a single event by asking that all parties should be able to meet that condition, for all possible contingencies defining the partition contract. It would be helpful to exemplify immediately the possible difference; we do so considering again Example 1

Example (1a) $T=\{a,b,c\}$ and $S=\{a,b\}$; moreover let $N=2$, $P^i(t)=\{t\}$, for $t=a,b$ and $P^i(c)=S$ with $i=1,2$. Finally, let $\Pi=\{\{a\},\{b\}\}$. It is easy to see that Π is observable by the parties but not verifiable by a court that would have the same parties' possibility correspondence. As we previously argued, at $t=c$ the court may be said to have too narrow a view on the objectively possible contingencies failing to explain, for instance, why it neither feels sure of *a* nor of *b*, even though considers both of them as being possible.

4. The Main Characterisation Results

It is worth starting from a benchmark case, the one in which $C=Z$, namely where the set of states spanned by the court's information processing abilities coincides with the

[8] In what follows, we shall also introduce observability on the part of the court; the definition in this case would be completely analogous and concern (one IP) the court, rather than the *N* parties.

contract-subjective state space. Because of Assumption 1, in what follows C and S will be used interchangeably.

4.1 The case of C=Z

In this, to an extent, extreme case the following theorem provides a full characterisation of verifiable contracts.

Theorem 1 *Contract $\Pi\in\Pi(C)$ is verifiable if and only if $C=Z=T$.*

Proof Since in what follows we only refer to the court, to economise on notation superscript $N+1$ will be omitted from the relevant epistemic operators.

Assume $\Pi\in\Pi(C)$ to be verifiable, namely that for all $\Pi(c)$ in the partition contract Π is $V\Pi(c)=T$. We first show that $C\subseteq T$ and then that $T-C=\varnothing$. Indeed, since

$$V\Pi(c)=(\Pi(c)\cap K\Pi(c)\cap KK\Pi(c))\cup(\neg\Pi(c)\cap\sim K\Pi(c)\cap K\sim K\Pi(c))= K\Pi(c)\cup\sim K\Pi(c)=T$$

and noticing that $V\Pi(c)$ is the union of two disjoint subsets of T, for all $t\in T$ it must either be that

$$t\in\Pi(c)\cap K\Pi(c)\cap KK\Pi(c)=K\Pi(c) \qquad (*)$$

or

$$t\in\neg\Pi(c)\cap\sim K\Pi(c)\cap K\sim K\Pi(c)=\sim K\Pi(c) \qquad (**)$$

However, if (*) is true then $P(t)\subseteq K\Pi(c)\subseteq T$ while if (**) is true it is $P(t)\subseteq\sim K\Pi(c)\subseteq T$ and since $Z=\cup_{t\in T}P(t)$ it follows that $C=Z\subseteq T=\Omega$.

Suppose then that $T-C\neq\varnothing$. From (*) we obtain that $K\Pi(c)\subseteq\Pi(c)\subseteq T$ while from (**) that $\sim K\Pi(c)\subseteq\neg\Pi(c)\subseteq T$ which taken together imply $K\Pi(c)=\Pi(c)$ and $\sim K\Pi(c)=\neg\Pi(c)$. Then it is

$T\text{-}C=\cap_{c\in C}\sim K\Pi(c)$ and for all $t\in T\text{-}C$ and $c\in C$, as $\sim K\Pi(c)=K\sim K\Pi(c)$, it will have to be $P(t)\subseteq C\text{-}\Pi(c)$ entailing $P(t)\subseteq\cap_{c\in C}(C\text{-}\Pi(c))=\varnothing$ which is impossible and so $C=T$.

Assume now $C=T=\Omega$ and, considering contract Π, take $P(t)=\Pi(c)$, for all $t\in\Pi(c)$ and $c\in C$. Hence, for all $\Pi(c)$ we have $K\Pi(c)=\Pi(c)$, $KK\Pi(c)=\Pi(c)$, $\sim K\Pi(c)=\neg\Pi(c)$ and $K\sim K\Pi(c)=\neg\Pi(c)$ so that $V\Pi(c)=T$.

Hence, when all the "agents' view of the world" is the same, the only way for any contract to be verified is that it should specify contingencies that can truly obtain and only those. Any kind of mis-specification might either prevent the drawing of correct conclusions or their appropriate motivation on the part of the court

An analogous characterisation can now be put forward as far as observability is concerned.

Theorem 2 *Contract $\Pi\in\Pi(C)$ is observable if and only if $C\subseteq T$.*

Proof If Π is observable then for all $c\in C$ and $i\in N$ it is $\Pi(c)\subseteq K^i\Pi(c)\subseteq T$ and $\neg\Pi(c)\subseteq\sim K^i\Pi(c)\subseteq T$ entailing $C\subseteq T$.

Assume instead $C\subseteq T$; then parties whose possibility correspondences, for all $t\in\Pi(c)$ and $c\in C$, satisfy $P^i(t)=\Pi(c)$ and, for all $t\in T\text{-}C$, satisfy $P^i(t)=C$ would make the conclusion hold true.

Taking theorems 2 and 3 together we immediately obtain the following further characterisation, linking verifiability and observability, the simple proof of which is omitted.

Corollary 1 *If contract $\Pi \in \Pi(C)$ is verifiable then it is observable.*

An interpretation of the above result could be that since under specification of *T* does not necessarily prevent observability of the contract parties can, in a sense, allow themselves *(afford)* to be unaware that they may be considering an incomplete list of contingencies (states).

It may be worth to further characterise a contract, based on *C*, which would be observable by the court but not verifiable. Example (1a) suggests how we could proceed to supply what is missing for verifiability. In that case, when the true state is $t \notin \Pi(c)$, the court is not only unable to take the correct decision but in general a decision. She has only a partial (though correct) view of the true world and when an unforeseen state obtains she is unable to recognise it. It is this possibility to decide that could be meaningful to introduce and through which we now formalise the gap between observability and verifiability.

Definition 4 *(Decidable Contract) Contract $\Pi \in \Pi(C)$ is decidable if there exists a court such that $P^{N+1}(t) \subseteq \Pi(c)$ for some $c \in C$ and all $t \in T$.*

In words, we define a contract to be decidable if a court can be found that always thinks one of the events in the contract to have obtained. Having introduced decidability we can now state the following result.

Lemma 1 *Contract $\Pi \in \Pi(C)$ is verifiable if and only if for the court is observable and decidable*

Proof Suppose $\Pi \in \Pi(C)$ is verifiable; then, by Theorem 1, $C=Z=T$ and for all $c \in C$ a court can be found such that $K\Pi(c)=\Pi(c)$ so that Π is observable and decidable. Instead, if $\Pi \in \Pi(C)$ is observable then $\Pi(c) \subseteq K\Pi(c)$ and $\neg\Pi(c) \subseteq \sim K\Pi(c)$; hence $K\Pi(c)=\Pi(c)$ and $\sim K\Pi(c)=\neg\Pi(c)$. Moreover, decidability implies $\cap_{c \in C} \sim K\Pi(c)=\varnothing$ so that $\cup_{c \in C}\Pi(c)=T$ and verifiability follows.

We now pass on to consider the more general case where the main result of the paper will be stated.

4.2 The case of $C \neq Z$

Prior to formulating the principal characterisation, it may help the intuition to consider verifiability of a contract in the extreme case where $Z \cap T=\varnothing$. It is not surprising to realise that no contract, in such a situation, could be verified. This immediately suggests that a necessary condition for verifiability to occur is that Z and T must overlap. The following theorem establishes the needed overlapping and proper inclusion order, between relevant spaces, that entail a full characterisation of verifiability.

Theorem 4 *i) Contract $\Pi \in \Pi(C)$ is verifiable if and only if $Z \subseteq T$. ii) If contract $\Pi \in \Pi(C)$ is verifiable and $C \cap T \neq \varnothing$ then $C \cap T \cap Z \neq \varnothing$ and it can not be $Z \subseteq C \cap T$.*

Proof i) If verifiability holds then the conclusion is immediate by the same reasoning as in Theorem 1. Consider instead $Z \subseteq T$ so constructed. Let $P(t)=T \cap \Pi(c)$ for all $t \in T \cap \Pi(c) \neq \varnothing$ and $P(t)=Z-C \neq \varnothing$ for all $t \in T-C$; then the result follows. If $C \cap T=\varnothing$ it is easy to see that all $\Pi(c)$ are verifiable.

ii) Assume $\Pi \in \Pi(C)$ to be verifiable, $C \cap T \neq \varnothing$ and $C \cap T \cap Z = \varnothing$. Then, since by verifiability it is $Z \subseteq T$, it follows that $K\Pi(c) = \varnothing$ and $\sim K\Pi(c) = T$, for all $\Pi(c)$. Moreover, by assumption there exists a $c^* \in C$ such that $T \cap \Pi(c^*) \neq \varnothing$ for which $V\Pi(c^*) = \neg\Pi(c^*) \cap \sim K\Pi(c^*) \cap K \sim K\Pi(c^*) = \neg\Pi(c^*) \cap T \cap T = T - \Pi(c^*) \subset T$; therefore a contradiction arises and the first conclusion follows. The proof that it can not be $Z \subseteq T \cap C$ is simple and omitted.

As for the non-specificity of point (ii), in particular the one concerning the relationship between $C \cap T$ and Z, the following example illustrates how indeed there may be no inclusion hierarchy between the two.

Example 2. Let $T = \{a,b,c,d\}$, $C = S = \{a,b,e\}$ and $\Pi = \{\{a,b\},\{e\}\}$; moreover suppose $P^{N+1}(a) = \{a\} = P^{N+1}(b)$, $P^{N+1}(c) = \{c\} = P^{N+1}(d)$ so that $Z = \{a,c\}$. It is easy to see that Π is verifiable with no inclusion order between $C \cap T$ and Z being the case.

Admittedly, the above theorem provides a relatively weak characterisation, in the specific sense that it does not put much structure on the relation between $C \cap T$ and Z. The coming theorem will show that with the finest contract partition the conclusion, in this particular direction, may be made more precise.

Theorem 5 *If $\Pi \in \Pi(C)$ is verifiable, and $\Pi(c) = \{c\}$, for all $c \in C$, then $T \cap C \subseteq Z \subseteq T$, with $T \cap C = Z$ if and only if $T \subset C$.*

Proof If for all $c \in C$ is $V\Pi(c)=T$ then $Z \subseteq T$. As $\Pi(c)=\{c\}$ it must be $K\Pi(c)=T \cap \Pi(c)$ and so $T \cap C=\cup_{c \in C}(T \cap \Pi(c))$; it also has to be $P(c)=\{c\}$ and the first part follows. The second part is easy to prove and omitted.

We are now capable to comment on the full characterisation of verifiable contracts in case of observability, namely when $C \subseteq Z$. What we notice is an interesting discontinuity between the case of equality and that of strict inclusion. When $C=Z$, a situation in which agents to a contract have the same conception of the world, $C=Z$ must coincide with T; instead, when $C \subset Z$ the court's state space may be different from T. We interpret this as a situation in which to verify an observable contract the court's information processing ability must be such that she should, correctly, imagine at least the states spanned by the contract contingencies.

5. Conclusions

In the paper we provided a characterisation of observability and verifiability, two fundamental and wide spread notions in the theory of contracts. Typically, in the literature, both observability and verifiability appear to refer to correct detection of the events-contingencies specified in the contract. However, in the work, we conceived this correct detection to be different in the two cases. Indeed, a sense in which observability seems to be intended evokes, when they obtain, self-evidence of contract contingencies. However, when no contract contingency occurs no request, beyond that of not drawing the wrong conclusion that a contract event has occurred, is apparently imposed upon parties' information processing skills. Alternatively, when a true state does not belong to the contract

space, correct detection of it would not necessarily be asked to parties. Consequently, parties may find themselves in the position of not believing that any contract event obtained without necessarily understanding that this is due to none of them having occurred.

This can not evidently be the case when verifiability is considered. In case of conflict among parties, the court should not only emit a verdict but also properly motivate it. Therefore, if a state not contemplated in the contract were to be the true one, the court would have to properly explain to parties that the case has to be dismissed just because no contract contingency applies. The main result of the paper says that for a contract to be verified the court's state space must be included in the true state space. This agrees with the intuition; roughly speaking, no correct detection of contract events and proper motivation could be put forward unless the court imagines states that are always truly possible. Moreover, when a contract is composed of single state contingencies, the court must necessarily consider as being possible those contract states which are true.

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