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Efficiency and Equilibrium in The Electronic Mail Game; The General Case

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ABSTRACT

In the Email Game (Rubinstein, AER 89) noisy information channels may prevent efficient coordination, even when the game is almost common knowledge. In the paper we show that this is the case whenever message failure probabilities are not sufficiently different. Quite intuitively, the extent of the difference is governed by the game payoffs, and in particular by the purely mixed Nash equilibrium strategy of one of the two coordination games to be played. This is because, conditional to having observed one's type, a player's beliefs on the opponent's choices are governed by the reliability of communication channels.

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EFFICIENCY AND EQUILIBRIUM IN THE ELECTRONIC MAIL GAME; THE GENERAL CASE

1. Introduction

In a fascinating paper Rubinstein (1989) investigated the consequence of a condition of *almost common knowledge* in the Electronic Mail Game (EMG) that two individuals are playing. The EMG is a Bayesian game where players are differently informed on the relevant strategic form coordination game, which is chosen by nature out of two possible ones. The depth (degree) of *almost common knowledge* concerning the game that is being played is governed by a computer based communication technology through which the two players exchange email messages. The main conceptual, and to some extent puzzling, point of the Osborne and Rubinstein (1994) version of the model is that no matter how *deep* knowledge of the game players might have, the unique Nash Equilibrium (NE) of the EMG is a pair of strategies specifying that each player chooses, at all types (and so states), the same action that would be chosen at the unique equilibrium with no exchange of information. Namely, unless the communication protocol would lead to common knowledge of the game, which by construction in the EMG occurs with probability zero, no or abundant informational exchange would induce the same equilibrium. At first, the theoretical feature of equilibrium uniqueness is remarkable as it seems to be lacking intuitive appeal, especially when the number of messages sent back and forth by the two players is very high. However, related experimental work (Camerer, 1999) appears instead to suggest that the finding may be less counterintuitive than it would seem. More specifically, the empirical evidence collected indicates that the unique NE may in fact be conceived as the final outcome of some learning process. Indeed, subjects involved in this very same strategic situation for a sufficiently high number of repetitions eventually exhibited choices consistent with Rubinstein's equilibrium.

From the point of view of possible real life applications of the model, implemented to enhance efficient coordinated choices, the following consideration is unavoidable. If *rational* players consider setting up such device they should also anticipate that it would be pointless as it would entail the same choices of the no communication case. Hence a natural question to pose is whether or not the same communication protocol could in some way accommodate agents' coordination, and if yes how.

A first positive answer in this direction was provided by Rubinstein (1989) himself. Indeed in his work he shows how, operating on the technological side, a more intuitive outcome could arise. In particular he suggests that, under appropriate conditions, imposing a commonly known upper bound to the number of exchanged messages might induce two NE and, in one of them, informational exchange has a role in enhancing more appropriate coordination. Recent, independent, work by Binmore and Samuelson (2000) pointed out that if informational exchange is either voluntary, or else costly or both then efficient coordination could take place.

In this paper, departing in a *minimal* way from the original version of the EMG, we are interested in investigating the possibility of improved coordination within a very general communication-technology framework. As we shall see, this will allow us to identify a theoretical key point underlying Rubinstein's results. More specifically, as in Rubinstein the parameter ε, with $1 \geq \varepsilon \geq 0$, represents the probability for *both* players' computers that an email message will fail reaching the other machine, we shall consider here the situation in which players may (possibly correctly) perceive computers to have different failure probabilities. It may be worth mentioning that such analysis was mainly motivated by the following two basic intuitions: in Rubinstein's framework no other NE, of the single games defining the EMG, plays a role and, for the result to obtain, no specific relationship is asked to hold between the communication-technology parameters and payoffs.

We shall see that the results provide a fully satisfactory answer to the above observations Indeed, roughly speaking, what we find is that informational exchange may possibly entail more desirable coordination if and only if *i)* for one of the two individuals, the probability of a successful message is greater than the purely mixed strategy equilibrium probability, in one of the two games defining EMG, of the more rewarding-risky action and *ii)* the opponent has a "sufficiently less reliable" computer in sending messages.

Nonetheless, even in this more general context, uniqueness of the NE found by Rubinstein is still pervasive in terms of probability values; what we are now capable of however is to put it into a broad perspective that allows us to understand its nature. More specifically, uniqueness in Rubinstein could be interpreted as a particular manifestation (when message failure probabilities are equal) of a more general phenomenon that obtains within a well defined subset of the space of message failure probabilities, given by the unit square. Outside the above subset there could be multiple equilibria, including that in which players coordinate "almost perfectly" (namely with the exception of just one state of the world) on the Pareto efficient NE, regardless of the game chosen by nature.

The results appear also to suggest that a relevant notion of almost common knowledge, needed for proper coordination, may not be so much one related to the number of messages exchanged by players but rather a notion linked to the closeness of message error probabilities to their values characterising common knowledge of the game played. Alternatively, what seems to matter is not so much the depth of the beliefs hierarchy concerning the game but rather beliefs about actions taken by the opponent. As these are supported by the reliability of communication channels our findings are not surprising. Moreover, this view might also be corroborated by "payoff continuity" considerations. More specifically, both players' expected payoff associated to the equilibrium strategy profile where, at all types, agents choose actions as in the efficient equilibrium with common knowledge converge to this last equilibrium expected payoff when the relevant message failure probability tends to the value entailing common knowledge of the game.

The paper is structured as follows. In Section 2 we define the Generalised Electronic Mail Game, formulate necessary and sufficient conditions for uniqueness (multiplicity) of Nash Equilibria, discuss ex-ante efficiency, argue that the notion of almost common knowledge could be related to message failure probabilities rather than to the number of exchanged messages and provide an alternative interpretation for the model and results. Section 3 concludes the paper.

2. The Generalised Electronic Mail Game (GEMG)

2.1 The Game

We refer to the version of the EMG appearing in Osborne and Rubinstein (1994), where two individuals (*I* and *II*) have to play one of the two coordination games depicted in Fig. 1 below

	A	B
A	M, M	1, -L
B	-L, 1	0, 0

G_a ; probability 1-p

	A	B
A	0, 0	1, -L
B	-L, 1	M, M

G_b ; probability p

$L>M>1$; $p<1/2$

Figure I

States of nature *a* and *b* are the only two possible ones; depending upon the state one of the two games will be played. It will either be Game *a* (G_a), with probability *1-p*, or Game *b* (G_b) with probability *p*. Notice that while G_a has only one (pure strategy) NE, the profile *(A,A)*, G_b has three NE; the pure strategy profiles *(A,A)* and *(B,B)* and the purely mixed profile where both players choose *A* with probability $q=(M-1)/(L+M-1)<1/2$. Hence all equilibria are symmetric and, in playing them, individuals have to coordinate on the same action; moreover they are Pareto rankable with *(B,B)* being the most and *(A,A)* the least preferred, among the three, by both players. Notice also that *(A,A)* is risk dominant.

After the game has been chosen only *I* knows with certainty (observes) which one it is. The two players have computers on their desks to communicate; we indicate them as C_i, with $i=I,II$.

The communication protocol is as follows. If G_a obtains then no message is exchanged between the two machines. If instead G_b obtains then C_I automatically sends an email message to C_{II}; in Rubinstein's EMG the message has probability $1>\varepsilon>0$ of failing to reach the other machine. If the first message arrives then C_{II} will automatically reply by sending a confirmation message with the same probability $1>\varepsilon>0$ of not getting through. If C_I receives this message it will in turn send a confirmation (of a confirmation) message, still with $1>\varepsilon>0$ failure probability and so on. Messages are independent of each other. With probability one communication eventually stops; the only uncertainty is when. Once stopped, on their computer screens players will privately read the number of messages sent by their own machines. Then the EMG is a Bayesian game in which the communication technology specifies a common type-space T, the set of natural numbers, namely $T=\{0,1,2,...\}$ given by the number of messages appearing on the screen at the end of the communication exchange. If $S=\{A,B\}$ is the pure strategies space, in G_a and G_b (for both agents), then a strategy for player i in the Bayesian game is a function $\delta_i:T\rightarrow \Delta(S)$, where $\delta_i(t)$ indicates the probability with which player i chooses A at type t, with $i=I,II$. The state space Ω will then be a subset of $\Omega=T^2$, with the generic state $\omega\in\Omega$ defined by a pair of possible types $\{t_I,t_{II}\}$, such that either $t_I=t_{II}$ or $t_I= t_{II}+1$.

To simplify notation, from now on a strategy in the game will be written as

$$\delta_i=\Sigma_{t\in T}\, \delta_i(t)I(t_i=t)$$

where $I(t_i=t)$ is the standard indicator function. So, for example $\delta_i=I(t_i=0)$ is the strategy for player i specifying to play A at type $(t_i=0)$ and B at all other types.

According to Rubinstein, with probability one the game played will not be common knowledge; nonetheless, it could be of *almost common knowledge* should the number of messages appearing on the screens be sufficiently high.

2.2 Nash Equilibria and Communication Technology

We start by recalling Rubinstein's main result; below, $0<\varepsilon_I,\varepsilon_{II}<1$[1] indicate (respectively) the failure message probabilities of C_I and C_{II}. From now on we shall refer to the EMG, where ε_I and ε_{II} may possibly differ, as the Generalised Electronic Mail Game (GEMG). However, before stating the first proposition it is worth pointing out explicitly that the following assumption holds (in an informal way) throughout the paper.

Assumption *The probabilities $0\leq\varepsilon_I(t),\varepsilon_{II}(t)\leq 1$ are common knowledge between players.*

We shall see below that since in Rubinstein probabilities are equal their value does not play any role in the main result: due to this, no assumption like the above one would be needed in that particular case. However, as we hinted at in the introduction, if players are interested in efficient coordination then they should make sure to agree on probabilities being sufficiently different.

Proposition 1 (Osborne-Rubinstein, 1994) *If $\varepsilon_{II}=\varepsilon=\varepsilon_I$ the unique Nash Equilibrium of the GEMG is the pair of strategies $\delta_I=\delta^*=\delta_{II}$ where $\delta^*=\Sigma_{t\in T}I(t_i=t)$.*

Rubinstein shows that playing *A*, on the part of both agents, is the unique NE also in absence of the email communication protocol; exchanging information under the above setting then appears useless to enhance, in equilibrium, more desirable coordination when playing G_b. The questions we then want to tackle are *i)* whether or not, and if yes under what requisites, with the possibility of different message failure probabilities $0<\varepsilon_I,\varepsilon_{II}<1$ other equilibria could emerge in the GEMG and *ii)* in case discuss their relative efficiency. In what follows, as long as some conditions on error probabilities are satisfied, we shall provide a positive answer to the first question.

Below we start focusing on what we consider to be, in terms of the possibility of more efficient coordination, a main consequence of introducing different probabilities.

Proposition 2 *(1) The pair of strategies* $\delta_I=\delta^{**}=\delta_{II}$, *where* $\delta^{**}=I(t_i=0)$, *is a Nash Equilibrium of the GEMG if and only if* $\varepsilon_{II}>(1-q)\varepsilon_I/q(1-\varepsilon_I)$ *and* $\varepsilon_I<q$.

(2) The pair of strategies $\delta_I=\delta^{***}=I(t_I=0)+I(t_I=1)$ *and* $\delta_{II}=\delta^{**}$ *is a Nash Equilibrium of the GEMG if and only if* $\varepsilon_I>(1-q)\varepsilon_{II}/(1-\varepsilon_{II})q$ *and* $\varepsilon_{II}<q$.

Proof We shall only discuss sufficiency as necessity is immediate. *(1)* First consider player *I*. *i)* As in the EMG, at $t_I=0$ playing *A* is strictly dominant for *I*. *ii)* At $t_I=1$ she is uncertain on the true state being either the pair of types *(1,0)* or *(1,1)*. Hence if *II* plays δ^{**}, namely chooses *A* when observes $t_{II}=0$ and *B* elsewhere, the (conditional to the type) expected payoff of *I* when playing the (mixed strategy) $(\delta_I(1),1-\delta_I(1))\in\Delta(S)$, where as we said $\delta_I(1)$ is the probability of choosing *A* at type $t_I=1$, is given by

$$E\Pi_I(\delta_I(1)\,|\,t_I=1)= [p\varepsilon_I(0\delta_I(1)-(1-\delta_I(1))L) + p(1-\varepsilon_I)\varepsilon_{II}\,(1\delta_I(1)+(1-\delta_I(1))M)]/[p\varepsilon_I+ p(1-\varepsilon_I)\varepsilon_{II}] = [\delta_I(1)\,(\varepsilon_I L - (1-\varepsilon_I)\varepsilon_{II}\,(M-1)) + (1-\varepsilon_I)\varepsilon_{II}\,M - \varepsilon_I L]/[\varepsilon_I+ (1-\varepsilon_I)\varepsilon_{II}].$$

As $\varepsilon_{II}>(1-q)\varepsilon_I/(1-\varepsilon_I)q$ the term $(\varepsilon_I L-(1-\varepsilon_I)\varepsilon_{II}(M-1))$ in the numerator of the above expression is strictly negative; moreover, since $\varepsilon_I<q$ there exists ε_{II} in the interval $(0,1)$ satisfying the first inequality. In this case the optimal $\delta_I(1)$ is $\delta_I(1)^{**}=0$ which proves this part. *iii)* If $t_I>1$ then it is clearly optimal for *I* to play $\delta_I(t_I)^{**}=0$ against *II* choosing δ^{**}.

Consider now player *II*. *i)* Again, as in the EMG, it is easy to see that at $t_{II}=0$ playing *A* is best reply against any choice of *I* *ii)* At $t_{II}\geq 1$ it is obvious that playing *B*, for *II*, is best reply against *I* playing δ^{**}.

(2) Start with *I*. *i)* If $\varepsilon_I>(1-q)\varepsilon_{II}/q(1-\varepsilon_{II})$ and $\varepsilon_{II}<q$ then the conditions of point *(1)* can not, strictly, hold; hence it is optimal for *I* to choose $\delta_I(1)^{***}=1$. *ii)* At any $t_I>1$ playing *B* is optimal for *I* against *II* choosing δ^{**}. Take now player *II*. By a reasoning analogous to that of point (1) it can be easily verified that at $t_{II}\geq 1$ action *B* is best reply for *II* against *I* playing δ^{***} and the proof is complete.

When conditions in point (1) of the above proposition hold it is $\varepsilon_{II}>\varepsilon_I$; viceversa when those in point (2) are true. Namely improved coordination can only obtain if the message failure probability of C_I is *sufficiently* different from that of C_{II}. Is not enough that one machine is *highly* reliable; what is also needed is that the other should be *relatively* less so. Discussion on why this has to be the case will be postponed until Corollary 2 below. A consequence of the finding could be that, for more desirable coordination to be achieved, mutual knowledge of the game might suffice; in any case, the common knowledge efficient outcome can never be achievable.

Indeed, for example in state *(1,0)*, at the equilibrium $(\delta^{**},\delta^{**})$ players would choose the action profile *(B,A)* while at $(\delta^{***},\delta^{**})$ the profile *(A,A)*.

It is worth noticing that the conditions in the above proposition entail a whole class of equilibria, indeed an infinite number. This is stated informally by the following Corollary.

Corollary 1 *If conditions in point (1) of Proposition 2 hold then any pair of strategies* $\delta_i=\Sigma_{t=0,..,t^*}I(t_i=t)$, *with* $i=I,II$, *is a NE of the GEMG. If conditions in point (2) of Proposition 2 hold then any pair of strategies* $\delta_I=\Sigma_{t=0,..,t^*+1}I(t_I=t)$ *and* $\delta_{II}=\Sigma_{t=0,..,t^*}I(t_{II}=t)$ *is a NE of the GEMG, where* $t^*=0,1,2,..$

In words, under the same set up of Proposition 2 there exist equilibria where agents coordinate efficiently when either both observe at least t^* messages, or *I* sends at least t^* and *II* at least t^*-1 messages. However, waiting for a high number of messages to coordinate on *B* does not seem to be a *natural* way to proceed. In case of an infinite number of equilibria, considerations relative to their *ex-ante efficiency* may be a useful tool to formalise the above observation.

Corollary 2 *(1) At any* $t_I\geq 1$, *if* $\varepsilon_{II}>(1-q)\varepsilon_I/q(1-\varepsilon_I)$ *and* $\varepsilon_I<q$ *then is* $z=P(t_{II}=t_I-1 \mid t_I)=\varepsilon_I/[\varepsilon_I+\varepsilon_{II}(1-\varepsilon_I)]<q$ *and* $z'=P(t_I=t_{II} \mid t_{II})=\varepsilon_{II}/[\varepsilon_{II}+\varepsilon_I(1-\varepsilon_{II})]>q$.

(2) At any $t_{II}\geq 1$, *if* $\varepsilon_I>(1-q)\varepsilon_{II}/(1-\varepsilon_{II})q$ *and* $\varepsilon_{II}<q$ *then is* $z'<q$ *and* $z>q$.

The above result makes it explicit the very conceptual point for players to choose *B* when $t_i\geq 1$, $i=I,II$. Take, for example, point *(1)* and consider *I* at $t_I=1$; at that type she is uncertain on whether $t_{II}=0$ or $t_{II}=1$. If *II* plays δ^{**} then *I* knows that *II* can either be of type (she would choose) *A* with probability *z* or *B* with probability *(1-z)*. More explicitly, from the point of view of *I* it is *as if II*, in G_b, is playing the purely mixed strategy $(z,1-z)$ with the, obvious, interpretational proviso that z is a technological, rather than behavioural, parameter[2]. Hence, playing B is strictly optimal for I if and only if the "mixed strategy" z is no greater than the NE purely mixed strategy of choosing A, namely q. Analogous considerations would hold for point (2). If $\varepsilon_I=\varepsilon_{II}$ then $z=1/(2-\varepsilon)=z'>1/2$ and the above requisites are never satisfied. Rubinstein hints at this last point as the crucial one in the EMG inducing coordination failure at game G_b. As $q<1/2$, the analysis pursued indicates that $1-z>1/2$ ($1-z'>1/2$) is certainly a necessary however not sufficient condition for $(\delta^{**},\delta^{**})$ ($(\delta^{***},\delta^{**})$) to be an equilibrium in the GEMG; indeed, it has to be that $(1-z)>(1-q)>1/2$ ($(1-z')>(1-q)>1/2$).

Below we see that these conditions are in fact much more meaningful; in particular, they provide a full characterisation for the pair (δ^*,δ^*) to be the unique NE of the GEMG.

Proposition 3 *The pair* (δ^*,δ^*) *is the unique NE of the GEMG if and only if* $z>q$ *and* $z'>q$.

Proof Immediate. Indeed, as for sufficiency, let $t_I^*\geq 1$ and $t_{II}^*\geq 1$ be the lowest types such that, respectively, $\delta_I(t_I^*)<1$ and $\delta_{II}(t_{II}^*)<1$ at a NE. Then it is either $t_I^*-1=t_{II}^*$ or $t_I^*=t_{II}^*$. In both cases, by a reasoning analogous to that in the proof of Proposition 2 it is easy to see that either $z\leq q$ or $z'\leq q$. Assume instead $z>q$ and $z'>q$; then it is also simple to check that no NE can accommodate players choosing B with strictly positive probability at some type.

Clearly, (δ^*,δ^*) is always a NE of the GMEG; therefore, Proposition 3 is suggesting that communication technology could be thought of as a (possibly inefficient) device to select among multiple equilibria. Furthermore, the interplay between payoffs (in particular the purely mixed strategy equilibrium) in G_b and technology is now completely clear and, what is more, decisive for efficient coordination. Consequently, as payoffs vary the subset of the unit square space of technological parameters entailing equilibrium uniqueness (multiplicity) changes. In particular, as $M \to 1$ the area inducing multiple equilibria shrinks as a result of the fact that action A, in this case, tends to become a weakly dominating strategy. Instead, if $M \to L$ then $q \to (L-1)/(2L-1)$; since q is increasing in M this bound identifies the largest possible area for multiple equilibria. This is because it is when $M \to L$ that coordination on B is most attractive; as a result, there is a stronger incentive to do so. An alternative interpretation of this could be in terms of risk dominance. As $M \to L$ the profile (A,A) clearly remains risk dominant in G_b; the extent of its dominance however decreases as M increases. The same type of interpretative key might also be used when $L \to \infty$ so that $q \to 0$; in this case coordination on B becomes, in a sense, *infinitely* risky with the area of multiplicity tending to disappear. However, this is true when M and L are not related; clearly, in general this may not be so. Indeed, for example, suppose $L=aM$, with $a>1$ and finite; then $q=[(L/a)-1]/[(L/a)+L-1]$ so that as L gets large $q \to 1/(a+1)$ and the above conclusion would evidently no longer hold.

Before discussing efficiency it may be interesting to identify the unique NE with purely mixed strategies, at all $t_I, t_{II} \geq 1$, for both players.

Proposition 4 *The pair of strategies* $\delta_I=\delta'=I(t_I=0)+\Sigma_{t\geq 1}\, qI(t_I=t)$ *and* $\delta_{II}=\delta''= I(t_{II}=0)+\Sigma_{t\geq 1}\, \delta_{II}(t)I(t_{II}=t)$*, where* $\delta_{II}(t)=-\delta_{II}(t-1)z/(1-z)+q/(1-z)$*, is a Nash Equilibrium of the GEMG if and only if* $\varepsilon_{II}>(1-q)\varepsilon_I/(1-\varepsilon_I)q$ *and* $\varepsilon_I<q$*.*

Proof (Sufficiency) Following the same reasoning of Proposition 2 action A is optimal for both players *I* and *II*, when the state is $(0,0)$, against any choice on the part of the opponent. Then take first player *I*. It is easy to see that, for all $t=1,2,..$, any $\delta_I(t)\in(0,1)$ is best reply against $\delta_{II}(t)$ and $\delta_{II}(t-1)$, when linked by the recursive relation $\delta_{II}(t)= - \delta_{II}(t-1)z/(1-z) + q/(1-z)$ Consider now player *II*. Analogously any $\delta_{II}(t)\in(0,1)$, with $t=1,2,..$, is best reply against choices on the part of *I* following the pattern $\delta_I(t+1)= - \delta_I(t)z'/(1-z') + q/(1-z')$, with $\delta_I(1)\in(0,1)$. Hence, the pair (δ', δ'') is an equilibrium. Necessity is evident.

In the above result notice that as $z<1/2$ it is $z/(1-z)<1$ so that for large enough *t* strategy $\delta_{II}(t)$ approximates *q*, the purely mixed strategy equilibrium of the stage game G_b. Instead, since $z'>1/2$ it is $z'/(1-z')>1$ so that as *t* gets large $\delta_I(t)$ would diverge from *q* unless it was equal to it for all $t=1,2,\ldots$ To conclude, not counter intuitively, when an equilibrium with purely mixed strategies at all non zero types obtains, as *t* gets large it tends to coincide with the purely mixed strategy equilibrium of G_b.

2.3 Ex-Ante Efficiency

As under the conditions of Proposition 2 the GEMG has multiple equilibria it is interesting to briefly investigate their ex-ante efficiency. We shall do so, on the one hand, by comparing players' expected payoffs at (δ^*,δ^*) with that of alternative equilibria and, on the other hand, by discussing possible rankings of equilibria based on their efficiency.

As far as the first point is concerned it appears worth dwelling shortly on the matter since, in principle, is not obvious that (δ^*,δ^*) would be ex-ante Pareto inferior to equilibria with coordination on B at some non zero type. For example, to be more explicit, $(\delta^{**},\delta^{**})$ induces efficient coordination at all states except state $(1,0)$ where, in G_b, player II chooses A and I chooses B getting the lowest stage game payoff of $-L$. Coordination failure at that state could, in a sense, be interpreted as a *price* paid by I to enhance coordination when the number of exchanged messages is higher. As player I would obtain $-L$ with probability $p\varepsilon_I$, intuitively she may be willing to bear the risk of losses at state $(1,0)$ only if ε_I is sufficiently low. If $(\delta^{**},\delta^{**})$ could at all be Pareto superior to (δ^*,δ^*) this would likely happen under a similar condition. Below we show that this is indeed true and that, moreover, the upper bound q for ε_I suffices.

Proposition 5 *If* $(\delta^{**},\delta^{**})$ *and* $(\delta^{***},\delta^{**})$ *are NE of the GEMG then* (δ^*,δ^*) *is not ex-ante Pareto optimal.*

Proof Immediate. Let $E\Pi_i(\delta_I, \delta_{II})$ be player's i expected payoff when players choose, respectively, δ_I and δ_{II}. Then,

$$E\Pi_I(\delta^{**}, \delta^{**}) = (1-p)M - p\varepsilon_I L + p(1-\varepsilon_I)\varepsilon_{II}M + p(1-\varepsilon_I)(1-\varepsilon_{II})\varepsilon_I M + p(1-\varepsilon_I)^2(1-\varepsilon_{II})\varepsilon_{II}M + p(1-\varepsilon_I)^2(1-\varepsilon_{II})^2\varepsilon_I M + .$$

$$= M - p\varepsilon_I(L+M)$$

while

$$E\Pi_I(\delta^*, \delta^*) = (1-p)M$$

It is easy to see that $E\Pi_I(\delta^{**}, \delta^{**}) > E\Pi_I(\delta^*, \delta^*)$ if and only if $\varepsilon_I < M/(L+M)$. By the same reasoning it is simple to check that $E\Pi_{II}(\delta^{**}, \delta^{**}) > E\Pi_{II}(\delta^*, \delta^*)$ for all $0<\varepsilon_I,\varepsilon_{II}<1$; as $M/(M+L)>q$ the result follows. Similar considerations hold when comparing $(\delta^{***}, \delta^{**})$ with (δ^*, δ^*).

The above proposition simply says that reaching an ex-ante (self-sustaining) agreement on both of them playing *B*, even when a minimal number of messages is exchanged (according to the definition of δ** and δ***), is always beneficial to the players. Clearly, (δ**,δ**) would be preferable for *II* and (δ***,δ**) for *I*. It is simple to see that equilibria, induced by the conditions of Proposition 2 and in which agents wait for a higher number of messages before coordinating on *B*, are clearly less preferred to either (δ**,δ**) or (δ***,δ**), by at least one player. Hence, in so far as efficiency is concerned, there appear to be a natural ranking suggesting players to coordinate on a minimal number on messages sent, depending upon the relative machines reliability. This is what observation on how people appear to interact in reality seems to suggest; based on this the consideration could be forwarded that efficiency may be an explicit or implicit criterion used by agents in order to select among mulyiple equilibria. To the extent that failure message probabilities allow so, players' coordination on *B* achieved by economising on the number of messages exchanged is simply more *ex-ante* efficient.

2.4 Type-Dependent Technology

Let us now consider the very general case where ε_i:T-$\{0\}\rightarrow[0,1]$, so that $\varepsilon_i(t)$ is computer C_i message error probability at its *tth* message. The uniqueness result can now be generalised, in full analogy with Proposition 3, as follows.

Proposition 6 *The pair (δ*,δ*) is the unique NE of the GEMG if and only if* $\min_{t\geq1} z(t)>q$ *and* $\min_{t\geq1} z'(t)>q$ *where* $z(t)=\varepsilon_I(t)/[\varepsilon_I(t)+\varepsilon_{II}(t)(1-\varepsilon_I(t))]$ *and* $z'(t)=\varepsilon_{II}(t)/[\varepsilon_{II}(t)+\varepsilon_I(t+1)(1-\varepsilon_{II}(t))]$.

This general framework is now capable to encompass Rubinstein's result on the possibility of coordination on B when a commonly known maximum number of messages is imposed. In particular if $N \geq 1$ is the sum of messages sent by both machines, then $N=t_I+t_{II}$; letting $t_I=t$ it is either $N=2t-1$, if N is odd, or $N=2t$ when N is even, with $t \geq 1$.

Corollary 3 (Rubinstein 1989) *i) (N odd) Let* $\varepsilon_I(t)=\varepsilon$, *with* $0<\varepsilon<1$, *for* $t<(N+1)/2$, $\varepsilon_I((N+1)/2)=1$ *and* $\varepsilon_I(t)\in[0,1]$ *for* $t>(N+1)/2$; *moreover let* $\varepsilon_{II}(t)=\varepsilon$ *for* $t<(N+1)/2$ *and* $\varepsilon_{II}(t)\in[0,1]$ *for* $t \geq (N+1)/2$. *Then if* $\varepsilon<q$ *the pair of strategies* $(\delta_{(N+1)/2},\delta_{(N-1)/2})$, *where* $\delta_k=\Sigma_{t<k}I(t_i=t)+\Sigma_{t>k}\delta(t)I(t_i=t)$, *for* $t=0,1,2,..$; $k=1,2..$*and* $\delta_0=I(t_i=0)+\Sigma_{t>0}\delta(t)I(t_i=t)$, *with* $\delta(t)\in[0,1]$, *is a NE of the GEMG. ii) (N even) Let* $\varepsilon_I(t)=\varepsilon$ *for* $t \leq N/2$, *and* $\varepsilon_I(t)\in[0,1]$ *for* $t>N/2$; *moreover let* $\varepsilon_{II}(t)=\varepsilon$ *for* $t<N/2$, $\varepsilon_{II}(N/2)=1$ *and* $\varepsilon_{II}(t)\in[0,1]$ *for* $t>N/2$. *Then if* $\varepsilon<q$, *the pair of strategies* $(\delta_{N/2},\delta_{N/2})$ *is a NE of the GEMG.*

The general principle underlying the above result is as before. For example, consider $N>1$ odd; the condition $\varepsilon<q$ is in fact $z'((N-1)/2)<q$ as $z'((N-1)/2)=p\varepsilon(1-\varepsilon)^{N-2}/[\, p\varepsilon(1-\varepsilon)^{N-2}+ p(1-\varepsilon)^{N-1}]= \varepsilon$. The same reasoning would hold for N even; analogously, in that case, would be $z(N/2)=\varepsilon$. Intuitively, failure with probability one of a message entails, at a certain type, the possibility for either z or z' to be strictly lower than the purely mixed strategy NE of playing A, which would otherwise be precluded because of the equality of probabilities.

2.5 Almost Common Knowledge

The analysis conducted so far may suggest a possible alternative view of almost common knowledge (*ack*) with respect to the one proposed by Rubinstein. Indeed, as in the EMG common knowledge arises when $\varepsilon_I=\varepsilon=\varepsilon_{II}=0$, it could be argued that the relevant notion of *ack* should somehow refer to the values of ε_I and ε_{II} rather than to the number of messages delivered. Alternatively, in some sense, may privilege *quality* of the transmitted information rather than *quantity*, or else concern actions and not the length of beliefs hierarchy of the game. In order to articulate more on the issue consider the connection between information partitions and common knowledge in some relevant extreme instances.

i) $\varepsilon_I=0$ and $0<\varepsilon_{II}<1$. In this case possible states can only be of the kind $\{(t,t)\}$, with $t=0,1,2..$; hence, players' information partitions are the finest ones and so is their meet. Then event $\{b\}$, defined as the set of states reflecting nature choice of G_b, is clearly $\{b\}=\cup_{t=1,2...}\{(t,t)\}$ implying that according to Aumann's definition (Aumann, 1976), $\{b\}$ can be common knowledge also at $\{1,1\}$, namely when the lowest possible number of messages is exchanged. We take this as a possible indication that common knowledge, and relevant *ack*, may have more to do with the reliability of messages than with their number.

ii) Let instead $\varepsilon_{II}=0$ and $0<\varepsilon_I<1$. Possible states are now $\{(0,0)\}$ and of the kind $\{(t+1,t)\}$, with $t=0,1,2.$ Hence, $\{b\}=\cup_{t=0,1,2...}\{(t+1,t)\}$ and it can clearly be common knowledge even at state $\{(2,1)\}$.

It has been noticed (Binmore and Samuelson, 2000) that in ordinary, every day life, conversations individuals may frequently agree on joint course of actions using just two messages; typically, the first would communicate coordination terms while the second simply confirms that the first message was understood. On the basis of the above considerations this could be justified in more than one way. For example, it may not be common knowledge that the first message gets successfully through (even if it does), while success of confirmation message is common knowledge. Alternatively, it may be common knowledge that the first message has positive failure probability while it is common knowledge that confirmation goes through successfully. In both instances we would interpret coordinated actions undertaken by players as being common knowledge.

The following two could be taken as further arguments in support of the view that we proposed on *ack* .

(Threshold for ack) While the notion based upon messages would leave undetermined the number of exchanged messages needed to speak of *ack*, that founded on message error probabilities may instead identify a critical value below which *ack* of G_b could be triggered. More specifically, the alleged threshold could be represented by q as it provides, either to ε_I or ε_{II}, the (necessary) upper bound for efficient coordination to obtain.

(Payoff Continuity) It is simple to see that $E\Pi_{II}(\delta^{**}, \delta^{**}) = M - p\varepsilon_I(M-1)$ which, as well as $E\Pi_I(\delta^{**}, \delta^{**})$ is independent of ε_{II} and tends to M when ε_I tends to zero, the efficient equilibrium outcome with common knowledge of the game. In words, the payoffs associated to the equilibrium where, at all types, players choose as in the efficient equilibrium with common knowledge of the game, are continuous in ε_I as it tends to the value entailing common knowledge of G_b.

2.6 An Alternative Interpretation

Differences in message failure probabilities could be justified by introducing interpretative elements alternative to those in Rubinstein's model, in so far as how the informational exchange would take place between the two players. As well as in Binmore-Samuelson (2000), subjective aspects could be introduced in the framework together with objective elements (machine reliability). Consider the following simple example. Suppose it is common knowledge that *i)* $0<\varepsilon<1$ is both players' machine message failure probability and *ii)* machines do not reply automatically to messages received rather players decide, every time they receive a message, whether to do so or not. In particular, (say) player *I* replies to every message she receives (included that from *nature*) with probability one while *II* answers only with probability $0<\lambda<1$, independently of machines failure probabilities. Hence, while $\varepsilon_I=\varepsilon$ remains the probability that *II* does not receive a message sent by *I*, the probability that *I* will not receive a message from *II* is now $\varepsilon_{II}= (1-\lambda)+\lambda\varepsilon$ which, given the assumptions, is strictly greater than ε_I. In particular, if λ satisfies the inequality $\lambda<[q-\varepsilon]/[(1-\varepsilon)^2q]$, and moreover $\varepsilon<q$, then conditions of point (1) in Proposition 2 will be met entailing multiple equilibria. In other words, the symmetry assumption of machine reliability could be retained, for instance in absence of discriminatory elements, and yet multiplicity could arise as a result of sufficiently different personal attitudes towards answering messages.

3 Conclusions

We believe that a main conclusion delivered by the paper is that in the Electronic Mail Game efficient coordination could obtain when the technological parameters, governing the reliability of communication channels, and the payoff structure of the coordination games to be played are appropriately related. In particular, the purely mixed strategy Nash Equilibrium will play a decisive role in calibrating machines reliability and, in equilibrium, will represent the crucial dividing line between the possibility of efficient and inefficient coordination. Such results appear to suggest what we consider to be the other main (general) consideration, to our knowledge still to be investigated in more complex frameworks, that imperfect communication networks (not leading to common knowledge of the relevant games) set up with the aim of inducing proper coordination must have well defined and commonly known characteristics of reliability, explicitly linked to the payoff structure of the game(s). The intuition is that imperfect communication systems, generating uncertainty on the opponents' types, induce (meta) second order lotteries on the (first order mixed) strategies available to players. Then, at an equilibrium, is not surprising to find that for desirable coordination to obtain the reliability of communication channels should be above a certain minimum level connected to the payoffs of the relevant game *via* the purely mixed strategy equilibrium. Indeed, in the game support to personal beliefs formation is provided by message failure probabilities.

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[1] The more general case of $0 \leq \varepsilon_I(t), \varepsilon_{II}(t) \leq 1$, with type-dependent probabilities taking any value in the closed interval $[0,1]$ will be considered in paragraph 2.4.

[2] Notice that z too could be seen as chosen by players, when able to control the error probability messages of the communication protocol. However, also under this interpretation z would not be determined unilaterally by *II* but rather jointly by both agents.