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Università degli Studi di Siena
DIPARTIMENTO DI ECONOMIA POLITICA

MASSIMO A. DE FRANCESCO

On Bertrand Competition under not so Large an Excess of Total Capacity

n. 320 - Aprile 2001

Abstract - We consider a homogeneous product market where, given their capacities, existing firms compete in prices. First, pricing at the constant short-run average-marginal cost – i. e., Bertrand outcome - is shown to be a Nash equilibrium of the static price game provided total capacity is sufficiently higher than the quantity demanded at a price equal to marginal cost; most importantly, the minimum amount of excess capacity that is required is quite modest when the one-firm concentration ratio is sufficiently small. Second, a study of repeated price decisions is carried out in a context where less than "fully rational" firms aim in each period at maximising current profits. The convergence result hinted at by Bertrand for a duopoly is shown to extend straightforwardly to the *n*-firm case: prices converge to short-run average cost under iterated best responses as well as under milder restrictions on the learning process. These results suggest that, in an unconcentrated homogenous-product industry, self-interested behaviour can easily prove "destructive" to the firms – making them not covering anything towards their fixed costs under even a modest excess of total capacity.

Keywords: Bertrand competition, iterated best-responses, learning.

JEL: C72, D43

Massimo De Francesco, Dipartimento di Economia Politica, Università di Siena

1. Introduction

It is well known how Bertrand (1883), in his critical review of Cournot, argued that prices must equal marginal costs in a duopoly. In fact he sketched a disequilibrium analysis suggesting that, so long as prices were higher than marginal costs, they would fall due to continuous attempts by each duopolist to undercut the rival's price. Thus, similarly as had previously been done by Cournot (1838, pp. 90-91) in the context of quantity setting firms, Bertrand provided an earlier analysis of how a Nash equilibrium might emerge through repetition of the same game.

On the other hand, and again quite similarly to Cournot, Bertrand analysis as well as Edgeworth (1925) more extended one may be criticised in view of their not allowing for much learning – each duopolist keeps on holding static expectations on the rival's price, in spite of prices being in fact evolving over time. (However, as has been suggested for Cournot (see, for example, Fudenberg and Levine, 1998, p. 10), the analogous methodology employed by Bertrand and Edgeworth might be interpreted in the sense that the duopolists take turns setting prices, so each one legitimately takes the rival's current price as given at the previous level when it is his turn to set the price.)

While disequilibrium processes have been studied extensively in the Cournot setting (see, for example, Theocharis, 1959, Fisher, 1961, and Hahn, 1962), this has not been the case in the Bertrand setting. (A partial exception is Qin and Stuart, 1997, where the two settings are combined in the sense that, at each date, each firm is allowed to choose between acting as a Bertrand or a Cournot competitor).[1] In a step towards filling this gap, the present paper shows that the convergence of prices to the constant average/marginal cost envisaged by Bertrand for a duopoly extends straightforwardly to the *n*-firm case, under iterated best responses as well as under milder restrictons on the learning process. This result parallels a not dissimilar one recently obtained in an evolutionary game setting by Hehenkamp, Qin, and Stuart (1999). Incidentally, it is worth noting that our result is in contrast with a claim by these authors that no convergence result is possible in the Bertrand setting under the iterated-best response process (p. 218).

The paper is organised as follows. Section 2 considers a static game for an industry where a given number of price setting firms produce a homogeneous output at constant short-run average/marginal costs up to their fixed capacity. We take into account that the price is actually not a continuous variable,[2] though the smallest feasible fraction of the money unit is realistically assumed to be insignificantly small. The necessary and sufficient condition for pricing at marginal cost to be an equilibrium is shown to be that total capacity must exceed the quantity demanded at a price just above marginal costs, to an extent that depends positively on the share of total capacity pertaining to the largest firm. Quite importantly, however, even a modest excess of capacity is enough when the one-firm concentration ratio is sufficiently small. Within the set of (nearly equivalent) equilibria obtaining under the excess capacity condition, the one we select has all firms charging the lowest possible price above marginal costs, this being the only equilibrium not involving weakly dominated strategies.

The condition on excess capacity is assumed to hold throughout Section 3, which envisages a repeated price game where the firms freely compete in prices, aiming at maximising expected profits for the current period. We first prove that, under static expectations on competitors' prices, convergence to the selected equilibrium of the static game obtains whatever the number of firms. More specifically, convergence is involved in the following property of the process of iterated best responses: so long as some current prices are higher than (the lowest possible price above) marginal cost, next-period highest price will be lower than the highest current price. This same property, and hence convergence, is subsequently shown to still hold under a milder restriction on the learning process, whatever it may be: it suffices that at each date each firm believes sufficiently many firms to keep on charging prices *not higher* than last-period highest price.

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[1] In Qin-Stuart model this choice is made under static expectations about the actions currently being made by competitors. So the model is a generalised one of iterated best responses.

[2] This fact has recently received considerably more attention than it was customarily the case (see, among others, Chaudhuri, 1996, Chowdhuri, 1999, Hehenkamp and Leininger, 1999, Hehenkamp, Qin, and Stuart, 1999; see also Vives, 1999, n. 2, p. 368).

The final section hints at a possible direction for further research. Indeed, the results above, concerning the nature and stability of the equilibrium of the price game under enough excess capacity, suggest that unrestrained price competition may easily become "ruinous" to the firms in an unconcentrated industry: in the pursuit of their individual interest the firms may easily end up making losses equal to their fixed costs as a result of even modest "mistakes" in capacity building, mistakes which in turn are almost unavoidable if for no other reason that investment decisions are made independently by the firms. The question then arises of how is it that such an outcome is seldom observed. Several answers can of course be made, some of them not mutually exclusive. A possible explanation – the one we focus on in the concluding section - is that it is precisely to avoid these consequences that firms may have developed alternative codes of behaviour, such as pricing on the basis of long-run average costs.

2. Pricing at marginal costs as a Nash equilibrium of the static game, under not so large an excess of total capacity

We consider an industry where n firms produce with given capacities a homogeneous output. $N = \{1,...,i,...,n\}$ denotes the set of firms, q_i and $\overline{q}_i$ firm i's output and capacity, respectively, $Q = \sum_{i \in N} q_i$ and $\overline{Q} = \sum_{i \in N} \overline{q}_i$ total output and capacity. Each firm incurs a fixed cost of $k\overline{q}_i$ that is "sunk" and produces at a constant marginal cost of c any amount up to $\overline{q}_i$. Each firm i sets its price p_i to maximise profits, and hence quasi-rents $(p_i - c) \times q_i$, given its single-valued expectation over $p_{-i} = (p_1,..., p_{i-1}, p_{i+1},..., p_n)$, the strategy profile adopted by the other firms. We recognise that the price is not truly a continuous variable. As in Chowdury (1999, p. 208), the set of pure strategies available to each firm may then be written as $P = \{p_0, p_1,..., p_l,...\}$, where $p_0 = 0$, $p_l = p_{l-1} + \varepsilon$, and ε stands for the smallest feasible fraction of the money unit; for the sake of realism ε is taken to be insignificantly small. We assume a nonincreasing demand function $D(p)$, with $D(p) > 0$ for $p < \overline{p}$ and $D(p) = 0$ for $p \geq \overline{p}$, where $\overline{p} > c + \varepsilon$. Let $D_i(p_i, p_{-i})$ denote the demand facing firm i at the given vector of price calls. Firms produce on demand, hence $q_i = min\{D_i(p_i, p_{-i}), \overline{q}_i\}$ so long as $p_i \geq c$. Concerning the allocation of demand when lower-priced firms are facing an excess of demand, it is assumed that rationing occurs according to the surplus-maximising rule.[3] Further, when several firms are charging the same price, each is assumed to take a share of the total demand available for all of them in proportion of its capacity (Allen and Hellwig, 1986, p. 179). In view of these assumptions, any firm *i* that is charging the highest price faces a residual demand

(1) $$D_i(p_i = \hat{p}, p_{-i}) = max\left\{0, \left(D(\hat{p}) - \sum_{j \notin H} \overline{q}_j\right) \times \frac{\overline{q}_i}{\sum_{j \in H} \overline{q}_j}\right\},$$

where $H = \{j : p_j = \hat{p}\}$ denotes the set of firms that are quoting the highest price $\hat{p}$.

"Excess capacity" is assumed throughout, in the sense that

(2) $$\overline{Q} \geq D(c),$$

where $D(c)$ is total demand at a price equal to marginal cost. Thus $p = c$ at a competitive equilibrium: at this price total supply is any amount between 0 and $\overline{Q}$, and it can be presumed that the firms willingly meet the demand $D(c)$. As the next proposition makes clear, (2) is necessary but not sufficient for equality between prices and marginal cost to represent an equilibrium of the price game.

Proposition 1. *A vector of price calls* $(p_1,..., p_n) = (c,...,c)$ *is an equilibrium if and only if*

(3) $$\sum_{j \neq i} \overline{q}_j \geq D(c + \varepsilon) \qquad \text{for all } i \in N.$$ [4]

[3] For more on the surplus-maximizing and proportional rationing rules see, for example, Vives, 1999, pp.124-125.

Proof

(*Necessity*) Let $\sum_{j \neq i} \overline{q}_j < D(c+\varepsilon)$ for some $i \in N$. Then, faced with $p_{-i} = (c,...,c)$, any such i would obtain positive quasi-rents by quoting $p_i \in \{c+\varepsilon,...,\overline{p}-\varepsilon\}: D_i(p_i, p_{-i} = (c,...,c)) > 0$.

(*Sufficiency*) With $\sum_{j \neq i} \overline{q}_j \geq D(c+\varepsilon)$ for all $i \in N$, replying any $p_i > c$ to $p_{-i} = (c,...,c)$ would make firm *i*'s output fall to zero, thus leaving the firm with zero quasi-rents just as when it charges $p_i = c$ and revenues equal variable costs. *QED*

Inequalities (3) can actually be expressed in terms of a single condition – a relationship between the minimum required degree of excess capacity and a simple measure of industry concentration. Let *h* denote the share of total capacity pertaining to the largest firm – a variant of the one-firm concentration ratio. Clearly, inequalities (3) are met if and only if $\overline{Q} - h\overline{Q} \geq D(c+\varepsilon)$, or

(4) $$\left(\overline{Q} - D(c+\varepsilon)\right) / D(c+\varepsilon) \geq h/(1-h).$$

The minimum degree of excess capacity that is involved in (4) is the lower as the one-firm concentration ratio is the lower: while total capacity must be at least twice as large as $D(c+\varepsilon)$ when *h*=1/2, it suffices that total capacity be at least one-third higher than $D(c+\varepsilon)$ when *h*=1/4, one-ninth higher when *h*=1/10, and one-nineteenth higher when *h*=1/20. This suggests that (4) is likely to be met in an unconcentrated industry when demand is sufficiently depressed. Hence, the event of firms just covering variable costs at an equilibrium of the static price game appears to be a concrete possibility. Strangely enough this point, though implied in an analogous condition obtained by Vives (1986, p. 115) for the case of equally sized firms,[5] has so far gone unnoticed.

The following result deals with equilibria of the price game other than $(p_1,..., p_n) = (c,...,c)$.

[4] With equally sized firms and the price as a continuous variable, inequalities (3) would read as $\overline{q} \geq D(c)/(n-1)$, where $\overline{q}$ stands for the firm's capacity (see Brock and Scheinkman, 1985, p. 373, and Vives, 1986, p. 114).

Proposition 2

Assume inequalities (3) to hold. Then:

(a) the price vector $(p_1,...,p_n)=(c+\varepsilon,...,c+\varepsilon)$ *is an equilibrium;*

(b) no price vector $(p_1,...,p_n)>(c+\varepsilon,...,c+\varepsilon)$ *can be an equilibrium;*

(c) any price vector $(p_1,...,p_n)>(c,...,c)$ *with* $p_i=c$ *for some i is an equilibrium if and only if* $\sum_{j\neq i:p_j=c}\overline{q}_j \geq D(c+\varepsilon)$ *for all* $i:p_i=c$.

Proof

(a) At $(p_1,...,p_n)=(c+\varepsilon,...,c+\varepsilon)$ every firm *i* earns a negligible quasi-rent of $\varepsilon\times D(c+\varepsilon)\times(\overline{q}_i/\overline{Q})$. No firm would gain from a unilateral deviation: its quasi-rents would fall to zero if either quoting just less or anything more than rivals (in the latter case, due to (3) and *D*(*p*) being nonincreasing).

(b) To prove our case we show that, at $(p_1,...,p_n)>(c+\varepsilon,...,c+\varepsilon)$, a firm $i\in H$ has not replied optimally to p_{-i}. The position of $i\in H$ depends on whether $\sum_{j\notin H}\overline{q}_j\geq D(\hat{p})$ or $\sum_{j\notin H}\overline{q}_j<D(\hat{p})$. In the former case, $i\in H$ has no residual demand left whereas it would have earnt positive quasi-rents by setting $p_i\in\{c+\varepsilon,...,\hat{p}-\varepsilon\}: D_i(p_i,p_{-i})>0$. As to the second case, note that, in view of (3) and of *D*(*p*) being nonincreasing, it can only arise if $\#H>1$. Most importantly, then $0<D_i(p_i=\hat{p},p_{-i})=(D(\hat{p})-\sum_{j\notin H}\overline{q}_j)\times(\overline{q}_i/\sum_{j\in H}\overline{q}_j)<\overline{q}_i$ for $i\in H$.[6] It is easily understood that any $i\in H$ should have quoted a lower price. For example, quoting just less than $\hat{p}$ would make demand for firm *i* be $D_i(p_i=\hat{p}-\varepsilon,p_{-i})=\left(D(\hat{p}-\varepsilon)-\sum_{j\notin H:p_j<\hat{p}-\varepsilon}\overline{q}_j\right)\times\left(\overline{q}_i/(\sum_{j\notin H:p_j=\hat{p}-\varepsilon}\overline{q}_j+\overline{q}_i)\right)$. This can be shown to

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[5] In our notation, Vives shows that, with any $\overline{Q}>D(c)$, $(p_1,...,p_n)=(c,...,c)$ is an equilibrium for *n* large enough. This is obviously implied by (4): with *h*=1/*n* (the case of equally sized firms analysed by Vives), (4) can be written as $n\geq\overline{Q}/[\overline{Q}-D(c)]$.

[6] That $(D(\hat{p})-\sum_{j\notin H}\overline{q}_j)\times(\overline{q}_i/\sum_{j\in H}\overline{q}_j)<\overline{q}_i$ follows straighforwardly from (2) and $D(p)$ being nonincreasing.

be strictly higher than $\left(D(\hat{p})-\sum_{j\notin H}\bar{q}_j\right)\times\left(\bar{q}_i/\sum_{j\in H}\bar{q}_j\right)$, with either $p_j<\hat{p}-\varepsilon$ for all $j\notin H$ or $p_j=\hat{p}-\varepsilon$ for some $j\notin H$. (While less immediate in the latter case, [7] this proposition is easily established in the former case, where $\left(D(\hat{p}-\varepsilon)-\sum_{j\notin H:p_j<\hat{p}-\varepsilon}\bar{q}_j\right)\times\left(\bar{q}_i/(\sum_{j\notin H:p_j=\hat{p}-\varepsilon}\bar{q}_j+\bar{q}_i)\right)=D(\hat{p}-\varepsilon)-\sum_{j\notin H}\bar{q}_j$.) This being so, by quoting just less than $\hat{p}$ firm i would obtain quasi-rents of $(\hat{p}\text{-}\varepsilon-c)\times min\left\{\left(D(\hat{p}-\varepsilon)-\sum_{j\notin H:p_j<\hat{p}-\varepsilon}\bar{q}_j\right)\times\left(\bar{q}_i/(\sum_{j\notin H:p_j=\hat{p}-\varepsilon}\bar{q}_j+\bar{q}_i)\right),\bar{q}_i\right\}$ which, by negligibility of ε, is always higher than $(\hat{p}-c)(D(\hat{p})-\sum_{j\notin H}\bar{q}_j)\times(\bar{q}_i/\sum_{j\in H}\bar{q}_j)$, the quasi-rents obtained by charging $\hat{p}$.

(c) (*Necessity*) Assume $(p_1,...,p_n)>(c,...,c)$ with $p_i=c$ for some i. If $\sum_{j\neq i:p_j=c}\bar{q}_j<D(c+\varepsilon)$ for some $i:p_i=c$, any such i has failed to make a best reply because it would have earnt positive quasi-rents by setting $p_i\in\{c+\varepsilon,...,\bar{p}-\varepsilon\}:D_i(p_i,p_{-i})>0$.

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[7] With $p_j=\hat{p}-\varepsilon$ for some $j\notin H$, firm *i* would compete with any such *j* when turning to $p_i=\hat{p}-\varepsilon$. First of all, note that $\left.\frac{d\left\{\left(D(\hat{p}-\varepsilon)-\sum_{j\notin H:p_j<\hat{p}-\varepsilon}\bar{q}_j\right)\times\left(\bar{q}_i/(\sum_{j\notin H:p_j=\hat{p}-\varepsilon}\bar{q}_j+\bar{q}_i)\right)\right\}}{d\left\{\sum_{j\notin H:p_j=\hat{p}-\varepsilon}\bar{q}_j\right\}}\right|_{\sum_{j\notin H}\bar{q}_j=constant}\geq 0$ if and only if $D(\hat{p}-\varepsilon)\leq\bar{q}_i+\sum_{j\notin H}\bar{q}_j$. Assume this condition is met. Then it follows immediately that $\left(D(\hat{p}-\varepsilon)-\sum_{j\notin H:p_j<\hat{p}-\varepsilon}\bar{q}_j\right)\times\left(\bar{q}_i/(\sum_{j\notin H:p_j=\hat{p}-\varepsilon}\bar{q}_j+\bar{q}_i\right)>(D(\hat{p})-\sum_{j\notin H}\bar{q}_j)\times(\bar{q}_i/\sum_{j\in H}\bar{q}_j)$, given that this inequality has already been shown to hold when $\sum_{j\notin H:p_j<\hat{p}-\varepsilon}\bar{q}_j=\sum_{j\notin H}\bar{q}_j$. Assume now instead $D(\hat{p}-\varepsilon)>\bar{q}_i+\sum_{j\notin H}\bar{q}_j$. Then the minimum of $D_i(p_i=\hat{p}-\varepsilon,p_{-i})$ would obtain when $p_j=\hat{p}-\varepsilon$ for all $j\notin H$. This minimum is $D(\hat{p}-\varepsilon)\times\bar{q}_i/(\sum_{j\notin H}\bar{q}_j+\bar{q}_i)$, which is strictly higher than $(D(\hat{p})-\sum_{j\notin H}\bar{q}_j)\times(\bar{q}_i/\sum_{j\in H}\bar{q}_j)$, due to $D(\hat{p}-\varepsilon)>\bar{q}_i+\sum_{j\notin H}\bar{q}_j$.

(*Sufficiency*) At $(p_1,...,p_n) > (c,...,c)$ with $p_i = c$ for some *i* and $\sum_{j \neq i: p_j = c} \overline{q}_j \geq D(c+\varepsilon)$ for all $i: p_i = c$, all firms are earning zero quasi-rents and no one would profit from a unilater deviation: as for *i*: $p_i = c$, raising the price would make its ouput fall to zero; as for *i*: $p_i > c$, it should lower its price to *c* in order to obtain a positive demand. *QED*

Propositions 1 and 2 are summarised by saying that under conditions (3) – or, what is equivalent, under (4) - at a pure-strategy equilibrium of the price game either all or sufficiently many firms are pricing precisely at marginal costs (Propositions 1 and 2c), or all firms are charging the lowest possible price above marginal costs (Proposition 2a). Clearly, equilibria other than $(p_1,...,p_n) = (c,...,c)$ are almost equivalent to the latter in terms of strategies and outcomes. Therefore one is approximately correct when saying that output is sold at marginal costs and firms earn zero quasi-rents at any equilibrium, just as under perfect competition.

At any rate, we may select among these nearly equivalent equilibria by excluding that firms will ever play weakly dominated strategies. Since $p_i = c$ is a weakly dominated strategy (as noted, for example, by Kreps, 1990, pp. 446-447, and Vives, 1999, p. 368), then we are left with a unique solution for the price game, i. e., $(p_1,...,p_n) = (c+\varepsilon,...,c+\varepsilon)$. Incidentally, the case for deleting weakly dominated strategies is particularly compelling here: indeed, charging *any* price $p_i \in \{c+\varepsilon, c+2\varepsilon,..., \overline{p}-\varepsilon\}$ yields no less than zero quasi-rents, no matter what $p_{-i} > (c,...,c)$, while yielding positive quasi-rents for some $p_{-i} > (c,...,c)$.

3. A generalisation of Bertrand adjustment process

Inequalities (3) are assumed to hold throughout this section, where we study the dynamics of prices over a sequence of plays, starting from vectors of price calls that are not equilibria of the static price game. This task will be accomplished by a methodology similar to Bertrand (1883) and Edgeworth (1925): in each period the firms are assumed to set prices to maximize current profits given their expectations on rivals' current prices. Though such an attitude might of course be rationalised in terms of myopic firms, this assumption is not strictly necessary. It may rather be that firms have no clear idea of how rivals will subsequently react to their current course of action or they may even presume no reaction at all: in either case they would be justified in looking at the immediate consequences of their actions. As a matter of fact, the second view was often entertained with regard to an industry or "group" with a sufficiently large number of firms, each with a limited capacity and in close competition with all the others rather than with a particular subgroup of them. In such a context it appeared legitimate to envisage the single producer as ignoring any reaction on the part of rivals to its current move; as Chamberlin put it, "a price cut, for instance, which increases the sales of him who made it, draws inappreciable amounts from the markets of each of his many competitors, achieving a considerable result for the one who cut, but without making incursions upon the market of any single competitor sufficient to cause him to do anything he would not have done anyway." (Chamberlin, 1962, p. 83; see also Kaldor, 1935, p. 35)

It should also be stressed that the firms are assumed to be uncontrained by any notion of what would be a "proper" or "fair" way to behave – a point to be touched upon in the final section. Let $(p_1(t),..., p_n(t))$ denote the vector of prices and $\hat{p}(t)$ the highest price in period *t*, as chosen as date *t*. We make provisionally the same assumption of static expectations that is usually viewed to underly Bertrand disequilibrium analysis. Thus, for all *i* and *j*, $p_j^{\,i}(t+1) = p_j(t)$, where $p_j^{\,i}(t+1)$ denotes the single-valued expectation of firm *j*'s price in period *t*+1, as held by firm *i* at date *t*+1. The following result generalises to the *n*-firm case Bertrand's argument of a tendency of prices to marginal costs.

Proposition 3

Let $(p_1(0),..., p_n(0)) > (c+\varepsilon,..., c+\varepsilon)$ *at the initial date. Under static expectations* $(p_1(t),..., p_n(t)) = (c+\varepsilon,..., c+\varepsilon)$ *at all* $t \geq t^*$, *for t* large enough.*

Proof

With $(p_1(t),..., p_n(t)) > (c+\varepsilon,..., c+\varepsilon)$, under static expectations each firm finds it optimal to quote at $t+1$ a price strictly lower than $\hat{p}(t)$ (see the proof of Proposition 2b). Hence

$(p_1(t),...,p_n(t)) > (c+\varepsilon,...,c+\varepsilon)$ implies $(p_1(t+1),...,p_n(t+1)) \geq (c+\varepsilon,...,c+\varepsilon)$ and $\hat{p}(t+1) < \hat{p}(t)$; as a result, there exists some future date *t** such that $(p_1(t^*-1),...,p_n(t^*-1)) > (c+\varepsilon,...,c+\varepsilon)$ and $(p_1(t^*),...,p_n(t^*)) = (c+\varepsilon,...,c+\varepsilon)$. Further, since at *t** each firm has made a best reply to the other firms' choices, under static expectations $(p_1(t),...,p_n(t)) = (c+\varepsilon,...,c+\varepsilon)$ for all *t*>*t**. *QED*

Assuming static expectations is inadequate, however, because it does not allow for much learning on the part of firms. What is unsatisfactory is that, in spite of prices being constantly evolving over the adjustment process, each firm is supposed to believe with certainty that rivals' prices will remain unchanged in the next period. Further, static expectations involve systematic biases: for example, one may immediately check that during the adjustment process the price currently charged by last-period highest priced firm(s) is sistematically overestimated.

However, convergence of prices to marginal costs does not quite require lack of uncertainty, let alone static expectations. To show this, rather than studying some specific learning procedure we propose a restriction that would be *sufficient* for the learning procedure, whatever it may be, to involve convergence. Let $B^i(t+1)$ denote firm *i*'s set of beliefs at date *t*+1 on the prices the other firms are about to quote, $\beta_j^{\,i}(t+1) \in B^i(t+1)$ firm *i*'s belief about firm *j*'s price, $S_j^{\,i}(t+1)$ the support of $\beta_j^{\,i}(t+1)$, i.e., the set of prices that *i* believes will be quoted with positive probability by *j* in period *t*+1, $p_j^{\,i}(t+1)$ any $p_j(t+1) \in S_j^{\,i}(t+1)$, and $p_{-i}^{\,i}(t+1)$ any $p_{-i}(t+1) \in \prod_j S_j^{\,i}(t+1)$. The following result is easily obtained.

Proposition 4

Let $(p_1(0),...,p_n(0)) > (c+\varepsilon,...,c+\varepsilon)$ *at the initial date. A sufficient condition for* $(p_1(t),...,p_n(t)) = (c+\varepsilon,...,c+\varepsilon)$ *at* $t \geq t^*$, *for t* large enough, is* $B^i(t+1)$ *being such that*

(5) $$\sum_{j:S_j^{\,i}(t+1)\subseteq\{c+\varepsilon,\dots,\hat{p}(t)\}} \overline{q}_j \geq D(\hat{p}(t)) \text{ for all } i\in N$$

whenever $(p_1(t),\dots,p_n(t)) \geq (c+\varepsilon,\dots,c+\varepsilon)$.

Proof

As a matter of fact, (5) turns out to be sufficient for $(p_1(t),\dots,p_n(t)) > (c+\varepsilon,\dots,c+\varepsilon)$ to imply $\hat{p}(t+1) < \hat{p}(t)$, just as under static expectations. Let $Pr\left(p_{-i}^{\;i}(t+1): \sum_{j:p_j^{\,i}(t+1)<\hat{p}(t)} \overline{q}_j \geq D(\hat{p}(t)) \right)$ and $Pr\left(p_{-i}^{\;i}(t+1): \sum_{j:p_j^{\,i}(t+1)<\hat{p}(t)} \overline{q}_j < D(\hat{p}(t)) \right)$ denote the probabilities, according to firm *i*'s beliefs, that the prices quoted by its rivals be such that $\sum_{j:p_j^{\,i}(t+1)<\hat{p}(t)} \overline{q}_j \geq D(\hat{p}(t))$ and $\sum_{j:p_j^{\,i}(t+1)<\hat{p}(t)} \overline{q}_j < D(\hat{p}(t))$, respectively. Further, denote by $q_i\left((t+1)\middle| p_i(t+1), p_{-i}^{\;i}(t+1)\right)$ firm *i*'s output at *t*+1 conditional on its charging $p_i(t+1)$ and the other firms charging $p_{-i}^{\;i}(t+1)$, and by $E\left(q_i(t+1)\middle| p_i(t+1), B^i(t+1)\right)$ firm *i*'s expected output at *t*+1 conditional on its charging $p_i(t+1)$ and on *i*'s set of beliefs. With $p_i(t+1) = \hat{p}(t)$, *i*'s expected quasi-rents can be written as

$$(\hat{p}(t)-c)\times E\left(q_i(t+1)\middle|\hat{p}(t), B^i(t+1)\right) = (\hat{p}(t)-c)\times \sum Pr\left(p_{-i}^{\;i}(t+1)\right)\times\left(q_i(t+1)\middle|\hat{p}(t), p_{-i}^{\;i}(t+1)\right)$$

With $p_i(t+1) = \hat{p}(t)$, firm *i* has no residual demand left at $p_{-i}^{\;i}(t+1): \sum_{j:p_j^{\,i}(t+1)<\hat{p}(t)} \overline{q}_j \geq D(\hat{p}(t))$. Hence the expression above reduces to

$$(\hat{p}(t)-c)\times E\left(q_i(t+1)\middle|\hat{p}(t), B^i(t+1)\right) = (\hat{p}(t)-c)\times$$
$$\times\left\{\sum Pr\left(p_{-i}^{\;i}(t+1): \sum_{j:p_j^{\,i}(t+1)<\hat{p}(t)} \overline{q}_j < D(\hat{p}(t)) \right)\times\left(q_i(t+1)\middle| \hat{p}(t), p_{-i}^{\;i}(t+1): \sum_{j:p_j^{\,i}(t+1)<\hat{p}(t)} \overline{q}_j < D(\hat{p}(t)) \right)\right\}.$$

The point is that, even when $p_{-i}^{\;i}(t+1)$ is such that $\sum_{j:p_j^{\;i}(t+1)<\hat{p}(t)} \overline{q}_j < D(\hat{p}(t))$ - an event which, in view of (5), may only occur jointly with $\sum_{j:p_j^{\;i}(t+1)\leq\hat{p}(t)} \overline{q}_j \geq D(\hat{p}(t))$ - there are prices lower than $\hat{p}(t)$ that will yield higher quasi-rents to *i*: for example, in light of the proof of Proposition 2b one such price is $\hat{p}(t)-\varepsilon$.

Thus, $(p_1(t),...,p_n(t)) > (c+\varepsilon,...,c+\varepsilon)$ implies $(p_1(t+1),...,p_n(t+1)) \geq (c+\varepsilon,...,c+\varepsilon)$ and $\hat{p}(t+1) < \hat{p}(t)$, what suffices to conclude that $(p_1(t^*),...,p_n(t^*)) = (c+\varepsilon,...,c+\varepsilon)$ at some future date t^*. Further, (5) is clearly sufficient for it to be $(p_1(t),...,p_n(t)) = (c+\varepsilon,...,c+\varepsilon)$ for all $t > t^*$. *QED*

One may want to assess the reasonableness of the sufficient condition just discussed. Consider a period in which $(p_1(t),...,p_n(t)) > (c+\varepsilon,...,c+\varepsilon)$. In view of $D(p)$ being nonincreasing, of (3) and the definition of $\hat{p}(t)$, price vectors $(p_1(t),...,p_n(t)) > (c+\varepsilon,...,c+\varepsilon)$ obviously involve $\sum_{j\neq i:p_j(t)\leq\hat{p}(t)} \overline{q}_j \geq D(\hat{p}(t))$ for all $i \in N$. Thus (5) amounts to saying that each firm expects sufficiently many firms to *keep on* charging no more than last-period highest price. This seems not unreasonable: after all, with $(p_1(t),...,p_n(t)) > (c+\varepsilon,...,c+\varepsilon)$, the firms which quoted $\hat{p}(t)$ at date *t* regret not having charged a lower price in the face rivals' prices; also, the firms which quoted less than $\hat{p}(t)$ understand that their quasi-rents would have been zero if they had charged more than $\hat{p}(t)$. Furthermore, note that restriction (5) is never contradicted by the resultant experience, given that $p_i(t+1) < \hat{p}(t)$ for all $i \in N$ when $(p_1(t),...,p_n(t)) > (c+\varepsilon,...,c+\varepsilon)$. Thus, it seems at least not unreasonable that beliefs, however formed and revised in light of experience, will keep consistent with (5) during the entire adjustment process.

Consider now a period in which $(p_1(t),...,p_n(t)) = (c+\varepsilon,...,c+\varepsilon)$. Having excluded that firms will ever play weakly dominated strategies, then inequality (5) means that all or sufficiently many firms are believed by each firm to keep on quoting $c+\varepsilon$ in *t*+1. This again is not unreasonable given that at *t* each firm has replied optimally to the other firms' choices.

I just want to add that learning procedures consistent with (5) are not difficult to envisage. The following one, in the spirit of the model of anticipatory learning proposed by Selten (1988), develops the previous remarks about the plausibility of (5). After observing $(p_1(t),...,p_n(t))$, each firm i may at t+1 anticipate that each $j \neq i$ is about to quote a price that is a best response to $(p_{-j}(t))$; then firm i chooses a price that is a best response to the anticipated strategy profile of its opponents. It follows straightforwardly from the proof of Proposition 3 that, with $(p_1(t),...,p_n(t)) \geq (c+\varepsilon,...,c+\varepsilon)$, each firm i anticipates $p_j^i(t+1) < \hat{p}(t)$ for all $j \neq i$, which obviously meets condition (5).

4. A possible interpretation of the results

We have shown that in a homogeneous product industry Bertrand outcome is a concrete possibility when the firms act self-interestedly: if the industry is unconcentrated, even a modest excess of total capacity is sufficient for prices to equal short-run constant average-marginal costs at an equilibrium of the static price game; furthermore, the tendency of prices to marginal costs takes place also when the assumption of static expectations is replaced by milder, and apparently not implausible, restrictions on beliefs about rivals' prices.

Needless to say, these results are at variance with reality, where prices often stay above short-run marginal costs in spite of firms operating quite below full capacity at prevailing prices. The model is useful, however, in that it may give a valid clue as to which assumptions might be modified to obtain predictions more in line with actual experience. Certainly the event of total capacity being excessive – to an extent such that (4) is met - can hardly be exaggerated: investment decisions are made independently by the firms and based on anticipations on demand conditions that may prove too optimistic; apart from this, under variable demand conditions periods of capacity underutilisation are almost unavoidable if total capacity must be large enough to meet the most favourable realisations of demand which can possibly occur. Thus, in an industry selling a homogeneous product and characterised by constant short-run marginal costs up to capacity,[8] unrestrained price competition would expose the firms to a substantial risk of bankruptcy in that they would often end up collecting nothing against their fixed costs. Of course, if there were some product differentiation prices would not fall to marginal costs. However, if the elasticity of substitution among the competing products is sufficiently high and the costs related to capacity sufficiently large, the difference would be one of degree rather than of substance: under unfavourable demand conditions the margin of prices over short-run marginal costs would be too low at a Nash equilibrium of the price game to make a significant contribution toward fixed costs.

As G. B. Richardson (1965) pointed out some time ago, actual experience with such a "ruinous" outcome of price competition may have led the firms to resist from making price reductions when demand falls short of their capacity.[9] In particular, they may rather price at normal cost or full cost, i. e., based on an estimate of long-run average cost at a standard rate of capacity utilisation. Though this is not the place to elaborate on normal cost or full cost pricing in a detailed fashion, a few related questions are worth mentioning.

First, precisely because this alternative pricing rule conflicts with the individual interest of the firm, it seems that adherence to it must rely on some institutional arrangements and/or the willingness to forgo a profit opportunity in order to behave properly. Both kinds of circumstances have in fact been recognised by proponents of normal or full cost pricing. The role of customary notions of fairness emerged, for example, from the results of the interviews which formed the empirical basis of Hall and Hitch (1939) classic article on full cost. According to Hall and Hitch, "a study of the replies confirms the existence of a strong tradition…that price 'ought' to equal full cost. This tradition is accounted for to some extent by an idea of fairness to competitors and is undoubtedly one of the reasons for the adherence to the full cost policy" (p. 21). [10] As to institutional arrangements which have been interpreted, at least in part, as a means to stabilising prices in the face of short-run fluctuations of demand, they include open-price

[8] Constancy of marginal costs in the short run appears not to be inconsistent with the empirical evidence. For example, according to an interview study over a sample of 200 firms, 48 percent of respondents held their "variable costs of producing additional units" to be constant and 40 percent to decrease when output increases (Blinder, 1994, p. 141).

[9] "Competitive activity of this particular kind (which we may term short-run price competition) is so obviously destructive that entrepreneurs have a very strong incentive to develop codes of behaviour capable of preventing it. The simplest of such codes would consist in each firm maintaining the price of its product when capacity exceeds demand, in the expectation that its rivals would act likewise." (Richardson, 1965, p. 441).

agreements (Richardson, 1967) as well as resale-price maintenance (Deneckere, Marvel, and Peck, 1997; Flath and Nariu, 2000).

Finally, it seems important to distinguish between two different roles that price rigidity can in principle perform. Compared to the case of flexible prices, price rigidity may just stabilise ex-post profits in the face of fluctuations in capacity utilisation or it may also increase expected profits (net of risk-premium). Proponents of full-cost or normal-cost pricing appeared to lean toward the first view.[11] Interestingly enough, these two different roles were carefully distinguished between by Keynes when, after the *General Theory*, he reconsidered the issue of price behaviour over the cycle. Then he vividly referred to the businessmen notion of a "rightly ordered competition", consisting "in a proper pressure to secure an adjustment of prices to changes in long-period average cost",[12] and also to the necessity of distinguishing "between price agreements for maintaining prices in right relation to average long-period cost and those which aim at obtaining a monopolistic profit in excess of average long-period cost" (Keynes, 1939, p. 47).

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[10] Quite similarly, Richardson noted: "Firms would refrain, as Marshall pointed out some time ago, from 'spoiling the market'. This restraint would have as its motive the fear of retaliation, a sense of common interest with rival sellers or even a vague feeling, almost of a moral kind, that this was 'the right thing to do'." (1965, pp. 441-442)

[11] For example, Hall and Hitch suggested that competitive pressures by potential entrants prevent incumbent firms from agreeing to raise prices above full cost ("if prices are in the neighbourhood of full cost, they are not raised by actual or tacit agreement because it is thought that, while this would pay in the short run, it would lead to an undermining of the firms by new entrants in the long run" (Hall and Hitch, 1939, p. 22).

[12] "Indeed, it is rare for anyone but an economist to suppose that price is predominantly governed by marginal cost. Most business men are surprised by the suggestion that it is a close calculation of short-period marginal cost or of marginal revenue which should dominate their price policies. They maintain that such a policy would rapidly land in bankruptcy anyone who practised it." (Keynes, 1939, p. 46).

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