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A Model of Bertrand-Edgeworth Competition in the Labour Market

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Abstract - This paper formalises, for the labour market, the traditional view that competition among the sellers leads to a fall in prices so long as there is excess supply. First we show that, at a Nash equilibrium of the one-period game, wages are set equal to the Walrasian wage. Then, similarly as in Edgeworth's analysis of duopoly, we take workers as engaging repeatedly in Bertrand competition, each one seeking at every date to make a best reply to the wages that all other workers are expected to quote. A quite reasonable condition on expectations is shown to be sufficient in order for the sequence of disequilibria to converge to Walrasian equilibrium.

JEL Classification: C72, J23, J3.

Keywords: Labour market, Bertrand-Edgeworth competition, Walrasian equilibrium.

1. Introduction

It has long been recognised that prices and nominal wages often remain more or less stable under excess supply in the goods and labour markets. Though several theories have been proposed in the last decades to explain this fact, the usual picture of the functioning of markets with many sellers is still that competition over the price leads to a general decrease in prices in the presence of excess supply. Though often implicitly, the argument underlying the "traditional" view is that each individual seller would have an incentive to reduce his own price because of the highly elastic demand curve he perceives to be facing at the "market" price. The market price itself decreases as a result and the process keeps on going until excess supply has disappeared. When applying this argument to the labour market, one should recognise that excess supply of labour usually means that there are workers fully unemployed as well as workers fully employed rather than partial employment of all the labour force. The demand curve facing the individual worker should then be viewed as relating the worker's wage to his perceived hiring probability, given the "market" wage. A highly elastic demand curve should then imply that under involuntary unemployment it pays the worker to undercut the "market" wage because a negligible sacrifice of the wage would would assure him of employment.

The traditional view referred to above - in fact embodying a sort of dynamic Bertrand competition among sellers of a homogeneous product – does not seem to have ever been incorporated into a formal model showing how precisely, for any arbitrary number of sellers, prices would be driven toward the market clearing level when starting from a situation of excess supply. This gap may help explain the persistence of some criticisms concerning validity, on its own premises, of such a view. In his influential analysis of price adjustment Arrow started noting that, in disequilibrium, different sellers have presumably quoted different prices so it it is unclear what is meant by the "market price", at which the demand curve confronting the individual seller is supposed to be perfectly elastic (Arrow, 1959, p. 46). However, the main difficulty in the notion of perfecty elastic demand curves would be a different one, as shown by the following passage: "Suppose we have a situation which conforms in all the aspects of homogeneity of output and multiplicity of firms to the usual concept of perfect competition, but in which the aggregate supply forthcoming at the 'market' price exceeds the demand at that price. Then the individual firm cannot sell all it wishes to at the market price; i. e., when supply and demand do not balance, even in an objectively competitive market, the individual firms are in the position of monopolists as far as the imperfect elasticity of demand for their products is concerned" (p. 46).

The main point raised by Arrow was to be subsequently taken over by disequilibrium and conjectural equilibrium theorists. Authors like Barro (1972, p. 17), Barro-Grossman (1971, p. 85, n. 10), Hahn (1977, pp. 33-34), and Negishi (1977, p. 501) all accepted that the notion of a horizontal demand curve for the seller can no longer be maintained in a situation of excess supply. Assuming instead that the seller would then perceive a downward sloping demand curve, Negishi and Hahn were able to derive conjectural equilibria with excess supply in both the output and the labour market: prices and nominal wages do not decrease at these equilibria because of the imperfect elasticity of the individual demand curve perceived by individual sellers, which makes it unrewarding to any firm to charge a lower price and to any worker to demand a lower wage.[1]

The alleged inconsistency between a situation of excess supply and the notion of a perfectly elastic demand curve for the individual seller rests on interpreting this curve as describing a situation in which each seller believes he would be able to sell any amount at the "market" price. It is unclear,

[1] Interestingly enough, both Negishi and Hahn in subsequent work supplemented their (and Arrow's) earlier criticism of the notion of a perfectly elastic demand curve by pointing informally to independent circumstances as responsible for a sufficient *inelasticity* of individual demand curves as conjectured by sellers. As regards the output market, Negishi stressed imperfect information on prices: customers currently buying from other firms may not be informed of a price reduction made by a seller, so the latter may perceive demand for his output not to increase substantially if reducing the price (Negishi, 1979, p. 88). As to the labour market, both Negishi and Hahn insisted on institutional arrangements constraining the firms. More specifically, an employer may not be allowed to pay different wages to his employees and/or it may be impossible or very costly to replace entirely his workforce with workers demanding lower wages; as a result, a worker may perceive his hiring probability not to increase substantially if offering to work for less than workers who are currently employed (Negishi, 1979, p. 92; Hahn, p. 288). The very fact that these new arguments have subsequently been introduced may reveal the emergence of some doubts about the adequacy of their (and Arrow's) initial kind of criticism to the notion of a perfectly elastic demand curve.

however, why the traditional view under discussion should require such a strict interpretation. A different, and apparently less stringent, interpretation appears instead to be involved, namely, that each firm (each worker) would obtain a very large increase in demand (certainty of being hired) by asking a price (a wage) just a bit lower than competitors'. Such a notion can be found, for example, in Stigler when he works out a numerical example to compute elasticity of demand for a firm which charges just a bit less than the (uniform) price charged by all other firms (1966, p. 91).[2] As a second example, the following definition of the perfectly elastic demand curve is quite remarkable, in that it contemplates the possibility of a multiplicity of prices facing the competitive firm: according to Friedman, "a firm is competitive if the demand curve for its output is infinitely elastic with respect to its own price for some price and all outputs, given the prices charged by all other firms…" (Friedman, 1953, p. 35).[3]

The analysis carried out in the present paper provides what may be viewed to be a reasonable formalisation, for the labour market, of the traditional view under discussion. This formalisation is made for its own sake and also for the purpose of understanding the episodes of intense price or wage competition which after all are not so rare. Furthermore, it may help making the right questions when trying to understand what lies behind wage rigidity. (Another paper will analyse the goods market with a similar perspective.)

We analyse a simple labour market in which one firm is confronted with n wage setting workers. Although the game structure is built on a previous contribution by Weibull (1987), our main purpose is rather to obtain theoretical predictions about the dynamic of wages under disequilibrium. Thus, in a sense, our analysis is closer in spirit to Bertrand-Edgeworth analysis of price dynamics in a duopoly (Bertrand, 1883; Edgeworth, 1925), where each duopolist in every period chooses the price

[2] Needless to say, once all firms *are assumed* to sell at the same price it follows straightforwardly that, with a sufficiently large number of firms, the demand curve facing any of them is almost horizontal. This demand curve would show at different levels of the firm's output the market price, as determined according to the total demand curve for the commodity given the outputs of all the other firms. This is the way in which the horizontal demand curve is usually presented in textbooks (see, for example, Stigler, 1967, pp. 89-90).

that is optimal to him given the rival's price.[4] Similarly, we take workers as engaging repeatedly in Bertrand competition, each one seeking to make in every period a best reply to the wages he expects to be quoted by the other workers. Equilibrium of the one-period game is easily seen to involve market clearing. Then it is shown that, under mild assumptions on expectations, wages would actually decline over time if starting from a level above the market-clearing wage. The circumstance behind this result turns out to be precisely that the market is "competitive", in the sense of Friedman: given the wages each worker expects on the part of others, his hiring probability is zero so long as his wage is higher than some critical level while jumping to one when determined at that level (or just below that level); from this it follows that, so long as some of the wages demanded by others are higher than the market-clearing wage it pays the worker to underbid some of them to assure employment to himself.

The paper is organised as follows. Section 2 presents a model of the labour market in which both the workers and the firm are wage takers, the wage being determined by an auctioneer. Section 3 assumes workers as wage setters and determines the outcome of the labour market as a (subgame perfect) equilibrium of a two-stage game, in which workers first make once and for all their wage calls whereupon the firm chooses which and how many workers to hire. Compared with earlier work on this subject (Weibull,1987; Solow 1990, pp. 53-56)[5] we make further progress in two respects. First, we also deal with the case in which the Walrasian wage (the market-clearing wage in the model of section 2) is higher than the reservation wage, whereas only the case of equality between the two was considered by Weibull and Solow. Second, we provide complete proofs of propositions about

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[3] Further, while aknowledging that "no observed demand curve will ever be precisely horizontal, so the estimated elasticity will always be finite", he added soon after: "The relevant question always is whether the elasticity is 'sufficiently' large to be regarded as infinite…" (p. 36).

[4] Edgeworth assumed that each duopolist takes the rival as currently charging the same price as in the last period. Thus, the argument was carried out on the assumption of static price expectations, or, alternatively, on the assumption that duopolists take turns setting their prices. Incidentally, Bertrand too was concerned with the dynamics of prices.

[5] Cfr. also De Francesco (1993).

equilibria of this game.[6] The result of the model is that equilibrium entails workers quoting the Walrasian wage, so wages are the same as in the model of section I. [7]

The close scrutiny of the wage game carried out in section 3 will put us in a position to analyse the evolution of the structure of wages over time: section 4 establishes convergence of wages to the Walrasian wage, under mild and quite reasonable conditions on wage expectations.

2. A model of the labour market with wage taking agents

For simplicity we assume that there is one firm only in the labour market, producing a single output under decreasing returns to labour. Output is thus written $Y = F(L)$ with $F(0) = 0, F'(L) > 0, F''(L) < 0$, with Y and L denoting quantities of output and labour, respectively. (It should be emphasised that the assumption of decreasing returns to labour – implying a notional demand for labour that increases as the real wage decreases – is made only to follow to the most usual treatment. The results obtained are much more general; for example, one might as well assume constant returns to labour up to some maximum level of output that the firm can obtain given its capital stock.) $N = \{1, \ldots, i, \ldots, n\}$ denotes the set of (identical) workers. Worker *i*'s utility is written $U_i = U(c_i, l_i)$, where c_i and l_i denote worker *i*'s income and hours of working, respectively. For the levels of c_i and l_i considered below, preferences are assumed to be represented by the following Von Neumann-Morgenstern utility function $U_i = U\left[c_i + m(\bar{l} - l_i)\right]$, with $U' > 0, U'' \leq 0, m > 0$, where $\bar{l}$ denotes worker's maximum working hours. The marginal rate of substitution between consumption and leisure is thus constant at m for whatever amount of labour

[6] For example, to show that workers quoting more than the Walrasian wage cannot be an equilibrium Weibull only considers situations in which all workers have quoted the same wage (Weibull, 1987, pp. 23-24).

[7] With the Walrasian wage equal to the reservation wage, other equilibria are shown to exist too: however, they entail the same outcome (or approximately so) in terms of workers' utilities and total employment as the equilibrium in which all workers are quoting precisely the Walrasian wage.

and consumption. For this assumption not to be wholly unreasonable, $\bar{l}$ may be viewed as fixed by law at a non exhausting level. Further, the employed worker is taken to have no discretion as to the actual working time (i. e., $l_i = \bar{l}$ for the employed). Taking $\bar{l}$ as the physical unit of labour, worker i's wage income coincides with the worker's wage rate, denoted w_i. Worker i's utility is thus $U_i = U(w_i)$ when employed and $U_i = U(b+m)$ when unemployed ($b \geq 0$ being any income benefit to the unemployed). Thus $w^r = b + m$, where w^r denotes the reservation wage; at $w_i = w^r$ the worker is indifferent between working and not working, whereas at $w_i > w^r$ ($w_i < w^w$) he prefers working (not working).

We first assume workers and the firm to be wage takers, with an auctioneer quoting the wage. Denoting total labour supply as $L^s(w)$, it is $L^s(w) = 0$ for $w < w^r$, $L^s(w) = n$ for $w > w^r$, and $L^s(w) \in \{1, 2, \ldots, n\}$ for $w = w^r$.

At any level of employment $L°$, due to the fixed working time the marginal product of labour is the extra output provided by one additional worker, i. e., $MP(L°) = F(L°+1) - F(L°)$. For any $w < MP(0)$, a profit-maximizing level of employment, denoted $L(w)$, is such that $MP(L(w)) \leq w \leq MP(L(w)-1)$. Thus $L(w)$ - and hence labour demand, denoted as $L^d(w)$ - decreases in a stepwise fashion as the wage increases, as in Fig. 1. $L(w)$ is a function except when $w = MP(L°)$ for some $L°$, in which case $L(w) \in \{L°, L°+1\}$. At any such wage we take labour demand to be $L^d(w) = L° + 1$.

Let ε be the minimum amount by which the wage may be changed; we take ε to be negligibly small. Walrasian equilibrium obtains at the wage equating labour demand and supply. All workers are employed at Walrasian equilibrium to the extent that $MP(n-1) \geq w^r$; with $MP(n-1) < w^r$, there are workers voluntarily unemployed at the Walrasian wage, then equal to the reservation wage. With $MP(n-1) > w^r$ there is a small range of indeterminacy as to the Walrasian wage since then any $w \in \left[\max\{w^r,\ MP(n)+\varepsilon\}, MP(n\text{-}1)\right]$ is consistent with clearing of the labour market. In such a case, for simplicity we identify the Walrasian wage with the highest value in that range. The Walrasian wage-employment pair, (w^w, L^w), is thus determined by either of the following two conditions:

$$\left(w^w = MP(n-1),\ L^w = n\right) \qquad \textit{if} \qquad MP(n-1) \geq w^r$$
$$\left(w^w = w^r,\ L^w :\ MP(L^w) < w^r \leq MP(L^w - 1)\right) \qquad \textit{if} \qquad MP(n-1) \leq w^r$$

Figures 1 and 2 depict Walrasian equilibria ot the two types.

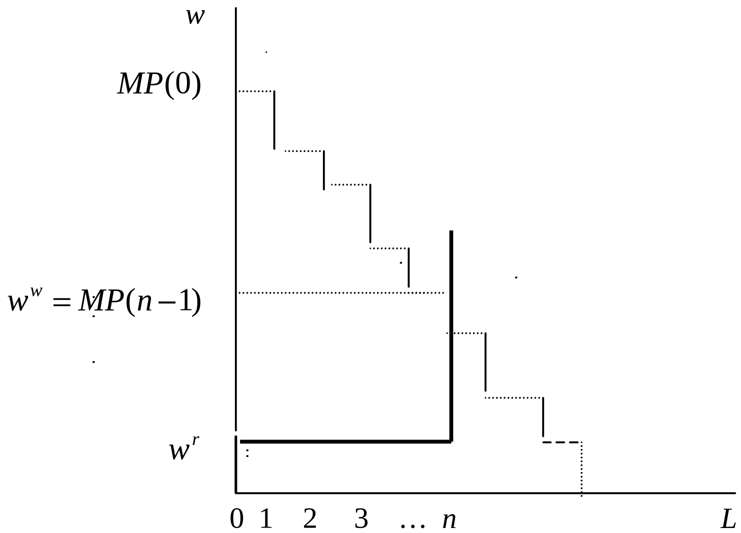

Fig. 1

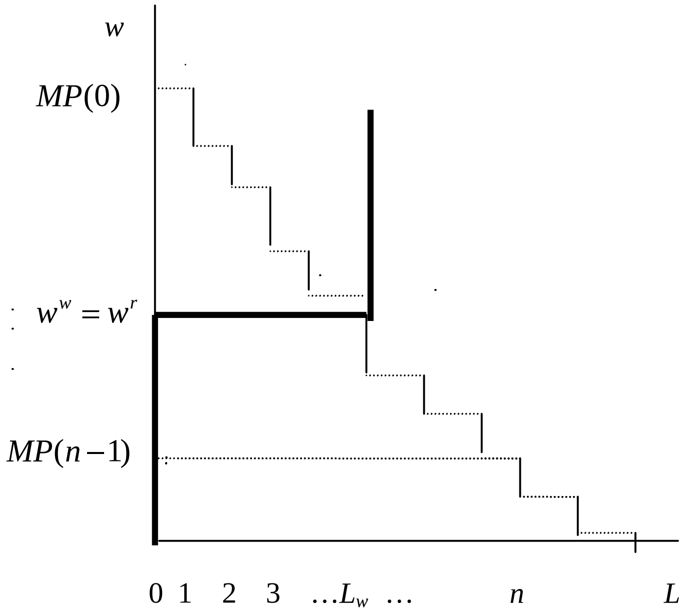

Fig.2

3. Market clearing as an equilibrium outcome with wage setting workers

Take now workers to be wage setters, each quoting independently and simultaneously his own wage. Given the vector of wage calls $\boldsymbol{w} = (w_1, \ldots, w_n)$, the firm chooses which workers to hire. A "hiring decision" can be represented as an *n*-component vector, $\boldsymbol{H} = (h_1, \ldots, h_i, \ldots, h_n)$**,** with $h_i = 1$ or $h_i = 0$, for any $i \in N$, according as to whether worker i is hired or not. (We borrow this formalization from Weibull (1987).) A hiring decision gives rise to an employment level $L_H = \boldsymbol{H}\ \boldsymbol{I}$, where $\boldsymbol{I}$ is a (column) unit vector. Finding optimal hiring decisions is trivial when $w_i > MP(0)$ for all $i \in N$, in which case $L_H = 0$, and when $w_i < MP(n-1)$ for all $i \in N$, in which case $L_H = n$. Turning to the case in which $w_i < MP(0)$ and $w_j > MP(n-1)$ for some $i, j \in N$, we identify three critical dimensions of a hiring decision, L_H, w^{h_e}, and w^{l_u}, these being, respectively, the resulting employment level, the highest wage among those quoted by workers hired (if any) and the lowest wage among those quoted by workers not hired (if any). In fact, three conditions are necessary and sufficient for the hiring decision to be optimal to the firm: (i) $w^{h_e} \leq MP(L_H - 1)$, for profits would otherwise be higher by laying off the most expensive worker hired; (ii) $w^{l_u} \geq w^{h_e}$, for labour costs would otherwise decrease, and hence profits would increase, by replacing the least expensive worker not hired for the most expensive worker hired; (iii) $w^{l_u} \geq MP(L_H)$, for profits would otherwise increase by hiring the least expensive worker not hired. Sufficiency of these conditions is also obvious. Consider a hiring decision meeting conditions (i) to (iii). Condition (ii) assures that the hiring decision minimises the total wage bill at the associated level of employment. Consequently, if the hiring decision did not maximise profits this would mean that profits would increase either by assuming the least expensive among unemployed workers or by laying off the most expensive among hired workers. But neither can happen, in view of (i) and (iii), respectively, hence the hiring decision under discussion actually maximises profits. It is worth noting that, with $w_i < MP(0)$ and

$w_j > MP(n-1)$ for some $i, j \in N$, it is $0 < L_H < n$ at an optimal hiring decision: indeed, $L_H = n$ would violate condition (i) whilst $L_H = 0$ would violate condition (iii).

A strategy for the firm is a rule specifying for each vector of wage calls the probability distribution over the set of hiring decisions. We are concerned with subgame-perfect equilibria, so the firm strategy must be such that, at any vector of wage calls, it chooses only among profit-maximizing hiring decisions. With some workers quoting the same wage there may exist several profit-maximizing hiring decisions giving rise to the same level of employment L_H; they have in common $w^{h_e} = w^{l_u}$ and only differ in terms of the identity of workers hired among those quoting a wage equal to w^{h_e}. We assume that, whenever there are several profit-maximizing hiring decisions, all of them implying the same L_H, the firm's strategy dictates to pick any of them with the same probability. For example, suppose all workers have quoted the same wage $w : w \neq MP(L)$ *for any* $L \leq n; MP(n-1) < w < MP(0)$. Then all hiring decisions giving rise to a level of employment $L_H : MP(L_H) < w < MP(L_H - 1)$ are optimal and there are $\binom{n}{L_H}$ of them; clearly each worker has the same hiring probability, L_H / n.

At wage calls such that $w_i = MP(L^\circ)$ for some $i \in N$ and $L^\circ < n$, it may be that at some optimal hiring decisions $L_H = L^\circ$ and $w^{l_u} = MP(L_H)$. When this is so, there also exist hiring decisions involving an employment level of $L_H = L^\circ + 1$. Hereafter it is assumed that, among all optimal strategies, the one selected by the firm consists in picking with equal probability any of the optimal hiring decisions giving rise to the higher level of employment. Incidentally, this implies that, at any vector of wage calls, the level of employment is a single-valued function, denoted below as $L_H(\boldsymbol{w})$. For example, let $w_i = MP(L^\circ)$ for all $i \in N$, with $L^\circ < n$. Then, in spite of hiring decisions involving $L_H = L^\circ$ being optimal, the firm chooses any of those involving $L_H = L^\circ + 1$, which are also optimal. Hence $L_H(\boldsymbol{w}) = L^\circ + 1$ and each worker is hired with probability $(L^\circ + 1)/n$.

A major task of this paper is to show that any equilibrium of the game exhibits – precisely or approximately – Walrasian features. To this aim we first establish the following proposition.

Proposition 1. With either $w^w > w^r$ or $w^w = w^r$, the Walrasian vector of wage calls $\boldsymbol{w}^w = (w^w, ..., w^w)$ is an equilibrium of the game.

Proof. First of all, it is obviously $L_H(\boldsymbol{w}^w) = L^w$. It is easily understood that quoting $w_i = w^w$ is in fact a best reply to strategy profile $\boldsymbol{w}_{-i}{}^w = (w^w, ..., w^w)$ for any $i \in N$. Denoting by $EU_i(\boldsymbol{w})$ worker *i*'s expected utility at wage calls $\boldsymbol{w}$, $EU_i(\boldsymbol{w}^w) = U(w^w)$. Our point is first made for the case $w^w > w^r$. Then $EU_i(w_i > w^w, \boldsymbol{w}_{-i}{}^w) = U(w^r) < U(w^w)$ since only workers other than *i* would then be hired – such a hiring decision would meet conditions (i) to (iii) above; on the other hand, $EU_i(w_i < w^w, \boldsymbol{w}_{-i}{}^w) = U(w_i) < U(w^w)$. Next consider the case $w^w = w^r$. Here, $EU_i(w_i \geq w^r, \boldsymbol{w}_{-i}{}^w) = U(w^r)$,[8] whilst $EU_i(w_i < w^r, \boldsymbol{w}_{-i}{}^w) = U(w_i) < U(w^r)$[9]: thus, $\boldsymbol{w}^w$ is again an equilibrium since any wage $w_i \geq w^r$ is in fact a best reply to $\boldsymbol{w}_{-i}{}^w$. *QED*

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[8] Quoting $w_i = w^w$ in response to $\boldsymbol{w}_{-i} = (w^w, \ldots, w^w)$ would yield expected utility $EU_i = U(w^w)(L^w / n) + U(w^r)(1 - L^w / n)$, which reduces to $EU_i = U(w^w) = U(w^r)$ given that $w^w = w^r$. On the other hand, replying $w_i > w^w$ would yield $EU_i = U(w^r)$ since then worker *i* would certainly not be hired.

[9] Hereafter it is assumed that a worker cannot refuse working at the terms he asked. Hence, if offered a job when quoting a wage lower than w^r he ends up with a utility less than $U(w^w)$, the reservation one. While not strictly necessary, this assumption will simplify the discussion in a few cases.

The next step is to show that a vector of wage calls $\boldsymbol{w}^\circ \neq \boldsymbol{w}^w$ is generally not an equilibrium. We first dispose of vectors $\boldsymbol{w}^\circ : w_i{}^\circ < w^w$ for some i, with either $w^w > w^r$ or $w^w = w^r$. One easily understands that, with $w^w > w^r$, by quoting any wage not higher than w^w a worker is certainly hired no matter the value of $\boldsymbol{w}_{-i}{}^\circ$. Thus, for any $i : w_i{}^\circ < w^w$, $EU_i(\boldsymbol{w}^\circ) = U(w_i{}^\circ) < U(w^w)$ whereas $EU_i(w_i = w^w, \boldsymbol{w}_{-i}{}^\circ) = U(w^w)$, showing that any such worker has not made a best reply. With $w^w = w^r$, one should understand that, among workers quoting a wage lower than w^r there are some who have a positive hiring probability; thus, $EU_i(\boldsymbol{w}^\circ) < U(w^r)$ for any such worker whereas it would be $EU_i(w_i = w^r, \boldsymbol{w}_{-i}{}^\circ) = U(w^r)$.

We also easily exclude wage calls $\boldsymbol{w}^\circ : w_i{}^\circ > MP(0)$ for all i, for $EU_i(\boldsymbol{w}^\circ) = U(w^r)$ for all i, while $EU_i(w_i = MP(0), \boldsymbol{w}_{-i}{}^\circ) = U(w_i) > U(w^r)$.

There remain to consider wage calls $\boldsymbol{w}^\circ > \boldsymbol{w}^w$, with $w_i{}^\circ \leq MP(0)$ for some i. As will become apparent soon, a key role in the argument below is played by a particular wage worker *i* might reply to $\boldsymbol{w}_{-i}{}^\circ$, i. e., the highest wage making *i* be hired with unit probability in the face of $\boldsymbol{w}_{-i}{}^\circ$. This wage, denoted by $w_i{}'$, may on reflection take on either of two possible values according as to whether $p_i(\boldsymbol{w}^\circ) < 1$ or $p_i(\boldsymbol{w}^\circ) = 1$, where $p_i(\boldsymbol{w}^\circ)$ denotes *i*'s hiring probability at $\boldsymbol{w}^\circ$: $w_i{}' = \max\left[MP\left(L_H(\boldsymbol{w}^\circ)\right), w^{h_e} - \varepsilon\right]$ for any $i : p_i(\boldsymbol{w}^\circ) < 1$ and $w_i{}' = \min\left[MP\left(L_H(\boldsymbol{w}^\circ) - 1\right), w^{l_u} - \varepsilon\right]$ for any $i : p_i(\boldsymbol{w}^\circ) = 1$.

The discussion of wage calls $\boldsymbol{w}^\circ > \boldsymbol{w}^w$ is better organised by dealing with the cases $w^w > w^r$ and $w^w = w^r$ in turn.

To start with, we assume $w^w > w^r$. With $\boldsymbol{w}^\circ > \boldsymbol{w}^w$, there clearly exists some worker *j* for whom $p_j(\boldsymbol{w}^\circ) < 1$ and hence $EU_j(\boldsymbol{w}^\circ) = U(w_j{}^\circ) p_j(\boldsymbol{w}^\circ) + U(w^r)(1 - p_j(\boldsymbol{w}^\circ)) < U(w_j{}^\circ)$. The next two results state that, with either $0 < p_j(\boldsymbol{w}^\circ) < 1$ or $p_j(\boldsymbol{w}^\circ) = 0$, worker *j* has not replied optimally to $\boldsymbol{w}_{-j}{}^\circ$, the optimal reply being insted $w_j{}'$.

Lemma 2.1. With $w^w > w^r$, at $\boldsymbol{w}^\circ > \boldsymbol{w}^w$ any *j*: $0 < p_j(\boldsymbol{w}^\circ) < 1$ has failed to make a best reply, this being instead $w_j{}' = w_j{}^\circ - \varepsilon$.

Proof. It must preliminarily be noted that, in view of condition (ii) above, there exists some $j: 0 < p_j(\boldsymbol{w}^\circ) < 1$ if and only if $\boldsymbol{w}^\circ$ is such that $\#\{i : w_i{}^\circ < w_j{}^\circ\} < L_H(\boldsymbol{w}^\circ) < \#\{i : w_i{}^\circ \leq w_j{}^\circ\}$. Necessity of these conditions is easily established. If $L_H(\boldsymbol{w}^\circ) \leq \#\{i : w_i{}^\circ < w_j{}^\circ\}$, then it must be $h_j = 0$ at any optimal hiring decision and hence $p_j(\boldsymbol{w}^\circ) = 0$: indeed, if some optimal hiring decision involved $h_j = 1$ when $L_H(\boldsymbol{w}^\circ) \leq \#\{i : w_i{}^\circ < w_j{}^\circ\}$, then, corresponding to any such decision, we would have that a worker quoting $w_j{}^\circ$ is hired in spite of there being at least one worker quoting a wage lower than $w_j{}^\circ$ who is out of work, what violates condition (ii). Likewise, if $L_H(\boldsymbol{w}^\circ) \geq \#\{i : w_i{}^\circ \leq w_j{}^\circ\}$, then $p_j(\boldsymbol{w}^\circ) = 1$ as it would be $h_j = 1$ at any optimal hiring decision: indeed, assuming on the contrary that $h_j = 0$ at some optimal hiring decision when $L_H(\boldsymbol{w}^\circ) \geq \#\{i : w_i{}^\circ \leq w_j{}^\circ\}$ would imply that, corresponding to these decisions, a worker quoting $w_j{}^\circ$ is out of work while there is at least one worker quoting a wage higher than $w_j{}^\circ$ who is hired, again contradicting condition (ii). Sufficiency is also easily established. By condition (ii), inequalities $\#\{i : w_i{}^\circ < w_j{}^\circ\} < L_H(\boldsymbol{w}^\circ) < \#\{i : w_i{}^\circ \leq w_j{}^\circ\}$, which obviously require $\#\{i : w_i{}^\circ = w_j{}^\circ\} > 1$, imply $w^{h_e} = w^{l_u} = w_j{}^\circ$ at an optimal hiring decision.[10] This in turn implies that $h_j = 1$ at some optimal hiring decisions while $h_j = 0$ at others. Therefore $0 < p_j(\boldsymbol{w}^\circ) < 1$ since every optimal hiring decision is chosen with positive probability. Notice, further, that inequalities $\#\{i : w_i{}^\circ < w_j{}^\circ\} < L_H(\boldsymbol{w}^\circ) < \#\{i : w_i{}^\circ \leq w_j{}^\circ\}$ imply $MP(L_H(\boldsymbol{w}^\circ)) < w_j{}^\circ \leq MP(L_H(\boldsymbol{w}^\circ) - 1)$.[11] Consequently, $w_j{}' = \max\left[MP(L_H(\boldsymbol{w}^\circ)), w^{h_e} - \varepsilon\right] = w_j{}^\circ - \boldsymbol{\varepsilon}$.

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[10] Given that $L_H(\boldsymbol{w}^\circ) < \#\{i : w_i{}^\circ \leq w_j{}^\circ\}$, admitting $w^{h_e} > w_j{}^\circ$ would imply that some worker charging more than $w_j{}^\circ$ is hired in spite of there being workers unemployed among those quoting wages up to $w_j{}^\circ$, which contradicts condition (ii). A similar contradiction would arise if assuming $w^{l_u} < w_j{}^\circ$, this time in view of $L_H(\boldsymbol{w}^\circ) > \#\{i : w_i{}^\circ < w_j{}^\circ\}$. At the same time, $L_H(\boldsymbol{w}^\circ) < \#\{i : w_i{}^\circ \leq w_j{}^\circ\}$ and $L_H(\boldsymbol{w}^\circ) > \#\{i : w_i{}^\circ < w_j{}^\circ\}$ immediately rule out the possibility of $w^{l_u} > w_j{}^\circ$ and $w^{h_e} < w_j{}^\circ$, respectively.

[11] Recall that among workers quoting $w_j{}^\circ$ some are employed while others are unemployed. Then, if it were $w_j{}^\circ > MP(L_H(\boldsymbol{w}^\circ) - 1)$ it would have paid the firm to dismiss one of the former, thus making employment decrease below $L_H(\boldsymbol{w}^\circ)$. Likewise, if it were $w_j{}^\circ < MP(L_H(\boldsymbol{w}^\circ))$ the firm would have made higher profits by hiring one of the latter, thus raising employment above $L_H(\boldsymbol{w}^\circ)$;

All this being said, we can now see that quoting $w_j{}'$, thereby achieving a unit hiring probability (at the expense of some other worker quoting $w_j{}^\circ$), is the best reply for worker *j*, provided ε is sufficiently small. In fact, $EU_j(w_j{}', \boldsymbol{w}_{-j}{}^\circ) = U(w_j{}^\circ - \varepsilon)$ whereas $EU_j(\boldsymbol{w}^\circ) = U(w_j{}^\circ) p_j(\boldsymbol{w}^\circ) + U(w^r)(1 - p_j(\boldsymbol{w}^\circ))$, so that $EU_j(w_j{}', \boldsymbol{w}_{-j}{}^\circ) > EU_j(\boldsymbol{w}^\circ)$ if and only if $U(w_j{}^\circ - \varepsilon) - U(w^r) > p_j(\boldsymbol{w}^\circ)\big(U(w_j{}^\circ) - U(w^r)\big)$. With $w^w > w^r$, sufficient smallness of ε guarantees that the last condition is met. Further, $w_j{}'$ is easily understood to be in fact a best reply and not just better than $w_j{}^\circ$: on the one hand, $EU_j(w_j > w_j{}^\circ, \boldsymbol{w}_{-j}{}^\circ) = U(w^r)$, as worker *j*'s hiring probability would fall to zero if quoting a wage higher than the current one; on the other hand, $EU_j(w_j \le w_j{}', \boldsymbol{w}_{-j}{}^\circ) = U(w_j)$. *QED*

Wage calls $\boldsymbol{w}^\circ > \boldsymbol{w}^w$ may imply $p_j(\boldsymbol{w}^\circ) = 0$, and hence $EU_j(\boldsymbol{w}^\circ) = U(w^r)$, for some *j*. By now it should be clear that such a situation arises if and only if $L_H(\boldsymbol{w}^\circ) \le \#\{i : w_i{}^\circ < w_j{}^\circ\}$, where $w_j{}^\circ$ now denotes the wage quoted by any $j : p_j(\boldsymbol{w}^\circ) = 0$. A scrutiny of the wage choice made by any such worker gives the next result.

Lemma 2.2. With $w^w > w^r$, at $\boldsymbol{w}^\circ > \boldsymbol{w}^w$ any *j*: $p_j(\boldsymbol{w}^\circ) = 0$ has failed to make a best reply, this being instead $w_j{}' = \max\left[MP\left(L_H(\boldsymbol{w}^\circ)\right), w^{h_e} - \varepsilon\right]$.

Proof. First of all, notice that, at $\boldsymbol{w}^\circ > \boldsymbol{w}^w$, $L_H(\boldsymbol{w}^\circ) < L^w = n$ and hence $MP\left(L_H(\boldsymbol{w}^\circ)\right) \ge w^w$. Thus, for any *j*: $p_j(\boldsymbol{w}^\circ) < 1$, $w_j{}' = \max\left[MP\left(L_H(\boldsymbol{w}^\circ)\right), w^{h_e} - \varepsilon\right] \ge w^w$ and $EU_j(w_j{}', \boldsymbol{w}_{-j}{}^\circ) = U(w_j{}') > U(w^r)$ as $w^w > w^r$. As a result, for any *j*: $p_j(\boldsymbol{w}^\circ) = 0$, $EU_j(w_j{}', \boldsymbol{w}_{-j}{}^\circ) > EU_j(\boldsymbol{w}^\circ)$ given that $EU_j(\boldsymbol{w}^\circ) = U(w^r)$. Again, it must be understood that $w_j{}'$ is a best reply to $\boldsymbol{w}_{-j}{}^\circ$ and not just a better reply than $w_j{}^\circ$. In particular, quoting $w_j{}'$ is better than quoting any $w_j > w_j{}'$, with either $w_j{}' = MP\left(L_H(\boldsymbol{w}^\circ)\right)$ or $w_j{}' = w^{h_e} - \varepsilon$. In the former case,

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and employment would have also been increased in the limit case in which $w_j{}^\circ = MP(L_H(\boldsymbol{w}^\circ))$, given that, by assumption, at equal profits the firm chooses the higher level of employment.

$p_j(w_j, \boldsymbol{w}_{-j}°) = 0$ and thus $EU_j(w_j, \boldsymbol{w}_{-j}°) = U(w^r)$ for all $w_j > w_j'$.[12] In the latter case, in which $w_j' + \varepsilon = w^{h_e}$, $p_j(w_j > w_j' + \varepsilon, \boldsymbol{w}_{-j}°) = 0$ and thus $EU_j(w_j > w_j' + \varepsilon, \boldsymbol{w}_{-j}°) = U(w^r)$, while $0 < p_j(w_j = w_j' + \varepsilon, \boldsymbol{w}_{-j}°) < 1$[13] and thus, by Lemma 2.1, $EU_j(w_j = w_j' + \varepsilon, \boldsymbol{w}_{-j}°) < EU_j(w_j = w_j', \boldsymbol{w}_{-j}°)$. *QED*

Before drawing the implications of the results achieved so far, it is worth for later use to give another result, relating to any *i*: $p_i(\boldsymbol{w}°) = 1$.

Lemma 2.3. Let $w^w > w^r$. At $\boldsymbol{w}° > \boldsymbol{w}^w$, for any *i*: $p_i(\boldsymbol{w}°) = 1$ the best reply is $w_i' = \min\left[MP(L_H(\boldsymbol{w}°) - 1), w^{l_u} - \varepsilon\right]$.

Proof. Let there be some *i*: $p_i(\boldsymbol{w}°) = 1$. First of all, with $\boldsymbol{w}° > \boldsymbol{w}^w$ and $w^w > w^r$, it follows immediately from $w_i' \geq w_i°$ that $EU_i(w_i = w_i', \boldsymbol{w}_{-i}°) = U(w_i') > U(w^r)$. We now show that $EU_i(w_i \geq w_i' + \varepsilon, \boldsymbol{w}_{-i}°) < U(w_i')$ with either $MP(L_H(\boldsymbol{w}°) - 1) \leq w^{l_u} - \varepsilon$ or $MP(L_H(\boldsymbol{w}°) - 1) > w^{l_u} - \varepsilon$. In the former case, $p_i(w_i, \boldsymbol{w}_{-i}°) = 0$ and thus $EU_i(w_i, \boldsymbol{w}_{-i}°) = U(w^r)$ at all $w_i \geq w_i' + \varepsilon$ because the firm would reduce employment by one unit laying off worker *i* as soon as $w_i > MP(L_H(\boldsymbol{w}°) - 1)$. As to the latter case: first of all, $p_i(w_i, \boldsymbol{w}_{-i}°) = 0$ and thus $EU_i(w_i, \boldsymbol{w}_{-i}°) = U(w^r)$ at $w_i > w_i' + \varepsilon = w^{l_u}$, as worker *i* would then be displaced by a worker quoting w^{l_u}; second, $0 < p_i(w_i, \boldsymbol{w}_{-i}°) < 1$ at $w_i = w_i' + \varepsilon = w^{l_u}$ - worker *i* would now compete for a job with worker(s) already quoting w^{l_u} - and thus, by Lemma 2.1, $EU_i(w_i = w_i' + \varepsilon, \boldsymbol{w}_{-i}°) < EU(w_i = w_i', \boldsymbol{w}_{-i}°))$. *QED*

The next proposition follows immediately from Proposition 1 and Lemmas 2.1 and 2.2.

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12 When $w_j' = MP(L_H(\boldsymbol{w}°)) > w^{h_e}$, at vectors of wage calls $(w_j > w_j', \boldsymbol{w}_{-j}°)$ the level of employment remains unchanged and exactly the same workers are hired as at $\boldsymbol{w}°$ given that $w_j > MP(L_H(\boldsymbol{w}°)) > w^{h_e}$.

13 When $w_j' = w^{h_e} - \varepsilon$, at vector $(w_j = w^{h_e}, \boldsymbol{w}_{-j}°)$ the level of employment remains the same as at $\boldsymbol{w}°$ and j competes for jobs with workers already quoting w^{h_e} given that $w_j = w^{h_e} > MP(L_H(\boldsymbol{w}°))$.

Proposition 2. With $w^w > w^r$ the Walrasian vector of wage calls $\boldsymbol{w}^w$ is the unique equilibrium.

Now we turn to the case in which $w^w = w^r$. It will be shown that, while uniqueness of equilibrium does not literally hold in this case, other equilibria are nonetheless very "close" to $\boldsymbol{w}^w$ and involve the same consequences (or approximately so) as $\boldsymbol{w}^w$ on workers' expected utility and total employment.

As a first step, the next two propositions make, with regard to vectors $\boldsymbol{w}^\circ >> \boldsymbol{w}^w$, similar points as Lemmas 2.1 and 2.2.

Lemma 3.1. With $w^w = w^r$, at $\boldsymbol{w}^\circ >> \boldsymbol{w}^w$ any $j\!: 0 < p_j(\boldsymbol{w}^\circ) < 1$ has normally failed to make a best reply, this being instead $w_j{}' = w_j{}^\circ - \varepsilon$. The only exception arises at $\boldsymbol{w}^\circ$ "so close" to $\boldsymbol{w}^w$ that $w_j{}^\circ = w^r + \varepsilon$ for $j: 0 < p_j(\boldsymbol{w}^\circ) < 1$.

Proof. As to the general point, by reviewing the proof of Lemma 2.1 one can check that it also applies with $w^w = w^r$, so long as $w_j{}^\circ > w^r + \varepsilon$ for j: $0 < p_j(\boldsymbol{w}^\circ) < 1$. As to the exception, notice, first of all, how close to $\boldsymbol{w}^w$ are vectors $\boldsymbol{w}^\circ >> \boldsymbol{w}^w$ such that $w_j{}^\circ = w^r + \varepsilon$ for $j: 0 < p_j(\boldsymbol{w}^\circ) < 1$. These vectors are all $\boldsymbol{w}^\circ >> \boldsymbol{w}^w$: $\#\{j : w_j{}^\circ = w^r + \varepsilon\} > L_H(\boldsymbol{w}^\circ)$. Bearing in mind that $MP(L^w) < w^r \le MP(L^w - 1)$ when $w^w = w^r$, inequality $\#\{j : w_j{}^\circ = w^r + \varepsilon\} > L_H(\boldsymbol{w}^\circ)$ in turn is met at wage calls such that $\#\{j : w_j{}^\circ = w^r + \varepsilon\} > L^w$, as these wage calls imply $L_H(\boldsymbol{w}^\circ) = L^w$ whenever $MP(L^w) < w^r < MP(L^w - 1)$ while implying $L_H(\boldsymbol{w}^\circ) = L^w - 1$ in the limit case in which $w^r = MP(L^w - 1)$; as to this limit case, inequality $\#\{j : w_j{}^\circ = w^r + \varepsilon\} > L_H(\boldsymbol{w}^\circ)$ is met as well at wage calls such that $\#\{j : w_j{}^\circ = w^r + \varepsilon\} = L^w$, also implying $L_H(\boldsymbol{w}^\circ) = L^w - 1$. Now, at these wage calls, $EU_j(\boldsymbol{w}^\circ) = U(w^r + \varepsilon) \times \left(L_H(\boldsymbol{w}^\circ)/\#\{j : w_j{}^\circ = w^r + \varepsilon\}\right) + U(w^r) \times \left(1 - L_H(\boldsymbol{w}^\circ)/\#\{j : w_j{}^\circ = w^r + \varepsilon\}\right)$ for $j: 0 < p_j(\boldsymbol{w}^\circ) < 1$, hence $EU_j(\boldsymbol{w}^\circ)$ is (just a bit) higher than $EU_j(w_j = w_j{}' = w^r, \boldsymbol{w}_{-j}{}^\circ) = U(w^r)$ or $EU_j(w_j > w_j{}^\circ, \boldsymbol{w}_{-j}{}^\circ) = U(w^r)$. *QED*

Lemma 3.2. With $w^w = w^r$, at $\boldsymbol{w}^\circ >> \boldsymbol{w}^w$ any *j*: $p_j(\boldsymbol{w}^\circ) = 0$ has failed to make a best reply, this being, as a norm, w_j'.

Proof. First of all, it must be noted that $\boldsymbol{w}^\circ >> \boldsymbol{w}^w$ implies $w^{h_e} \geq w^r + \varepsilon$ and hence $w_j' = max\left[MP(L_H(\boldsymbol{w}^\circ)), w^{h_e} - \varepsilon\right] \geq w^r$ for *j*: $p_j(\boldsymbol{w}^\circ) < 1$. More specifically, it is normally $w_j' > w^r$ at $\boldsymbol{w}^\circ >> \boldsymbol{w}^w$; when this is so, one can easily check that the proof of Lemma 2.3 applies so that quoting w_j' is a best reply for any *j*: $p_j(\boldsymbol{w}^\circ) = 0$. As to wage calls $\boldsymbol{w}^\circ >> \boldsymbol{w}^w$ for which $w_j' = w^r$ for *j*: $p_j(\boldsymbol{w}^\circ) < 1$, notice that $w_j' = w^r$ requires $w^{h_e} = w^r + \varepsilon$ and $MP(L_H(\boldsymbol{w}^\circ)) \leq w^r$. The last two conditions in turn are met at $\boldsymbol{w}^\circ >> \boldsymbol{w}^w : \#\{i : w_i^\circ = w^r + \varepsilon\} \geq L^w$, what implies $L_H(\boldsymbol{w}^\circ) = L^w$ and hence $MP(L_H(\boldsymbol{w}^\circ)) < w^r$ whenever $MP(L^w) < w^r < MP(L^w - 1)$; in the limit case in which $w^r = MP(L^w - 1)$, the conditions under discussion are met at $\boldsymbol{w}^\circ >> \boldsymbol{w}^w : \#\{i : w_i^\circ = w^r + \varepsilon\} \geq L^w - 1$, implying $L_H(\boldsymbol{w}^\circ) = L^w - 1$ and $MP(L_H(\boldsymbol{w}^\circ)) = w^r$. Corresponding to these wage calls, $p_j(\boldsymbol{w}^\circ) = 0$ and thus $EU_j(\boldsymbol{w}^\circ) = U(w^r)$ for any $j : w_j^\circ > w^r + \varepsilon$. For any such worker $EU_j(w_j', \boldsymbol{w}_{-j}^\circ) = U(w^r) = U_j(\boldsymbol{w}^\circ)$, so worker *j* gains nothing from achieving a unit hiring probability. On the other hand, for any such worker it is optimal to reply $w_j = w^{h_e} = w^r + \varepsilon$: indeed, $EU_j(w_j = w^{h_e}, \boldsymbol{w}^\circ_{-j})$ is a bit higher than $U(w^r)$, as $0 < p_j(w_j = w^{h_e}, \boldsymbol{w}_{-j}^\circ) < 1$.[14] *QED*

In view of Lemmas 3.1 and 3.2 the next result can be established.

Proposition 3. With $w^w = w^r$, among vectors $\boldsymbol{w}^\circ >> \boldsymbol{w}^w$ the only equilibrium is $\boldsymbol{w}^\circ : w_j^\circ = w^r + \varepsilon$ for all $j \in N$, yielding each worker just a bit more than Walrasian utility $U(w^r)$.

Proof. (*Sufficiency*) it is easy to understand that at $\boldsymbol{w}^\circ : w_j^\circ = w^r + \varepsilon$ for all $j \in N$, each worker has made a best reply. On the one hand,

[14] Since $w^{h_e} > MP(L_H(\boldsymbol{w}^\circ))$, employment remains unchanged when worker $j : w_j^\circ > w^r + \varepsilon$ turns to quoting w^{h_e}, so this worker would now be competing for a job with workers already quoting w^{h_e}.

$EU_j(\boldsymbol{w}^\circ) = U(w^r+\varepsilon)\times(L_H(\boldsymbol{w}^\circ)/n) + U(w^r)\times(1-L_H(\boldsymbol{w}^\circ)/n)$ is just a bit higher than $U(w^r)$; on the other hand, $EU_j(w_j, \boldsymbol{w}_{-j}{}^\circ) = U(w^r)$ with either $w_j > w^r+\varepsilon$, as $p_j(w_j > w^r+\varepsilon, \boldsymbol{w}_{-j}{}^\circ) = 0$, or $w_j = w^r$. Incidentally, it is worth noting that at this equilibrium $L_H(\boldsymbol{w}^\circ) = L^w$ in the normal case in which $MP(L^w) < w^r < MP(L^w-1)$ while $L_H(\boldsymbol{w}^\circ) = L^w - 1$ in the limit case in which $w^r = MP(L^w-1)$.

(*Necessity*) To start with, notice that, at $\boldsymbol{w}^\circ >> \boldsymbol{w}^w$, $p_j(\boldsymbol{w}^\circ) < 1$ for *j*: $w_j{}^\circ = \hat{w}^\circ$, where $\hat{w}^\circ$ denotes the highest wage call at $\boldsymbol{w}^\circ$. At vectors $\boldsymbol{w}^\circ >> \boldsymbol{w}^w$: $\#\{i : w_i{}^\circ = \hat{w}^\circ\} = 1$ it is clearly $p_j(\boldsymbol{w}^\circ) = 0$ for *j*: $w_j{}^\circ = \hat{w}^\circ$, which suffices, by Lemma 3.2, to rule out any such $\boldsymbol{w}^\circ$ from being an equilibrium. So we are left with vectors $\boldsymbol{w}^\circ >> \boldsymbol{w}^w$: $\#\{j : w_j{}^\circ = \hat{w}^\circ\} > 1$. In order for $0 < p_j(\boldsymbol{w}^\circ) < 1$ for *j*: $w_j{}^\circ = \hat{w}^\circ$, it must be $\#\{i : w_i{}^\circ < \hat{w}^\circ\} < L_H(\boldsymbol{w}^\circ) < \#\{i : w_i{}^\circ \le \hat{w}^\circ\}$. By Lemma 3.1, any *j*: $w_j{}^\circ = \hat{w}^\circ$ has only made a best reply if $\hat{w}^\circ = w^r + \varepsilon$. This is turn can only happen at $\boldsymbol{w}^\circ : w_j{}^\circ = w^r + \varepsilon$ for all $j \in N$. *QED*

There remain to discuss vectors $\boldsymbol{w}^\circ > \boldsymbol{w}^w$: $w_i{}^\circ = w^w$ *for some i*. The main result is conveyed by the next proposition.

Proposition 4. With $w^w = w^r$, among vectors of wage calls $\boldsymbol{w}^\circ > \boldsymbol{w}^w$: $w_i{}^\circ = w^w$ *for some i*, the only equilibria are $\boldsymbol{w}^\circ > \boldsymbol{w}^w : \#\{i : w_i{}^\circ = w^r\} > L^w$ or - in the limit case in which $w^r = MP(L^w-1)$ - $\boldsymbol{w}^\circ > \boldsymbol{w}^w : \#\{i : w_i{}^\circ = w^r\} \ge L^w$. At any such equilibrium each worker gets an expected utility of $U(w^r)$, just as at the Walrasian vector $\boldsymbol{w}^w$.

Proof. (*Necessity*) *See Appendix*. (All vectors $\boldsymbol{w}^\circ > \boldsymbol{w}^w$: $w_i{}^\circ = w^w$ *for some i* apart from those under discussion here are examined in the appendix, showing that none of them can be an equilibrium.)

(*Sufficiency*) At $\boldsymbol{w}^\circ > \boldsymbol{w}^w : \#\{i : w_i{}^\circ = w^r\} > L^w$,[15] we have $L_H(\boldsymbol{w}^\circ) = L^w$, $w^{h_e} = w^r$, $MP(L_H(\boldsymbol{w}^\circ)) < w^r$, and $p_j(\boldsymbol{w}^\circ) = 0$ for $j : w_j{}^\circ > w^r$. Any *j*: $p_j(\boldsymbol{w}^\circ) = 0$, for whom

[15] By the way, such vectors exist to the extent that $L^w < n-1$.

$EU_j(\boldsymbol{w}^\circ) = U(w^r)$, has already made a best reply, given that $EU_j(w_j = w_j', \boldsymbol{w}_{-j}{}^\circ) = U(w^r - \varepsilon) < U(w^r)$ while $EU_j(w_j = w^r, \boldsymbol{w}_{-j}{}^\circ) = U(w^r)$. As to $i : w_i{}^\circ = w^r$, it is $0 < p_i(\boldsymbol{w}^\circ) = L^w / \#\{i : w_i{}^\circ = w^r\} < 1$ and $EU_i(\boldsymbol{w}^\circ) = U(w^r)$. Clearly worker *i* would be as well off as at $\boldsymbol{w}^\circ$ if quoting $w_i > w^r$, as $p_i(w_i > w^r, \boldsymbol{w}_{-i}{}^\circ) = 0$.

The argument runs likewise to show that, - in the limit case in which $w^r = MP(L^w - 1)$ - any vector of wage calls $\boldsymbol{w}^\circ > \boldsymbol{w}^w : \#\{i : w_i{}^\circ = w^r\} \geq L^w$ is an equilibrium. *QED*

In view of Propositions 1, 3 and 4 it should be clear that, while there exists a multiplicity of equilibria of the wage game when $w^w = w^r$, all of them exhibit (precisely or approximately) Walrasian features in terms of workers' utility and total employment.

4. Convergence to Walrasian equilibrium

Now we turn to the issue of how a Nash equilibrium of the static game might emerge in a repeated interaction between wage setting workers. To keep things simple, this task is accomplished under the assumption that $w^w > w^r$. Then, according to the analysis above, it is only when all workers have quoted the Walrasian wage that everyone is satisfied with his own wage decision given the others'. Now, let $\boldsymbol{w}(t) > \boldsymbol{w}^w$, $\boldsymbol{w}(t)$ denoting the vector of wage calls in period *t*. (The event of $\boldsymbol{w}(t)$ having some component lower than w^w is immediately discarded in view of the analysis in the previous section (p. 9.) The question is whether $\boldsymbol{w}(t)$ converges to $\boldsymbol{w}^w$ as *t* goes on.

When taking his own current wage decision, each worker is here assumed to look only at the immediate consequences. Thus each worker takes his current wage decision based on the wages he expects to be currently quoted by others. These single valued expectations are assumed to depend somehow on wages previously quoted. Further, each worker is assumed to have perfect information about wages quoted by others in the last period.

We first establish convergence of wages to the Walrasian wage under static expectations. Let $\boldsymbol{w}(t) > \boldsymbol{w}^w$ in some initial period *t*. Under static expectations, each worker quotes in *t*+1 what is in period t his best reply to the wages then quoted by the other workers. Consequently $\boldsymbol{w}(t+1) \neq \boldsymbol{w}(t)$, given that at least one worker has not made a best reply in period t. In particular, look at any worker $j: p_j(\boldsymbol{w}(t)) < 1$. In view of Lemmas 2.1 and 2.2, $w_j(t+1) = w_j'(t) = max\lfloor MP(L_H(\boldsymbol{w}(t)), w^{h_e}(t) - \varepsilon \rfloor$, where $w_j'(t)$ denotes the highest wage worker j might have replied to $\boldsymbol{w}_{-j}(t)$ in period t consistently with a unit hiring probability. Therefore $w_j(t+1) < w_j(t)$ for any $j: p_j(\boldsymbol{w}(t) < 1$. Turn now to any worker $i: p_i(\boldsymbol{w}(t)) = 1$. In view of Lemma 2.3, $w_i(t+1) = w_i'(t) = min\lfloor MP(L_H(\boldsymbol{w}(t)) - 1), w^{l_u}(t) - \varepsilon \rfloor$, where $w^{l_u}(t)$ denotes the lowest wage quoted in period t among workers not hired. Notice that $w^{l_u}(t) - \varepsilon < w_j(t)$,[16] where $w_j(t)$ denotes the wage quoted by any worker $j: p_j(\boldsymbol{w}(t)) < 1$. As a result, $w_i(t+1) = w_i'(t) < w_j(t)$ for any $i: p_i(\boldsymbol{w}(t)) = 1$ and $j: p_j(\boldsymbol{w}(t)) < 1$.

The simple implication of all the above is that $\hat{w}(t+1) < \hat{w}(t)$ when $\boldsymbol{w}(t) > \boldsymbol{w}^w$, where $\hat{w}(t)$ and $\hat{w}(t+1)$ denote the highest wage calls in periods t and t+1, respectively. Likewise, $\hat{w}(t+2) < \hat{w}(t+1)$ if $\boldsymbol{w}(t+1) > \boldsymbol{w}^w$, and so on. Therefore, the Walrasian vector of wage calls will be reached, sooner or later.

Of course, workers keeping on holding static expectations in disequilibrium is hardly acceptable, in that the resulting systematic errors are bound to get noticed. For example, one can see that in the process just described wage calls currently being made by workers who lastly faced a lower-than-one hiring probability are over-estimated by the other workers. However, one by no means needs to stick to static expectations to obtain convergence to the Walrasian wage vector. As we now show, the

[16] It must be understood that this inequality holds with either $p_j(\boldsymbol{w}(t)) = 0$ or $0 < p_j(\boldsymbol{w}(t)) < 1$. In the former case, $w^{l_u}(t) \leq w_j(t)$ is implicit in the very definition of $w^{l_u}(t)$; in the latter case, $w^{l_u}(t) - \varepsilon < w_j(t)$ follows again from the definition of $w^{l_u}(t)$ as soon as one recalls that there are workers quoting $w_j(t)$ who are unemployed along with others who are employed.

property - *sufficient* for convergence – that $\hat{w}(t+1) < \hat{w}(t)$ when $\boldsymbol{w}(t) > \boldsymbol{w}^{w}$ can be retained under a much more general assumption.

To start with, note that $\hat{w}(t) > MP(L_H(\boldsymbol{w}(t)))$ when $\boldsymbol{w}(t) > \boldsymbol{w}^{w}$ (implying $L_H(\boldsymbol{w}(t)) < n$), otherwise profits would clearly be increased by hiring one additional worker. Let $E_j \boldsymbol{w}_{-j}(t+1)$ denote the vector representing worker *j*'s expectations about the wages quoted in *t*+1 by all $i \neq j$; its generic component, denoted $E_j w_i(t+1)$, represents worker *i*'s wage call according to *j*'s expectations. We have this result.

Proposition 5. A *sufficient* condition for $\hat{w}(t+1) < \hat{w}(t)$ when $\boldsymbol{w}(t) > \boldsymbol{w}^{w}$ is that $E_j \boldsymbol{w}_{-j}(t+1) : \#\{i \neq j : E_j w_i(t+1) \leq \hat{w}(t)\} \geq L_H(\boldsymbol{w}(t))$ for all *j*.

Proof. Before proceeding, it is worth noting that the condition under discussion is actually a very weak one in a situation of excess supply: in simple words, it states that each worker expects that a sufficiently large number of workers - not less than the number of workers previously employed - does not raise their own wages *above* the highest wage call made in period *t*.

The proposition is proved by showing that under this condition $w_j(t+1) < \hat{w}(t)$ for all *j*. To start with, notice that, by Lemmas 2.1, 2.2, and 2.3, it always pays any worker *j* to quote the highest wage consistent with a unit hiring probability, whatever $E_j \boldsymbol{w}_{-j}(\ ) \geq \boldsymbol{w}^{w}$. Next we show that $p_j\left(w_j = \hat{w}(t), E_j \boldsymbol{w}_{-j}(t+1)\right) < 1$ with $E_j \boldsymbol{w}_{-j}(t+1) : \#\{i \neq j : E_j w_i(t+1) \leq \hat{w}(t)\} \geq L_H(\boldsymbol{w}(t))$, where $p_j\left(w_j = \hat{w}(t), E_j \boldsymbol{w}_{-j}(t+1)\right)$ denotes worker *j*'s hiring probability in period *t*+1, as (correctly) perceived by *j* conditional on wage calls $\left(w_j = \hat{w}(t), E_j \boldsymbol{w}_{-j}(t+1)\right)$ in *t*+1. Suppose, contrary to the claim, that $p_j\left(w_j = \hat{w}(t), E_j \boldsymbol{w}_{-j}(t+1)\right) = 1$, so worker *j* might consider quoting, say, $w_j = \hat{w}(t)$.

Consistently with this conjecture, $p_i\left(w_j = \hat{w}(t), E_j\mathbf{w}_{-j}(t+1)\right)=1$ for all $i: E_j w_i(t+1) \le \hat{w}(t)$, $p_i(\ )$ denoting worker i's hiring probability as (correctly) anticipated by j conditional on the stipulated wage calls. Recalling that $\#\{i \ne j : E_j w_i(t+1) \le \hat{w}(t)\} \ge L_H(\mathbf{w}(t))$, this in turn implies that total employment, as (correctly) anticipated by j under the stipulated wage calls, is $L_H\left(w_j = \hat{w}(t), E_j\mathbf{w}_{-j}(t+1)\right) \ge L_H(\mathbf{w}(t))+1$. However, as already noted, $\hat{w}(t) > MP(L_H(\mathbf{w}(t))$; further, it must be $MP(L_H(\mathbf{w}(t)) \ge MP\left(L_H\left(w_j = \hat{w}(t), E_j\mathbf{w}_{-j}(t+1)\right)-1\right)$ in order for $L_H\left(w_j = \hat{w}(t), E_j\mathbf{w}_{-j}(t+1)\right) \ge L_H(\mathbf{w}(t))+1$. The implication would be $\hat{w}(t) > MP\left(L_H\left(w_j = \hat{w}(t), E_j\mathbf{w}_{-j}(t+1)\right)-1\right)$, thus contradicting condition (i) for profit maximization, for the firm would make higher profits by laying-off worker *j*. The conjecture that $p_j\left(w_j = \hat{w}(t), E_j\mathbf{w}_{-j}(t+1)\right)=1$ must then be rejected to avoid this contradiction. Therefore, $w_j'(t+1) < \hat{w}(t)$, $w_j'(t+1)$ denoting the highest wage perceived by *j* as yielding him a unit hiring probability in the face of $E_j\mathbf{w}_{-j}(t+1)$, and thus $w_j(t+1) = w_j'(t+1) < \hat{w}(t)$, where $w_j(t+1)$ is the wage actually quoted by *j* in *t*+1. As a result, when this condition on expectations holds for all *j*, $w_j(t+1) < \hat{w}(t)$ for all *j* and thus $\hat{w}(t+1) < \hat{w}(t)$ when $\mathbf{w}(t) > \mathbf{w}^w$. *QED*

5. Conclusions

The simple model presented above has formalised, for the labour market, the traditional view that competition among the sellers would drive prices down as long as there is excess supply. It has been shown that, under the assumptions of the model, each worker has in fact an incentive (at least in the short term) to try to secure employment to himself by asking a wage sufficiently low given the wages he expects to be quoted by all other workers. From this we have been able to derive a sufficient condition on expectations in order for the level of wages to fall whenever it is higher than the Walrasian wage: in fact, it suffices that each worker expects a sufficiently large number of workers to be quoting, today, a wage that is *not higher* than the *highest* wage that was quoted yesterday. This condition is quite reasonable given that some workers are involuntary unemployed.[17]

As already emphasised, our analysis is close in spirit to Edgeworth's dynamic model of price competition in a duopoly. One difference should be emphasised, though. Edgeworth analysis may be interpreted in the sense that each seller has static expectations over the price currently being quoted by the rival, what has been criticised on grounds that it leads to continuous errors in expectations (Shubik, 1959, p. 92; Brown Kruse, Russenti, Reynolds, and Smith, 1994, p. 351).[18] In this respect, however, many would perhaps agree that the difficulty to be avoided is to adopt any expectation regime which makes current values being persistently *overestimated* or *underestimated*, for the way in which expectations are formed will be modified as soon as the resulting biases are noticed. We avoid such a difficulty by providing a sufficient condition for the convergence result which does not require static expectations or any simple formula relating expected wages to past wages. Of course, so long as the vector of wage calls differs in some component from the Walrasian wage, expectations have clearly been disappointed for at least some worker; this is an unavoidable feature of disequilibrium, and a quite acceptable one provided the sign of the error does not remain unchanged over time for every individual.

Finally, we want to suggest that the model above, if properly modified, may also help understand why wages often remain stable in the presence of involuntary unemployment. One kind of explanation which has recently attracted considerable attention emphasises the role of social conventions, according to which undercutting the prevailing wage is "unfair" (Akerlof, 1980; Kahneman, Knetsch, and Thaler, 1986; Solow, 1990, pp. 48-49; Fehr, Kirchsteiger, and Riedl, 1993). As soon as social conventions of this kind are introduced into the model, undercutting the prevailing wage to gain employment may no longer be in the best interest of the worker, for violating widely accepted standards of fairness might entail external as well as internal sanctions. The model presented above, on the other hand, may give insights about the rationale of similar social conventions, just by showing what would happen in case they were absent. Consider, for example,

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[17] For the sake of brevity, this analysis of convergence has been carried out only for the case in which the Walrasian wage is higher than the reservation wage; however, it should not be difficult to see that convergence would obtain as well when the Walrasian wage is equal to the reservation wage.

the case in which the Walrasian wage is equal to the reservation wage. Then it will be readily understood that, from the workers' vantage point, a situation in which all workers refrain from wage undercutting – all of them quoting some wage higher than the reservation wage and thus bearing the risk of being involuntary unemployed - is better than Walrasian equilibrium, in which all workers are quoting the reservation wage and all unemployment is voluntary. Under this or similar circumstances,[19] the workers might have learnt from experience that wage competition for jobs would be against their common interest. It seems quite possible that social conventions according to which it is "unfair" to undercut the prevailing wage arose out of this perception.

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[18] The assumption of static expectations on the prices (or quantities) currently being chosen by the rivals is also made by Quin and Stuart (1997) in their generalised oligopolistic model in which, at every date, each firm chooses among Cournot and Bertrand strategies.

[19] Provided the elasticity of labour demand is sufficiently low, all workers quoting some wage higher than the Walrasian wage may be better, for each worker, than Walrasian equilibrium even when the Walrasian wage is higher than the reservation wage (De Francesco, 1993).

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Appendix

Proof of Proposition 4 (Necessity)

What has to be shown is that, with $w^w = w^r$, *no vector* $\boldsymbol{w}^\circ > \boldsymbol{w}^w : w_i{}^\circ = w^w$ for some i *other than those considered in Proposition 4 can be an equilibrium.*

Consider first vectors $\boldsymbol{w}^\circ > \boldsymbol{w}^w : 0 < \#\{i : w_i{}^\circ = w^w\} < L^w - 1$. *Then* $p_j(\boldsymbol{w}^\circ) < 1$ *for at least some* $j : w_j{}^\circ > w^r$. *As to any* $j : p_j(\boldsymbol{w}^\circ) = 0$, *he has not replied optimally, the best reply* *being normally* $w_j{}'$ *and in any case a wage lower than* $w_j{}^\circ$. [20] *Turning to any* $j : 0 < p_j(\boldsymbol{w}^\circ) < 1$, *it must be noted that, with* $\boldsymbol{w}^\circ > \boldsymbol{w}^w : 0 < \#\{i : w_i{}^\circ = w^w\} < L^w - 1$, *it can either be* $L_H(\boldsymbol{w}^\circ) < L^w - 1$ *or* $L_H(\boldsymbol{w}^\circ) \in \{L^w - 1, L^w\}$. *When the former obtains the best reply is* $w_j{}' = w_j{}^\circ - \varepsilon$ *for* $j : 0 < p_j(\boldsymbol{w}^\circ) < 1$ *and the same holds, with a few exceptions, in the latter case too.* [21]

Wage calls $\boldsymbol{w}^\circ > \boldsymbol{w}^w : \#\{i : w_i{}^\circ = w^w\} = L^w$ cannot, as a rule, be equilibria. While any $j : w_j{}^\circ > w^w$ has made a best reply, this is not normally the case with any $i : w_i{}^\circ = w^w$. For any such worker $p_i(\boldsymbol{w}^\circ) = 1$ and thus $w_i{}' = \min\left[MP\left(L^w - 1\right), w^{l_u} - \varepsilon\right] \geq w_i{}^\circ$. To the extent that $w_i{}' > w^r$ worker *i* has clearly failed to make a best reply. The only case in which this is not so is when $MP\left(L^w - 1\right) = w^r$ rather than $MP(L^w) < w^r < MP(L^w - 1)$; then $w_i{}^\circ$ is a best reply and thus $\boldsymbol{w}^\circ$ is an equilibrium, as already noted in Proposition 4.[22]

There remain wage calls $\boldsymbol{w}^\circ > \boldsymbol{w}^w : \#\{i : w_i{}^\circ = w^w\} = L^w - 1$, entailing $L_H(\boldsymbol{w}^\circ) \in \{L^w - 1, L^w\}$. If $L_H(\boldsymbol{w}^\circ) = L^w - 1$ then $EU_j(\boldsymbol{w}^\circ) = U(w^r)$ for any $j : w_j{}^\circ > w^w$. As a rule, any such worker should have replied $w_j{}' = \max\left[MP\left(L_H(\boldsymbol{w}^\circ)\right), w^{h_e}\right] = MP(L^w - 1)$; indeed,

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[20] The special case occurs when $w_j{}' = w^r$, i. e., when it is both $MP\left(L_H(\boldsymbol{w}^\circ)\right) \leq w^r$ and $w^{h_e} = w^r + \varepsilon$. Then, for any $j : p_j(\boldsymbol{w}^\circ) = 0$, the best reply is $w_j = w^{h_e} < w_j{}^\circ$ rather than $w_j{}'$; indeed, $0 < p_j(w_j = w^{h_e}, \boldsymbol{w}_{-i}{}^\circ) < 1$ and thus $U(w^r) < EU_j(w_j = w^{h_e}, \boldsymbol{w}_{-i}{}^\circ) < U(w^r + \varepsilon)$, whereas $EU_j(w_j = w_j{}', \boldsymbol{w}_{-i}{}^\circ) = EU_j(\boldsymbol{w}^\circ) = U(w^r)$.

[21] The exceptions occur at $\boldsymbol{w}^\circ > \boldsymbol{w}^w : 0 < \#\{i : w_i{}^\circ = w^w\} < L^w - 1$ such that $L_H(\boldsymbol{w}^\circ) \in \{L^w - 1, L^w\}$ and $w_j{}^\circ = w^r + \varepsilon$ for $j : 0 < p_j(\boldsymbol{w}^\circ) < 1$. In this special case, in which $w^r \geq MP\left(L_H(\boldsymbol{w}^\circ)\right)$, it is $U(w^r) < EU_j(\boldsymbol{w}^\circ) < U(w^r + \varepsilon)$ for $j : 0 < p_j(\boldsymbol{w}^\circ) < 1$ while $EU_j(w_j = w_j{}', \boldsymbol{w}_{-i}{}^\circ) = U(w^r)$. Even in this case $\boldsymbol{w}^\circ$ would not be an equilibrium, given that, for any $i : w_i{}^\circ = w^w$, it is $EU_i(\boldsymbol{w}^\circ) = U(w^r) < EU_i(w_i = w^r + \varepsilon, \boldsymbol{w}_{-i}{}^\circ) < U(w^r + \varepsilon)$.

with $w^w = w^r$ it is normally $MP(L^w) < w^r < MP(L^w - 1)$ and hence $EU_j(w_j', \boldsymbol{w}_{-j}{}^\circ) = U(w_j') > U(w^r)$.[23] Now assume instead $L_H(\boldsymbol{w}^\circ) = L^w$. By condition (i) and recalling that $\#\{i : w_i{}^\circ = w^w\} = L^w - 1$, it is then $MP(L^w - 1) \geq w^{h_e} > w^r$. One thus understands that, in the present circumstances, it is inevitably $n > L^w$ because, with $w^w = w^r$, it would be $n = L^w$ only when $w^w = MP(L^w - 1)$, which is impossible in view of $MP(L^w - 1) > w^r = w^w$. Consequently there exists some worker *j*: $p_j(\boldsymbol{w}^\circ) < 1$. As to any $j : w_j{}^\circ > w^{h_e} > w^r$, for whom $p_j(\boldsymbol{w}^\circ) = 0$, he has clearly failed to make a best reply, this being $w_j' = max[MP(L_H(\boldsymbol{w}^\circ)), w^{h_e} - \varepsilon] = max[MP(L^w), w^{h_e} - \varepsilon] = w^{h_e} - \varepsilon$ so long as $w^{h_e} - \varepsilon > w^r$.[24] Turn next to any $j : 0 < p_j(\boldsymbol{w}^\circ) < 1$. This is the situation faced by any $j : w_j{}^\circ = w^{h_e}$ whenever $\#\{j : w_j{}^\circ = w^{h_e}\} > 1$. By the standard argument, *j* has normally failed to make a best reply, this being instead $w_j' = max[MP(L_H(\boldsymbol{w}^\circ)), w^{h_e} - \varepsilon] = w^{h_e} - \varepsilon$ so long as $w^{h_e} > w^r + \varepsilon$. Even in the special case in which $w^{h_e} = w^r + \varepsilon$, however, $\boldsymbol{w}^\circ$ is not an equilibrium: as already seen, any $i : w_i{}^\circ = w^r$ should have quoted $w_i = w^{h_e}$.

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[22] What if $MP(L^w - 1) > w^r$ and $\boldsymbol{w}^\circ > \boldsymbol{w}^w : \#\{i : w_i{}^\circ = w^w\} = L^w$ is such that $w^{l_u} = w^r + \varepsilon$? In such a case too $w_i' = \min[MP(L^w - 1), w^{l_u} - \varepsilon] = w_i{}^\circ$ for $i : w_i{}^\circ = w^w$. The best reply is now $w_i = w^{l_u}$, as it is $0 < p_i(w_i = w^{l_u}, \boldsymbol{w}_{-i}{}^\circ) < 1$ and hence $U(w^r) < EU_i(w_i = w^{l_u}, \boldsymbol{w}_{-i}{}^\circ) < U(w^{l_u})$.

[23] In the limit case in which $MP(L^w - 1) = w^r$ the worker under consideration has already made a best reply since then $EU_j(w_j', \boldsymbol{w}_{-j}{}^\circ) = U(w^r)$. Even in this case $\boldsymbol{w}^\circ$ is not an equilibrium, though, since any $i : w_i{}^\circ = w^w$ should have replied a higher wage. Indeed, worker *i*'s best reply is $w_i' = \min[MP(L_H(\boldsymbol{w}^\circ) - 1), w^{l_u} - \varepsilon] = \min[MP(L^w - 2), w^{l_u} - \varepsilon]$ whenever $w_i' > w^r$; in the special case in which $w^{l_u} = w^r + \varepsilon$ and hence $w_i' = w^r$, the best reply is w^{l_u} (similarly as was argued in the previous footnote).

[24] Similarly as was seen in the two previous footnotes, in the special case in which $w^{h_e} = w^r + \varepsilon$ the worker under consideration should have replied $w_j = w^{h_e}$.