QUADERNI

Università degli Studi di Siena
DIPARTIMENTO DI ECONOMIA POLITICA

LUIGI M. TOMASINI

A Model of Intercountries Income Distrubution

n. 315 - Marzo 2001

Abstract - Consider a system of N countries which exchange goods and services. The result of an interaction between two countries is determined by the wealth of each country and by its bargaining power expressed in appropriarte units. We find the wealth w as a function of b (bargaining power) and after some mathematics we obtain the bargainning power b as a function of w . This allows us to find the distribution function of wealth. As in a steady state the wealth of a country does not vary with time, from the distribution of wealth we can compute the distribution of income among countries.

The main result: Assuming the distribution of bargaining power is normnal, the distribution of wealth and income which depends on it is highly skewed : very few countries will be wealthy and very many poor.

Luigi M. Tomasini, Dipartimento di Economia Politica, Università di Siena

A MODEL OF INTERCOUNTRIES INCOME DISTRIBUTION

1 . Economic relations are, essentially, exchange relations. It is, generally, postulated that two or more economic entities will take part to an exchange if and only if this interelation is mutually advantageous. Such a schema does not consider the bargaining power each economic entity possesses.This deficiency is, obviously, connected with the difficulty of measuring bargaining power differences so to rank entities accordingly.

Assuming that we were able to rank economic entities according to their bargaining power,we could identify the number of economic entities which,at a given time,possess a given "level of bargaining power".

In this paper it is suggested that the distribution of wealth, and hence income, among economic entities depends on the distribution of bargaining power.

The model can be described in the following way: There is a universe of economic entities that interact among themselves, each in possession of two different attributes: bargaining power levels and wealth. The entities interact in pairs at equal intervals of time and ,as all interactions take place,by assumption in the same interval of time,each entity has an equal number of interactions per unit of time. In such a process one will lose and the other will gain wealth and this according to how much wealth the poorer of the two already has and to the level of bargaining power possessed by each of them. Therefore,after a number of interactions there will be a change in the distribution of wealth within the system.

2 . Consider a system of N, $N \geq 2$,countries which exchange goods among them. Each country can be described by two variables: the stock of wealth (w) and, the level of bargaining power (b).

Countries interact in pairs at equal interval of time; therefore each country has the same number of exchanges per unit of time. In such a process a country will lose while the other will gain wealth and this according to how much wealth the gainer has and to the difference in bargaining power.

Let w_1 , w_2 and b_1 , b_2 be the stock of wealth and the bargaining power level respectively, of countries 1 and 2, before exchange takes place and let α, ($\alpha > 0$) be an exchange coefficient in the interaction.

Consider the case where $w_1 < w_2$ and $b_1 \leq b_2$. After exchange takes place, country 1 will have a stock of wealth

(1) $w_1 = w_1 + \alpha (b_1 - b_2) w$

while country 2 will have a stock of wealth

(2) $w_2 = w_2 - \alpha (b_1 - b_2) w_1$

For values of $b_1 > b_2$, country 1 gains while country 2 loses. The opposite is true for $b_1 < b_2$. This implies that the country that gains in the exchange process is that one that has a higher level of bargaining power. From eqs (1) and (2) we see that gains and losses are, by assumption, proportional to the least wealthy country.

3 . Let us assume,for mathematical adhockery, to have b vary from $-\infty$ to $+\infty$ with mean zero. If $b_i < 0$ ($i = 1,\ldots,N$) , as in (1) and in (2) , then the i.th country has a propensity to lose in the exchange process. The opposite is true for $b_i > 0$.

Let p (b) db be the fraction of countries whose b is between b and b + db, so that

(3) $$\int_{-\infty}^{+\infty} p(b)db = 1$$

where p (b) is assumed to be normally distributed over the N countries and is symmetric with respect to $b = 0$, i.e.,

(4) $$p(b) = (1/\sqrt{2\pi}).e^{-b^2/2}$$

which implies that very few countries have a high level of bargaining power and that very few have a zero one.

If p (b) is defined as in (4) and given that

(5) $$\int_{b}^{\infty} b.e^{-b^2/2} db = 1/2 \int_{b^2}^{\infty} e^{-b^2/2} d(b^2) = e^{-b^2/2}$$

then

(6) $$\int_{b}^{\infty} b.p(b)db = p(b).$$

It should be pointed out that (1) and (2) make mathematically possible for a country to lose more than the stock of wealth it has. (The possibility of a country indebtedness makes such a case plausible). This requires, however, large absolute values of b' s which are, because of (3),very unlikely since $p(b) \to 0$ when $|b| \to \infty$.

If all the interactions have the same length T per unit of time the i.th country with a level b_i of bargaining power, interacts 1/T other countries. Of those, (1/T).p(b) $d\bar{b}$ have a level of bargaining power between $\bar{b}$ and $\bar{b} + d\bar{b}$. The i.th country will lose to all countries with $\bar{b} < b$.

Given (1), the total loss per unit of time will be

$$(7)\ \frac{\alpha}{T}\int_b^{\infty} w(b)(\bar{b}-b)p(\bar{b})d\bar{b},$$

while the total gain from countries with $\bar{b} < b$ will be

$$(8)\ -\frac{\alpha}{T}\int_{-\infty}^{b} w(\bar{b})(\bar{b}-b)p(\bar{b})d\bar{b}.$$

Combining (7) and (8) we find

$$(9)\ -\frac{\alpha}{T}\int_{-\infty}^{b} w(\bar{b})(\bar{b}-b)p(\bar{b})d\bar{b} - \frac{\alpha}{T}\int_b^{\infty} w(b)(\bar{b}-b)p(\bar{b})d\bar{b} = 0$$

which says that in a steady state the algebraic sum of all gains per unit of time through exchange must equate zero for each country.

4 . Now let us define

$$(10)\ \beta = \alpha / T;\ s(b) = \int_b^{\infty} p(\bar{b})d\bar{b}.$$

Differentiating (9) with respect to b, given (6) we have:

$$(11)\ \int_{-\infty}^{b} w(\bar{b})p(\bar{b})d\bar{b} + w(b)\,s(b) + (dw(b)/db)\left[b \cdot s(b) - p(b)\right] - \beta = 0$$

Differentiating this expression with respect to b, we obtain

$$(12)\ \left[p(b) - p.s(b)\right](d^2w(b)/db^2) - 2s(b)\left[dw(b)/db\right] = 0$$

Define now $dw(b)/db = v(b)$ and let

$$(13)\ w(b) = \int_0^b v(b)db + P$$

where P is an integration constant.

Rearranging (12) we have now:

$$(14)\ [dv(b)/db] - [2s(b)/p(b) - b.s(b)]\, v(b) = 0$$

For the integral of eq (14) we find

$$(15)\ v(b) = Q\int_0^b \exp\left\{\int_0^\psi (2s(x)dx)/(p(x) - sx(x))\right\} d\psi$$

where Q is an integration constant.

(To determine P and Q, let $w'(b) = dw(b)/db$, then from (16) we note that

$$P = w(0)\ ; \quad Q = w'(0).$$

Now let QF (b) indicate the right-hand side of (15) and let $F'(b) = (dF(b)/db)$ then

$$F(O) = 0 \quad ; \quad F'(O) = 1$$

Therefore, (16) can be written

$$w(b) = w(O) + w'(O)\ F(b)$$

which is a solution of (12).)

Introducing this into (13), we have

$$(16)\ w(b) = P + Q\int_0^b \exp\left\{\int_0^\psi (2s(x)dx)/(p(x) - xs(x))\right\} d\psi$$

which is a solution of (12).

5. Solving (16) for b we obtain the level of bargaining power as a function of w. Introducing this into (4), we find the distribution function of wealth (the number of countries whose stock of wealth is between, w and w + dw).

As in a steady state the wealth of a country is invariant with respect to time, from the distribution of wealth we can find the distribution of national incomes.

Let n (y) be an income distribution function, i.e. n (y) dy gives us the number of countries with income between y and y + dy.

Therefore, we have

$$(17) \quad N = \int_0^\infty n(y)dy$$

where N is the total number of countries and

$$(18) \quad Y = \int_0^\infty yn(y)dy$$

where Y is the world total income.

The fraction of all countries that have an income less than or equal to Y is given by

$$(19) \quad (1/N) \int_0^y n(y)dy$$

while,

$$(20) \quad (1/Y) \int_0^y yn(y)dy$$

represents the fraction of the world total income obtained by this fraction of countries.

As both fractions are functions of y, eliminating it we obtain a relation which gives us the fraction of all countries who receive a given fraction of the world total income.

Therefore assuming, as we do, that the distribution of bargaining power among countries is normal, the countries distribution of wealth and income, which depends on those levels is highly skewed. Infact as a result of interactions formalized in (1) and (2) very few countries are wealthy and very many poor.

References

1. LANDAHL, H.D. (1950), Note on a Mechanism of the Distribution of Wealth, Bulletin of Mathematical Biophysics, vol.12.

2. TOMASINI, L. (1969), A Model of Distribution of Wealth, Cowles Foundation Paper.