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Coordination in an Email Game Without Mutual Knowledge

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Abstract - The paper presents a version of the EMAIL Game, originally proposed by Rubinstein (AER,1989), in which efficient coordination is shown to obtain even when the relevant coordination game is not mutual knowledge. In the model investigated a mediator is introduced in such a way that the two individuals are symmetrically informed on the game chosen by nature, rather than asymmetrically as in Rubinstein. As long as the message failure probability is sufficiently low, with the upper bound being a function of the game payoffs, conditional beliefs on the opponent's actions can allow players to coordinate on the more rewarding-risky choice.

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1. **Introduction**

In a remarkable paper Rubinstein (1989) proposed a game theoretic version of a coordination problem, posed also in the Artificial Intelligence literature (Gray,1978) where was made known as the “Coordinated Attack” (Halpern and Moses,1990). Besides the interest in the matter *per se*, the problem attracted much attention because of its *puzzling* result (Nash Equilibrium (NE) in the game theoretic version). More specifically, in the Osborne and Rubinstein (1994) version of the model there is a unique and (ex-post) possibly inefficient outcome independently of the amount of information on the game that the two individuals might have. Alternatively, coordination may fail even when the game to be played is *almost common knowledge*, which in Rubinstein corresponds to a *sufficiently high* number of informative messages exchanged between the two individuals.

A distinguishing feature of the model is the initial informational asymmetry, existing between the two parties, on the action that they should coordinate upon. More specifically, in Rubinstein’s framework only one individual is initially *informed* by nature on the game to be played; she, in turn, informs the opponent *via* electronic mail. In the “Coordinated Attack“ problem this corresponds to the *general*, deciding the time of attack, being in perfect communication with one half of the army; indeed, typically he is viewed as being part of one of the two halves.

Since the communication protocol may not entail efficient use of the available information, namely players at the unique NE would choose actions as if no information on the game was exchanged, the possibility of alternative arrangements leading to more appropriate use of such information might, and should, be considered by the individuals involved. In the paper we discuss a simple variation of the original model, where a *mediating figure* (between nature and players) is introduced in such a way that the game to be played could still be individually, but never mutually, known. The motivation behind it is the following. If in Rubinstein *almost common knowledge* of the game is not enough for efficient coordination to obtain, then the explanation for such failure should not plausibly be looked for in this degree (depth) of interactive knowledge but rather elsewhere. In particular, our general intuition was that a main role must be played by the reliability of communication channels, since that is what supports the formation of individuals' beliefs concerning the opponent's possible actions. As is widely known, in a game with multiple equilibria beliefs on the opponent's actions play a fundamental role in selecting among them, even when the game is common knowledge. *A fortiori*, their importance is reinforced in absence of common knowledge. The model studied in the paper permits us a systematic investigation on the role of beliefs concerning actions (and the technology supporting them), since beliefs on the game to be played do not even allow it to be mutual knowledge. The main result of the work confirms our conjecture: if the reliability (message success probability) of communication channels is *sufficiently high*, with the critical threshold value being determined as a function of the game payoffs, then efficient coordination may obtain[1].

Among the schemes one could have thought of, in the paper we concentrate on the following simple example. The two players in the game will delegate a *third* (party) computer the task of being, separately, informed on the game chosen by nature[2]. We can imagine that this change might take place as the informed player may deliberately

[1] Within the original Rubinstein's framework, by allowing the two computers message failure probabilities to be different, Dimitri (2000) obtains an analogous result.

choose to forgo her informational advantage to favour an alternative arrangement that may provide both individuals with the possibility of more efficient coordination. Once *informed*, the players' machines will then be engaged in a one-to-one informational exchange only with the third computer, not between themselves, according to the Rubinstein's procedure. In the Coordinated Attack this could translate into the *general* being separated from the army altogether, initially sending the same message independently and simultaneously to the two halves.

Hence the number of messages sent by each player's machine, to the third one, is compatible with any number of messages sent by the opponent's computer to the mediating figure. At all states of the world, this would entail lack of mutual knowledge of the game and yet more efficient coordination may follow. Then, in the model the number of messages privately observed will serve only as a correlation device for players' choices since, unlike what happens in Rubinstein, they can reveal no information on the opponent's knowledge of the game.

2. The Model

2.1 The Electronic Mail Game (EMG)

We recall here the EMG version of Osborne and Rubinstein (1994), where two individuals (*I* and *II*) have to play one of the two coordination games depicted in Fig. 1 below.

[2] Should the third party be an individual we assume her preferences to be such that she has no

	A	B
A	M, M	1, -L
B	-L, 1	0, 0

G_a ; probability 1-p

	A	B
A	0, 0	1, -L
B	-L, 1	M, M

G_b ; probability p

$L>M>1$; $0<p<1/2$

Fig. 1

Nature can either be in state *a* or *b*. In state *a* individuals play game G_a, with probability $1-p$, while in state *b* they play game G_b, with probability p. In G_a there is only one NE; the pure strategy profile (A,A). In G_b instead there are three NE; the pure strategy profiles (A,A), (B,B) and the purely mixed profile where both players choose A with probability $q=(M-1)/(L+M-1)<1/2$. The equilibrium (B,B) is Pareto optimal while (A,A) is risk dominant.

Once the game is *chosen* by nature only I will be informed of it. The two players have computers on their desks to communicate; we indicate them as C_i, with $i=I,II$. The communication protocol is as follows. If G_a obtains then no message is exchanged between the two machines. If instead G_b obtains then C_I automatically sends an email message to C_{II} with probability $1>\varepsilon>0$ of not getting through. If the first message arrives then C_{II} automatically replies by sending a confirmation message with the same probability $1>\varepsilon>0$ of not getting through. If C_I receives this message in turn sends a

incentive to lie.

confirmation (of a confirmation) message, still with $1>\varepsilon>0$ error probability and so on. Messages are independent of each other. With probability one communication eventually stops; the only uncertainty concerns when it will. When this happens, players will privately read on their computer screens the number of messages sent by their own machines. Then the EMG is a Bayesian game in which the communication technology specifies a common type-space T, the set of naturals $T=\{0,1,2,...\}$, given by the possible number of messages appearing on the screen at the end of the communication exchange. If $S=\{A,B\}$ is the pure strategies space, in G_a and G_b (for both agents), then a strategy for player i in the Bayesian game is a function $\delta_i:T\rightarrow\Delta(S)$, where $\delta_i(t)$ indicates the probability with which player i chooses A at type t, with $i=I,II$. The state space Ω will then be a subset of $\Omega=T^2$, with the generic state $\omega\in\Omega$ being defined by a pair of possible types (t_I,t_{II}), such that either $t_I=t_{II}$ or $t_I= t_{II}+1$.

To simplify notation, from now on a strategy in the game will be written as

$$\delta_i=\Sigma_{t\in T}\ \delta_i(t)I(t_i=t)$$

where $I(t_i=t)$ is the standard indicator function. So, for example $\delta_i=I(t_i=0)$ is the strategy for player i specifying the choice of A at type $(t_i=0)$ and of B at all other types.

2.2 The Mediated Electronic Mail Game (MEMG)

By MEMG we mean an EMG modified in the following way. Introduce a third computer C_{III} in the framework and let notation (C_i, C_j), with $i\neq j$ and $i,j=I,II,III$, stand for the communication connection (channel) between C_i and C_j. In the pair, C_i will always indicate the computer starting the communication, namely the one sending the first message between the two. Consider now the following communication protocol. If nature chooses state *a* then C_{III} sends no message to C_j, with $j=I,II$. If nature chooses *b* then C_{III} automatically sends, separately, one message to C_I and one to C_{II}. If C_j receives a message then it automatically replies to C_{III}. If C_{III} receives a confirmation message from C_j then automatically replies to it with a message and so on. All messages, as in Rubinstein, have the same failure probability $1>\varepsilon>0$.

In words, for the two (separate) connections (C_{III}, C_I) and (C_{III}, C_{II}) the informational exchange is like in the EMG; the main difference is that in the MEMG player *I* has no initial informational advantage with respect to *II*. Indeed, this is what delegation to a third computer (party) of the task of (simultaneously) informing both of them entails here. This being the case, it is easy to see that now $\Omega=T^2$, since any pair (t_I,t_{II}) would have strictly positive probability to obtain[3].

Agent *I* information partition is then as follows. For every $t=0,1,2,..$each information set is a collection of pairs of the kind (t,n), where $n=0,1,2,..$ Similarly, still for all $t=0,1,2,..$, player *II* would not distinguish between pairs of the kind (n,t). The two figures below provide a graphical illustration of the two players' information partitions, respectively, in the EMG and MEMG. As Ψ_j stands for player *j* information partition, $\Psi_j(t)$ represent her information set when the type is *t*, with $j=I,II$; rows indicate information sets for player *I* while columns for player *II*.

[3] Notice that a complete description of the state would now be a triple of the kind (t_I,t_{II},t_{III}) where t_i indicates the total number of messages sent by C_i, with $i=I,II,III$. Since often in the analysis the values of t_I and t_{II} would suffice to simplify notation, unless otherwise indicated, in most of the paper we shall refer only to them.

$\Psi_I ; \Psi_{II}$	$\Psi_{II}(0)$	$\Psi_{II}(1)$	$\Psi_{II}(2)$	$\Psi_{II}(3)$	$\Psi_{II}(4)$	………
$\Psi_I(0)$	*(0,0)*					………
$\Psi_I(1)$	*(1,0)*	*(1,1)*				………
$\Psi_I(2)$		*(2,1)*	*(2,2)*			………
$\Psi_I(3)$			*(3,2)*	*(3,3)*		………
$\Psi_I(4)$				*(4,3)*	*(4,4)*	………
………	………	………	………	………	………	………

Players' Information Partitions in EMG

Fig 2.

$\Psi_I ; \Psi_{II}$	$\Psi_{II}(0)$	$\Psi_{II}(1)$	$\Psi_{II}(2)$	$\Psi_{II}(3)$	$\Psi_I(4)$	………
$\Psi_I(0)$	*(0,0)*	*(0,1)*	*(0,2)*	*(0,3)*	*(0,4)*	………
$\Psi_I(1)$	*(1,0)*	*(1,1)*	*(1,2)*	*(1,3)*	*(1,4)*	………
$\Psi_I(2)$	*(2,0)*	*(2,1)*	*(2,2)*	*(2,3)*	*(2,4)*	………
$\Psi_I(3)$	*(3,0)*	*(3,1)*	*(3,2)*	*(3,3)*	*(3,4)*	………
$\Psi_I(4)$	*(4,0)*	*(4,1)*	*(4,2)*	*(4,3)*	*(4,4)*	………
………	………	………	………	………	………	………

Players' Information Partitions in MEMG

Fig 3.

2.3 The Main Result

For completeness, below we recall Rubinstein's finding.

Proposition 1 *(Osborne and Rubinstein, 1994) In the EMG the pair of strategies,* $\delta_I=\delta^*=\delta_{II}$ *where* $\delta^*=\Sigma_{t\in T}I(t_j=t)$*, with* $j=I,II$*, is the unique Nash Equilibrium.*

In the MEMG a multiplicity of equilibria is instead possible. In particular, the following theorem presents the main result of the paper. More specifically, it formalises the existence of an equilibrium where both players choose, at all types, actions as in the efficient equilibrium of the EMG with common knowledge of the game played.

Theorem *The pair of strategies,* $\delta_I=\delta^{**}=\delta_{II}$*, where* $\delta^{**}=I(t_j=0)$*, with* $j=I,II$*, is a Nash Equilibrium of the MEMG if and only if* $P(t_{II}=0 \mid t_I=t)=\varepsilon = P(t_I=0 \mid t_{II}=t)<q$*, for all* $t\geq 1$*.*

Proof *(1)* Consider player *I's* behaviour at her possible types. *i*) Suppose $t_I=0$; then, since C_{III} sends separate messages to C_I and C_{II} its (marginal) probability is, as in Rubinstein's model, $P(t_I=0)=(1-p)+p\varepsilon = (1-p)+p\varepsilon^2+p\varepsilon(1-\varepsilon)$ where the three terms are the probabilities, respectively, of the triples of types $(0,0,0)$, $(0,0,2)$ and $(0,t_{II},t_{III})$, with $t_{II}\geq 1$ and either $t_{III}=t_{II}+1$ or $t_{III}=t_{II}+2$, the union of which gives event $t_I=0$. Hence if *II* plays δ^{**}, namely chooses *A* when observing $t_{II}=0$ and *B* elsewhere, the (conditional to the type) expected payoff of *I* when playing the (mixed strategy) $(\delta_I(0),1-\delta_I(0))\in\Delta(S)$, where as we said $\delta_I(0)$ is the probability of choosing *A* at type $t_I=0$, is given by

$$E\Pi_I(\delta_I(0))= \{(1-p)[\delta_I(0)M-(1-\delta_I(0))L]+p\varepsilon^2[\delta_I(0)0-(1-\delta_I(0))L]+p\varepsilon(1-\varepsilon)[\delta_I(0)+(1-\delta_I(0))M]\}/P(t_I=0)=$$

$$=\delta_I(0)[(1-p)(M+L)+p\varepsilon^2L-p\varepsilon(1-\varepsilon)(M-1)]/ P(t_I=0)+g(\varepsilon)$$

where $g(\varepsilon)$ is a function independent of $\delta_I(0)$. Thus choosing A is optimal for I, namely $\delta_I(0)=1$, if and only if $(1-p)(M+L)+p\varepsilon^2L>p\varepsilon(1-\varepsilon)(M-1)$, which is always true as $p<1/2$ and $\varepsilon(1-\varepsilon)<1$. *ii)* Assume now $t_I=t\geq 1$. By a consideration analogous to that for $t_I=0$, it is easy to see that its marginal probability is $P(t_I=t)= p\varepsilon(1-\varepsilon)^{2t-1}+ p\varepsilon(1-\varepsilon)^{2t} = p\varepsilon(1-\varepsilon)^{2t-1}(2-\varepsilon)$. It then follows that

$$E\Pi_I(\delta_I(t))=P(t_{II}=0 \mid t_I=t)[\delta_I(t)0-(1-\delta_I(t))L]+ P(t_{II}\geq 1 \mid t_I=t)[\delta_I(t)+(1-\delta_I(t))M]=$$

$$=\delta_I(t)[P(t_{II}=0 \mid t_I=t)L - P(t_{II}\geq 1 \mid t_I=t)(M-1)]+h(\varepsilon)$$

where $h(\varepsilon)$ is a function independent of $\delta_I(t)$. Hence, action B is optimal for I, i.e. $\delta_I(t)=0$, if and only if $P(t_{II}=0 \mid t_I=t)<q$. But

$$P(t_{II}=0 \mid t_I=t)=P(t_{II}=0,t_I=t)/P(t_I=t)=P((t,0,t+1)\cup(t,0,t+2))/P(t_I=t)$$

from which

$$P(t_{II}=0 \mid t_I=t) = [p\varepsilon^2(1-\varepsilon)^{2t-1}+ p\varepsilon^2(1-\varepsilon)^{2t}] / p\varepsilon(1-\varepsilon)^{2t-1}(2-\varepsilon) =\varepsilon$$

showing that δ^{**} is best reply for I against player II choosing δ^{**} if and only if $\varepsilon<q$.

(2) Since player's II situation is perfectly symmetric, her reasoning will be exactly the same at all types and the result follows.

The above theorem shows the possibility for the pair $(\delta^{**},\delta^{**})$ to be a Nash Equilibrium of the MEMG and so for both players to choose action B, the efficient equilibrium when they play G_b, even if at no state the game is mutual knowledge. When states are $(t,0,t_{III})$ or $(0,t,t_{III})$, with $t=0,1,2,$.and $t_{III}\geq 2$ (namely when at least one of the two very initial messages is not successful), the game is G_b but at the equilibrium $(\delta^{**},\delta^{**})$ there is coordination failure (as in $(0,0,2)$ players choose the pair of actions (A,A) while in the other states either the pair (B,A) or the pair (A,B)). It is then natural to ask whether players would be ex-ante better off at the Rubinstein's equilibrium of the EMG[4] or at the above equilibrium of the MEMG. The following proposition provides the answer.

Proposition 2 *If the pair $(\delta^{**},\delta^{**})$ is a Nash Equilibrium of the MEMG then it is ex-ante Pareto superior to the Nash Equilibrium (δ^{*},δ^{*}) in the EMG.*

Proof. Let $E\Pi_i(\delta_I, \delta_{II})$ be player's i expected payoff (with $i=I,II$) when players choose, respectively, δ_I and δ_{II}. Then, in the MEMG the equilibrium $(\delta^{**},\delta^{**})$ provides player i (the reasoning of both players is the same) with the following expected payoff.

$$E\Pi_i(\delta^{**},\delta^{**}) = (1-p)M + 0p\varepsilon^2 - Lp\varepsilon(1-\varepsilon)+ 1p\varepsilon(1-\varepsilon)+ Mp(1-\varepsilon)^2$$

Since $E\Pi_i(\delta^{*},\delta^{*})=(1-p)M$ then $E\Pi_i(\delta^{**},\delta^{**})>E\Pi_i(\delta^{*},\delta^{*})$ if and only if

$$p\varepsilon(1-\varepsilon)(1-L)+ p(1-\varepsilon)^2M>0$$

namely $\varepsilon < M/(L+M-1)$ which when $(\delta^{**},\delta^{**})$ is a Nash Equilibrium holds true.

Hence, as long as the failure message probability is less than q, players would be better off by coordinating on action B whenever observing at least one message. With respect to the EMG the introduction of a mediating figure, sending separate messages to each player, renders each individual's type uninformative with respect to the opponent's type. Though message failure probabilities are equal, it is now possible to obtain multiple equilibria as long as communication channels are *sufficiently* reliable. Alternatively,

[4] Clearly (δ^{*}, δ^{*}) would also be an equilibrium in the MEMG.

unlike what happens in the original Rubinstein's model, the probability value now counts.

3 Conclusions

In the Mediated (version of the) Electronic Mail Game that we have investigated, indirect informational exchange can enhance an equilibrium with more efficient coordination if the message error probability ε is sufficiently low, with the upper bound being determined by the purely mixed strategy Nash Equilibrium of one of the two coordination games that individuals have to play. For each player, the conditional (to her own type) probability on the opponent's type clearly depends on ε. As, in equilibrium, such probability represents a player's belief on the opponent choosing a specific action, then it is not surprising that the optimal choice could depend upon the value of ε. To conclude, the example presented in the paper appears to suggest that when information exchanged by the individuals on the game to be played is noisy, as long as the reliability of communication channels appropriately supports personal beliefs on the choices available to the other, individual knowledge of a game may suffice for efficient coordination to obtain.

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