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Inequality of Opportunity in Income and Functionings

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INEQUALITY OF OPPORTUNITY IN FUNCTIONINGS AND INCOME

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Abstract. We analyse inequality of opportunity sets, studying individual disparity in income and functionings. We modify the framework of the Lancaster's consumer theory, in order to capture this multidimensional extention of the standard inequality analysis.

1. Introduction

Since the pioneering work of Atkinson [1], a lot of research has taken income as the space for evaluating inequality. However, the evaluation of individuals' well-being must cover a wider range of spaces. The more the dimensions of inequality, the more heterogeneity across individuals is important for the evaluation of well-being. The analysis of disparities across individuals has to be further expanded to more dimensions than income alone. Our aim is to study the 'sources of di¤erences' in well-being and present a social evaluation of well-being with concern for situations of deprivation. Individuals may di¤er not only in budget constraints but also for their ability to transform income into opportunity sets in the space of functionings. To this extent we exploit some similarities of the Sen's concept of functionings with the notion of characteristics of Lancaster's consumer theory [1]. Provided that goods are inputs of the individual process of consumption, they are conceived as instruments for achieving functionings. In order to capture the individuals' heterogeneity in terms of personal capacity to transform a basket of goods into functionings, we extend the Lancaster's diagrammatic representation of characteristics of goods to the space of functionings produced by the access to a set of commodities. The e¢cient frontier stemming from the Lancaster's theoretical framework is thought of as the consumption technology of an hypothetical individual able to optimally use all goods.

Our theoretical framework then sets inequality of opportunities across individuals in the dimensions of income and functionings and focuses on di¤erences in well-being to be traced back to individuals' heterogeneity as for objective and subjective conditions of life. This links our approach to

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[1]See Lancaster [6].

both notions of freedom appearing in the economic literature: 1) "freedom of choice" and 2) "freedom as achievement", that is to the procedural and consequentialist aspects of freedom, respectively.

Some important distinctive characters of our theoretical approach are to be emphasized. As for the …rst notion, in stressing autonomy of decision and immunity from encroachment by others, the "freedom of choice view" considers that only exogenous observable restrictions on choice sets accessible to the individual may constrain "freedom of choice". On the contrary, we consider the whole range of objective and subjective limitations to individuals' functionings. Besides, this "objectivist" approach concentrates on the cardinality of the available alternatives (just to count the number of elements in the set re‡ects the value of the range of choice),[2] and on diversity across elements of choice (the variety of options is enriched also by goods with different characteristics and by complementarities among them).[3] Our focus is instead on di¤erences in the shape and in the level of e¢cient frontiers in the space of functionings across individuals.

As for the second notion, granted that any individual may well be interested in an additional option, the "freedom as achievement view" stresses that to express a choice mainly means to attach importance to its consequences. We agree with two tenets of this "subjectivist approach". First, that any additional good is preferred in the anticipation of its capacity to foster the achievement of the individuals' plans of life, and "di¤erences in age, gender, special talents, disability, proneness to illness, etc., can make two di¤erent persons have quite divergent substantive opportunities even when they have the very same commodity bundle".[4] Second, and consequently, individuals' preferences are crucial in the evaluation of opportunity sets, and a shift from the space of goods to the space of the ful…llment of well-being must happen in order the evaluation of individual well-being be sensible.[5]However, we underline that there is something that logically preceedes the moment in which preferences can be expressed. To be concerned with the choice of the functionings that the opportunity set permits to achieve, the individual must be …rst concerned with his own personal accessibility, both for income and functionings, to this additional option. Freedom relates to the individual's opportunity set as the possibility to transform that speci…c bundle of goods which a person can a¤ord (both in terms of budget and accessibility) into several functionings.

Therefore, in this paper individual well-being depends on the improvement both of "freedom of choice" and "freedom of achievement", provided

[2]See Pattanaik and Xu [8] for an outstanding axiomatic derivation of the number-counting method of opportunity set evaluation.

[3]Several theories of opportunity sets based on diversity and complementarity are reviewed in Sugden [14].

[4]Sen and Foster [4], p.209.

[5]"Freedom is considered in the evalutative space of capabilities, not in the space of the means of freedom (e.g. real income, wealth, opulence, primary goods, or resources)" (Sen [9], pg 33).

that a preference expressed over the goods represents the attempt to reach a better alternative in terms of the desired combination of functionings (capability). A low income might combine with the price level in such a way that the individual is impeded to enjoy a higher level of well-being. The utilization of those goods that are accessible to the individual according to his objective and subjective conditions, and that, given income and prices, appear in his consumption frontier, allows a person to be or to do something.

The next step is the evaluation of individual well-being. The problem of ranking opportunity sets of measurable functionings then arises. Some comparisons can be made in terms of set inclusion, as long as the increasing of the 'menu' from which an individual can choose will not decrease his functionings. Nevertheless, whenever a set is not entirely included in the other, we need to go beyond such trivial ordering. In order to avoid possible paradoxes, we …rst consider the individual's ability to reach minimally acceptable levels of functionings. We apply an evaluation functional corrected by a target as an exogenous threshold (a sort of multidimensional poverty line) and order individual opportunity sets by considering individuals who are ranked above or below such a target. Finally, we introduce in the standard setup of ethical inequality measurement, proposed by Atkinson [1], the non-welfarist informations about individual opportunity sets. We present a relative index of deprivation, which conveys an evaluation of the inequality in terms of opportunities of transforming income in functionings. The main di¤erence with the Atkinson-Kolm-Sen class of indices is that it takes into account a target below which individuals are not considered capable of achieving a "su¢cient" standard of living.

Our paper is organized as follows. In section 2, we show empirical evidence which testi…es how important the various situations of disadvantage are in determining inequality. Then, in section 3, we argue that Lancaster's appraisal of the characteristics of goods can be extended to the Sen's "theory of capabilities". In section 4, we analyse a situation of "inequality of opportunity in income and functionings", showing how to rank individuals' opportunity sets. We conclude by suggesting a variation in the Atkinson's index of inequality, which incorporates speci…c situations of deprivation.

2. Empirical evidence

In this section, we aim at showing how much complementarities among di¤erent dimensions of inequality are important in assessing individual well-being. By using data and answers to questionaires as appearing in the sample of the European Community Households Panel (ECHP), we give an empirical account for Italy of the following remark by Sen:

> "[...], there may be some accentuation of inequality due to the "coupling" of (i) income inequality and (ii) unequal advantages in converting incomes into capabilities, the two together intensifying the problem of inequality in terms of opportunity - freedoms. Those who are disabled, or ill, or old or otherwise

> handicapped may have, on one hand, problems in earning a decent income, and on the other, also face greater di¢culties in converting income into capabilities to live well. The same factors that may make a person unable to …nd a good job and a good income may put the person at a disadvantage in achieving a good quality of life even for the same job and the same income".(Sen, [9] pg. 536)

The …rst part of the Sen's reasoning deals with the possible link between "disadvantaged" conditions of life and income. Disadvantaged conditions could worsen income distribution. In Table 1, we compute income distribution by household heads and then the percentage of households in each decile for di¤erent couples of conditions of "unequal advantage" as for education and health (See legend in Table 1).

Table 1. Italy - ECHP wave 4 - 1996: Household's head deciles

	d1	d2	d3	d4	d5	d6	d7	d8	d9	d10
E1-H1	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,34	0,33	0,83
E1-H2	0,18	0,39	0,00	0,36	0,17	0,00	0,34	0,50	0,50	0,83
E1-H3	2,51	2,12	3,58	2,51	4,35	3,67	5,33	8,72	13,25	21,43
E2-H1	0,54	0,19	0,54	0,54	0,35	0,87	0,00	0,17	0,66	0,33
E2-H2	0,36	1,35	1,61	1,26	1,57	1,05	2,23	1,51	2,15	2,16
E2-H3	16,13	15,83	19,68	20,29	23,65	28,32	32,65	34,56	35,10	37,04
E3-H1	7,89	4,83	6,08	3,05	4,35	4,72	3,78	3,69	3,31	1,83
E3-H2	6,63	10,04	7,87	9,69	7,65	5,94	3,61	4,87	3,48	3,32
E3-H3	65,77	65,25	60,64	62,30	57,91	55,42	52,06	45,64	41,23	32,23
	100	100	100	100	100	100	100	100	100	100

Legend:
E: Highest level of general or higher education level completed
H: Are you hampered in your daily activities by any physical or mental illness or disability?

E1 = ISCED 5-7 recognized 3rd level	H1 = yes, severely
E2 = ISCED 3 second stage of secondary level	H2 = yes, to some extent
E3 = ISCED 0-2 less than second stage	H3 = no

It is easy to check from the data that income inequality is very high for the worst couple of "disadvantaged conditions" (very low education and very bad health: E3 - H1). The percentage of poor in this category is 7.89, while for the rich it is only 1.83. To a lesser extent, but with very high percentages, the same happens for cases of single disadvantage (high education and very bad health: E1 - H1 and very low education and very good health: E3 - H3). In particular, it can be noted that the very poor (…rst decile) and the poor (second decile) appear in row E3 - H3 with percentages that are very much higher than those of the rich's (nineth and tenth deciles). The percentages are just the opposite when we consider an increase in education with the same (good) health (E2 - H3). The inversion in results captures the importance of education: in the class E2-H3, out of their respective totals

the rich more than double the poor. Overall, this evidence con…rms the …rst part of Sen's remark: disadvantaged conditions may severely worsen income inequality.

Table 2 shows the distribution of percentage deciles for di¤erent levels of education and health across the three social-economic areas in which Italy is usually divided.

Table 2. Italy - ECHP wave 4 - 1996: Household's head deciles (N=North; C=Centre; S=South)

	d1	d2	d3	d4	d5	d6	d7	d8	d9	d10
E1-H1 N	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,17	0,17	0,33
E1-H1 C	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,17	0,00	0,17
E1-H1 S	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,17	0,33
E1-H2 N	0,00	0,00	0,00	0,18	0,17	0,00	0,00	0,17	0,17	0,33
E1-H2 C	0,18	0,19	0,00	0,00	0,00	0,00	0,17	0,17	0,33	0,17
E1-H2 S	0,00	0,19	0,00	0,18	0,00	0,00	0,17	0,17	0,00	0,33
E1-H3 N	0,90	0,39	0,72	0,90	1,57	1,05	1,89	2,86	5,14	9,65
E1-H3 C	0,72	0,58	0,54	0,54	1,40	1,22	0,86	2,18	3,48	5,99
E1-H3 S	0,72	1,16	2,33	1,08	1,40	1,40	2,58	3,70	4,64	5,82
E2-H1 N	0,54	0,19	0,18	0,36	0,17	0,35	0,00	0,00	0,33	0,17
E2-H1 C	0,00	0,00	0,18	0,00	0,00	0,17	0,00	0,17	0,33	0,00
E2-H1 S	0,00	0,00	0,18	0,18	0,17	0,35	0,00	0,00	0,00	0,17
E2-H2 N	0,18	0,19	0,18	0,36	0,70	0,35	1,20	0,50	0,83	1,66
E2-H2 C	0,00	0,39	0,54	0,36	0,35	0,35	0,52	0,67	0,33	0,33
E2-H2 S	0,18	0,77	0,89	0,54	0,52	0,35	0,52	0,34	1,00	0,17
E2-H3 N	4,49	3,87	4,65	6,47	10,12	13,11	14,95	14,45	19,73	24,63
E2-H3 C	2,33	2,51	5,72	7,19	5,58	7,17	8,25	9,58	8,46	8,49
E2-H3 S	9,52	9,28	9,48	6,65	8,03	8,04	9,45	10,42	6,97	3,99
E3-H1 N	1,80	1,55	1,61	1,26	2,09	2,97	2,75	2,18	1,49	1,66
E3-H1 C	1,44	2,13	2,33	0,36	1,05	1,05	0,69	1,34	1,00	0,17
E3-H1 S	4,67	1,16	1,97	1,44	1,22	0,70	0,34	0,17	0,83	0,00
E3-H2 N	1,08	2,32	2,33	3,42	3,66	2,62	1,72	1,68	2,16	1,83
E3-H2 C	1,26	1,93	1,79	2,52	1,57	1,40	1,03	1,68	0,66	1,16
E3-H2 S	4,31	5,80	3,76	3,78	2,44	1,92	0,86	1,51	0,66	0,33
E3-H3 N	10,59	16,83	16,28	23,20	21,82	25,52	27,84	27,23	27,20	18,97
E3-H3 C	12,75	12,77	12,34	14,21	17,98	14,34	14,78	11,76	9,95	8,82
E3-H3 S	42,37	35,78	32,02	24,82	17,98	15,56	9,45	6,72	3,98	4,33
	100,00	100,00	100,00	100,00	100,00	100,00	100,00	100,00	100,00	100,00

A divided pro…le between the "advanced" North and the "backward" South turns out for the worst couple of "disadvantaged conditions" (very low education and very bad health: E3 - H1) and the intermediate situation of very low education and very good health (E3 - H3). These results emphasise the importance of objective conditions in shaping disparities in subjective conditions. By regionally articulating the information relative to objective conditions, the conclusion that disadvantaged conditions clearly combine with income inequality is strengthened.

An analogous intepretation of results applies to Table 3,

Table 3.a. Italy - ECHP wave 4 - 1996
Main source of income, education and health by household deciles

	d1	d2	d3	d4	d5	d6	d7	d8	d9	d10
Y1-E1-H1	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,17	0,00	0,17
Y1-E1-H2	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,17	0,00	0,33
Y1-E1-H3	1,43	1,16	0,90	1,24	2,61	2,24	2,92	6,71	10,45	14,48
Y1-E2-H1	0,00	0,00	0,18	0,35	0,35	0,17	0,00	0,00	0,33	0,33
Y1-E2-H2	0,00	0,19	1,08	0,18	1,22	0,52	0,34	0,50	1,33	0,83
Y1-E2-H3	4,30	6,40	10,31	12,77	14,43	17,21	22,34	24,16	22,55	21,96
Y1-E3-H1	0,00	0,00	0,36	0,35	0,52	0,52	0,17	0,50	0,83	0,33
Y1-E3-H2	1,25	1,94	1,63	0,89	0,70	1,03	0,69	1,17	1,00	0,67
Y1-E3-H3	15,05	21,12	22,42	23,05	19,30	22,03	20,79	16,28	16,75	12,98
Y2-E1-H1	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,17	0,00	0,00
Y2-E1-H2	0,18	0,00	0,00	0,18	0,00	0,17	0,17	0,00	0,17	0,00
Y2-E1-H3	0,90	0,97	1,08	0,71	0,87	1,20	1,03	1,01	1,49	2,66
Y2-E2-H1	0,18	0,00	0,18	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Y2-E2-H2	0,00	0,00	0,00	0,18	0,00	0,17	0,34	0,17	0,17	0,33
Y2-E2-H3	5,20	4,46	4,52	2,66	3,83	5,51	3,61	3,69	4,48	4,99
Y2-E3-H1	0,36	0,00	0,36	0,00	0,35	0,17	0,00	0,00	0,00	0,00
Y2-E3-H2	0,72	0,58	1,08	0,35	0,52	0,69	0,34	0,34	0,17	0,33
Y2-E3-H3	17,74	11,63	9,76	7,98	9,04	7,92	6,87	6,21	3,98	4,16
Y3-E1-H1	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,17	0,67
Y3-E1-H2	0,00	0,39	0,00	0,00	0,17	0,00	0,17	0,34	0,17	0,50
Y3-E1-H3	0,00	0,00	0,54	0,35	0,17	0,17	0,86	0,84	0,66	3,16
Y3-E2-H1	0,18	0,19	0,18	0,18	0,00	0,69	0,00	0,17	0,33	0,00
Y3-E2-H2	0,00	0,97	0,18	0,89	0,35	0,17	1,20	0,84	0,66	1,00
Y3-E2-H3	1,97	1,74	2,35	2,30	3,30	4,48	4,47	4,36	5,97	6,99
Y3-E3-H1	4,12	3,49	4,16	2,13	2,78	3,27	2,41	2,68	2,16	1,00
Y3-E3-H2	2,15	6,20	4,16	7,09	5,57	3,61	2,23	2,52	2,16	1,83
Y3-E3-H3	17,38	24,22	18,81	24,29	24,52	20,65	20,27	18,46	17,58	10,48
Y4-E1-H1	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Y4-E1-H2	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Y4-E1-H3	0,00	0,00	0,00	0,00	0,17	0,00	0,17	0,00	0,00	0,17
Y4-E2-H1	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Y4-E2-H2	0,00	0,00	0,00	0,00	0,00	0,00	0,17	0,00	0,00	0,00
Y4-E2-H3	0,72	0,00	0,00	0,00	0,00	0,00	0,00	0,34	0,33	0,00

Table 3.b. Italy - ECHP wave 4 - 1996:
Main source of income, education and health by household deciles

	d1	d2	d3	d4	d5	d6	d7	d8	d9	d10
Y4-E3-H1	0,00	0,19	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Y4-E3-H2	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Y4-E3-H3	1,08	0,19	0,54	0,35	0,17	0,52	0,17	0,50	0,17	0,33
Y5-E1-H1	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Y5-E1-H2	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Y5-E1-H3	0,00	0,00	0,36	0,00	0,00	0,00	0,17	0,00	0,17	0,17
Y5-E2-H1	0,18	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Y5-E2-H2	0,18	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Y5-E2-H3	0,18	0,19	0,18	0,00	0,00	0,00	0,17	0,34	0,00	0,00
Y5-E3-H1	2,87	0,78	0,90	0,53	0,70	0,34	1,03	0,50	0,33	0,33
Y5-E3-H2	1,61	0,58	0,54	0,89	0,17	0,34	0,34	0,50	0,17	0,17
Y5-E3-H3	1,97	1,94	1,45	0,53	0,87	0,34	0,52	0,17	0,00	0,17
Y6-E1-H1	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,17	0,00
Y6-E1-H2	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Y6-E1-H3	0,18	0,00	0,00	0,18	0,35	0,00	0,00	0,17	0,33	0,67
Y6-E2-H1	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Y6-E2-H2	0,18	0,19	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Y6-E2-H3	1,08	0,58	0,18	0,35	0,35	0,52	0,86	0,34	0,50	2,00
Y6-E3-H1	0,00	0,00	0,00	0,00	0,00	0,00	0,17	0,00	0,00	0,17
Y6-E3-H2	0,36	0,00	0,18	0,00	0,00	0,34	0,00	0,17	0,00	0,17
Y6-E3-H3	3,23	0,97	1,63	0,89	0,87	0,17	0,86	1,34	1,49	2,33
Y0-E1-H1	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Y0-E1-H2	0,00	0,00	0,00	0,18	0,00	0,00	0,00	0,00	0,00	0,00
Y0-E1-H3	0,00	0,00	0,00	0,71	0,17	0,00	0,34	0,00	0,17	0,17
Y0-E2-H1	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Y0-E2-H2	0,00	0,00	0,00	0,35	0,00	0,00	0,00	0,00	0,00	0,00
Y0-E2-H3	2,69	2,52	2,35	1,95	1,74	1,20	1,20	1,34	1,33	1,16
Y0-E3-H1	0,54	0,00	0,36	0,18	0,00	0,17	0,00	0,00	0,00	0,00
Y0-E3-H2	0,54	0,78	0,36	0,35	0,70	0,52	0,00	0,17	0,00	0,00
Y0-E3-H3	9,32	5,43	6,69	4,43	3,13	2,93	2,58	2,68	1,33	1,83
	100,00	100,00	100,00	100,00	100,00	100,00	100,00	100,00	100,00	100,00

Legend: Y = main source of income 1 = wages and salaries 4 = unemployment 0 = no income
2 = self-employment 5 =other social benefits 3 = pensions 6 = capital income

in which the main source of income has been added to each couple of education and health levels. One can observe that individuals in the category "any other social bene…ts" (Y5) with "low education" (E3) and "bad health" (H1) show up in decreasing percentages across deciles going from the poor to the rich.

The second part of Sen's reasoning deals with the impact on income inequality of the individuals' heterogeneity in transforming income in capabilities, independently from initial income disparities.

It would be very di¢cult to …nd in the ECHP sample answers which may convey an unambiguous information with regard to e¤ective well-being in terms of combinations of functionings (capabilities). Thus, we are forced to …nd out indirect answers. We follow the strategy to consider the information about the region of residence as an indicator of the objective conditions of life and the information about the individual "living alone" or "living in a family" (respectively "single" and "no single" in Table 4) as a signi…cant aspect of the subjective conditions of life. Therefore, the second part of the Sen's whole remark - "the same factors..." - has been veri…ed by relating income distributions (the households' deciles) for di¤erent conditions of life ("single" and "no single") to the joint impact of objective conditions (the residence in three di¤erent parts of Italy, going from the "advanced" North to the "backward" South) and subjective conditions (one or more di¤erent sources of income). We concentrate the comment of Table 4 on the main results. The data seem to indicate that a much more unequal income distribution occurs: i) for an individual living as a single (subjective condition) under "social transfer" in Northern Italy environment (objective condition) than for an individual in the same situation in the Southern Italy; and ii) for an individual living in a family with a sole labour income in the Southern Italy environment than an individual in the same situation in the Northern Italy one. The reason for this tentative conclusion is the following. Granted that the homogenisation conducted by transforming the data in "equivalent income" [6] equates "living in the family" with "living alone", no di¤erence between percentages appearing in the two decile distributions ("single" and "no single") should come out.

It may be contended that data can be biased due to possible ‡aws in correcting for "equivalence scale". However, we think that the wide di¤erences which have been found cannot be traced back to "equivalent income"

[6] As a …rst approximation, we consider the equivalent income of a family as the sum of incomes divided by the square root of the household's members.

distortions alone.

Table 4. Italy - ECHP wave 4 - 1996: Income source by household deciles

single	d1	d2	d3	d4	d5	d6	d7	d8	d9	d10
K C	0,58	0,14	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
K N	0,43	0,00	0,00	0,00	0,29	0,00	0,00	0,00	0,00	0,00
K S	1,30	0,14	0,15	0,00	0,00	0,00	0,00	0,00	0,00	0,00
K+ST C	0,14	0,57	0,15	0,29	0,29	0,14	0,15	0,73	0,15	0,15
K+ST N	0,58	0,71	1,46	1,03	2,34	0,58	0,87	1,02	0,58	0,88
K+ST S	0,00	0,29	0,59	0,29	0,29	0,29	0,15	0,00	0,00	0,15
L C	0,43	0,57	0,15	0,00	1,61	0,58	1,46	2,04	0,73	0,74
L N	0,72	0,14	0,00	0,29	2,19	2,46	3,35	2,63	1,17	1,62
L S	0,86	0,14	0,29	0,00	0,88	0,87	1,16	2,04	1,17	0,44
L+K C	0,00	0,14	0,00	0,15	0,00	0,00	0,29	0,00	0,00	0,74
L+K N	0,29	0,00	0,00	0,15	0,29	0,00	0,15	0,29	0,00	0,00
L+K S	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
L+K+ST C	0,14	0,00	0,15	0,00	0,00	0,14	0,00	0,15	0,44	0,88
L+K+ST N	0,72	0,00	0,00	0,29	0,44	0,29	0,58	2,63	1,31	2,21
L+K+ST S	0,43	0,00	0,00	0,00	0,00	0,14	0,00	0,29	0,58	0,44
L+ST C	0,14	0,14	0,00	0,00	0,00	0,00	0,15	0,00	0,29	0,00
L+ST N	0,00	0,14	0,00	0,29	0,58	0,43	1,16	0,73	0,15	0,29
L+ST S	0,29	0,14	0,00	0,15	0,15	0,14	0,15	0,15	0,00	0,00
ST C	2,16	5,14	2,78	1,76	0,73	1,30	0,44	1,02	0,29	0,44
ST N	4,32	9,27	6,73	5,74	6,13	5,93	3,93	1,17	0,88	0,29
ST S	4,90	8,84	1,76	2,79	1,75	0,87	0,87	0,29	0,44	0,00
non single										
K C	0,58	0,14	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
K N	0,72	0,14	0,00	0,00	0,00	0,00	0,00	0,00	0,15	0,15
K S	1,59	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
K+ST C	0,29	0,71	1,02	0,44	0,88	0,72	1,16	0,44	0,58	0,59
K+ST N	0,86	1,00	1,76	0,88	2,04	3,47	2,77	1,90	1,75	2,35
K+ST S	0,14	1,00	1,61	1,32	0,88	0,72	0,44	0,00	0,29	0,29
L C	6,05	5,14	8,93	9,85	9,05	9,26	5,97	8,91	7,30	5,29
L N	5,33	5,99	6,15	9,12	11,53	12,01	13,83	14,45	15,33	12,94
L S	21,18	23,40	23,57	15,44	10,66	9,41	8,30	7,30	7,15	4,41
L+K C	1,30	0,57	1,17	2,79	2,92	2,89	3,06	3,21	3,94	6,76
L+K N	0,72	0,57	2,20	3,53	2,34	4,34	4,51	6,28	10,95	15,88
L+K S	6,77	2,71	3,07	2,50	2,04	2,75	2,91	3,36	2,63	3,38
L+K+ST C	0,72	0,43	0,44	1,47	1,46	2,32	1,89	2,77	1,75	4,56
L+K+ST N	0,14	0,00	0,44	1,32	1,90	3,04	4,66	5,11	9,49	12,35
L+K+ST S	1,59	1,85	1,61	0,59	1,61	1,74	2,62	2,92	2,19	2,65
L+ST C	1,87	3,00	2,93	4,85	5,69	6,80	6,70	7,88	8,18	3,53
L+ST N	1,73	2,28	2,93	3,68	6,72	7,53	13,10	12,41	14,31	11,03
L+ST S	7,64	8,13	10,25	8,38	7,30	8,54	5,82	4,23	3,07	2,50
ST C	3,17	4,42	5,42	4,12	4,38	1,88	3,06	1,02	1,02	0,88
ST N	3,31	3,71	5,71	8,97	5,99	5,50	3,49	1,75	1,17	0,88
ST S	15,85	8,27	6,59	7,50	4,67	2,89	0,87	0,88	0,58	0,29
TOTAL	100	100	100	100	100	100	100	100	100	100

Legend: K = capital income; L = labour income; ST = social transfers; N = Northern Italy; C = Central Italy; S = Southern Italy

Something more is embedded in these data. Our feeling is that the higher concentration of workers (L as the only source of income) in the "low income" deciles of "no single" in Southern Italy and of "single" in Northern

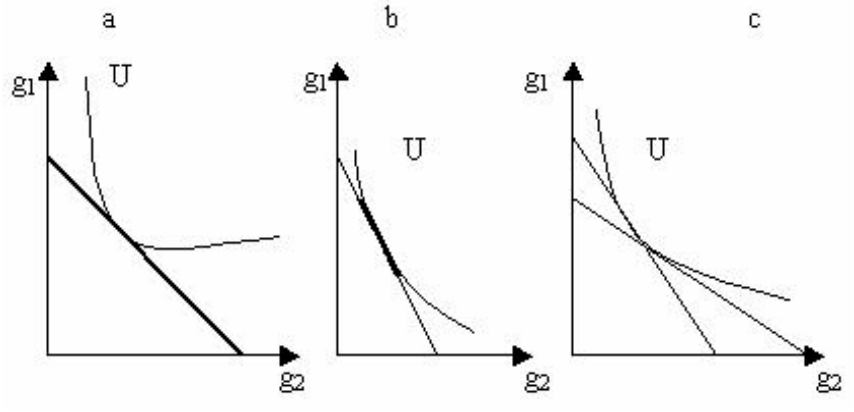
Italy, with just the opposite happening as for "social transfers" individuals (ST as the only source of income), may have an explanation in terms of di¤erent objective conditions of life stemming from the "cultural and social environment". The sociological account is that public assistance for the young unemployed and the retired people in the Northern Italy and the redistribution inside the family of the sole breadwinner's income in the South of Italy are not su¢cient to put individuals in the condition to apply for a job. In the South, there is a relatively much higher number of people out of the labour market (think to the much lower women's "participation rate") in the low and "unique" income households. In the North, there is a relatively much higher number of singles in need, due to mismatch in the labour market. We interpret these results as a clue that disadvantaged objective and subjective conditions of life, also associated with a very low income, may be an obstacle to the peculiar functioning "matching labour demand".

3. The theoretical framework

In the "freedom of choice" literature, a special attention is dedicated to the analysis of "freedom" as individual opportunities. Scholars adopted two alternative approaches in order to answer the fundamental question: "Does freedom of choice relative to an opportunity set depend on the individual preferences on elements of the set?" Depending on the answer, we can distinguish an objectivist and a subjectivist approach. The former is followed, among others, by Pattanaik and Xu [8], and Gravel, Laslier and Trannoy [5]. By contrast, Sen [12] , [11], [10] criticizes this view and proposes a way for including individual preferences in the evaluation of freedom of choice. As Sudgen [14] has observed, both views can be legitimated by reasonable arguments. In Figure 1, we summarize the two approaches by standard consumer theory, using the space of two marketable goods, with regular preferences and budget constraint.

In Fig.1 we apply these di¤erent approaches in the case of a decision maker subject to a linear budget constraint, with regular preferences on two marketable goods.

Fig. 1



Following the "objectivist" approach, the budget constraint identi…es the opportunity set available for an individual, given his income and market prices. According to graph a, a rational individual that makes use of his freedom of choice deviating from the optimal choice, represented by the tangency point, su¤ers from a loss of utility.

A subjectivist approach could identify freedom of choice with the dimension of the equivalence class of options, available for a given level of utility. For instance, in the graph b, the individual has a continuous set of optimal solutions and, consequently, a freedom of choice wider than in the situation described by graph a. In the third graph, for strictly convex indi¤erence curves, di¤erent combinations of income and prices are available. In this case, two di¤erent choices are optimal for a given utility level.

The previous example suggests a way to link freedom of choice with the relation of inclusion. In fact, both subjectivist and objectivist approaches accept the basic idea that a set gives more opportunities then its proper subsets. In our analysis, we state the Set-Inclusion Axiom after de…ning a binary relation of inclusion β between subsets of the vector space R^n_+; as follows.

De…nition 1. Given the sets $X; Y \mu R^n_+$, $Y \beta X$ if $8y 2 Y$; with $y = (y_1; :::; y_n)$, there exists a vector $x = (x_1; :::; x_n) 2 X$ such that $x_i \, ¸ \, y_i$ for $i = 1; :::; n$.

Axiom 1 (I-inclusion). For $X; Y \mu R^n_+$, if $Y \beta X$; then $X \%_I Y$.

We interpret the relation $\%_I$ as 'X provides at least as freedom of choice as Y'. A problem arises when two sets contain vectors that are not comparable by using the relation $\%_I$. In order to solve this problem, we introduce the concept of dominance in terms of freedom of choice as follows:

De…nition 2. A binary relation D de…ned on the subsets of R^n_+, is a dominance relation if it is a complete preordering and it extends the ordering b. Analitically:

i) 8 X, Y µ R^n_+;we have X D Y or Y D X;
ii) X b Y) X D Y.

Axiom 2 (D-dominance). For X; Y µ R^n_+, if Y D X then X $\%_D$ Y, which means that X dominates Y in terms of freedom of choice according to the relation D.

In the following, we study individual freedom of choice in the space of functionings. We will complete the partial preordering generated from the relation of inclusion in the space of functionings by a total preordering based on arguments independent on preferences. We start by considering as income inequality is an important source for individual di¤erences in accessing to market goods. Nevertheless, for a given income, others important factors a¤ect the individual capacity in transforming income in well-being. As the capacity of transforming goods in personal achievements depends on the individual process of consumption, both subjective and objective circumstances can limit individuals in exploiting some commodities.[7] For instance, let us imagine two twins with perfectly identical preferences and with the same income. The …rst twin lies in bed by such a severe pathology that he cannot even move. Even if both twins desire and are able to buy the same goods, many goods cannot contribute to well-being of the handicapped brother. His real capacity in transforming goods in functionings is inferior with respect to the healthy twin.

In order to capture this issue, we propose to revise Lancaster's consumption theory in order to de…ne and compare e¢cient frontiers of functionings.

In Lancaster's approach, marketable goods as food, cars, bikes, etc. are inputs of the individual process of consumption. Goods and combinations of goods produce many characteristics (outputs), as "have a nutrient composition", "have an easy driving" etc., on which individual preference relations are de…ned. The consumption process comes in two steps:[8]

1. for a given vector of prices and income, an e¢cient frontier is determined by choosing a combination of inputs maximizing the quantity of produced characteristics;
2. individual preferences then select a speci…c point on the e¢cient frontier of characteristics.

Lancaster claims that the e¢cient frontier is identical for all individuals.

[7] By contrast, the heterogeneity of preferences was studied, for instance, by Stigler and Becker [13] and Broome [2]. The latter advocates a precise distinction between some causes of preferences (as age, social status, handicaps etc.) assumed as parameters, and the object of preferences, that is the set on which an individual can really make a choice.

[8] We present a simpli…ed version of the linear characteristics model. See Lancaster [6] for more details.

We extend the linear characteristic model quoted above. We suppose that there exists a linear relation between the characteristics provided by goods and the quantity of functionings achieved by individuals.

3.1. A linear functionings model.

3.1.1. Goods and characteristics. Let R^C_+ and R^G_+ be the euclidean space of C characteristics and G goods respectively. In a primitive economy the number of characteristics is higher than equal to the number of goods. In a more realistic framework, the number of goods exceeds the number of characteristics, so that G > C.We assume a linear function transforming goods in characteristics. We write $z = Bx$; where B is a matrix representing the consumption technology and $z \in R^C_+$ and $x \in R^G_+$. A given vector in the C-space can be produced by different combinations of goods. Efficient choice leads consumers to purchase goods in combinations minimizing the costs of production of the desired characteristics. For a price vector p and an income y, a consumer solves the following maximization problem:

$$\max U(z) \quad \text{s.c.t.} \quad \begin{cases} px \leq y \\ z \leq Bx \\ \text{and } z; x \geq 0. \end{cases}$$

Where $U : R^C_+ \to R$ is a concave utility function. In figure 2 we consider the case of two characteristics, z_1 and z_2, produced by four factor goods: $x_1; :; x_4$. Let $p \in \Delta^3$ (the simplex of dimension 3) be the vector of prices and y the level of income. The linear technology transforming goods in characteristics is summarized by two equations:

$$z_1 = \sum_s b_{1s} x_s$$

and

$$z_2 = \sum_s b_{2s} x_s.$$

The line Ox_s represents the set of characteristics accessible by using only good s. Its shape is given by the ratio between the production coefficients b_{1s} and b_{2s}. If the individual only buys units of good s, she produces $\frac{y}{p_s} b_{1s}$ units of z_1 and $\frac{y}{p_s} b_{2s}$ units of z_2.

The opportunity set (including inefficient points) in the space of characteristics is given by the convex set generated by the convex combination of vectors $\left(\frac{y}{p_s} b_{1s}; \frac{y}{p_s} b_{2s}\right) \in R^2_+$; with s = 1; :::; 4; and the vector origin O. Efficient opportunities are contained in the upper boundary of this convex set. Some remarks will be useful in the following:

1. All the characteristics' combinations inside the frontier are inefficient.

2. For given prices and consumption technology, an increase in the income level produces a proportional expansion of the characteristics' e¢cient frontier.
3. If a new good appears and has a price su¢ciently low, the e¢cient frontier can change.

In our example, the hatched line shows the e¤ect of the introduction of the good x_4 in the choice set A. The change in the e¢cient frontier represents an improvement of opportunities in the space of characteristics for all individuals able to utilize the new good. This expansion of the opportunity set can also increase the individuals' welfare. Nevertheless, in some cases (see …g.2), individual welfare may not be modi…ed by adding a new good.

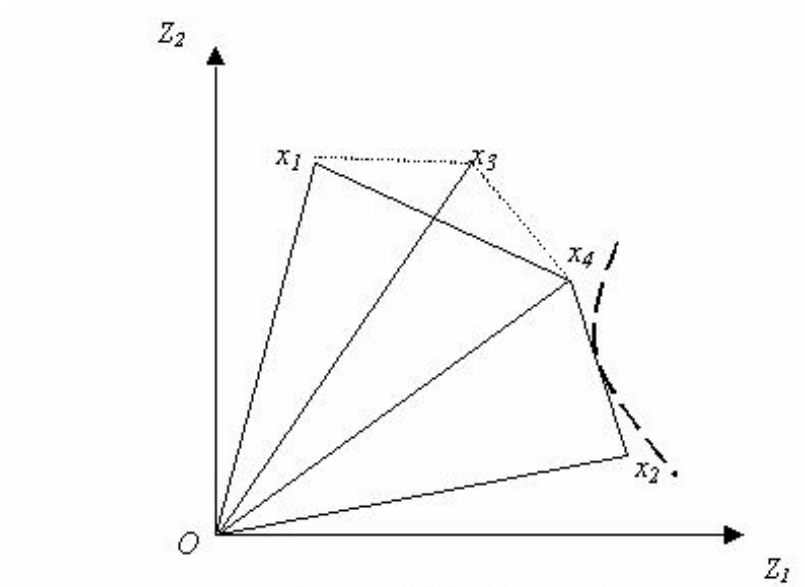


Figure 2: The efficient frontier

In what follows, we adopt the Lancaster's model described above in order to study the relation between consumption technology and opportunity sets of functionings. We assume a linear relation between characteristics and functionings. For instance, education can be viewed as a linear function of the characteristics of books. Obviously two persons can obtain di¤erent levels of education from the same books. In order to simplify this framework, we suppose:

i) the unit of measure is the same for characteristics and functionings.

ii) We assume that Lancaster's e¢cient frontier in terms of characteristics represents the best opportunities available in terms of functionings. It depicts the e¢cient combination of functionings achieved by an ideal individual able to optimally exploit all goods.

iii) Individuals can be classi…ed in groups, characterized by the same type of consumption technology. Individuals belonging to the same group embody the same e¢cient frontier but they can be endowed with di¤erent income.

For a given income, we cannot always order e¢cient frontiers by using the inclusion relation $\supseteq$. When e¢ciency frontiers cross, we consider the Lancaster's e¢cient frontier as the convex combination of the best opportunities available by real individuals.

3.1.2. Individuals and functionings. Let us consider a population of N individuals. We suppose that the prices of goods are …xed and identical for all. People are distinguished by income and consumption technology. For a given income, according to assumption iii), we have $V \cdot N$ classes of individual consumption technology. For each type $v \in V$, B^v is the matrix representing the consumption technology gap between type v and the Lancaster's matrix B de…ned above.

De…nition 3. For each $B^v = (b^v_{zx})$ and a given $B = (b_{zx})$,

$$b^v_{zx} \cdot b_{zx} \quad \forall z \text{ and } \forall x$$

and the consumption technology of individuals of type v is given by:

$$b_{zx} - b^v_{zx} \quad \forall z \text{ and } \forall x.$$

If, for some good x; we have $b_{zx} = b^v_{zx}\ \forall z$, individual of type v cannot transform good x into any functioning. On the other hand, individual of type v does not exploit the good x if the vector of characteristics generated by x falls short inside the characteristics frontier obtained by using other goods. The vector $(b_x - b^v_x) \in R^C$ represents the characteristics obtained by an individual of type v consuming one unit of good x. Denoted with $\|\cdot\|$ the Euclidean norm, the scalar $\frac{y}{p_x}\|b_x - b^v_x\|$ represents the distance from the origin of the vector of functionings obtained by a type v individual who only buys good x. Let us call subjective price, the personal value of a good given by its price adjusted by individual productivity. For the good x and individual type v, it is denoted as $\frac{p_x}{\|b_x - b^v_x\|}$. In such a case, a loss in consumption e¢ciency has the same e¤ect of an increasing in market price. For people of type v, when the subjective price of good x is too high, such a good does not belong to the set of goods generating the e¢cient frontier.

The opportunity set in the space of functionings for an individual of type v, endowed by an income y, is the compact convex subset $H_{vy} \subseteq R^C$. Analitically:

$$H^{vy} = (z^v_1; ...; z^v_C)$$

where:

$$z^v_h = \sum_i z^v_{hi};\ \forall \sum \in 4^{G-1};$$
$$z^v_{hi} = \frac{y}{p_i}(b_{hi} - b^v_{hi}):$$

The set inclusion relation between e¢cient frontiers establishes a trivial order in comparing opportunity sets in the space of functionings.

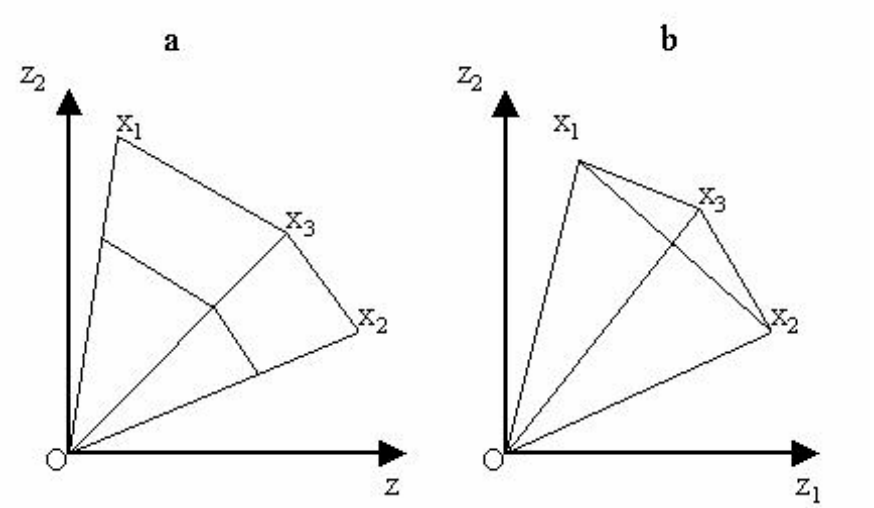


Fig.3 Two cases of inclusion between efficient frontiers

In …g.3-a we present two individuals with the same consumption technology, but di¤erent incomes. The graph 3-b shows two individuals with equal income, but individual 1 does not exploit a third good in her consumption process.

3.2. Ordering the opportunity sets. The set inclusion ordering is a too weak criterion in evaluating opportunities of individuals who di¤er both for income and consumption technology. All cases with intersecting e¢cient frontiers are indeed not rankable.

In order to go beyond such a criterion, we introduce a reference point t (target) in the space of functionings R^C. We suppose such a target as exogenous.[9] The distance between the target and individual e¢cient frontiers is de…ned as follows.

De…nition 4. The distance between H^{vy} and the (target) vector t is given by:

$$d(H^{vy}; t) = \min \{kz; tk \quad 8z \ 2H^{vy}\},$$

where kz; tk is the euclidean distance.

For a given level of income, the distance $d(H^{vy}; t)$ has two important properties:

1. It ranks all types of consumption technologies;

[9] It can be …xed by a social planner or obtained by some agreement among individuals.

2. It can be measured (in terms of income) as the monetary transfer that moves the e¢cient frontier H^{vy} along the target vector t.

Let us consider in …gure 4 two individuals a and b who are identical in income, and with crossing e¢cient frontiers. We can imagine that good x_1 is country residence, x_2 books, x_3 town residence, and x_4 concerts, while the functionings taken into account are z_1 education and z_2 health. Suppose that individual a is a single parent, who cannot go out and attend concerts but has to look after her child. Instead, individual b has got a severe emphysema by smoking. The polluted air of the town is very dangerous for his health, so that he cannot leave in a town dwelling.

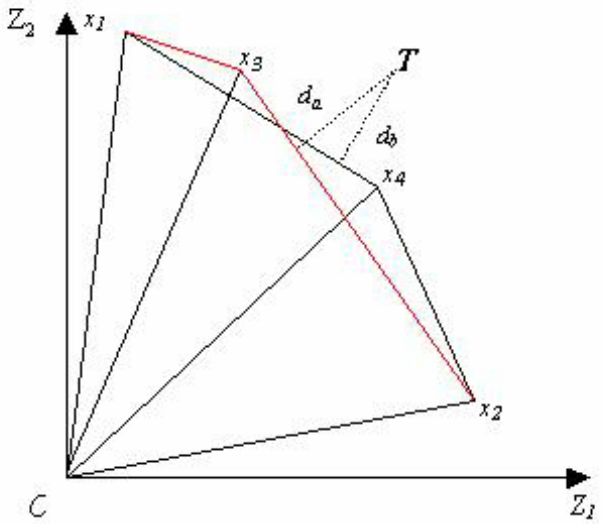


Figure 4: Efficient frontiers for different consumption technology and equal incomes

We compute the distances d_a and d_b between their e¢cient frontiers and the social target in terms of functionings. Even if individuals have the same income, they …nd a constraint to e¤ective well-being in their di¤erent consumption technology.

For a given income, we rank all types of consuption technology by using a complete preordering D generated by the distance between opportunity sets and the social target.

Given an income y, let V be the set of all v individual opportunity sets in the space R^C.

De…nition 5. We de…ne a complete preordering D on V as:

$$d(H_{iy}; t) \cdot d(H_{jy}; t) \;)\; H_{iy} \; D \; H_{jy} \quad \text{for all} \quad H_{iy}; H_{jy} \; 2 \; V.$$

As the binary relation D is a dominance relation, we state the following:

Axiom 3 (t-dominance). For any $H_{iy}; H_{jy} \in V$, if $H_{iy} \supset H_{jy}$ then $H_{iy} \succeq_t H_{jy}$, which means that H_{iy} dominates in terms of freedom of choice H_{jy}, given a target t.

As the preordering $\succeq_t$ evaluates as equivalent all efficient frontiers containing the target, in order to avoid trivial situations, we consider a low income level such that no one can get the target.

As noticed above, a comparison of individual situations characterised by functionings and income inequality may generate paradoxes. In the situation as in figure 5, if the social target is the vector of functionings t , the efficient frontier of the poorest individual is at a smaller distance from the target than the efficient frontier of the richest one.

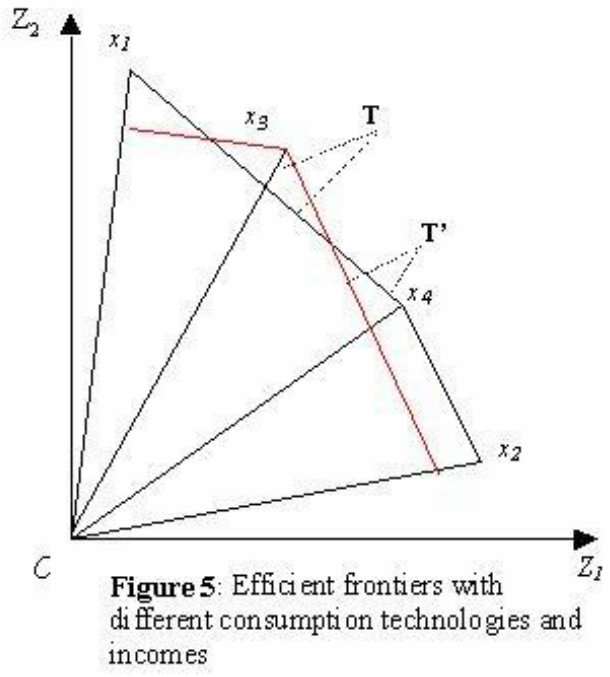


Figure 5: Efficient frontiers with different consumption technologies and incomes

Therefore, a social planner interested in the achievement of a target may find difficult to reach a correct evaluation of the individual well-being when implementing a principle of justice in terms of functionings.

In order to solve such a paradoxical situation, in the next section, we suggest a procedure which consists in calculating the amount of income that each individual needs in order to reach the target. Such an information will then be introduced in an "ethical inequality index".

4. An inequality index based on income and deprivation in functionings

In the standard set-up of ethical inequality measurement, proposed by Atkinson[1], the social evaluation of individual well-being is given by an increasing and concave (real valued) transformation of the income $U(¢)$.[10]

If the target represents the minimal level of functionings, required by an hypothetical social planner for each individual, personal well-being is computed by taking into account the deprivation in achieving such a minimal level.

Let $P(v; y)$ be the bivariate individual distribution function of consumption technology and income. We interpret p_{vy} as the frequency of individuals with income y and consumption technology of type v. We denote with Y the set of all possible income values.

Let t be the exogenous social target. For an income y, we de…ne with $¥_{vy}$ the monetary transfer required for moving a v-type opportunity set along the target t.

We modify an utilitarian social welfare evaluation function by introducing a transformation $f(¢)$ of the di¤erence between the utility of individual income and the same utility after a monetary transfer su¢cient to bring the individual e¢cient frontier on the social target takes place.

Let $f : R_+ \to R_+$ be a continuous, strictly increasing and convex function, such that $f(0) = 0$; then the social evaluation function corrected by the distance of the individual e¢cient frontiers from a given target t is the following:[11]

$$U(y; v) = \sum_{v2V} \sum_{y2Y} p_{vy} \max\{0; U(y) - f[(U(y + ¥_{vy}) - U(y)]\}; \tag{4.1}$$

where $(y; v)$ are the individual vector of incomes and consumption technology.

The social function 4.1 shows how the impact on well-being of a compensation for deprivation depends on income and consumption technology. Suppose two di¤erent situations of deprivation, such that an equivalent compensation works. For instance, an individual has a lower income and a "better" consumption technology, and another one is richer but endowed with a less e¢cient consumption technology, as showed in the …g.6.

[10] In the following, we improperly use the term utility for $U(¢)$:

[11] Expression 4.1 is dealt from the disappointment functional studied in Loomes and Sugden[7] and Farina [3].

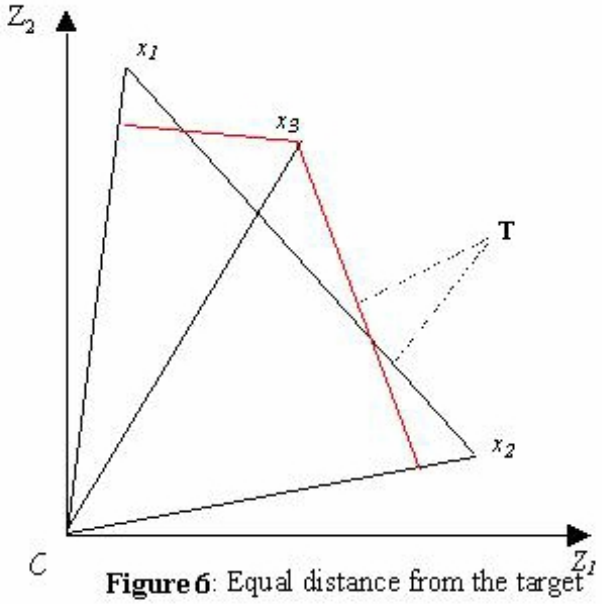


Figure 6: Equal distance from the target

We remark that, being $U(y)$ concave, the effect of the same monetary transfer to the different individuals on the social welfare function is greater for the lower levels of income. We then register that as the concavity of U emphasizes the dimension of inequality of income, the convexity of f emphasizes inequality in consumption technology. A social decision-maker can express aversion to inequality in income or opportunities by choosing particular functional forms for U and f.

Finally, we define a relative index of inequality corrected for deprivation in opportunities of functionings as follows:

$$A(y;v) = 1 - \frac{U(y;v)}{U(y^t;v)}$$

where $U(y^t;v) = \sum_{v \in V} \sum_{y \in Y} p_{vy} U(y + y_{vy})$ represents the social welfare obtained when all individuals receive a sum sufficient to reach the target.

5. Conclusions

We extend standard unidimensional measurement disparity across individuals to two dimensions: income and functionings. Given the income level, alternative individual opportunity sets of functionings are ranked, according to a social evaluation expressed by a distance from a target. A relative index of inequality, modified for taking into account both income and deprivation in terms of functionings, is derived to measure the welfare of a given distribution of income and functionings. A future extension of our contribution will concern the endogenisation of the target, obtaining a set of values by a voting procedure on an "optimal" level of functionings.

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