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Regime Dynamics in a Model of Inflation and Unemployment Fluctuations

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Abstract

In this paper we present a formal definition of the notions of economic regime and regime dynamics. Starting from these definitions, we discuss a multiple regime dynamic model generating an endogenous unemployment-price adjustment mechanism. Two different employment regimes are introduced and the regime dynamics properties of the model are analyzed. Specifically, we assume that the equations governing employment and prices dynamics undergo a discontinuous change in regime when a critical value of unemployment rate is reached. Depending on parameter values, we show that this model is capable of producing a rich variety of dynamic behavior, including complex irregular fluctuations. The main result of this paper is the representation of the regime dynamics via symbolic dynamics. In particular, we show that the regime dynamics of the model can be represented by a shift of finite type that depends on parameter values. In some particular cases, we can also have a representation via directed vertex graphs. An important consequence of this is the possibility of measuring the complexity of the model by using the topological entropy measure.

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1. Introduction

Very often, economic dynamic models are held to be represented by one system of differential or difference equations only, and not by two or more such systems. Sometimes this is a useful simplification, but in most instances, economic behavior is not necessarily governed by the same dynamics laws whatever values the state variables reach. The economic conditions determine the dynamic behavior and then qualitative changes in these conditions generate different relationships governing behavior. More and more this is recognized in the literature. If we agree in defining an economic regime as a set of rules and institutions that represents the whole economy and generates its qualitative dynamical behavior, changes in regime are particularly associated with changes in the economic dynamics generated by changes in the rules and/or institutions.

This paper is organized as follows. In section 2, I present a formal definition of the notions of regime and regime dynamics. In section 3, I discuss a dynamic model generating an endogenous unemployment price adjustment mechanism produced by Nickell (1987, 1988, 1990), Layard and Nickell (1986), and Layard, Nickell and Jackman (1991)[1] and extended in Day, Ferri and Greenberg (1993). Then, two different employment regimes are introduced (section 4) and the regime dynamics properties of the model are analyzed (section 5). Specifically, I assume that the equations governing employment and prices undergo a discontinuous change, or switch in regime, when a critical value of unemployment rate is reached. Depending on parameter values, I show that this model is capable of producing a rich variety of dynamic behavior, including complex irregular fluctuations. The main result of this paper is the representation of the regime dynamics via symbolic dynamics. In particular, the model is represented by different shifts of finite type that depends on parameter values. In some particular cases, we can also have a representation via directed vertex graphs. An important consequence of this is the possibility of measuring the complexity of the model by using the entropy measure.

2. Regimes and regime dynamics

Intuitively speaking, a dynamic regime is a qualitative behavior that can be clearly distinguished from other behaviors called, likewise, regimes. If we agree in representing different economic regimes with different models then an economy with multiple regimes must be represented with some kind of hyper-model. Thus, the equations representing this hyper-model must change when the economy change regime. We can formalize this fact with the following definition.

Definition: Given a dynamical system (D, f) and a partition $\{D_1, D_2, \ldots, D_n\}$of the domain D, a *regime* is a pair (D_i, f_i) $(i = 1, 2, \ldots, n)$ where $f_i = f\,|D_i$ is the restriction of the function f to the set D_i.

This definition reflects the fact that different regimes are described by different local models. Of course, when $n = 1$, we are in the standard one regime situation. To be interesting, a partition should slice the state space into (at least) two or more nonempty sets D_i.[2] The presence of multiple regimes provides alternatives: a variety of regimes is available to construct one's own history. In this case, it can be quite rich, for we have a twofold dynamics, one within a given regime and one across regimes. Their mixing can produce any kind of dynamic behavior. Regime changing corresponds to a form of structural change because is the model of the economy that is changing. So, in this work we will focus upon this dynamics across regimes (we will call it regime dynamics) to study structural changes in the economy.

[1] In Nickell's model, dynamics is shock dependent.

[2] Observe that $\phi_i\,(D_i)$ is not necessarily a subset of D_i. Thus, paths can traverse from one regime to another.

Regime dynamics is defined on the finite set of regimes and to represent it we make the next move: we label each regime by a symbol of an adequate alphabet and we describe the evolution of the economy in terms of regime changing with a symbolic sequence. This representation is called *coded dynamics* and is related (and partly overlaps) with the mathematical branch called *symbolic dynamics*. Such proximity often permits the use of well-established symbolic dynamics techniques, as we will illustrate in section 5. The use of these techniques makes the fundamental difference in our approach from the conventional approach in state space where state variables are vectors of real numbers.

In order to illustrate these concepts, I will analyze a simple macroeconomic model produced by Nickell (1987, 1988, 1990), Layard and Nickell (1986), and Layard, Nickell and Jackman (1991) and extended by Day, Ferri and Greenberg (1993) with the introduction of two different employment regimes. Depending on parameter values, this model can produce a rich variety of dynamic behavior, including irregular fluctuations. The coded-symbolic dynamics methods are used to represent in a simple and effective form the regime dynamics of the model.

Next section begins by summarizing the Nickell model and its properties. Then I describe the kind of (discontinuous) regime switching mechanism introduced in Day, Ferri and Greenberg (1993), I analyze the unemployment and price dynamics and their relationships with regime switching and business cycles, and derive the regime dynamics properties of the model.

3. The model

In this section I discuss a dynamic model generating an endogenous unemployment price adjustment mechanism. The understanding of this mechanism is one of the paradigms in economic theory and this is reflected by, for example, the important role of Phillips curve in macroeconomics. The model is described by the aggregate demand function

$$Y = e^{\alpha_0}\left(\frac{M}{P}\right)^{\alpha_1} G^{\alpha_2}\,, \tag{1}$$

where Y is aggregate output, M the supply of money, P the general price level, G autonomous expenditure, and by the production function

$$Y = BL^{\beta}\,, \tag{2}$$

where L is aggregate labor utilization. Pricing behavior in the goods and labor markets is represented by two equations determining the target price of output and desired real wage. The target price of output is given by

$$P = e^{\pi_0} W.\left(\frac{L}{Y}\right) D\,, \tag{3}$$

where e^{π_0} is the markup under conditions of full capacity and stable prices. The adjustment D for anticipated price changes and under-utilized capacity is given by

$$D = \left(\frac{P^e}{P}\right)^{\pi_1}\left(\frac{Y}{\hat{Y}}\right)^{b} = \left(\frac{P^e}{P}\right)^{\pi_1}\left(\frac{L}{\hat{L}}\right)^{\pi_2}, \tag{4}$$

where P^e is anticipated price, $\hat{Y}$ is potential output, $\hat{L}$ is the size of the labor force in terms of a standard work week and $\pi_2 = \beta.b$. The target real wage is

$$\frac{W}{P} = e^{\gamma_0} \frac{Y}{L}.H \ , \tag{5}$$

where Y/L is the average output of labor, e^{γ_0} is the target share given full employment and constant prices with $\gamma_0 < 0$. *H* is an adjustment term for the outcome of bargaining when prices are expected to change and unemployment exists given by

$$H = \left(\frac{P^e}{P}\right)^{\gamma_1} \left(\frac{L}{\hat{L}}\right)^{\gamma_2} . \tag{6}$$

In a full employment competitive equilibrium in which workers receive the marginal product, we must have: $e^{\gamma_0} = \beta$ and $e^{\pi_0} = 1/\beta$. Out of equilibrium the outcome depends on the relative strengths of management and labor of employment, which in turns depend on the influence of price expectations and the level of employment and capacity utilization as represented by the adjustment factors *D* and *H*. Then we can interpret π_1 and γ_1 as reflecting nominal inertia and π_2 and γ_2 as measuring real rigidity.

Taking natural logarithms in (1), (2), (3) and (5) (after the substitution of *D* and *H* in (3) and (5) respectively), using the fact that $\hat{Y} = B(\hat{L})^{\beta}$, and denoting by *p = Log(P), w = Log(W), l = Log(L), y = Log(Y), m = Log(M)* and *g = Log(G)*, we obtain the price equation

$$p = \pi_0 + w + l - y + \pi_1(p^e - p) + \pi_2(l - \hat{l}) , \tag{7}$$

the wage equation

$$w - p = \gamma_0 + y - l + \gamma_1(p^e - p) + \gamma_2(l - \hat{l}) , \tag{8}$$

the demand equation

$$y = \alpha_0 + \alpha_1(m - p) + \alpha_2 g \ , \tag{9}$$

and

$$\hat{y} - y = \beta(\hat{l} - l) = \beta.u \ , \tag{10}$$

Equations (7)-(10) form a complete static model that determines temporary equilibrium output, prices, real wage, and unemployment *u* for given values of *m*, *g* and p^e.

At a self-fulfilling price equilibrium at which $p^e = \tilde{p}$, we get the stationary states

$$\tilde{l} = \hat{l} - \frac{\pi_0 + \gamma_0}{\pi_2 + \gamma_2} \ , \ \tilde{u} = \hat{l} - \tilde{l} = \frac{\pi_0 + \gamma_0}{\pi_2 + \gamma_2} , \tag{11}$$

$$\tilde{y} = \hat{y} - \beta.\tilde{u} = \hat{y} - \beta.\frac{\pi_0 + \gamma_0}{\pi_2 + \gamma_2} , \tag{12}$$

$$\tilde{p} = m + \frac{\alpha_0 + \alpha_2 g - \tilde{y}}{\alpha_1}, \qquad (13)$$

$$\tilde{w} = \tilde{p} + \gamma_0 - \gamma_2 . \tilde{u} + (\beta - 1)\tilde{l} \;. \qquad (14)$$

Observe that with the hypothesis of $p^e = \tilde{p}$, the parameters π_1 and γ_1 reflecting nominal inertia do not appear at the equilibrium levels, but the real inertia ones π_2 and γ_2 do. We can particularly note that the equilibrium of unemployment $\tilde{u}$ depends on the importance of real inertia and on the extent to which the markup coefficient π_0 and the target wage share coefficient γ_0 depart from the competitive equilibrium values (of –Log β and Log β respectively).

4. Dynamics of the model with naive expectations

The dynamics of the model depends on the expectation formation. Let p_t^e be the price expected at the beginning of period *t* and suppose that

$$p_{t+1}^e = p_t \,, \qquad (15)$$

that is, the price estimate for the next period is simply the current price. This is called *naive expectations*.

Substituting (15) into equations (7) and (8) we obtain a standard Phillips curve representing the tradeoff between price increases and the rate of unemployment, which depends on parameters measuring both nominal and real rigidities.

$$\Delta p = p_{t+1} - p_t = -\frac{\pi_2 + \gamma_2}{\pi_1 + \gamma_1} u_{t+1} + \frac{\pi_0 + \gamma_0}{\pi_1 + \gamma_1} = -\frac{\pi_2 + \gamma_2}{\pi_1 + \gamma_1}\left(u_{t+1} - \tilde{u}\right). \qquad (16)$$

Using equations (7)-(10) and expectations hypothesis about prices, we obtain the first order difference equation representing the dynamic behavior of unemployment:

$$u_{t+1} = \left(\frac{\beta(\pi_1 + \gamma_1)}{\beta(\pi_1 + \gamma_1) + \alpha_1(\pi_2 + \gamma_2)} \right) u_t + \frac{\alpha_1(\pi_0 + \gamma_0)}{\beta(\pi_1 + \gamma_1) + \alpha_1(\pi_2 + \gamma_2)} = \lambda u_t + (1 - \lambda)\tilde{u} \,, \qquad (17)$$

where

$$\lambda = \frac{\beta(\pi_1 + \gamma_1)}{\beta(\pi_1 + \gamma_1) + \alpha_1(\pi_2 + \gamma_2)} \,. \qquad (18)$$

The dynamics of the model is very simple. If $0 < \lambda < 1$, (or equivalently if $\beta(\pi_1 + \gamma_1)$ and $\alpha_1(\pi_2 + \gamma_2)$ are positive) then the steady states $\tilde{u}$ and $\tilde{p}$ are asymptotically stable and there are no business cycles. If $\lambda \geq 1$ (or equivalently if $\alpha_1(\pi_2 + \gamma_2)$ is negative)[3] the adjustment

[3] Note that π_2, which measures the marginal influence of unemployment on price markup, could be negative. This is suggested if economies to scale occurred at relatively low output levels, but at higher levels diseconomies prevailed. If the change over is rather abrupt, then π_2 could also change rather abruptly as some output or unemployment level was reached.

process is unstable. In this case if the initial unemployment $u_0 > \tilde{u}$, unemployment and price level increase monotonically in a continuing inflationary depression. If $u_0 < \tilde{u}$, then unemployment and price level decrease monotonically in a deflationary boom.

To allow the possibility of generating irregular fluctuations, we can proceed in the way of empirical economists adding random shocks to the model but I will show that irregular fluctuations can also be brought only by the intrinsic forces represented in the model if we introduce the possibility of different unstable regimes of unemployment. Each regime is characterized by a particular adjustment mechanism (or model) and regime switches allows for complex dynamics. Regimes represent different dynamical behaviors and the transition from one regime may be rather abrupt. For example, at low inflation rates the expectations about future prices changes may not play a great role but when inflation becomes grater than certain levels, a qualitative change may occur in the economic mechanisms that allows people to protect themselves against the effects of future price changes. Following Day, Ferri and Greenberg (1993), I assume that when a threshold is passed, the reaction of either or both price and wage-settings strategies change, and that this threshold is represented by some value of the unemployment rate, u^s. This threshold divides Regime 1 of low employment from Regime 2 of high employment. This critical level u^s is not an equilibrium value of unemployment; it is just a level separating an economy with high unemployment rates from one with low rates. Let denote by $\pi_0^1, \gamma_0^1, \pi_1^1, \gamma_1^1, \pi_2^1$ and γ_2^1 the prevailing values of the parameters for unemployment rates below u^s and the corresponding model by

$$u_{t+1} = \lambda_1 u_t + (1-\lambda_1)\tilde{u}_1, \tag{19}$$

where

$$\lambda_1 = \frac{\beta(\pi_1^1 + \gamma_1^1)}{\beta(\pi_1^1 + \gamma_1^1) + \alpha_1(\pi_2^1 + \gamma_2^1)} \tag{20}$$

and

$$\tilde{u}_1 = \frac{\pi_0^1 + \gamma_0^1}{\pi_2^1 + \gamma_2^1}. \tag{21}$$

For unemployment rates above u^s, I denote the prevailing values of the parameters by $\pi_0^2, \gamma_0^2, \pi_1^2, \gamma_1^2, \pi_2^2$ and γ_2^2 and the corresponding model by

$$u_{t+1} = \lambda_2 u_t + (1-\lambda_2)\tilde{u}_2, \tag{22}$$

where

$$\lambda_2 = \frac{\beta(\pi_1^2 + \gamma_1^2)}{\beta(\pi_1^2 + \gamma_1^2) + \alpha_1(\pi_2^2 + \gamma_2^2)} \tag{23}$$

and

$$\tilde{u}_2 = \frac{\pi_0^2 + \gamma_0^2}{\pi_2^2 + \gamma_2^2}\,. \tag{24}$$

Then different set of equations for different specific situations govern the behavior of the economic system and though the separate elements of this model are linearly structured, the overall system is nonlinear. Then, this model is a clear example of the intuition of regime that we have in mind: regime coincides with local model and particularly in this case local models are linear.

If we define the function f by

$$f(u) = \begin{cases} \lambda_1 u + (1-\lambda_1)\tilde{u}_1 \;, \text{ if } u \le u^s \\ \lambda_2 u + (1-\lambda_2)\tilde{u}_2 \;, \text{ if } u > u^s \end{cases}, \tag{25}$$

the model is represented by the first-order difference equation

$$u_{t+1} = f(u_t)\,. \tag{26}$$

The map f is piecewise linear with a discontinuity at u^s and dynamics depends upon the values of the parameters λ_1, λ_2, $\tilde{u}_1$ and $\tilde{u}_2$.

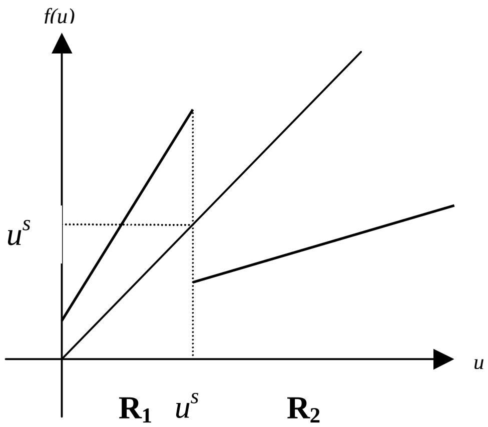


Figure 1: Graph of the map *f*

The intervals $R_1 = [0,u^s]$ and $R_2 = [u^s,+\infty]$ are respectively regimes 1 and 2 and f is linear in each regime. At u^s there is a discontinuity of f that allows for regime switching and as usual, regime dynamics is described by symbolic sequences in R_1 and R_2.

Observe that being linear in each regime, the map f could have two, one or zero fixed points. There is a great variety of possible dynamical scenarios for the model depending upon the angular coefficients of f at each regime and the quantity of fixed points but I am not going to analyze all these possibilities. I think that it is enough to show that we can generate a very complex regime dynamics. This will be the scope of the next section

5. Regime dynamics of the model

Let $z_1 = f(u^s) = \lambda_1 u^s + (1-\lambda_1)\tilde{u}_1$ and $z_2 = \lambda_2 u^s + (1-\lambda_2)\tilde{u}_2$. The difference between z_1 and z_2 measures the intensity of the jump from one regime to the other. It's easy to show that for a continuous of parameter values the interval $[z_2, z_1]$ is s trapping set: all trajectories must enter this interval and remain there. This is the first case I will analyze. In the second case I will show that there are parameter values where the introduction of regimes in the model do not affect the qualitative dynamics. Obviously, this is not an interesting case because it is no possible to change regime.

1. If $z_1 > u^s > z_2 \geq \tilde{u}_1$, $\tilde{u}_2 \notin (z_2, z_1)$ (i.e., $\tilde{u}_2 \geq z_1$ or $\tilde{u}_2 \leq z_2$) the interval $[z_2, z_1]$ is invariant by f and all trajectories with initial condition between $\tilde{u}_1$ and ũ2 become trapped in this interval. If, in addition, $\lambda_1 \leq 1$ and $\lambda_1 \leq 1$ there are not steady states and all trajectories become trapped in the interval $[z_2, z_1]$. In these cases the phase diagram of the dynamical system has the appearance shown in figure 2.

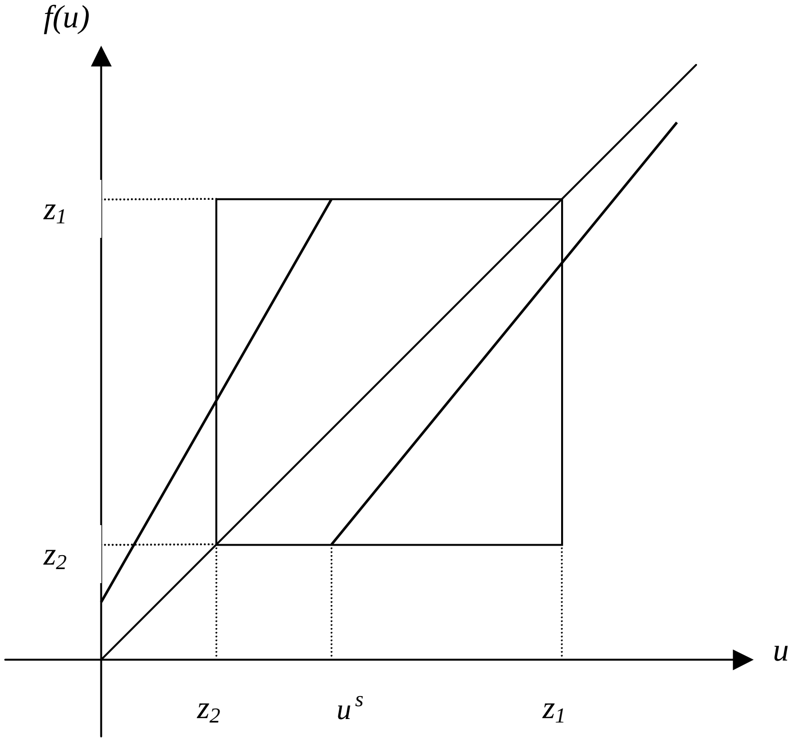


Figure 2: Graph of the map f; $z_1 > u^s > z_2 \geq \tilde{u}_1$, $\tilde{u}_2 \notin (z_2, z_1)$.

In these cases the interval $[z_2, z_1]$ is invariant by f and all trajectories with initial condition between $\tilde{u}_1$ and $\tilde{u}_2$ become trapped in this interval. If, in addition, $\lambda_1 \leq 1$ and $\lambda_1 \leq 1$ there are not steady states and all trajectories become trapped in the interval $[z_2, z_1]$.

Thus, for the selected parameter values all the interesting dynamics happens in the invariant interval $[z_2, z_1]$. Starting outside, all trajectories after finite iterations go toward $[z_2, z_1]$ and remain trapped there or diverge to infinity. Thus, I concentrate my attention on the dynamics in the invariant interval $[z_2, z_1]$. Observe that in this interval there are no fixed points of f (if we exclude the extreme cases where $\tilde{u}_2 = z_1$ or $\tilde{u}_1 = z_2$) and a point which begins in one regime after finite iterations must enter the other regime. Thus, fluctuations are the generic behavior. Figure 3 shows the graphs of map f restricted to the selected interval for different values of the parameters. In particular, graphs (a)-(d) illustrate some extreme cases are depicted where the regime dynamics can be well represented by a shift of finite type (and then by a directed graph and the associate matrix).[4] Figure 4 illustrates the transition graphs and transition matrices representing these particular cases.

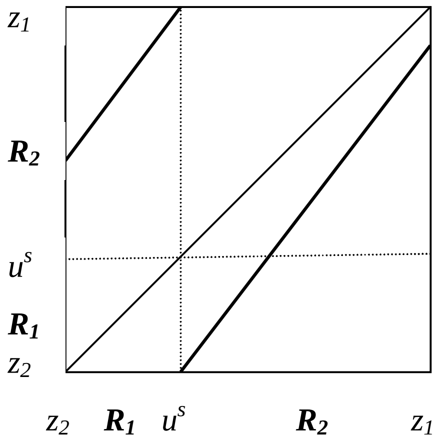
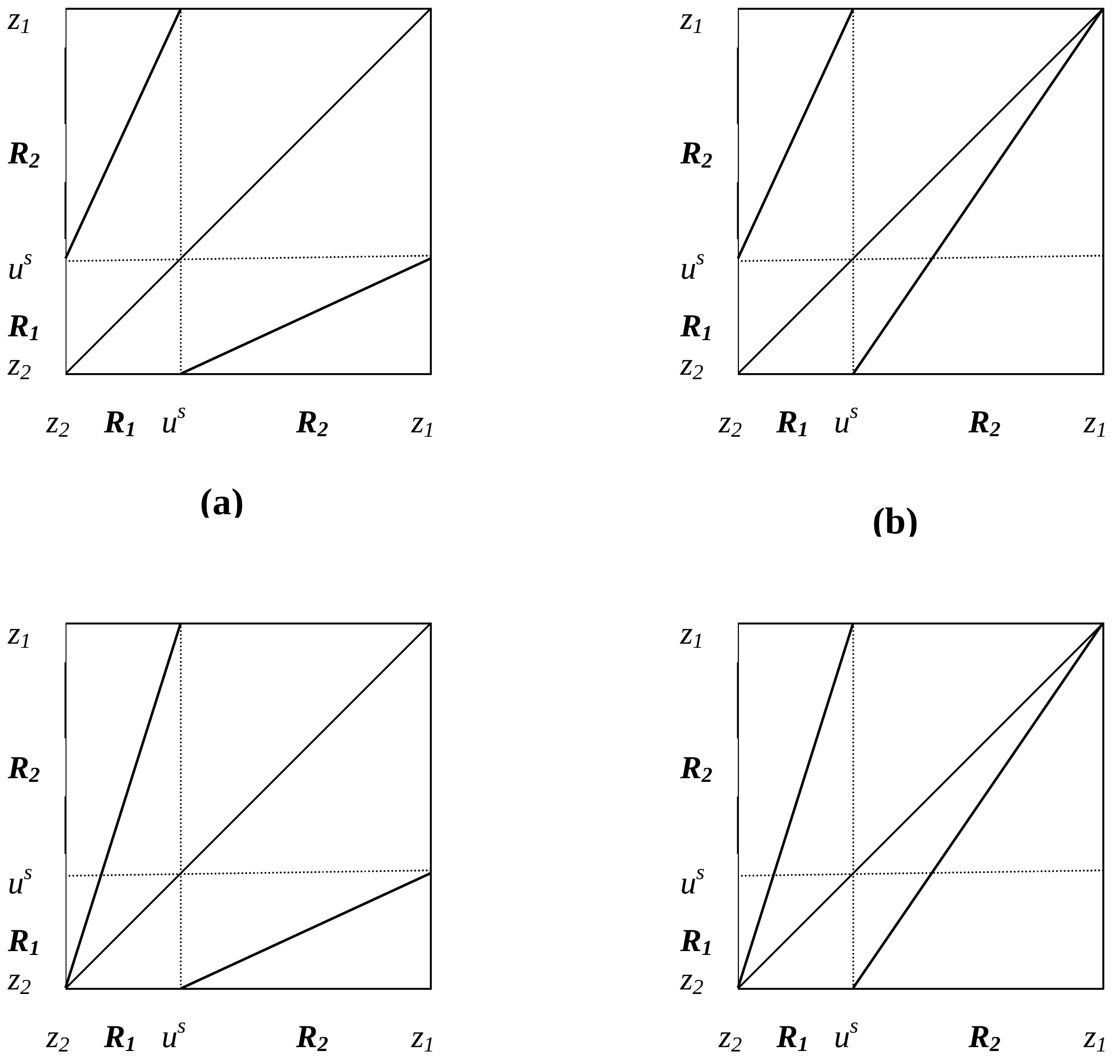
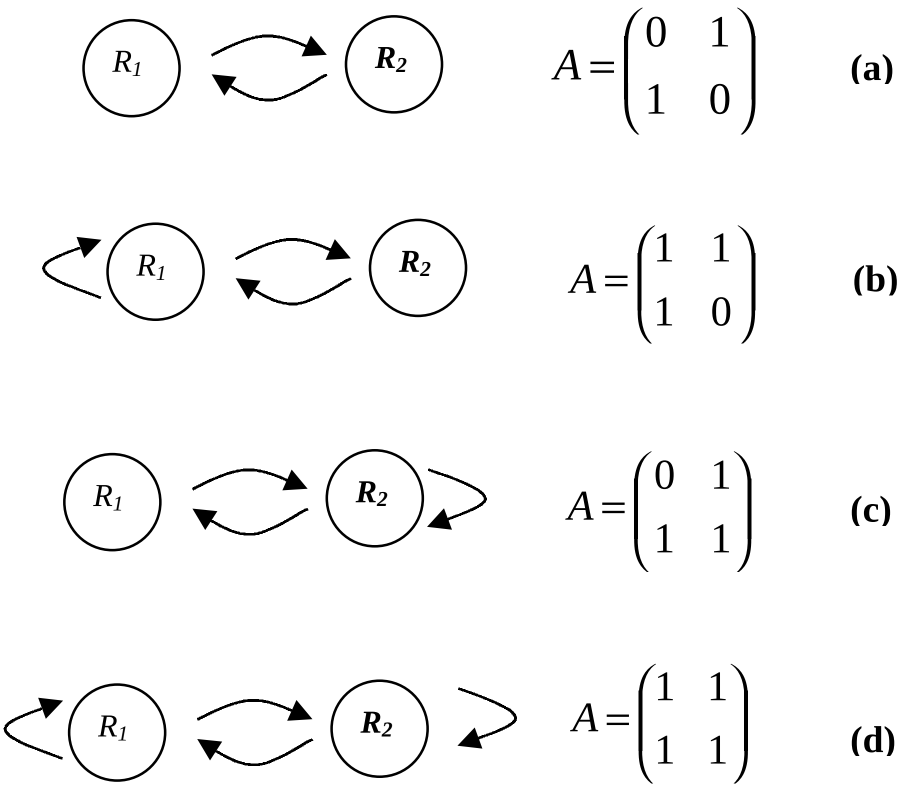


Figure 3: Graphs of the restriction of *f* to the invariant interval [z_2 , z_1] for different values of the parameters

All the interesting dynamics happens in this interval. In particular, cases (a)-(d) show the possibility of regime dynamics represented by a shift of finite type.



[4] See Brida (1999) for the proofs and more details on the symbolic representation.



$$A=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} \quad \textbf{(a)}$$

$$A=\begin{pmatrix}1 & 1\\ 1 & 0\end{pmatrix} \quad \textbf{(b)}$$

$$A=\begin{pmatrix}0 & 1\\ 1 & 1\end{pmatrix} \quad \textbf{(c)}$$

$$A=\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix} \quad \textbf{(d)}$$

Figure 4: Transition graph and adjacency matrix for the partition P = {R_1, R_2} and the map *f* restricted to the invariant interval. Cases (a)-(d) of figure 3.

A zero in the ij entry means that there is no arrow from i to j and a one means that there is an arrow from i to j, where i,j = R_1, R_2.

Case (a) is very simple: if the adjustment process starts at the high employment regime 2, unemployment decreases until the trajectory enters the interval (u^s, z_2]; at this point the next iterate cross the switching point u^s and low employment regime 1 is entered. Here begins to alternate regime. Then, starting at the invariant interval, the unique possibility for a trajectory is to alternate regime and then the only possible sequences of regimes are

$$R_1 R_2 R_1 R_2 \ldots R_1 R_2 \ldots = (R_1 R_2)^\infty \text{ and } R_2 R_1 R_2 R_1 \ldots R_2 R_1 \ldots = (R_2 R_1)^\infty$$

All punctual trajectories in this simple case are cycles of period 2, but for closer parameter values we can construct cyclical punctual trajectories of period k, with $k \geq 2$. These cases illustrate the possibility of stable cycles of unemployment and prices involving alternating periods of boom and inflation with periods of recession and deflation.

Cases (b)-(d) illustrate the possibility of very complex dynamics in the model. Here regime dynamics can be viewed like a Markov chain in two states and unstable nonperiodic fluctuations occur for almost all initial unemployment levels. It is clear that the complexity of each case is not the same; for example cases (b) and (c) have lower entropy than case (d).
(d) is surely the most complex case: regime dynamics is represented by the full shift in two symbols. In this case every sequence of regimes is possible and the unemployment process became unpredictable if we don't know the initial condition with infinite accurately.

In (b) the stage of every trajectory in regime R_2 is for exactly one period (of course, it can stay in regime R_2 for more than one period before being trapped in the invariant interval $[z_2,z_1]$). Then the possible symbolic sequences representing the regime dynamics can be constructed using the directed graph of Figure 4 (b): every R_2 must be followed by R_1, but R_1 can be followed by either symbols.
(c) is the symmetric case of (b) if we interchange the roles of regime R_1 and R_2.

I have described the dynamic behavior of these four extreme cases but; what can we say about regime dynamics in a generic case for the parameter values in question?
We can note that:
i) there are no stationary states, except when $\tilde{u}_2 = z_1$ or $\tilde{u}_1 = z_2$ where being unstable fixed points, they do not affect the dynamic behavior
ii) if a trajectory starts at the high employment regime 2, unemployment decreases governed by the second regime equation until the switching point is crossed and low employment regime 1 is entered.
iii) if a trajectory starts at the low employment regime 1, unemployment increases governed by the first regime equation until the switching point is crossed and high employment regime 2 is entered.

Then we have that generically regimes 1 and 2 are reversible and the adjustment process changes cyclically (in a periodic or non-periodic way) from one regime to the other. The duration within each regime depends on the parameter values and fluctuations could be periodic, quasi-periodic or irregular. In particular, it must be noted that the duration of the stage of a trajectory within regime R_i is non greater than the duration of the stage of the trajectory with initial condition z_j within regime R_i ($i, j = 1,2;\ i \neq j$). In a generic case, regime dynamics cannot be represented by a two vertex directed graph[5] like we did for the extreme cases, but we can state that the possibilities for regime switching are from the simplest case (a) where every regime sequence is periodic with switches between regime 1 and 2 to the more complex case (d) represented by the full shift where every regime sequence is possible including cycles of any order and non-periodic sequences. In spite of this, it is important to remark that in a generic case, regime dynamics can be represented in a symbolic way with a shift of finite type. In order to illustrate this point, let suppose (without loose of generality) that z_1 leaves regime R_2 at the *nth* iterate and z_2 leaves regime R_1 at the *mth* iterate; i.e.,

$$z_1, f(z_1), f^{\,2}(z_1), \ldots, f^{n-1}(z_1) \in R_2;\ f^{n}(z_1) \notin R_2 \text{ and } z_2, f(z_2), f^{\,2}(z_2), \ldots, f^{m-1}(z_2) \in R_1;\ f^{m}(z_1) \notin R_1.$$

This means that we are in a generic case between cases (a) and (d). Then, it is clear that

$$p_1 = (R_1)^{m+1} = R_1\, R_1 \ldots R_1 \ (m+1 \text{ times}) \text{ and } p_2 = (R_2)^{n+1} = R_2\, R_2 \ldots R_2 \ (n+1 \text{ times})$$

are forbidden "paths" for the symbolic dynamics representing our two regime dynamics. If we denote by $\mathfrak{F} = \{\, p_1, p_2\}$; then the *shift space (of finite type) of forbidden words* $\mathfrak{F}$, denoted by $\mathfrak{X}_{\mathfrak{F}}$, is the symbolic representation of all possible sequences of regime in our dynamic process. It should be noted that we cannot write down all these sequences because there is an uncountable infinite number of them. In spite of this, it is very simple to verify if certain sequence in the symbols R_1 and R_2 is a possible path representing the regimes traversed by a trajectory of our model: we have to check that this sequence contains no sub-sequences p_1 and p_2.

[5] See Brida (1999), Adler (1998) and Alligood, Sauer and Yorke (1997) for a discussion in graph representations of shifts of finite type.

At this point, we are able to discuss about the complexity of the model. The natural way to measure it is entropy. We know that with a two regimes model, we can get a maximal entropy level $h = log_2 2 = 1$ and a minimal entropy $h = 0$.

For the particular cases (a) – (d), being regime dynamics represented by a direct vertex graph and its transition matrix, it is simple to calculate the entropy. Here we have that $h = log_2 \lambda_M$, where λ_M is the largest positive eigenvalue (called the Perron-Frobenius eigenvalue) of the transition matrix M. Then, it is clear that in case (a) is $h = 0$, in case (d) is $h = 1$, and in cases (b) and (d) is $h = \log_2((1+\sqrt{5})/2)$. For other parameter values, the computation is not so easy. If m and n are the natural numbers such that z_1 leaves regime R_2 at the *nth* iterate and z_2 leaves regime R_1 at the *mth* iterate, then $h = h(n,m)$ increases monotonically with n and m from 0 to 1.

Finally, we will not describe the price dynamics of the model but, according to (16), it can be noted that fluctuations in u_t result in price fluctuations.

2. Suppose that $z_1, z_2 > u^s$ like the map sketched in graph (i) of figure 5. Here, a trajectory starting in regime 1 (and with initial condition grater than $\tilde{u}_1$) escapes into regime 2 and become trapped in this regime. But regime 2 is closed in the sense every trajectory starting there will not escape into regime 1. We cannot see paths taking from regime 2 to the other regime. Thus, regime sequences can only be of the form

$$R_1 R_1 \ldots R_1 R_2 \ldots R_2 R_2 \ldots = R_1 R_1 \ldots R_1 (R_2)^\infty ,$$

$$R_2 R_2 R_2 \ldots R_2 R_2 \ldots = (R_2)^\infty \text{ or}$$

$$R_1 R_1 R_1 \ldots R_1 R_1 \ldots = (R_1)^\infty .$$

We have the symmetric situation if we assume $z_1, z_2 > u^s$. This is the case of graph (ii) of Figure 5 where there are no paths taking from regime 1 to regime 2. In this case the characteristic regime sequences are of the form

$$R_2 R_2 \ldots R_2 R_1 \ldots R_1 R_1 \ldots = R_2 R_2 \ldots R_2 (R_1)^\infty ,$$

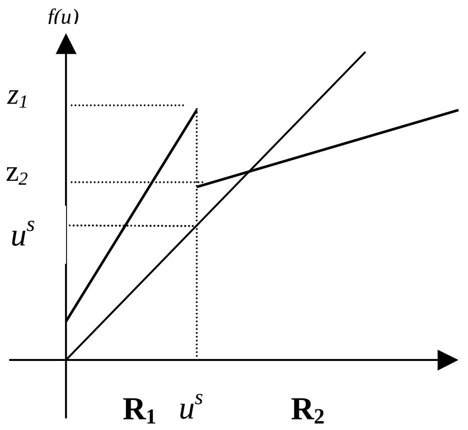
$$R_2 R_2 R_2 \ldots R_2 R_2 \ldots = (R_2)^\infty \text{ or}$$

$$R_1 R_1 R_1 \ldots R_1 R_1 \ldots = (R_1)^\infty .$$

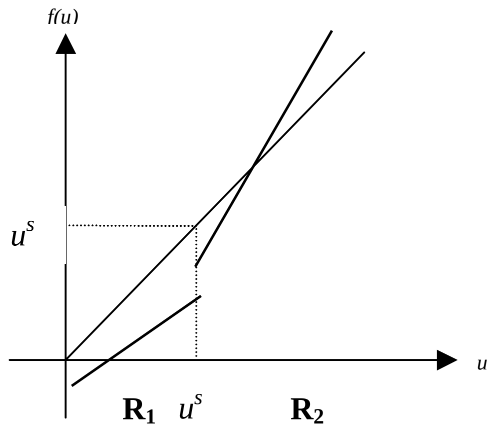
Now, let consider the case $z_2 > u^s > z_1$. A representative graph for the adjustment mechanism with parameter values verifying these conditions is shown in figure 5 (iii). For these parameter values we have that $f(R_1) \subseteq R_1$ and $f(R_2) \subseteq R_2$ *implying* that there are no paths taking from one to the other regime; i.e., both regimes are stable. Thus, regime sequences can be either of the form

$$R_2 R_2 R_2 \ldots R_2 R_2 \ldots = (R_2)^\infty \text{ or}$$

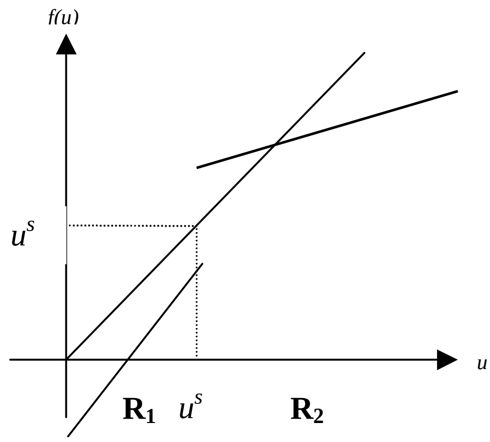
$$R_1 R_1 R_1 \ldots R_1 R_1 \ldots = (R_1)^\infty .$$



(i) $z_1, z_2 > u^s$



(ii) $z_1, z_2 < u^s$



(iii) $z_2 > u^s > z_1$

Figure 5: Some simple cases

(i) It is possible to go from R_1 to R_2 but not from R_2 to R_1
(ii) It is possible to go from R_2 to R_1 but not from R_1 to R_2
(iii) Each regime is a closed system: there is no possibility of changing regime.

Summarizing, we found that, for the parameter values of these section the introduction of regimes into the model cannot produce fluctuations and the same conclusions of simple dynamics of the original model can be stated. This is the consequence of the irreversibility property of both regimes. Other qualitative parameter conditions can be considered but all of them lead to similar dynamic behaviors of the results outlined above.

6. Conclusions

In this paper I have presented a simple macro model with two unemployment regimes, in which the active regime depends on whether the previous period's unemployment is above or below a threshold value. This multi-regime model can be viewed like a formalization of the economic intuition that different mechanisms govern economic behavior in different situations of state. In spite of the simplicity of the model, being linear in each regime with a discontinuous jump, it can give rise to aggregate fluctuations between employment and prices in absence of stochastic components. Thus, the introduction of two different regimes into the model can generate a quite different scenario than the simple dynamics of the original linear model. For a continuous of parameter values, the model is capable of generating cyclical, quasi-cyclical and chaotic fluctuations without changes in exogenous variables and according to this, the model represents an economy that undergoes business cycles. It was shown order to obtain periodic and nonperiodic fluctuations in the model, it is important that the direction of the variables in the two regimes must point to opposite directions.

It is important to remark that I confirmed in this model the analysis of the stability properties in economic models with regime switching of Honkapohja and Ito (1983). In this paper the authors have demonstrated that stability conditions for each subsystem (in my case, the equations representing the adjustment process in each regime) are neither necessary nor sufficient for overall stability. In fact, it was shown that for parameter values verifying $z_1 > u^s > z_2 \geq \tilde{u}_1$, $\tilde{u}_2 \notin (z_2, z_1)$, each subsystem has a stable steady state, but the model with two regimes has a cyclical behavior.

The use of symbolic dynamics techniques into the model has contributed to obtain a successful representation of the various types of regime dynamics. I have reviewed the qualitative properties of the model describing the very rich variety of possible evolutions in terms of sequences of regimes and in some particular cases it was shown how can we represent these evolutions by directed graphs and by shifts of finite type. It was shown that reversibility is a fundamental property to generate cycles. In terms of the directed graph representation of the two regime dynamics, reversibility is reflected by the presence of two arrows: one from R_1 to R_2 and one from R_2 to R_1. But is important to note that complex fluctuations cannot be generated only with reversible regimes (although it is a necessary condition). Another ingredient must also be present: a piece of regime R_1 (or R_2) must be mapped into the same regime. That is, at least one regime, let say R_1, can be decomposable into two non-trivial pieces E_1 and N_1 with where E_1 is mapped by f into R_2 and N_1 is mapped by f into R_1. In symbols, we can state this condition in the following way:

$$R_i = E_i \cup N_i \;;\; E_i \neq \varnothing \text{ and } N_i \neq \varnothing \quad \text{with} \quad f(E_i) \subseteq R_j \text{ and } f(N_i) \subseteq R_j. \qquad \text{(A)}$$

Summarizing, reversibility is a necessary and sufficient condition for periodic regime fluctuations and irregular fluctuations occurs when both regimes are reversible and at least one of them verifies condition (A).

Using Shannon entropy, we have founded out that the model is able to produce a rich variety of dynamic behavior with different complexity depending on parameter values. In fact, in case (d) we obtain the maximal entropy $h = log_2\, 2 = 1$ in a two regimes model and in cases (a) and 2 we have zero entropy (i.e., no simple dynamics). These are the extreme values, but the model can generate regime dynamics with entropy h, for all $0 \leq h \leq 1$. Then using this measure we are able not only to discriminate between simple or complex dynamics; we can talk about more or less complexity depending on the parameter values.

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