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Working Too Much in a Polluted World:
A North-South Evolutionary Model

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Abstract - This paper examines a simple North-South growth model where negative externalities may contribute to reinforce economic growth. Agents' welfare depends on three goods in the model: leisure, a common access renewable natural resource (one in each hemisphere) and a non-storable consumption good. Production and consumption of the latter good deplete the renewable natural resource. To protect against such environmental deterioration, agents may increase their labor supply in order to produce an additional amount of the consumption good to be used as a substitute for the depleted natural resource. The consequent growth in production and consumption may generate a further depletion of the natural resource. This may lead to a self-enforcing growth process in a polluted world where individuals work and produce "too much" (i.e. more than socially optimal). We examine the choices of the two hemispheres using a two-population evolutionary game where each agent chooses whether to work low or high. If an agent works low, she can consume the good only to satisfy basic needs (subsistence consumption). If the agent works high, she can consume an additional amount of the good as a substitute for the natural resource (substitution consumption). We assume that people who work high in the North can also have access to the Southern natural resource (e.g. they can afford a holiday in some developing country where natural resources are still relatively unpolluted), whereas the opposite is not true. We assume transboundary pollution, that is, the production of each hemisphere generates negative externalities both in the North and in the South, which determines the interdependence between the two hemispheres. We show that economic growth in the North and/or in the South may lead to stationary states that are Pareto dominated by states of the world with a lower level of production and consumption. Moreover, negative environmental externalities from the North to the South may foster growth in the South, which may have in turn feedback effects on growth in the North. Finally, we discuss possible welfare effects of transferring the environmental impact of Northern production to the South and show that such a policy may decrease welfare in both hemispheres.

J.E.L. classification: C70, D62, O13, Q20

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1 Introduction

The present paper examines the relationship between environmental self-protection choices and economic growth in the context of a North-South model. By environmental self-protection choices we mean choices that agents can do to protect against the deterioration of the environment they live in.[1] The argument is well-known in the environmental literature: many natural resources that were still freely available in developed economies some decades ago (e.g. meadows, woods, unpolluted air and water etc...) may deteriorate or become scarce as income grows. To counterbalance such a trend, people tend to replace consumption of these "free" environmental public goods with that of expensive private goods that may satisfy the same needs. Everyday life provides many examples of environmental self-protection choices. In most industrialized countries, for instance, people spend increasingly more on mineral water since tub water is often non-drinkable in many cities. Similarly, many beaches have become more and more dirty in the North, therefore Northern agents may prefer to buy an expensive holiday in some tropical paradise rather than go to the open access, polluted beach near home. Another example of self-protection choice is given by the increasing consumption of double windows in many Northern towns to protect against traffic noise.

The notion of self-protection choices is not new in the literature. Hirsch (1976) was the first to introduce the concept of defensive consumption, that is, consumption induced by growth negative externalities. The notion originally proposed by Hirsch concerned a wider set of choices than those induced by environmental deterioration. The concept, however, has become particularly popular in the environmental literature where there is a large debate on how Gross National Product should be corrected as a welfare measure to account for defensive expenditures and environmental depletion.[2]

Differently from the traditional literature, however, we set forth the idea that environmental defensive expenditures may contribute to a self-enforcing process of economic growth. Environmental deterioration, in fact, may induce agents to work harder to substitute previously free environmental goods with produced substitute goods. Production of substitute goods may further

[1] See, for instance, Hueting (1980), Leipert (1989), Leipert and Simonis (1989) and Cullino (1993) for alternative classifications of environmental defensive expenditures generated by these self-protection choices.

[2] See, for instance, Vincent (2000) and the special issue of the journal "Environment and Development Economics" on the most recent results of the research on this topic.

deplete the environment, which increases in turn production and consumption of substitute goods. Thus, the substitution mechanism of depleted natural resources with private goods might contribute to a self-feeding growth process: economic growth increases environmental degradation which, in turn, generates further growth. The idea that environmental negative externalities may promote economic growth was first introduced by Antoci and Bartolini (1999) in an evolutionary game context.[3] In this case, the authors show that agents' welfare may decrease as they increase their activity (and production) level. Bartolini and Bonatti (1999) have proved that such outcome does not depend on the hypothesis of bounded rationality underlying the evolutionary game model. Using, in fact, a neoclassical growth model with capital accumulation, they show that an "undesirable" (i.e. Pareto-dominated) growth process may result from a coordination failure among the agents even if agents are assumed to have perfect foresight. Finally, Antoci, Sacco and Vanin (2000) have extended the substitution mechanism described above to an economy where growth is reinforced by the substitution of "services" provided by social capital with private goods.[4]

The present work intends to contribute to this line of research by examining the substitution mechanism of environmental with private goods in a North-South evolutionary context. To fix ideas, think of sea pollution in the North. The increasing level of pollution of many Northern beaches may induce agents in the North to work harder for two reasons: to have access to a private swimming pool (the private substitution good) and to afford an expensive holiday in a country where beaches are still relatively clean. Since such countries are often developing countries, environmental degradation in industrialized countries may induce an increasing movement from the North to the South to enjoy unpolluted natural resources. However, if the number of Northern agents that go on holiday to the South is relatively high, this may increase the exploitation of natural resources and thus environmental degra-

[3] Some other theoretical contributions (Shogren and Crocker, 1991, Beltratti, 1996) also take environmental defensive expenditures explicitely into account, but they neglect their possible implications on economic growth. In Beltratti's model, for instance, labor is not a choice variable for the agents, therefore defensive expenditures cannot generate an increase in the production level.

[4] The increasing level of activity and the consequent lack of leisure that many people experience in most developed countries induce, for instance, many families to rent a colf or a baby-sitter. To afford these additional expenditures, however, people may be compelled to work (and produce) even more, which may further reduce the time that they have at disposal for the family.

dation in the South. If so, Northern agents have a lower incentive to work high and go on holiday to the South. Each agent's decision on how much to work thus depends on what other agents will do and the substitution mechanism due to pollution in the North leads to an increasing interdependence between environmental quality in the two hemispheres.

The paper has the following structure. Section 2 describes an evolutionary North-South model where each hemisphere's production adversely affects the environment both in the North and in the South. Production is determined by how much people work and agents can choose between two strategies: working low or high. Section 3 examines the dynamics of labor and production in one hemisphere assuming that all agents in the other hemisphere select a unique strategy (i.e. they all work either low or high). This analysis provides the basis for section 4 where the welfare analysis in the two hemispheres is performed also for the more realistic case where some agents work low and others work high within each hemisphere. Section 5 investigates some of the possible dynamics that may emerge from the model, focusing attention on the most interesting cases where a welfare-reducing growth process may occur. Section 6 examines the welfare effects of transferring the environmental impact of Northern production to the South. Finally, section 7 summarizes the main findings of the paper and suggests directions for future research.

2 The model

There are two hemispheres: North (N) and South (S).

There are three goods in the economy: leisure (l), a common access natural resource (E), and a non-storable produced good. The natural resource E is depleted by production of the good.

The produced good is produced by labor alone. Let us indicate with L the agents' labor supply.

Each agent in hemisphere j ($j = N, S$) is endowed with one unit of time that can be used for work or leisure and can decide whether to work low (L_l^j) or high (L_h^j). If the agent works low, she produces and consumes the fixed amount $\bar{Y}_1^j$ of the non-storable consumption good. In general, we can think of $\bar{Y}_1^j$ as a subsistence consumption level. If the agent works high, she produces and consumes $\bar{Y}_1^j + \bar{Y}_2^j$. In other words, if the agent works L_h^j, she produces and consumes an additional fixed amount $\bar{Y}_2^j$ of the good that can be used as a substitute for the depleted environmental resource. Only the agents that work high can thus afford substitution consumption.[5]

Let us indicate with $x \in [0,1]$ the portion of agents that choose to work high in the North. The total amount produced and consumed in the North at any time t will then be given by:[6]

$$Y^N(x) =: (\bar{Y}_1^N + \bar{Y}_2^N)x + \bar{Y}_1^N\ (1-x) = \bar{Y}_1^N + \bar{Y}_2^N\ x \tag{1}$$

Similarly, indicate with $z \in [0,1]$ the portion of agents that choose to work high in the South. Total production and consumption in the South is then equal to:

$$Y^S(z) =: (\bar{Y}_1^S + \bar{Y}_2^S)z + \bar{Y}_1^S\ (1-z) = \bar{Y}_1^S + \bar{Y}_2^S\ z \tag{2}$$

Let us indicate with $\overline{E}^N$ the endowment of the common access natural resource in the North when no production takes place. This amount is reduced by production and consumption in both hemispheres, therefore:

$$E^N(x,z) =: \overline{E}^N - \alpha[Y^N(x)] - \delta[Y^S(z)] \tag{3}$$

with $\overline{E}^N, \alpha, \delta > 0$.

Replacing $Y^N(x)$ and $Y^S(z)$ with equations (1) and (2) and collecting terms, we obtain:

$$E^N(x,z) = A - \alpha\ \bar{Y}_2^N\ x - \delta\ \bar{Y}_2^S\ z \tag{4}$$

where $A =: \overline{E}^N - \alpha\ \bar{Y}_1^N - \delta\ \bar{Y}_1^S$.

Mutatis mutandis, the same applies to the South:

$$E^S(x,z) =: \overline{E}^S - \gamma[Y^N(x)] - \beta[Y^S(z)] \tag{5}$$

with $\overline{E}^S, \gamma, \beta > 0$.

Substituting from (1) and (2), we get:[7]

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[5] The existence of an homogeneous produced good that satisfies both basic and environmental needs is assumed in the paper for analytical simplicity. Recalling the example above, however, one can think of two distinct goods, $\bar{Y}_1^j$ being the production of food that ensures agents' survival and $\bar{Y}_2^j$ the production of swimming pools that provide a substitute for the polluted sea.

[6] We set population size equal to 1 in both hemispheres.

[7] We assume $E^j(1,1) > 0$ $(j = N, S)$.

$$E^S(x,z) = B - \gamma\, \bar{Y}_2^N\, x - \beta\, \bar{Y}_2^S\, z \tag{6}$$

where $B =: \overline{E}^S - \beta\, \bar{Y}_1^S - \gamma\, \bar{Y}_1^N$.

Notice that the parameters A and B can be interpreted, respectively, as the Northern and Southern environmental stocks net of the subsistence activities (i.e. production and consumption of $\overline{Y}_1$) in both hemispheres. As equations (4) and (6) show, these stocks are further reduced by the defensive activities (i.e. production and consumption of $\overline{Y}_2$) in both hemispheres. We assume that natural resource E regenerates instantaneously, so that the previous equations hold at every time t.

Finally, suppose that all agents have the same utility function in both hemispheres. We assume that each agent has a logarithmic additively separable utility function that depends on three arguments: (i) leisure, (ii) (subsistence) consumption of the produced good ($\overline{Y}_1$) and (iii) environmental consumption. The latter can be consumption of the free natural resource E and (substitution) consumption of the produced good ($\overline{Y}_2$).[8] As mentioned above, if an agent works low she cannot afford such private consumption. Therefore, the utility function of an agent who works low in hemisphere j ($j = N, S$) will be:

$$U_l^j(x,z) = a \ln(1 - L_l^j) + b \ln \bar{Y}_1^j + \ln E^j(x,z) \tag{7}$$

where $a, b > 0$.

If an agent works high in hemisphere j she can enjoy both the environmental good in j and the substitute good that she produces. However, we assume that people who work high in the North can also enjoy part of the environment in the South (e.g. they can afford to spend their holidays at Maldives), whereas the opposite is not true (i.e. even if Southern agents work high they cannot afford a holiday in Saint Tropez). This asymmetry between the two hemispheres can be justified assuming that Northern agents are richer than Southern ones. The utility function of an agent who works high in the North will then be:

$$U_h^N(x,z) = a \ln(1 - L_h^N) + b \ln \bar{Y}_1^N + \ln \left[E^N(x,z) + cE^S(x,z) + d\, \bar{Y}_2^N \right] \tag{8}$$

[8] Observe that the stock E of the natural resource that enters the agents' utility function can be interpreted as a proxy for environmental quality or for the flux of goods and services freely provided by Nature that people use to satisfy their needs.

while that of an agent who works high in the South will be:

$$U_h^S(x,z) = a\ln(1-L_h^S) + b\ln \bar{Y}_1^S + \ln\left[E^S(x,z) + e\,\bar{Y}_2^S\right] \tag{9}$$

where $a, b, c, d, e > 0$.[9]

Subtracting (7) when $j = N$ from (8), we then obtain the payoff differential between working high and low in the North:

$\Delta U^N(x,z) =: U_h^N(x,z) - U_l^N(x,z) = a\ln\frac{1-L_h^N}{1-L_l^N} + \ln\frac{\left[E^N(x,z)+cE^S(x,z)+d\bar{Y}_2^N\right]}{E^N(x,z)}$

Similarly, subtracting (7) when $j = S$ from (9), the payoff differential in the South is equal to:

$\Delta U^S(x,z) =: U_h^S(x,z) - U_l^S(x,z) = a\ln\frac{1-L_h^S}{1-L_l^S} + \ln\frac{\left[E^S(x,z)+e\bar{Y}_2^S\right]}{E^S(x,z)}$.

We assume that if the payoff differential is positive, the number of people that work high will increase since working high provides a higher utility than working low. The opposite holds if the differential payoff is negative. Finally, if the payoff differential equals zero in hemisphere j people in that hemisphere are indifferent between working low or high, so that the population share that works low (high) keeps constant over time. Therefore, we can write:

$$\Delta U^N(x,z) \gtreqless 0 \Rightarrow \dot{x} \gtreqless 0 \quad \Delta U^S(x,z) \gtreqless 0 \Rightarrow \dot{z} \gtreqless 0 \tag{10}$$

Replacing $E^N(x,z)$ with (4) and $E^S(x,z)$ with (6), we can rewrite the inequalities of condition (10) as follows:

$$\begin{aligned} &d\,\bar{Y}_2^N + cB - (\overline{L}^N - 1)A + [\alpha(\overline{L}^N - 1) - c\gamma]\,\bar{Y}_2^N\,x + \\ &+[\delta(\overline{L}^N - 1) - c\beta]\,\bar{Y}_2^S\,z \gtreqless 0 \qquad \Rightarrow \qquad \dot{x} \gtreqless 0 \end{aligned} \tag{11}$$

$$e\,\bar{Y}_2^S + \left(\beta\,\bar{Y}_2^S\,z + \gamma\,\bar{Y}_2^N\,x - B\right)(\overline{L}^S - 1) \gtreqless 0 \qquad \Rightarrow \qquad \dot{z} \gtreqless 0 \tag{12}$$

where: $\overline{L}^N =: \left(\frac{1-L_l^N}{1-L_h^N}\right)^a$, $\overline{L}^S =: \left(\frac{1-L_l^S}{1-L_h^S}\right)^a$.[10]

[9] Notice from (9) that $E^S(x,z)$ and $\bar{Y}_2^S$ are perfect substitutes for Southern agents that work high, e being (the opposite of) their marginal rate of substitution. Similarly, from (8), $E^N(x,z)$, $E^S(x,z)$ and $\bar{Y}_2^N$ are perfect substitutes for Northern agents that work high, c and d being the marginal rates of substitution between the three goods.

[10] Note that $\overline{L}^N > 1$ and $\overline{L}^S > 1\ \forall a > 0$.

Hence, in each hemisphere the payoff differential ($\Delta U^N(x,z)$ and $\Delta U^S(x,z)$) has the same sign as the time derivative of the population share that works high in that hemisphere ($\dot{x}$ and $\dot{z}$, respectively). This property will turn out to be useful in the next paragraph to study the dynamics of growth. In our simple model without capital accumulation, in fact, economic growth in the two hemispheres is determined by how x and z evolve over time.

3 Dynamics of the economy

For the sake of simplicity, we can represent the dynamics of x and z by the so-called "replicator dynamics" (see, for instance, Weibull 1995):[11]

$$\begin{cases} \dot{x}= x(1-x)\Delta U^N(x,z) \\ \dot{z}= z(1-z)\Delta U^S(x,z) \end{cases} \tag{13}$$

The dynamic system (13) is defined in the square Q:
$Q =: \{(x,z) : 0 \leq x \leq 1, 0 \leq z \leq 1\}$.

All sides of this square are invariant, namely, if the pair (x,z) initially lies on one of the sides, then the whole correspondent trajectory also lies on that side. Note that the states $\{(x,z) = (0,0),(0,1),(1,0),(1,1)\}$ are fixed points of the dynamic system (13).[12]

In what follows we will denote with $Q_{x=0}$ the side where $x = 0$, with $Q_{x=1}$ the side where $x = 1$. Similar interpretations apply to $Q_{z=0}$ and $Q_{z=1}$. We will call Q_s the set of all four sides of Q.

We can distinguish four possible dynamic regimes along the side $Q_{x=i}$ (i = 0,1):

- point $z = 0$ is attractive and $z = 1$ repulsive (figure 1.a).[13] In this case we will say that there is l^S**-dominance** along $Q_{x=i}$ since working low

[11] Note that from (13) it follows that (10) holds $\forall x \in (0,1)$ and $\forall z \in (0,1)$.

[12] Dynamics (13) describe an adaptive process based on an imitation mechanism: every period part of the population changes its strategy adopting the more remunerative one (working high or low). For the imitation to occur, however, each strategy must be followed by a positive portion of individuals. This requirement is not met along the sides of Q where one of the two strategies is not followed by anyone in one hemisphere and cannot therefore be imitated. This explains why all sides are invariant and all vertex are fixed points of (13).

[13] In what follows, we will indicate attractive points with a full dot and repulsive points with an open dot in the diagrams.

is the dominant strategy in the South if $x = i$ (i.e. if everyone works either high or low in the North).

- point $z = 0$ is repulsive and $z = 1$ attractive (figure 1.b). In this case we will say that there is h^S**-dominance** along $Q_{x=i}$, working high being the dominant strategy in the South if $x = i$.
- both $z = 0$ and $z = 1$ are attractive and there is a repulsive fixed point along the side $Q_{x=i}$ (figure 1.c). We will denote this case as "**bistable dynamics**" along $Q_{x=i}$.
- both $z = 0$ and $z = 1$ are repulsive and there is an attractive fixed point along $Q_{x=i}$ (figure 1.d). We will denote this case as "**stable dynamics**" along $Q_{x=i}$.

Four similar cases can be identified along the sides $Q_{z=0}$ and $Q_{z=1}$.
Let us then examine the dynamics along each side of the square Q.

Proposition 1 ***Dynamics along*** $Q_{x=0}$:

1. if $B \leq B_0^* \equiv \frac{e\bar{Y}_2^S}{\overline{L}^S - 1}$, there is h^S-dominance
2. if $B \geq B_0^{**} \equiv \frac{e\bar{Y}_2^S}{\overline{L}^S - 1} + \beta\, \bar{Y}_2^S$, there is l^S-dominance
3. if $B_0^* \leq B \leq B_0^{**}$, there is a bistable dynamics. The repulsive fixed point along $x = 0$ is:

$$z_0^S = \frac{B(\overline{L}^S - 1) - e\, \bar{Y}_2^S}{\beta\, \bar{Y}_2^S\, (\overline{L}^S - 1)}$$

where subscript 0 of z denotes that the fixed point lies along $x = 0$.

Proof. See the Appendix. ■

The result of Proposition 1 is intuitively appealing. If B is relatively low (case 1 above), i.e. if the amount of natural resources that are left in the South after subsistence production of $\overline{Y}_1$ is relatively low, Southern people will decide to work high in order to replace consumption of the depleted environment with the substitution consumption of $\bar{Y}_2$. On the contrary, if natural resources that are left in the South after production of $\bar{Y}_1$ are relatively high (case 2 above), Southern people will then prefer to work low and enjoy their environment.[14] For intermediate cases, there exists a repulsive point z_0^S separating the attraction basin of $z = 0$ from that of $z = 1$. A similar interpretation applies when everyone is "workholic" (i.e. works high) in the North. In fact:

Proposition 2 ***Dynamics along*** $Q_{x=1}$:

1. if $B \leq B_1^* \equiv \frac{e\bar{Y}_2^S}{\bar{L}^S - 1} + \gamma\, \bar{Y}_2^N$, there is h^S-dominance

2. if $B \geq B_1^{**} \equiv \frac{e\bar{Y}_2^S}{\bar{L}^S - 1} + \gamma\, \bar{Y}_2^N + \beta\, \bar{Y}_2^S$, there is l^S-dominance

3. if $B_1^* \leq B \leq B_1^{**}$, there is a bistable dynamics. The repulsive fixed point along $x = 1$ is:

$$z_1^S = \frac{B - \gamma\, \bar{Y}_2^N}{\beta\, \bar{Y}_2^S} - \frac{e}{\beta(\overline{L}^S - 1)}$$

where subscript 1 of z denotes that the fixed point lies along $x = 1$.

Proof. Similar to the proof of Proposition 1, imposing condition (12) to hold as equality and setting $x = 1$ on the left-hand side. ■

Focusing attention on the bistable dynamics along $Q_{x=0}$ and $Q_{x=1}$, observe that:

$$z_0^S - z_1^S = \frac{\gamma\, \bar{Y}_2^N}{\beta\, \bar{Y}_2^S} > 0$$

This measures the reduction of the $z = 0$ attraction basin for the South as the North moves from a situation where everyone works low ($x = 0$) to a situation where everyone works high ($x = 1$).

Let us now turn to the dynamics of the North starting with the case when everyone is "lazy" (i.e. works low) in the South.

[14]Note that the ratio $\frac{\bar{Y}_2^S}{\bar{L}^S - 1}$ which enters B_0^* and B_0^{**} can be interpreted as a measure of the Southern agents facility to access the substitution consumption in terms of labor.

Proposition 3 ***Dynamics along*** $Q_{z=0}$:

Case 1: $\overline{L}^N - 1 > c\frac{\gamma}{\alpha}$:

1.1) if $A \leq A_0^* \equiv \frac{cB+d\bar{Y}_2^N}{\overline{L}^N-1}$, there is h^N-dominance

1.2) if $A \geq A_0^{**} \equiv \frac{cB+d\bar{Y}_2^N+[\alpha(\overline{L}^N-1)-c\gamma]\bar{Y}_2^N}{\overline{L}^N-1}$, there is l^N-dominance

1.3) if $A_0^* < A < A_0^{**}$, there is a bistable dynamics. In the repulsive fixed point along $z = 0$ we have:

$$x_0^N = \frac{A(\overline{L}^N - 1) - cB - d\,\bar{Y}_2^N}{[\alpha(\overline{L}^N - 1) - c\gamma]\,\bar{Y}_2^N}$$

where subscript 0 of x denotes that the fixed point lies along $z = 0$.

Case 2: $\overline{L}^N - 1 < c\frac{\gamma}{\alpha}$:

2.1) if $A \leq A_0^{**}$, there is h^N-dominance

2.2) if $A \geq A_0^*$, there is l^N-dominance

2.3) if $A_0^{**} < A < A_0^*$, there is a stable dynamics. In the attractive fixed point along $z = 0$ we have: $x = x_0^N$.

Case 3: $\overline{L}^N - 1 = c\frac{\gamma}{\alpha}$:

3.1) if $A < A_0^*$, there is h^N-dominance

3.2) if $A > A_0^*$, there is l^N-dominance

3.3) if $A = A_0^*$, the side $Q_{z=0}$ is a set of fixed points.

Proof. See the Appendix. ■

Although Proposition 3 shows a higher number of possible cases, the intuition is similar to that of Propositions 1 and 2. If the amount of natural resources that are left after production of $\bar{Y}_1$ is relatively low in the North ($A \leq \min(A_0^*, A_0^{**})$), Northern agents want to go on holiday to the South where the environment is better preserved, therefore they are induced to work high (h^N-dominance). Viceversa, if the amount of natural resources is relatively high in the North ($A \geq \max(A_0^*, A_0^{**})$), Northern agents do not have such an incentive, therefore they prefer to work low and enjoy more leisure (l^N-dominance).

Despite the similar interpretations of Propositions 1 to 3, the North differs from the South as it may also exhibit a stable dynamics (case 2). This is more likely to occur, the higher the ratio γ/α. To explain why this is the case, recall that γ and α measure the impact of Northern production on environmental resources in the South and in the North, respectively. If γ

is high with respect to α, an increase in the number of Northern agents that work high depletes the environment in the South relatively more than in the North. This makes the strategy h^N less attractive as it reduces the incentive of Northern agents to go on holiday to the South to enjoy a better environment as x increases. In other words, if Northern production damages the South more than the North, an individual may prefer to stay in the North rather than face a possibly expensive journey to the South where the environment may be equally or even more polluted at the end of the day.

Finally, let us examine the Northern dynamics when everyone is "workoholic" (i.e., works high) in the South.

Proposition 4 ***Dynamics along*** $Q_{z=1}$*:*

Case 1: $\overline{L}^N - 1 > c\frac{\gamma}{\alpha}$:

1.1) if $A \leq A_1^* \equiv \frac{cB+d\bar{Y}_2^N+[\delta(\overline{L}^N-1)-c\beta]\bar{Y}_2^S}{\overline{L}^N-1}$, there is h^N-dominance

1.2) if $A \geq A_1^{**} \equiv \frac{cB+d\bar{Y}_2^N+[\delta(\overline{L}^N-1)-c\beta]\bar{Y}_2^S+[\alpha(\overline{L}^N-1)-c\gamma]\bar{Y}_2^N}{\overline{L}^N-1}$, there is l^N-dominance

1.3) if $A_1^* < A < A_1^{**}$, there is a bistable dynamics. In the repulsive fixed point along $z = 1$ we have:

$$x_1^N = \frac{A(\overline{L}^N - 1) - cB - d\,\bar{Y}_2^N - \left[\delta(\overline{L}^N - 1) - c\beta\right]\bar{Y}_2^S}{\left[\alpha(\overline{L}^N - 1) - c\gamma\right]\bar{Y}_2^N}$$

where subscript 1 of x denotes that the fixed point lies along $z = 1$.

Case 2: $\overline{L}^N - 1 < c\frac{\gamma}{\alpha}$:

2.1) if $A \leq A_1^{**}$, there is h^N-dominance

2.2) if $A \geq A_1^*$, there is l^N-dominance

2.3) if $A_1^{**} < A < A_1^*$, there is a stable dynamics. In the attractive fixed point along $z = 1$ we have: $x = x_1^N$.

Case 3: $\overline{L}^N - 1 = c\frac{\gamma}{\alpha}$:

3.1) if $A < A_1^*$, there is h^N-dominance

3.2) if $A > A_1^*$, there is l^N-dominance

3.3) if $A = A_1^*$, the side $Q_{z=1}$ is a set of fixed points.

Proof. Similar to the proof of Proposition 3. ■

Let us compare the fixed points x_0^N and x_1^N of Propositions 3 and 4.

In Case 1 above ($\overline{L}^N - 1 > c\frac{\gamma}{\alpha}$), x_0^N and x_1^N are repulsive fixed points along the sides $Q_{z=0}$ and $Q_{z=1}$, respectively. Moreover, we have $x_0^N < x_1^N \quad iff$:

$$\overline{L}^N - 1 < c\frac{\beta}{\delta} \tag{14}$$

It follows that -if condition (14) applies- the attraction basin of $x = 0$ increases, while that of $x = 1$ decreases as we pass from $z = 0$ to $z = 1$. This is equivalent to say that Northern people find less convenient to work high so to spend their holidays in the South as the South becomes more productive. In fact, condition (14) implies that -ceteris paribus- the impact of Southern production on the environment in the South (measured by β) is relatively high with respect to its impact in the North (measured by δ). An increase in Southern production, therefore, makes the strategy h^N less attractive, inducing Northern people to work low and enjoy their own environment rather than work high to go to the South.

The opposite occurs in Case 2 above ($\overline{L}^N - 1 < c\frac{\gamma}{\alpha}$). In this case, in fact, x_0^N and x_1^N are attractive fixed points and it is $x_0^N < x_1^N \quad iff$:

$$\overline{L}^N - 1 > c\frac{\beta}{\delta} \tag{15}$$

If (15) holds, therefore, the quota x of agents that work high in the North grows as the activity level rises in the South (moving from $z = 0$ to $z = 1$). In other words, Northern people find now more convenient to work high and go on holidays to the South as the South increases its production, since the environmental impact of Southern production is relatively low in the South as compared to the North (β/δ being sufficiently low).

4 Welfare analysis

In this section we will examine the average welfare level in the two hemispheres at all possible dynamic regimes and for all possible values of x and z.The average welfare level in the North and in the South is equal to, respectively:

$$\overline{U}^N(x, z) =: xU_h^N(x, z) + (1 - x)U_l^N(x, z)$$

$$\overline{U}^S(x, z) =: zU_h^S(x, z) + (1 - z)U_l^S(x, z)$$

Observe that $\overline{U}^N(0,z) = U_l^N(0,z)$ and $\overline{U}^N(1,z) = U_h^N(1,z)$ are the Northern average welfare levels when everyone works low and high in the North, respectively. Similarly, for the South we have $\overline{U}^S(x,0) = U_l^S(x,0)$ and $\overline{U}^S(x,1) = U_h^S(x,1)$.

Let us first consider the case $z = 0$ (i.e. minimum production level in the South) and compare the Northern welfare levels at the fixed points with $x = 0$, $x = 1$ and $x = x_0^N$ (when existing). The following proposition applies.

Proposition 5 *If everyone works low in the South, it is:*

$$\overline{U}^N(0,0) > \overline{U}^N(1,0) \quad iff \quad A > A_0^w =: \frac{cB + d\,\bar{Y}_2^N - (\alpha + c\gamma)\,\bar{Y}_2^N}{\overline{L}^N - 1}$$

where: $A_0^w < A_0^*$, A_0^{**}.

Moreover, if there exists a fixed point with $x = x_0^N$, then it is always:

$$\overline{U}^N(0,0) > \overline{U}^N(x_0^N,0) > \overline{U}^N(1,0)$$

Proof. See the Appendix. ■

Proposition 5 above states that the fixed point $(0,0)$ Pareto-dominates the fixed point $(1,0)$ for the North even if there exists h^N-dominance along $z = 0$, provided A is sufficiently large, namely, provided the environmental stock that is left in the North after subsistence production and consumption of $\bar{Y}_1$ in both hemispheres is sufficiently high. In this case, economic growth in the North (i.e. an increase in the aggregate production level) reduces the Northern welfare as we pass from the repulsive point $(0,0)$ to the attractive point $(1,0)$. If so, the increase of production and consumption may lead to a Pareto worsening.[15]

If there is l^N-dominance along $z = 0$, the fixed point $(0,0)$ always Pareto-dominates the fixed point $(1,0)$ for the North since A is larger than A_0^* and A_0^{**} (see Proposition 3) that, in turn, are both larger than A_0^w. In this case, therefore, growth is always welfare-reducing.

[15] Observe that this somewhat surprising result may be partially affected by the structure of the present model. The model, in fact, focus attention on negative externalities only and disregards the possibility of positive externalities. Moreover, there is no coordination among agents in the model. Therefore, environmental degradation is bound to increase (reducing the welfare level) as income grows. We will comeback to this point in the concluding remarks.

The fixed point $(0,0)$ always Pareto-dominates the fixed point $(1,0)$ for the North also if there exists a stable or bistable dynamics along $z=0$. In the case of a stable dynamics, x_0^N is globally attractive in $Q_{z=0}$. Thus, as the economy moves from $x=0$ to $x=x_0^N$ we have again a welfare-reducing growth process. Finally, in the case of a bistable dynamics, x_0^N is repulsive in $Q_{z=0}$ and growth leads to a reduction in the welfare level if the economy moves from $x=x_0^N$ to the right towards $x=1$.

Summing up, when everyone works low in the South ($z=0$), growth is desirable in the North if and only if A is sufficiently low.

Similar propositions apply also for the other sides of Q (i.e. $Q_{z=1}$, $Q_{x=0}$, $Q_{x=1}$). It is easy to verify that, in general, necessary and sufficient condition for Northern (Southern) growth to be desirable is that A (B) is sufficiently low.

Let us now suppose that there is a stable dynamics regime in both $Q_{z=0}$ and $Q_{z=1}$, x_0^N and x_1^N being the attractive fixed points of $Q_{z=0}$ and $Q_{z=1}$, respectively. In this case the following proposition applies.

Proposition 6 *If there exist stable dynamics along $Q_{z=0}$ and $Q_{z=1}$, then:*

$$\overline{U}^N(x_1^N,1) < \overline{U}^N(x_0^N,0) \qquad \text{iff} \qquad \alpha\, \bar{Y}_2^N\,(x_1^N - x_0^N) + \delta\, \bar{Y}_2^S > 0 \qquad (16)$$

Proof. See the Appendix. ■

Proposition 6 implies that if $x_1^N > x_0^N$, namely, if more agents work high in the North as we pass from $Q_{z=0}$ to $Q_{z=1}$, then the Northern welfare decreases. Stated differently, if the South changes its working attitude from "lazy" ($z=0$) to "workholic" ($z=1$), the North will have to counterbalance it by reducing its activity and production level. If not, an increase in both Northern and Southern aggregate production may generate an "undesirable" (Pareto-dominated) growth process for the North.

So far we examined Northern and Southern welfare along the sides of Q, keeping one hemisphere's activity constant at its maximum or minimum level. Let us now investigate the welfare level in the two hemispheres in the whole set Q, namely, for all possible values of x and z.

Proposition 7 *Suppose that there exists a subset of Q where $\Delta U^N(x,z) < 0$, namely, l^N provides a higher utility than h^N. Then $\overline{U}^N(0,0)$ is the absolute maximum of $\overline{U}^N(x,z)$ in Q. Mutatis mutandis, the same applies to the South.*

Proof. See the Appendix. ■

Proposition 7 suggests that, whenever the dynamics of (13) is non trivial (i.e. $\dot{x}$ and $\dot{z}$ are not always positive in Q), the point $(0,0)$ Pareto-dominates any other possible state (x,z) in the North and/or in the South. In particular, if there exists a fixed point $(\widehat{x},\widehat{z})$ inside Q, then it is:
$\overline{U}^N(0,0) > \overline{U}^N(\widehat{x},\widehat{z})$ and $\overline{U}^S(0,0) > \overline{U}^S(\widehat{x},\widehat{z})$.

5 North-South interactions

For the sake of simplicity, we will restrict our attention to "robust" cases, that is, to the cases in which the fixed points are all hyperbolic.[16] Focusing attention on hyperbolic fixed points simplifies the analysis since the latter can be of three types only: attractive points ("sinks"), repulsive points ("sources") and saddle-points. It follows that the vertex of Q will be attractive points if and only if they are attractive along the sides they belong to. Thus, for instance, $(0,0)$ will be a sink if and only if it is an attractive point along both $Q_{x=0}$ and $Q_{z=0}$. Moreover, when there exist fixed points (other than the vertex) along the sides of Q, they must be sinks or saddle points if there is a stable dynamics, sources or saddle points if there is a bistable dynamics. As a consequence, we can infer the nature of a vertex or another fixed point along the sides of Q by simply looking at Propositions 1-4.

For space reasons, we will omit here a complete classification of all possible dynamics of (13) that can be obtained by combining Propositions 1-4. In what follows we will present the most interesting cases where an increase in the activity level may turn out to be "undesirable" (i.e. welfare-reducing). As shown below, the selection process of such cases is path-dependent, namely, it depends on the initial distribution of strategies.

Before moving onwards to these cases, let us first remember that the necessary condition for $(1,1)$ to be an absolute maximum of $\overline{U}^N(x,z)$ and $\overline{U}^S(x,z)$ is that both $\dot{x}$ and $\dot{z}$ are always positive in Q. If this condition is met, $(1,1)$ is globally attractive inside Q, as represented by the dynamics in

[16] By hyperbolic fixed points we mean fixed points that have real part of the eigenvalues different from zero. For our purpose, therefore, we have to exclude the cases where this condition is violated, that is: (i) $A \in \{A_0^*, A_0^{**}, A_1^*, A_1^{**}\}$ and $B \in \{B_0^*, B_0^{**}, B_1^*, B_1^{**}\}$ in Propositions 1-4; (ii) the lines $\Delta U^N(x,z) = 0$ and $\Delta U^S(x,z) = 0$ coincide or intersect at fixed points (x,z) on the boundary of Q.

figure 4. If, on the contrary, such condition is violated, the maximum activity level $(1,1)$ is never desirable for at least one hemisphere.[17]

Combining North and South strategy selection process, we can classify the dynamic regimes according to the number of coexisting strategies that are selected by the two hemisphere out of all four possible strategies (l^S, l^N, h^S, h^N). In this regard, three possible cases can be identified:

(a) only two out of the four possible strategies coexist. In other words, each hemisphere selects a unique strategy (either working high or low).

(b) three strategies coexist: the South selects only one strategy, while the North adopts both strategies (some Northern agents work high, others low).[18]

(c) all four strategies coexist: some people choose to work high and others low in each hemisphere.

5.1 Case a: two strategies coexist

In this case each hemisphere selects only one of the two strategies (l^j, h^j) with $j = N, S$ in the attractive fixed points. This implies a sort of imitation process among agents within each hemisphere so that all agents choose to work either low or high. The most interesting dynamics of this kind is the one represented in figure 5. Such case occurs if and only if there exists a bistable dynamics along all sides Q_s of Q, namely: $iff\ A_i^* < A < A_i^{**}$ and $B_i^* < B < B_i^{**}$ ($i = 0$,1). In this case all vertex of Q are sinks, all other fixed points along Q_s are saddle points and the fixed point inside Q is a source. As figure 5 shows, almost every trajectory will lead to a vertex of Q, where each hemisphere ends up choosing a unique strategy (either working low or high).[19] The vertex attraction basins are delimited by the stable manifolds of the saddle paths.

From the welfare analysis above, we know that each hemisphere achieve its highest and lowest welfare level in $(0,0)$ and $(1,1)$, respectively, with intermediate welfare levels in $(0,1)$ and $(1,0)$. Only one of the four possible vertex selected by the dynamics of Q implies, therefore, the maximum welfare level.

[17] The fixed point $(1,1)$, however, can still be locally attractive, as shown below.

[18] Obviously, the opposite cannot occur. As seen above, in fact, there cannot be stable dynamics of the South along the sides $Q_{x=0}$ and $Q_{x=1}$.

[19] We do not converge to the vertex in a zero measure set, given by the source inside Q and the stable manifolds of the saddle points along the sides of Q.

5.2 Case b: three strategies coexist

The most interesting example of this kind is represented in figure 6. In this case, the fixed point within Q and all four vertex are saddle-points, the fixed points along $Q_{x=0}$ and $Q_{x=1}$ are sources, while those along $Q_{z=0}$ and $Q_{z=1}$ are sinks. At the sinks $(x_0^N, 0)$ and $(x_1^N, 1)$ all Southern agents select the same strategy (working low and high, respectively), while lazy and hard workers coexist in the North (since $0 < x_0^N < 1$ and $0 < x_1^N < 1$). Since $x_1^N > x_0^N$, from the welfare analysis above it follows that both hemispheres are better-off in the sink $(x_0^N, 0)$ than in the sink $(x_1^N, 1)$:

$$U^j(x_0^N, 0) > U^j(x_1^N, 1) \qquad j = N, S$$

From Proposition 5, it also follows that the North is better-off in $(0, 0)$ than in $(x_0^N, 0)$. Similarly, the South is also better-off in $(0, 0)$ than in $(x_0^N, 0)$ since -ceteris paribus- its environment is less damaged from Northern production. Therefore, we have:

$$U^j(0, 0) > U^j(x_0^N, 0) \qquad j = N, S$$

Joining the last two inequalities, it yields:

$$U^j(0, 0) > U^j(x_0^N, 0) > U^j(x_1^N, 1) \qquad j = N, S$$

Both hemispheres achieve their highest welfare level in $(0, 0)$, but this point is not attractive in the present case, therefore neither the North nor the South maximizes its welfare level at the sinks and trajectories lead again to an undesirable growth outcome.

5.3 Case c: all strategies coexist

Sufficient condition for this case to occur is that we simultaneously have:

- l^S-dominance along $Q_{x=0}$
- h^S-dominance along $Q_{x=1}$
- h^N-dominance along $Q_{z=0}$
- l^N-dominance along $Q_{z=1}$

Observe that if the North is "workholic" ($x = 1$) the South also chooses to work high (there is h^S-dominance), while if the North is "lazy" ($x = 0$) the South also chooses to work low (there is l^S-dominance). This occurs because if the North is at its maximum activity level, the Southern environment is highly damaged by the North and the South is induced to make defensive choices producing the substitution good. The opposite occurs if the North is at its minimum activity level. Note that, as in cases (a) and (b) above, also in case (c) North and South achieve their maximum welfare at $(0, 0)$, that is, when they are both at their minimum activity level.

If the South is "lazy" ($z = 0$), working high is the dominant strategy in the North. This occurs because when $z = 0$ the Southern environment is little damaged by production in the same hemisphere and its quality is high with respect to that of the Northern environment, which induces Northern agents to work high to afford a holiday in the South. If the South is "workholic" ($z = 1$) the environmental impact of its production makes this incentive disappear and Northern people choose to work low (l^N-dominance).

The analysis along the sides suggests, therefore, that the South imitates the North, while the North does the opposite. This outcome is determined by the hypothesis of asymmetry between the hemispheres (Northern people can enjoy Southern environment, but not vice versa). Northern people are induced to work high when Southern people work low, since in this case the environment in the South is well preserved. Southern people are induced to work high when Northern people work high, as this tends to damage the environment in the South and calls for Southern production of substitute goods.

Figures 7 , 8 and 9 show three cases where all possible strategies coexist. In each of these figures all vertex are saddle-points and there are no other fixed points along Q_s. It follows that no trajectory will ever converge to a point along Q_s where all agents in one (or both) hemisphere select a unique strategy so that the other strategy can be ruled out. As it can be easily verified, the fixed point inside Q can be either attractive or repulsive and there can be a limit cycle in Q.[20]

Figure 7 shows the case of an attractive fixed point inside Q. In that point some people choose to work high and others low in each hemisphere and the population shares x and z keep constant.

[20] See the Appendix section "Stability of the fixed point inside Q" for a numerical example.

Figure 8 shows a case where the fixed point inside Q is repulsive and almost every trajectory converges to a limit cycle.[21] The population shares x e z "fluctuate" along the limit cycle without converging to any stationary value and none of the four strategies can be ruled out. The mechanism that generates such fluctuations is similar to the one described above. If z is initially low (i.e. the Southern activity level is low), the environment in the South is well preserved and Northern agents are induced to work high. As x increases, however, this damages the Southern environment, leading to an increase in z since more Southern agents work high to afford defensive expenditures. When z is high enough, working high is no longer the best strategy for Northern agents, therefore x decreases, which leads to a reduction in z as well and so on. In this case, therefore, an increase in the Northern activity level leads to a similar increase in the Southern one, whereas the opposite is not true.

Finally, figure 9 shows a case where the fixed point inside Q is repulsive and every trajectory starting inside Q tends to its sides without converging to any fixed point or limit cycle.

6 Comparative static analysis: transferring negative externalities to the South

One of the most debated issues in the environmental literature concerns the possibility that introducing green taxes and stricter environmental policies in the North may lead Northern industries to move polluting productions to the South where ecological regulations are less severe. To what extent such mechanism, known as environmental dumping, takes place in reality is still a matter of investigation in the empirical literature. In the case of worldwide problems like global warming, however, shifting polluting productions to the South may generate negative feedback effects on the North that counterbalance Northern ecological policies. The effects of environmental dumping can be examined in our model by simple comparative static analysis. If the most polluting productions move from the North to the South, the impact of Northern production on Northern environment (α in the model)

[21] The only trajectories that do not converge to the limit cycle are those along the sides of Q.

will fall, whereas its impact on Southern environment (γ) will rise.[22] The same applies if the North transfers to the South toxic wastes that result from its production. Similarly, if the North shifts to the South its exploitation of natural resources used as inputs in the production process, the environmental impact of Northern production is also transferred from the North to the South. Think, for instance, of deforestation induced in the South by paper production in the North, or of exploitation of biodiversity in the South for the pharmaceutical production in the North. All these cases may be analyzed by looking at the combined effect of reducing α and increasing γ in our model. A reduction in α improves Northern environmental quality. An increase in γ, on the contrary, worsens Southern environmental quality stimulating environmental defensive expenditures and thus also aggregate production, which has a negative feedback effect on Northern environmental quality. Hence, the final effect on Northern environment and welfare of transferring negative externalities to the South is a priori undetermined. The possible consequences of this policy can be described by looking again at the dynamics of case (a) above. Recall that in this case the fixed point $(0, 0)$ Pareto-dominates all other vertex, while $(1, 1)$ is Pareto-dominated by all of them. Both $(0, 0)$ and $(1, 1)$ are locally attractive since all sides of Q show a bistable dynamics. An increase in γ reduces -ceteris paribus- B and increases B_1^* (see Proposition 2). As γ increases, therefore, we can pass from bistable dynamics to h^S-dominance along the sides $Q_{x=1}$ and $Q_{x=0}$ (see figure 10). If so, the fixed point $(0, 0)$ with the highest welfare level is no longer attractive, while that with the lowest welfare level $(1, 1)$ is still a sink. Transferring the environmental impact of Northern production to the South, therefore, has ambiguous effects on the Northern welfare and, as shown in this case, it might end up decreasing welfare in both hemispheres.

[22] We are implicitely assuming that Northern industries spend or reinvest in the North the profits from producing in the South. Production in the South of Northern industries can be considered, therefore, as Northern production as it increases Northern income. If, on the contrary, the profits of this polluting activities remain in the South, environmental dumping will cause the impact of Southern production on its environment (β) to rise leaving γ unchanged.

7 Conclusions

Nowadays an increasing number of people make defensive expenditures to protect against deterioration of the environment they live in. This phenomenon, that leads to the substitution of common access depleted environmental goods with privately produced substitute goods, is becoming more and more frequent in modern industrialized economies. This observation has recently induced some studies to examine the relationship between environmental defensive expenditures and economic growth. The basic idea underlying these works is that negative externalities could contribute to a self-reinforcing growth process: environmental degradation induces individual defensive expenditures that raise the activity level which, in turn, may further increase environmental degradation.

The present paper builds on this literature by extending the analysis from a single population to a North-South evolutionary context. The aim of the paper is to investigate the possible feedback effects that environmental defensive expenditures may generate between the two hemispheres and their impact on growth and welfare in rich and poor countries. To examine this issue, we assume that agents can afford defensive expenditures only if they work high and start by analyzing the dynamics of labor and production in one hemisphere when all agents choose to work either low or high in the other hemisphere (section 3). The analysis of these "polar" cases provides the basis for the welfare analysis of each hemisphere (section 4) and for the dynamics that can be obtained by combining North and South strategy selection process (sections 5 and 6).

Growth is characterized in the model by "critical mass" or "imitation" effects. If a sufficiently high number of agents in one hemisphere choose to work high, the other agents living in that hemisphere are induced to choose the same strategy, with an overall growth in the activity and production level. If, on the contrary, the number of "lazy" agents is sufficiently high, the others will also be induced to work little. Critical mass effects may also occur across hemispheres. If the activity level is sufficiently low in the South, its environment will be rather well preserved. This may contribute to raise Northern production since Northern agents may decide to work harder to afford a holiday to the South. However, if the rise in Northern production is high enough, this may deplete the Southern environment, inducing in turn a growth process in the South since Southern people will now want to make defensive expenditures. These feedback effects may determine an undesirable (i.e. welfare-reducing) increase in the activity levels. Both hemispheres, in fact, may end up in a situation where everyone works “too much”: people work harder to protect against pollution, but they might be better-off by working less and enjoining a cleaner world. As shown in the paper, this outcome may occur both in “polar” cases (Propositions 5 and 6) and in the more realistic situation where some people work high and others low in each hemisphere. Calling x (z) the quota of population that works high and make defensive expenditures in the North (South), the solution where everyone works low ($x = 0$, $z = 0$) may Pareto-dominate any other (x, z) pair for both hemispheres (Proposition 7). North-South interactions may also generate limit cycles in the model. If Southern production is low, an increase in Northern production contributes to raise Southern production, but the consequent environmental degradation in the South lowers the incentive of Northern agents to work high, which reduces in turn Southern production and so on. Finally, we show that transferring the environmental impact of Northern production to the South (e.g. transferring polluting activities, production wastes, exploitation of natural resources...) may end up decreasing welfare in both hemispheres.

We are fully aware that the results obtained in this work may look provocative, but we believe that they might contribute to shed light on some aspects of economic growth that have generally be neglected in the literature. Much research, however, will be needed to deepen and improve the present analysis. In the first place, it would be interesting to see whether the same results hold by relaxing some of the simplifying assumptions of the model, such as no capital accumulation and instantaneous rate of regeneration for the environmental stock. In the second place, a larger scale of production is assumed to increase environmental degradation in the paper as it implies more wastes and higher exploitation of natural resources. Beyond this negative “scale” effect of growth on the environment, we would like to account in the future also for the positive environmental impact that economic growth may have via technological progress and a sector shift towards more environmental friendly activities, what Grossman (1995) calls the “technique” and “composition” effects of growth on the environment. Finally, while the choice of Northern agents to work high and spend their holidays in the South may damage the Southern environment through an increase in Northern production, it may also have a positive impact on the South. Northern tourism to the South, in fact, may play a crucial role both as an engine of growth and as a source of income for environmental protection investments in developing countries. These extensions of the future research seem particularly important to provide a more thourogh analysis of the critical mass effects that emerge in the paper.

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9 Appendix

Proof of Proposition 1

The generic expression for the fixed point z_0^S is obtained from condition (12), imposing it to hold as equality (i.e. setting $\Delta U^S(x,z) = 0$) and solving with respect to z when $x = 0$. Note that it is: $\frac{\partial \Delta U^S(x,z)}{\partial z} > 0$.

If case 1 applies ($B \leq B_0^*$), then $z_0^S \leq 0$. It follows that $\Delta U^S(x,z) > 0$ $\forall z \in [0,1]$, so that $z = 0$ is a repulsive, whereas $z = 1$ is an attractive fixed point (h^S-dominance). If case 2 applies ($B \geq B_0^{**}$), $z_0^S \geq 1$, therefore $\Delta U^S(x,z) < 0$ $\forall z \in [0,1]$ so that there is l^S-dominance. Finally, if case 3 applies ($B_0^* \leq B \leq B_0^{**}$), we have $z_0^S \in (0,1)$. In this case, z_0^S is a repulsive fixed point since $\Delta U^S(x,z)$ is negative to the left and positive to the right of z_0^S (bistable dynamics). ■

Proof of Proposition 3

The proof is similar to that of Proposition 1. The only difference is that when $z = 0$ we have that $\Delta U^N(x,z)$ is not strictly increasing in x. In fact, it is:

$\frac{\partial \Delta U^N(x,z)}{\partial x} \gtreqless 0 \leftrightarrow \overline{L}^N - 1 \gtreqless c\frac{\gamma}{\alpha}$.

If $\frac{\partial \Delta U^N(x,z)}{\partial x} > 0$ (case 1), $\Delta U^N(x,z)$ is positive to the right of the fixed point x_0^N and negative to its left. The opposite occurs when $\frac{\partial \Delta U^N(x,z)}{\partial x} < 0$ (case 2).

This property determines the stability analysis of the fixed point x_0^N. ■

Proof of Proposition 5

From (7) with $j = N$ and (4), we have:

$\overline{U}^N(0,0) = a \ln(1 - L_l^N) + b \ln \bar{Y}_1^N + \ln A$

Similarly, from (8), (4) and (6), it is:

$\overline{U}^N(1,0) = a \ln(1 - L_h^N) + b \ln \bar{Y}_1^N + \ln \left[A - \alpha \bar{Y}_2^N + c(B - \gamma \bar{Y}_2^N) + d \bar{Y}_2^N\right]$

It follows that:

$\overline{U}^N(0,0) > \overline{U}^N(1,0) \Leftrightarrow A > A_0^w$

Moreover, if there exists a fixed point $(x_0^N, 0)$ along $Q_{z=0}$, it must be:

$U_h^N(x_0^N, 0) = U_l^N(x_0^N, 0)$

This implies:
$\overline{U}^N(x_0^N,0) = U_l^N(x_0^N,0) = a\ln(1-L_l^N) + b\ln\bar{Y}_1^N + \ln\left(A - \alpha\,\bar{Y}_2^N\,x_0^N\right)$
which is always smaller than $\overline{U}^N(0,0)$ since $x_0^N > 0$.
Similarly, we can also write:
$\overline{U}^N(x_0^N,0) = U_h^N(x_0^N,0) = a\ln(1-L_h^N) + b\ln\bar{Y}_1^N + \ln[A - \alpha\,\bar{Y}_2^N +$
$+c(B - \gamma\,\bar{Y}_2^N\,x_0^N) + d\,\bar{Y}_2^N]$
which is always larger than $\overline{U}^N(1,0)$ since $x_0^N < 1$. Therefore, we have:
$\overline{U}^N(0,0) > \overline{U}^N(x_0^N,0) > \overline{U}^N(1,0)$ ■

Proof of Proposition 6

Observe that, since x_1^N is a fixed point, it is: $U_h^N(x_1^N,1) = U_l^N(x_1^N,1)$ and hence $\overline{U}^N(x_1^N,1) = U_l^N(x_1^N,1)$.

Similarly, for x_0^N we have: $\overline{U}^N(x_0^N,0) = U_l^N(x_0^N,0)$.

Condition (16) is then equivalent to $U_l^N(x_1^N,1) < U_l^N(x_0^N,0)$ and can be easily worked out replacing $U_l^N(x_1^N,1)$ and $U_l^N(x_0^N,0)$ with the correspondent expressions. ■

Proof of Proposition 7

Let us consider the case $\overline{L}^N - 1 > c\frac{\gamma}{\alpha}$, the proof for $\overline{L}^N - 1 \leq c\frac{\gamma}{\alpha}$ being completely analogous. When $\overline{L}^N - 1 > c\frac{\gamma}{\alpha}$, the locus $\Delta U^N(x,z) = 0$ (i.e. the set of points such that $\dot{x}= 0$) is a downward sloping line in (x,z). Moreover, it is $\Delta U^N(x,z) < 0$ (i.e. $\dot{x}< 0$) to its left and $\Delta U^N(x,z) > 0$ (i.e. $\dot{x}> 0$) to its right.[23] If it is $\dot{x}< 0$ in a subset of Q, therefore, two possible cases can occur:

(a) either the line $\Delta U^N(x,z) = 0$ crosses the interior of Q (see figure 2)

(b) or the line $\Delta U^N(x,z) = 0$ lies above and to the right of Q (see figure 3).

Consider case (a) above. We want to show that the Northern average utility is higher in $(0,0)$ than in any other point of Q. For this purpose, it is sufficient to prove that average utility in $(0,0)$ is higher than in any point of Q along the line $\Delta U^N(x,z) = 0$ (step 1), to its left (step 2) or to its right (step 3). The average utility level in $(0,0)$ is:

[23] This follows from the fact that $\frac{\partial \Delta U^N(x,z)}{\partial x} > 0$ when $\overline{L}^N - 1 > c\frac{\gamma}{\alpha}$.

$\overline{U}^N(0,0) = U_l^N(0,0) = a\ln(1-L_l^N) + b\ln \bar{Y}_1^N + \ln A$

Step 1:

Suppose $(\bar{x}, \bar{z}) \in \Delta U^N(x,z) = 0$. Then it is:

$U_h^N(\bar{x}, \bar{z}) = U_l^N(\bar{x}, \bar{z})$

which implies:

$\overline{U}^N(\bar{x}, \bar{z}) = U_l^N(\bar{x}, \bar{z}) = a\ln(1-L_l^N) + b\ln \bar{Y}_1^N + \ln\left(A - \alpha\, \bar{Y}_2^N \bar{x} - \delta\, \bar{Y}_2^S \bar{z}\right)$.

Since $\bar{x}$ and/or $\bar{z} > 0$, it follows that:

$$\overline{U}^N(0,0) > \overline{U}^N(\bar{x}, \bar{z}) \tag{17}$$

thus, average utility in $(0,0)$ is higher than in any point on the line $\Delta U^N(x,z) = 0$.

Step 2:

Consider now the set of points L that lie to the left of $\Delta U^N(x,z) = 0$. Being to the left of $\Delta U^N(x,z) = 0$, we have $\Delta U^N(x,z) < 0\ \forall (x,z) \in L$. Therefore, for every point in L, we have:

$U_l^N(x,z) > U_h^N(x,z)$

which also implies:

$$U_l^N(x,z) > \overline{U}^N(x,z) \tag{18}$$

since

$\overline{U}^N(x,z) - U_l^N(x,z) = \left[U_h^N(x,z) - U_l^N(x,z)\right] x < 0$.

Since $U_l^N(0,0) = \max_{(x,z)\in Q} U_l^N(x,z)$, from (18) it follows that:

$\overline{U}^N(0,0) = U_l^N(0,0) > U_l^N(x,z) > \overline{U}^N(x,z) \qquad \forall (x,z) \in L$.

Hence, average utility in $(0,0)$ is higher than in any point of Q that lies to the left of the line $\Delta U^N(x,z) = 0$.

Step 3:

Finally, take the set of points R that lie to the right of $\Delta U^N(x,z) = 0$. For every point $(x,z) \in R$, we have $\Delta U^N(x,z) > 0$, therefore, $U_h^N(x,z) > U_l^N(x,z)$, which implies:

$$U_h^N(x,z) > \overline{U}^N(x,z) \qquad \forall (x,z) \in R \tag{19}$$

Since $U_h^N(x,z)$ is a decreasing function of x and/or z, any point in R is Pareto dominated by any point $(\bar{x}, \bar{z})$ on the line $\Delta U^N(x,z) = 0$. Therefore, we can write:

$$\overline{U}^N(\bar{x}, \bar{z}) = U_h^N(\bar{x}, \bar{z}) > U_h^N(x,z) \qquad \forall (x,z) \in R \tag{20}$$

From (17), (19) and (20) it follows that:
$\overline{U}^N(0,0) > \overline{U}^N(x,z) \qquad \forall(x,z) \in R.$
Hence, average utility in $(0,0)$ is higher than in any point of Q that lies to the right of the line $\Delta U^N(x,z) = 0$.

Mutatis mutandis, the same proof applies to case (b) where the line $\Delta U^N(x,z) = 0$ is always above Q so that $\dot{x}< 0$ $\forall(x,z) \in Q$ (figure 3).[24] ■

Stability of the fixed point inside Q: a numerical example

Let us assume the following set of parameter values:
$c = d = e = 1$, $\beta = .1$, $\gamma = .3$, $\bar{Y}_2^S = 5$, $\bar{Y}_2^N = 10$, $\overline{L}^S = \overline{L}^N = 2$, $B = 8$.

For the dynamics along the sides of Q to hold, it must be $.5\delta + .25 < \alpha$ and $18 + 10(\alpha - .3) < A < 18 + 5(\delta - .1)$. It is sufficient, therefore, to set A equals to the mean of the upper- and lower-bound of the interval above, namely: $A = 16.25 + 2.5\delta + 5\alpha$.

With these parameter values, the fixed point has coordinates:
$x = 125\frac{1+4\alpha-22\delta}{\alpha-3\delta}$ and $z = 75\frac{4\alpha-1-2\delta}{\alpha-3\delta}$.

The determinant of the Jacobian matrix in that point is always positive and its trace has the same sign as that of the following expression: $g(\alpha, \delta) =: 51 - 42\delta - 146\alpha + 40\alpha^2$. Note that -ceteris paribus- if α is sufficiently high, the fixed point is repulsive. Also note that the trace can become negative for some given changes in the parameter values. If $\alpha = \frac{1}{3} + .5\delta$, in fact, we get $g(\frac{1}{3} + .5\delta, \delta) = 6.77 - 175\delta - 100\delta^2$. In this case, therefore, if δ is sufficiently high, the trace becomes negative (and the fixed point attractive). As it can be easily verified, in this numerical example there exists an Hopf bifurcation and thus a limit cycle when the trace changes its sign.

[24] To prove that this is the case, it is sufficient to inverte the role played by the payoffs of the strategies l^N and h^N in the proof above.

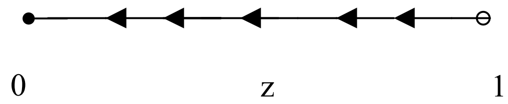

Figure 1.a: l^s-dominance

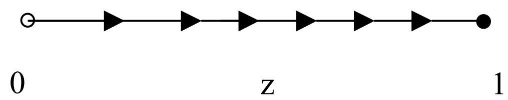

Figure 1.b: h^s-dominance

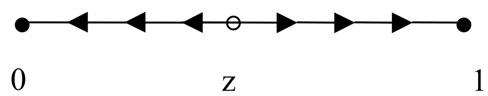

Figure 1.c: bistable dynamics

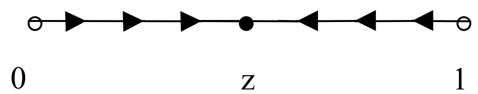

Figure 1.d: stable dynamics

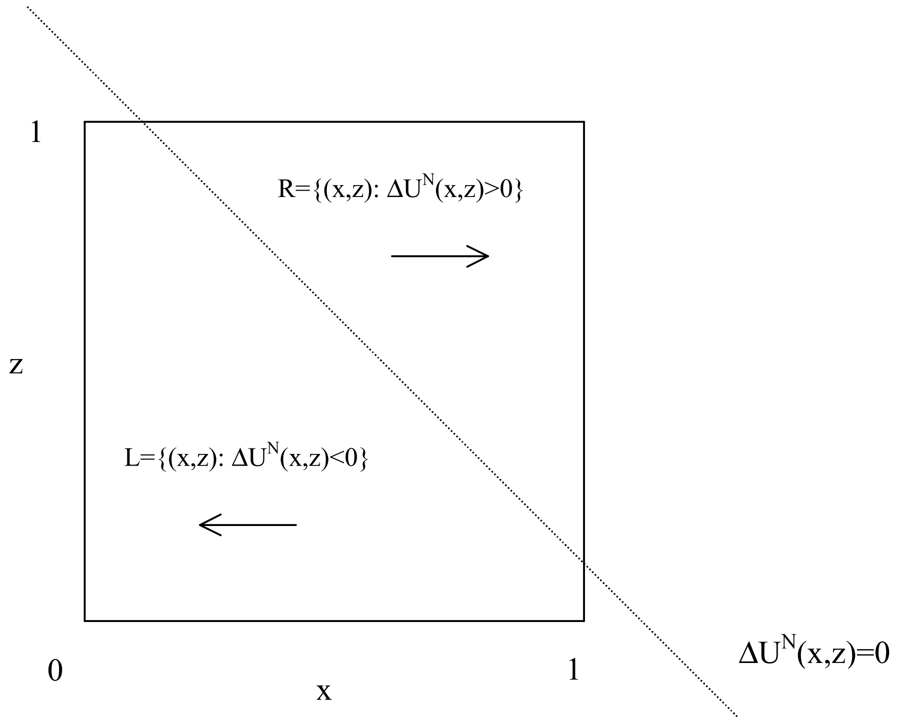

Figure 2

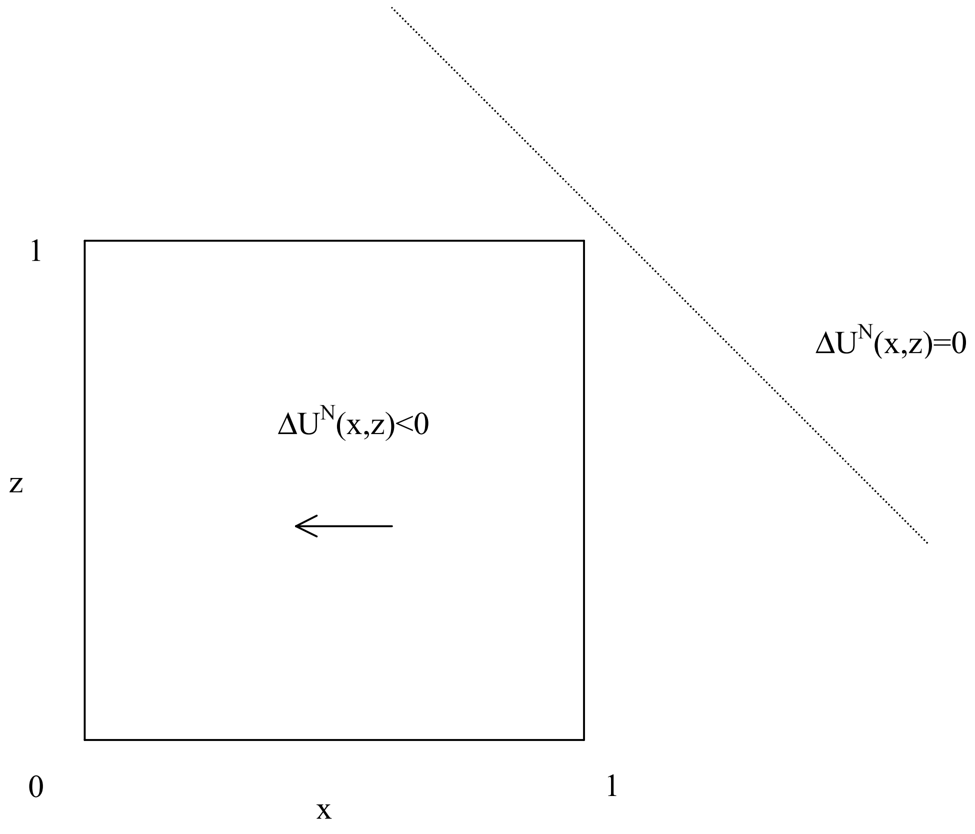

Figure 3

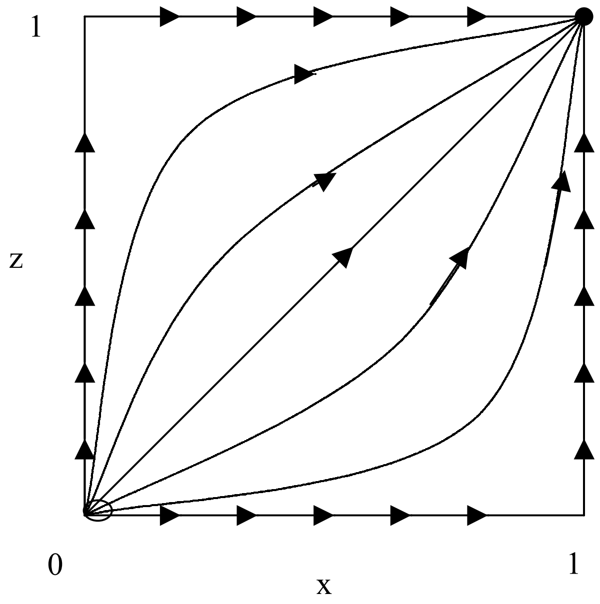

Figure 4

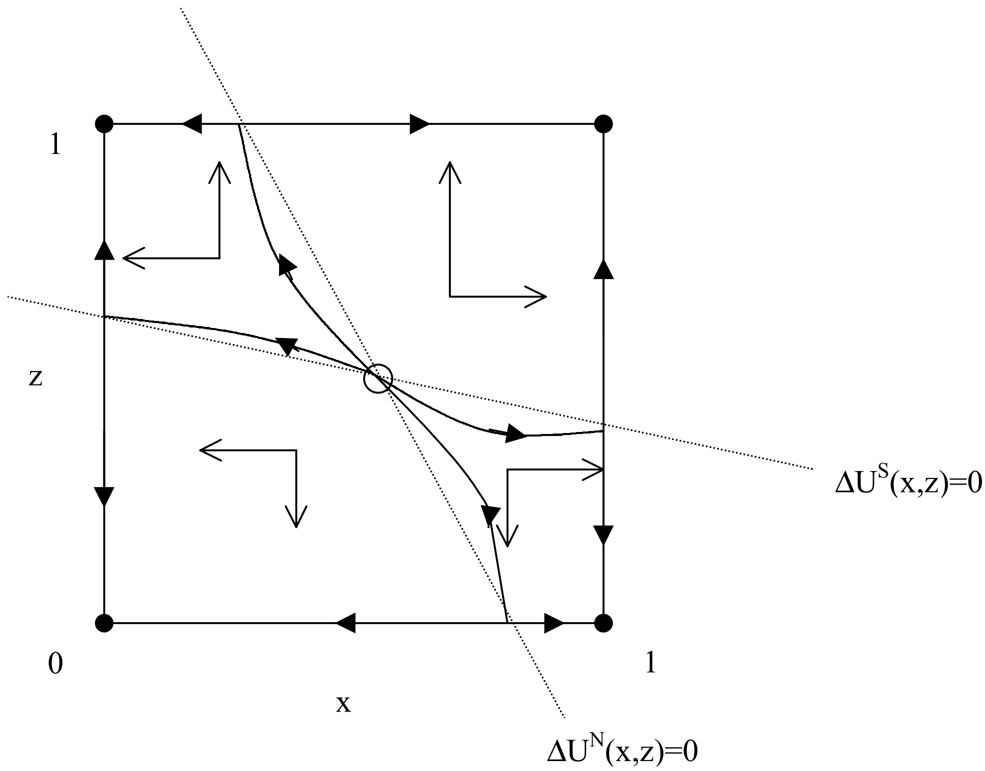

Figure 5

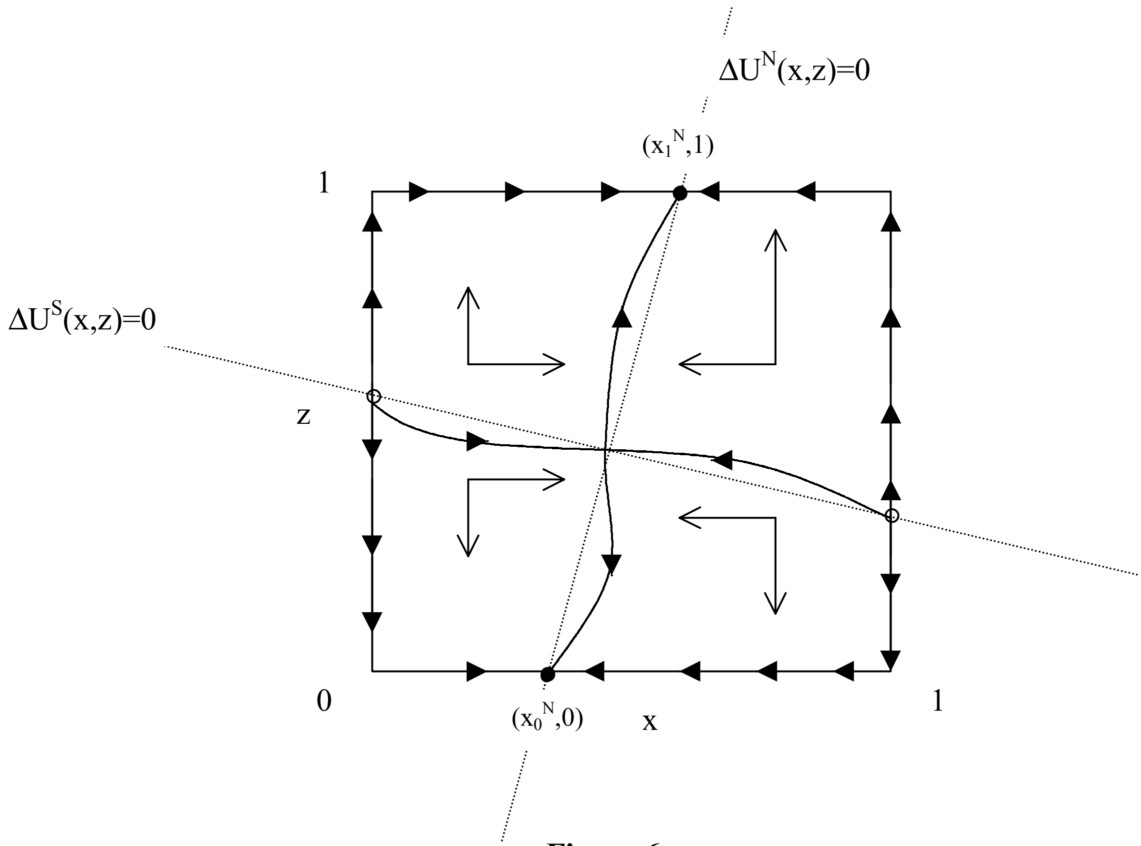

Figure 6

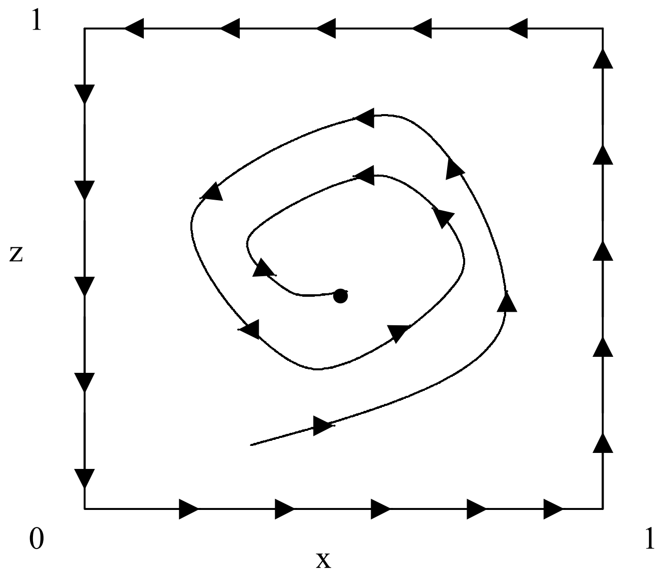

Figure 7

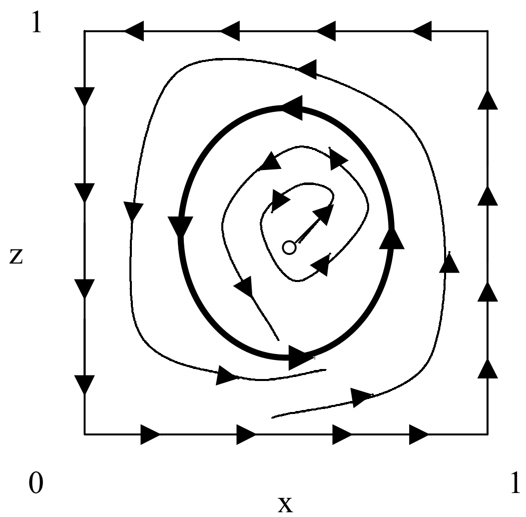

Figure 8

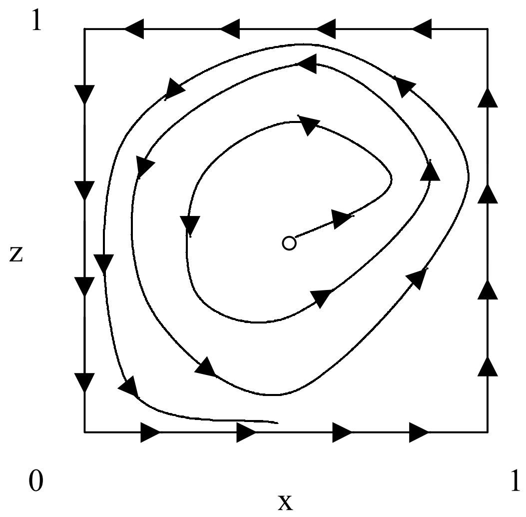

Figure 9

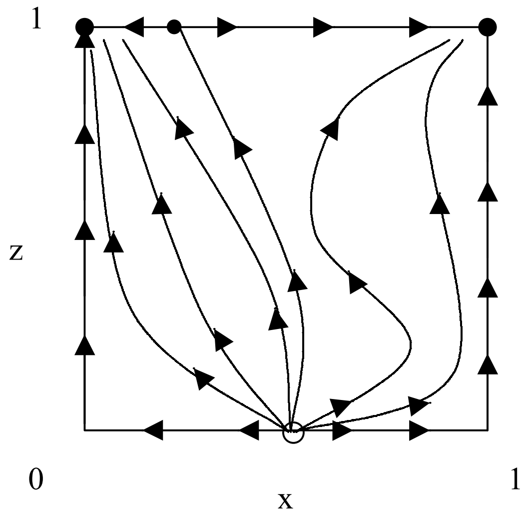

Figure 10