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R&D Models of Economic Growth and the Long-Term Evolution of Productivity and Innovation*

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Abstract

Beginning with Romer (1990), a first generation of endogenous R&D growth models with expanding variety or growing quality of intermediate inputs had a scale effect of R&D employment on productivity growth. C. Jones (1995) criticises this class of models on the ground that their prediction is widely at variance with the facts of R&D employment and productivity growth in the advanced countries over the last fifty years. He suggests a model which shares important features with Arrow's (1962) seminal paper on learning by doing. Growth is not endogenous, but, if population is growing, per capita output may persistently increase as a result of purposeful research effort, due to increasing returns to scale in the output sector.

More recently, a second generation of endogenous R&D growth models has appeared, in which the scale effect is eliminated and the simultaneous expansion of intermediate goods variety and quality occurs under conditions that make steady-state productivity growth depend on the ratio between intensive R&D employment and total employment Dinopoulos and Thompson (1998) , Peretto (1998), Howitt (1999)).

A unifying formal classification of the different types of R&D growth models is used in this paper to discuss how they face with the fact that not only R&D employment, but also the R&D employment *share* has risen dramatically in the advanced countries over the last fifty years. Depending on the model at hand, reconciling this fact with the facts of productivity growth requires different changes in the parameters that describe the 'production function of knowledge'. We try to characterise such changes and discuss their plausibility in the light of the literature on patents and productivity.

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1. Introduction

The ratio between the number of scientists and engineers engaged in research and development (R&D) and the level of total employment has increased dramatically in the U.S.A. and the advanced countries more generally in the second half of the twentieth century. Let us call this ratio $(1 - h_L)$, where h_L is the ratio between employment outside of R&D and total employment. In the U.S.A. $(1 - h_L)$ was nearly three time as large in 1993 than it was in 1950, with a pronounced upward fluctuation in the period 1960-1970 due to government-funded R&D. Jones (2000) estimates that from 1950 to 1993 an even larger rise of the researchers/employment ratio has been observed in the set of G-5 countries (France, West Germany, Japan, the United Kingdom and the United States). Although the numbers involved are very small (the level of the ratio is in any case quite close to zero), so that the time series is more exposed to indivisibility effects and measurement errors, the rise of $(1 - h_L)$ is highly systematic within the period and must be taken seriously.

It is quite striking how the observed dramatic rise of R&D employment did not show up in the productivity figures. As is well known, the growth rate of GDP per hour tended to decline in the advanced countries after the 'golden age' 1950-1970. The decline was less pronounced in the U.S.A. because this country did non enjoy the outburst of productivity from technological catching up after the second world war. For this reason the U.S. experience provides a more telling indication of the relation between R&D effort and productivity growth for a country located on the frontier of technological knowledge.

With the U.S. experience in mind, we shall refer to the stylised fact *(a)* of a large rise of the researchers/employment ratio and to the stylised fact *(b)* of a relatively constant (if compared to the rise mentioned under (a)) growth rate of GDP per hour in the second half of the twentieth century (on average 0.02 from 1950 to 1993)[2]. In the same period, the U.S. capital/output ratio and rate of interest showed an approximately horizontal (again, if compared to the rise under (a)) trend.

If we ask ourselfs how the facts *(a)* and *(b)* can be reconciled, two candidate explanations come to mind.

(i) There has been a fall in the average effect of innovations on measured productivity. This may be at least partly due to the fact that official statistics underrate the qualitative changes in goods and the improvement in their service characteristics (Nordhaus (1997)). Alternatively, or in addition to the previous cause, it may be the case that the rising well being associated with the rising per-capita income makes it increasingly difficult to produce the same proportional improvement in the service characteristics of goods. Hence, the productivity gain tends to fall in the more recent innovations. Robert Gordon (2000) compares the effects on well being of the 'new economy', to those produced by the great innovations during the second industrial revolution. He concludes that the effects of the former do not bear comparison with those of the latter.

(ii) A different, but compatible, line of explanation is a fall in the average productivity of R&D labour, as measured by the number of innovations per unit of research effort. A fall of this kind has certainly taken place, if the number of innovations is measured through the number of patents, granted or applied for (Griliches (1988), (1990)). Measures of this type are strongly biased not only by changes in the 'productive capacity' of institutional patent agencies (e. g. the U.S. Patent Office), but also by changes in the propensity to apply for a patent. Microeconomic studies (Lanjow and Schankermann (1999)) indicate that a lower fall of the productivity of R&D labour is obtained if the aggregate innovation output is obtained by weighting patents by means of indicators of their technological and economic importance. This is related to point *(i)* above.

The question discussed in this paper is how the R&D models developed within the recent revival of general-equilibrium-growth theory meet with the qualitative evidence presented above[3]. A similar question was addressed in an influential paper written by C. I. Jones and published in 1995. Jones observed how the R&D growth models developed to that date displayed a 'scale effect'

[2] See, for instance, Jones (2000).
[3] We shall not consider other families of models where growth is likewise driven by innovations, even less the huge microeconomic literature on R&D.

of the number of researchers on the growth rate of GDP per-capita. These models are criticised by Jones because the 'scale effect' is in striking contrast with the evidence. In the same paper he builds a model, which he defines *semi-endogenous*, where innovations are still the outcome of purposeful and costly R&D effort, but the steady-state growth rate of output per capita is completely determined by the technological parameters and the rate of growth of population. It is therefore independent of the level of population, of preferences, and of policy variables that do not affect technology. The family of R&D growth models with these properties is called here *non-endogenous*. By contrast, the endogenous R&D models of general-equilibrium growth are those where per-capita GDP growth depends upon preferences and/or policy variables generally.

The basic structure of the endogenous and non-endogenous general-equilibrium models of economic growth is discussed in part 2, 3 and 4 of this paper.

Partly as a reaction to Jones' critique, a second generation of endogenous R&D growth models has appeared in the late 1990's. In this second generation, beside 'intensive' innovations that increase the productivity of the intermediate good produced in their sector of application, there are 'extensive' innovations, that increase the number of intermediate goods. In steady-state equilibrium, the number of intermediate goods (hence of sectors) grows at the population growth rate n, so that, in steady state, the number of intensive-researchers per sector is constant. This implies that the 'scale effect' on the rate of growth disappears. In other words, there is a dilution of the 'scale effect' across the growing number of intermediate-good sectors.

A moment reflection reveals that the steady-state predictions of the second-generation endogenous and also of the non-endogenous R&D growth models are still in striking contrast with the evidence presented at the beginning of this introduction.

The dramatic long-term rise of the R&D employment share $(1 - h_L)$ reveals that the long-term growth path of the U.S. economy can not find a theoretical approximation through the hypothesis that the economy has been growing in the neighbourhood of a single steady-state path[4]. Perhaps, the observed long-term rise of $(1 - h_L)$ and the approximately constant rate of productivity growth are more consistent with the hypothesis of a sequence of transitions between different steady-state equilibria induced by a sequence of exogenous parameter changes. This issue is addressed in section 5.1 of part 5. Our conclusion here is that the non-endogenous model is more easily reconciled with the above interpretation of the evidence than the endogenous model, but the parameter change required to explain fact (a) above may be unplausibly large, at least in some range of the preference parameters.

[4] By definition, on a steady-state path the growth rate of every variable is constant for ever. Since the employment shares are bounded between zero and one, their unique admissible steady-state growth rate is zero.

In section 5.2, which hints at possible directions for future research, we broaden the scope of our view, in that the long-term rise of the R&D employment share is likened to the long-term fall of the agriculture employment share, or to the subsequent fall of the employment share of manufacturing industry in favour of services. We ask whether suitable modifications to the basic structure of the R&D general-equilibrium models of economic growth may move some steps towards a better understanding of the systematic association between growth and the composition of employment, hence, between growth and structural change.

The focus of this paper is on steady state results. It is however argued that every monotonic transitional dynamics, so restricted to reproduce the facts (a) and (b) above, is not easily reconciled with the predictions of the R&D growth models and of the endogenous model in particular. Eicher and Turnovsky (1999a), (1999b) show that non-monotonic transitional paths may exist in the non-endogenous growth models with two endogenously accumulating factors, knowledge A and capital K. In what follows the endogenously accumulating factors are capital K, intensive technical knowledge A and extensive technical knowledge N. To the best of my knowledge, a general analysis of the transition dynamics for the R&D growth models of this type is still lacking[5]. The discussion of how it may be relevant to the theme of this paper is left to future work.

An important caveat must be added. In what follows, the rigid supply orientation of the general-equilibrium models of economic growth is taken for granted and is not questioned. This is not because the author is not aware of the biases that are introduced when co-ordination problems or the stability of general equilibrium in the disequilibrium dynamics are disregarded. These issues are simply outside the scope of this paper. Still, in reading it, it is best to bear in mind what is implied by the seminal work by Jacob Schmookler on innovation and growth: the interest in the causes of the long-term growth of GDP *per capita*, as distinguished from the GDP *level*, is at best only a partial justification for the rigid supply orientation of general-equilibrium growth models.

2. A unifying representation of technology

In what follows we build a framework which embeds different views of the relation between output growth and the generation of new inputs, as may be encountered in R&D growth models. This is done under a number of simplifying assumptions about technology that still enable us to

[5] Peretto (1998) reports on the transition dynamics of an R&D growth model where the endogenously accumulating factors are only A and N. In the transition dynamics results of Aghion and Howitt (1998), pp. 109-115, the endogenous factors are A and K.

discuss usually neglected issues, such as the role of complementarities and the relation between technological compatibility and knowledge spillovers. The main simplifying assumption is that the service characteristics of final output *Y* are unchanged throughout, that *Y* can be either consumed or accumulated in the form of capital and that it is produced by means of intermediate goods and labour. The number of available intermediate goods N_t changes through time as a result of innovation activities.

Assume the number of service-characteristics types that exist in nature is finite. An intermediate good is a couple $(v, A_v) \in R_+^2$. *v* is the intermediate-good variety, which identifies a class of functions performed by the good, that is, a composition of the associated flow of service characteristics. For instance, a particular oil may serve mainly as a propeller, but partly also as a lubricant. A_v is the technological level, or generation, to which (v, A_v) belongs. In principle, we should expect that A_v has only an ordinal meaning, possibly with the further ordinal implication that later generations of a variety are also more productive. This is not, however, the interpretation we find in the new-growth literature, where A_v is an index leading to a cardinal productivity measure. The marginal product of (v, A_v) is a known time-invariant function of A_v (and possibly other variables). This leads to a time invariant production possibility frontier, describing the productive potential of every possible present and future combination of intermediate goods.

2.1 Production of material goods

Final output *Y* is produced by means of intermediate goods and labour by perfectly competitive firms, which, individually, face constant returns to scale. Following the R&D growth literature, we introduce a set of simplifying assumptions implying that at every date *t* only the highest (and latest) available technology level $A_{v,\,t}$ of each variety *v* is used. This will be the case since the value of the productivity gain from using the latest generation of a given variety *invariably* dominates the cost differential associated with the same choice.

The assumption is not fully realistic. Even granting that A_v amounts to a productivity index, we should in general expect that the flow of service characteristics associated with (v, A_v) depends upon the type and quantity of other intermediate goods with which (v, A_v) co-operates within a production activity[6]. If there are strong complementarities between different intermediate goods, it may be the case that the best-practice technology level of variety *v* at *t* may not be the highest available. Compatibility constraints may in fact imply that it is inefficient to use in the same activity

[6] If there are production externalities, this service flow may also depend upon the intermediate inputs participating in other production activities.

very distant technology levels of complementary varieties. Complementarities of his sort are simply ruled out in the R&D growth models.

In fact, these models assume a particular substitutability relation between intermediate goods, to the effect that they enter the production function in an additively separable form. Recalling our simplifying assumptions, the individual production function is:

$$Y_t = N_t^{\gamma} L_{Y,t}^{1-\alpha} [\int_{v=0}^{N_t} A_{v,t} x_{v,t}^{\alpha} \partial v] \tag{1}$$

where x_v is a quantity of the intermediate-good variety v and is L_Y labour employment in the production of final output. Thus, the marginal product at t of the intermediate good (v, $A_{v,t}$) is:

$N_t^{\gamma} L_{Y,t}^{1-\alpha} A_{v,t}^{\upsilon} x_{v,t}^{\alpha-1}$

It is independent of the inputs of the other intermediate goods, although it may depend, if $\gamma \neq 0$, on the total *number* of intermediate goods cooperating with it.

Intermediate goods are produced by local monopolists through a different set of activities. The reason why firms in the intermediate-good sector can not be perfectly competitive is quite robust (Arrow (1987) and (1998), Romer (1990)). The right to produce a new intermediate good involves an innovation cost that represents a fixed cost, because once the knowledge to produce a unit of a new good is acquired, it can be applied to the production of an indefinite number of units. If intermediate-goods production is otherwise subject to constant variable costs, we are faced with a clear case of increasing returns.

The input of the activity for producing one unit of *(v,A_v)* is a quantity of capital K which depends positively on the technology level A_v. To fix our ideas, K units of capital invested in the production of good *(v, A_v)* give rise to K/A_v^{ω} units of this good, where $\omega > 0$, thus implying that more capital intensive methods are required to produce intermediate goods of a later generation. For the sake of later reference we write:

$$K_{v,} = x_v A_v^{\omega} \tag{2}$$

Howitt (1999) adopts a similar increasing-capital-intensity assumption and claims that capital used in intermediate-goods production can be interpreted as human capital. The above specification implies that the average and marginal cost, in terms of final output, of producing *(v, A_v)* is $r A_v^{\omega}$, where r is the rental price of capital. Since we abstract from depreciation, r is also the rate of interest.

The monopoly output $x_{v,t}$ of variety v is:

$$x_{v,t} = \alpha^{2/(1-\alpha)} N_t^{\gamma/(1-\alpha)} L_{Y,t} r_t^{1/(\alpha-1)} A_{v,t}^{(1-\omega)/(1-\alpha)} \tag{3}$$

The monopoly profit from producing $x_{v,\,t}$ is:

$$\pi_{v,\,t} = \alpha(1-\alpha)\, N_t^{\gamma}\; L_{Y,t}^{(1-\alpha)}\, A_{v,\,t}\;\, x_{v,\,t}^{\;\alpha} \tag{4}$$

$1 > \omega$ implies that monopoly output is positively related to the technological advance $A_{v,t}$. Aghion and Howitt ((1998), chap. 12) and Howitt (1999) obtain a monopoly output which is uniform across varieties and independent of A, by imposing $\omega = 1$. We hold to the latter simplifying assumption to obtain:

$$x_{v,\,t} = \alpha^{2/(1-\alpha)}\, N_t^{\gamma/(1-\alpha)}\, L_{Y,t}\, r_t^{1/(\alpha-1)} = x_t \tag{5}$$

In equilibrium, final output Y is then:

$$Y_t = N_t^{\gamma}\; L_{Y,t}^{\;1-\alpha}\, N_t\, A_t\, x_{,t}^{\alpha} = \alpha^{2\alpha/(1-\alpha)}\, N_t^{(1-\alpha+\gamma)/(1-\alpha)}\, L_{Y,t}\, r_t^{1/(\alpha-1)}\, A_t \tag{6}$$

Where A_t is the average technology level across intermediate goods:

$$A_t = 1/N_t \left[\int_{v=0}^{N_t} A_{v,\,t}\;\partial v\right] \tag{7}$$

An equivalent equilibrium expression of Y_t is obtained by observing that, if h_K is the capital share employed in material, as opposed to knowledge, production, it must be the case that, in equilibrium we have $(h_{K,,\,t}\, K_t) / A_t = N_t\, x_t$. Hence:

$$Y_t = N_t^{\gamma}\, (h_{L,\,t}\, L_t)^{1-\alpha}\, N_t^{1-\alpha}\, A_t^{(1-\alpha)}\, (h_{K,\,t}\, K_t)^{\alpha} \tag{8}$$

It is then clear how the assumption $\gamma = \alpha - 1$ (see, for instance, Aghion and Howitt (1998), chapter 12) sterilises the effects of the growing number of varieties on final output, which result from the additively separable way in which the single varieties enter the production function. Where these effects are not sterilised, because $(1 - \alpha + \gamma) > 0$, we observe that the production function corresponding to a constant technology level contains a form of increasing returns due to specialisation, as measured by N. The best known example along these lines is probably Romer (1990), which assumes $\gamma = 0$.

Recalling that in steady state the rate of interest is constant, and the labour and capital shares employed in the (final and intermediate) output sector are also constant, equation (5) yields the steady-state-growth equation:

$$g_Y = g_L + [(1-\alpha+\gamma)/(1-\alpha)]\, g_{N.} + g_A \tag{9}$$

where g_i is the proportional instant rate of change of variable i. In particular, if following Romer (1990) we impose the restrictions $\gamma = 0$ and $g_A = 0$, the above relation boils down to $g_y = g_L + g_N$, where it is apparent that the growth rate of per-capita output is simply the growth rate in the number of specialised varieties.

2.2 Production of knowledge

2.2.1 Intensive innovations

An intensive innovation in sector v arriving in the interval $t + \partial t$ is the stochastic outcome of the innovation effort performed at t in this sector. The innovation contributes to shifting the technology frontier according to

$$\dot{A}_{t,Max} = (\delta / N_t) A_{t\,Max} \qquad (10)$$

and brings $A_{v,\,t}$ to the shifted frontier. Thus, access to the frontier technology level is available, but not costless, to every successful *intensive* innovator operating in sector v. The knowledge increment has elasticity +1 with respect to $A_{t\,Max}$ and elasticity $-$ 1 with respect to the number of sectors in the economy (Aghion and Howitt (1998), chap. 12). The idea is here that the higher the number of sectors, the lower the impact of an innovation in sector v on the technology frontier.

The Poisson arrival rate of an intensive innovation in sector v at t is:

$$\phi_{v,\,t} = \lambda\, (u_{L,\,v,\,t}\, L_t)^{\theta}\, (u_{K,\,v,\,t}\, K_t)^{\xi}\, A_{t\,Max}{}^{\chi} \qquad (11)$$

where $\xi > 0$, $\theta > 0$, λ is a constant, $u_{L,\,v}$, $u_{K,\,v}$ are the fractions of total labour and capital invested in intensive R&D on variety v.

The returns offered by the investment of *rival*-resources in intensive R&D are constant or decreasing, depending on $\theta + \xi = 1$ (Barro and Sala-I-Martin (1995), chap. 7), or $\theta + \xi < 1$. The second case arises if there is a congestion effect on the returns to R&D investment (Stokey (1995), Howitt (1999)), with the result that the larger the rival resources invested in research, the higher the probability that independent innovation efforts produce the same outcome.

The parameter χ is meant to capture how the arrival rate is affected by the frontier knowledge stock $A_{t\ Max}$. There are two main forces at work here and which act in opposite directions. Thus, we may split the parameter χ into two components:

$\chi = \chi_1 + \chi_2$.

χ_1 is the so called 'complexity effect': more advanced technology levels are progressively more difficult to discover as a result of the increasing complexity of the search activity. Thus, we have $\chi_1 < 0$. This is the assumption we find in a number of search-theoretic models of R&D-based economic growth (Jovanovic and Rob (1990), Stokey (1995), Kortum (1997)). Realistic as it may be, the positive the correlation between the technology-frontier index and the difficulty of search must be simply assumed and can not find a micro foundation *within* a formal framework which does not lend itself to consider the feed-back of innovations on the complexity of the search space.

The parameter $\chi_2 > 0$ captures the "standing on giants' shoulders" effect[7] (Caballero and Jaffe (1993)), which postulates that a higher frontier knowledge increases the probability of invention because an investment in intensive R&D creates the opportunity to exploit a knowledge spillover from the technology frontier to the innovators. This positive influence of knowledge on the innovation-success probability is distinct from and indeed adds to the influence of the stock of ideas on the size of the knowledge shift, which takes place *if* the innovation arrives (see (7) above). To this extent, it is unclear what are the grounds for assuming that the giants' shoulders effect is positive and is close in absolute magnitude to the complexity effect. We shall see nevertheless that the restriction $\chi = \chi_1 + \chi_2 = 0$ (or other equivalent condition) is characteristic of the R&D endogenous-growth models.

The main simplifying hypothesis introduced with (11) is that the success probability of intensive R&D on variety v is independent of the distribution of the local stocks $A_{v,\,t}$. Together with (7) this implies that the intensive research effort and the arrival rate are uniform across sectors. Other formulations (see, for instance, Barro and Sala-I.Martin (1995), chap. 7) relate the complexity effect and the giant's shoulders effect for sector v to the local stock $A_{v,\,t}$. The same property of a uniform equilibrium arrival rate is however imposed also in this case, by means of ad hoc restrictions introduced to this end.

Since intensive R&D is performed independently by the N sectors, the aggregate rate of intensive innovations is deterministic and equals

$$N_t\phi_{v,\,t} = N_t\,\lambda\,(u_{L,\,v,\,t}\,L_t)^{\theta}\,(u_{K,\,v,\,t}\,K_t)^{\xi}\,A_{t\,Max}{}^{\chi} \qquad (12)$$

Recalling (7),and the fact that the equilibrium research effort is uniform across sectors, we obtain that the overall shift of the technology frontier at time t resulting from the intensive R&D in the N sectors is:

$$\dot{A}_{t,Max} = \delta\,\lambda\,(u_{L,\,t}\,L_t\,/\,N_t)^{\theta}\,(u_{K,\,t}\,K_t\,/\,N_t)^{\xi}\,A_{t\,Max}{}^{\chi+1} \qquad (13)$$

where u_L and u_K are the aggregate labour and capital shares invested in intensive R&D.

2.2.2 Extensive innovations

An 'extensive' innovation is the introduction of a new variety v. On the assumption that there is an external effect such that the technical knowledge in the economy affects the technology level of a new variety, a-not-too-unplausible restriction is that the technology level distribution of a new variety corresponds to the technology level distribution across the existing varieties (Howitt (1999)). This implies that extensive innovations at t do not affect the average technology level in the economy A_t. An assumption to the same effect is that new varieties arriving at t have a deterministic technology level A_t (Peretto (1998)).

We assume that the extensive innovation effort is related to the creation of new varieties by the deterministic law:

$$\dot{N}_t = \beta\,(z_{L,\,t}\,L_t)^{\varepsilon} N_t^{\,\tau}\,(z_{K,\,t}\,K_t)^{\psi} A_t^{\,\nu} \equiv \phi_{N,\,t} \tag{14}$$

β is a constant, z_L is the fraction of total labour employed in extensive R&D. We impose the restriction $\varepsilon > 0$, $\psi > 0$, $\tau \geq 0$. The case $\varepsilon + \psi < 1$ indicates that there are decreasing returns with respect to the scale of the rival resources invested in extensive search. The restriction is referred to as the 'congestion hypothesis'. A positive τ bears the interpretation that a higher number of varieties amounts to a wider knowledge base in the economy as a whole and therefore facilitates the discovery of yet new varieties. If this is in itself quite plausible, far more questionable appear to be 'point restrictions' such as $\tau = 1$, or $\tau = 0$, as may be found, for instance, in the pure variety-extension model of Romer (1990) and in Peretto (1998), respectively.

The parameter ν indicates how the production of an extensive innovation flow $\dot{N}$ of technology level A is related to the size of the average technology index A. $\nu = 0$ (Peretto (1998)) states that the cost (in terms of rival resources invested in extensive R&D) of producing a given innovation flow $\dot{N}$ with average technology level A is independent of A. If $\nu > 0$ (< 0) this cost would be decreasing (increasing) in A. The restriction $\nu > 0$ fits with the idea that the growth of technical knowledge along the quality dimension goes hand in hand with a growing 'complexity' of technology, which has a positive effect on the ease with which new varieties are discovered. As before, since the present framework cancels from view the rising complexity of the technology space, the treatment of this feature can be at best evocative.

3. Steady-growth equations

[7] Cf. Merton (1965).

A steady state, or balanced-growth path, is a particular constant-growth path such that the growth rate of every variable is constant *for ever*. Since the factors employment shares can not exit the interval [0, 1], the definition immediately implies that the growth rate of such variables is zero on a balanced path.

The assumptions of section 2.2 imply that the ratio ($A_{t\ Max}$ / A_t) converges to $(1 + \delta)$[8]. Assuming that convergence has already taken place, (13) is written:

$$\dot{A}_t = \delta\, \lambda\, (u_{L,\,t}\, L_t / N_t)^{\theta}\, (u_{K,\,t}\, K_t / N_t)^{\xi}\, A_t^{\chi + 1} \tag{15}$$

Recalling that on a constant-growth path $\dot{A}_t$ and A_t grow at the same rate, using (8), (14) and (15) we write the steady-state growth equations:

$$g_A\, [-\chi] + (\xi + \theta)\, g_N - \xi\, g_K = \theta\, n \tag{16}$$

$$-\nu\, g_A + (1 - \tau)\, g_N - \psi\, g_K = \varepsilon\, n \tag{17}$$

$$-(1 - \alpha)\, g_A - (\gamma + 1 - \alpha)\, g_N + g_K\, (1 - \alpha) = (1 - \alpha)\, n \tag{18}$$

If we define the variables $k \equiv K/N$, $l \equiv L/N$, so that $g_K = g_k + g_N$, $n = g_l + g_N$, (16) – (17) – (18) yield the following system:

$$\begin{bmatrix} -\chi & 0 & -\xi \\ -\nu & 1-\tau-\varepsilon-\psi & -\psi \\ -(1-\alpha) & -(\gamma+1-\alpha) & 1-\alpha \end{bmatrix} \begin{bmatrix} g_A \\ g_N \\ g_k \end{bmatrix} = \begin{bmatrix} \theta\, g_l \\ \varepsilon\, g_l \\ (1-\alpha)\, g_l \end{bmatrix} \tag{19}$$

3.1 Endogenous R&D growth

Let [I – Γ] be the square matrix in the left-hand-side of (19). If [I – Γ] has a non zero determinant, the steady-state growth rates of A, N and K are fully determined by equations (19), hence by *technology*, given the exogenous growth rate of population. Thus Det [I – Γ] ≠ 0 states that preferences do not have any bearing on the speed of steady-state growth and policy measures by a government are equally uneffective, unless they are able to affect the technological parameters. It is then apparent how the crucial assumption of the endogenous R&D growth models is Det [I – Γ] = 0. In this case the coefficients in (19) are linearly dependent and additional equations are

[8] Cf. Aghion and Howitt (1998), p. 412.

necessary to determine the steady-state growth rates of the variables. One missing equation is derived from the firs-order conditions associated to the utility-maximisation problem:

$$Max: \int_{t=0}^{\infty} \frac{c_t^{1-\sigma} - 1}{1-\sigma} e^{-(\rho-n)t} \partial t$$

subject to the flow budget constraint that per-capita consumption at t c_t is not negative and is constrained by wage and interest income minus the accumulation of stocks at t[9]. ρ is the rate of time preference and $(1 / \sigma)$ is the constant inter-temporal elasticity of substitution.

In particular, the proportional growth rate of c_t must satisfy:

$g_c(t) = (r_t - \rho) / \sigma$

where c is per capita consumption, Obviously enough, in steady state $n + g_c = g_Y = g_K$.

The restriction Det $[I - \Gamma] = 0$ may be of course introduced in a number of ways. The standard practice of endogenous growth models with intensive R&D is to postulate the special case: $\chi = 0$ and $\xi = 0$ (see, for instance, Grossman and Helpman (1991), Aghion and Howitt (1992), Howitt (1999), Peretto (1998), Young (1998), Barro and Sala I-Martin (1995), chapter 7). This yields:

$$\dot{A}_t / A_t = \delta \lambda (u_{L,,t} L_t / N_t)^{\theta} \qquad (20)$$

As is also revealed by the first equation of system (19), with $\chi = \xi = 0$, consistency with steady state requires $g_N = n$, that is, $g_l = 0$. In particular, in the models where extensive innovations are not contemplated, so that N is constant, it is assumed that L is also constant and there is a *scale effect* of the intensive-research employment *level* on the growth rates of A and Y. This occurs in the pure quality expansion model of Grossman and Helpman (1991), Aghion and Howitt (1992) and Barro and Sala-I-Martin (1995) (chapter 7). Jones (1995) draws the attention on the lack of empirical corroboration for the hypothesis of a scale effect on the growth rate. In models with a growing population, equation (20) is reconciled with the lack of any scale effect on the steady-state rate of growth, by introducing special assumptions which make sure that L/N is constant (Howitt (1999)), or at least converges to a fixed steady-sate value (Peretto (1998), Young (1998)). With a simplified specification of equation (14) such that $\nu = 0$ and $\psi = 0$, the required restriction is:

$\tau + \varepsilon = 1$.

This implies:

$$\dot{N}_t / N_t = \beta z_{L,t}^{\varepsilon} (L_t / N_t)^{\varepsilon} \qquad (21)$$

[9] Cf. Barro and Sala-I-Martin (1995), charter 2.

and using the steady-state condition $g_N = n$, this yields

$$m\, z_L = (n / \beta)^{1/\varepsilon} \tag{22}$$

where m is the steady state value of L/N.

We now look at two different sets of steady-state solutions of the endogenous model, as specified above, which correspond to the possibility that: *(i)* the costs of one additional unit of labour effort invested in extensive or intensive R&D are identical; *(ii)* these costs are not identical.

We shall proceed under the simplifying assumption $\gamma = \alpha - 1$ (see equation (18)), so that $g_K = g_A + n$. Thus:

$$g_c = g_A = (r - \rho) / \sigma \tag{23}$$

3.1.1 Identical opportunity cost of effort in extensive and intensive R&D

Suppose the only cost of one additional unit of labour effort in extensive or intensive research is the forgone opportunity of obtaining the wage rate w by selling that unit in the labour market. This implies that the private instantaneous marginal returns from innovation effort in intensive and extensive R&D must be identical and equal to the wage rate w. With our production function (8) we have:

$$w = (1 - \alpha) h_L^{-\alpha} q^{\alpha} \mathrm{A} \tag{24}$$

where $q \equiv K/AL$.

$$[\phi_{v,\,t} / (u_{L,\,v,\,t}\, L_t)]\, V_{v,\,t} = \lambda\, (u_{L,,\,t}\, L_t / N_t)^{\theta - 1}\, V_t = w_t = [\phi_{N\,t} /(z_{L,\,,\,t}\, L_t)] V_{N,\,t} = \beta\, (z_{L,\,t}\, L_t / N_t)^{\varepsilon - 1}\, V_{N,\,t} \tag{25}$$

where $V_{v,\,t} = V_t$ is the expected value of a quality innovation in any sector v at time t, and $V_{N,\,t}$ is the expected value of an extensive innovation at time t.

Let $v_t \equiv V_t / \mathrm{A}_{t,\,Max}$ and $v_{N,\,t} \equiv V_{N,\,t} / \mathrm{A}_t$; in words, v_t and $v_{N,\,t}$ are the productivity adjusted values at time t of an intensive and extensive innovation, respectively.

From (24) and (25):

$$v_t = [(1 - \alpha) / \lambda(1 + \delta)]\, h_L^{-\alpha} q^{\alpha} (u_{L,,\,t}\, L_t / N_t)^{1 - \theta} \tag{26}$$

$$v_{N,\,t} = [(1 - \alpha) / \beta]\, h_L^{-\alpha} q^{\alpha} (z_{L,\,t}\, L_t / N_t)^{1 - \varepsilon} \tag{27}$$

Moreover, one obtains the asset equations[10]:

$$\partial v_t / \partial t = [r_t\, \phi_t]\, v_t - \pi_t \tag{28}$$

$$\partial v_{N,\,t} / \partial t = [r_t\, \phi_t]\, v_{N,\,t} - \pi_t \tag{29}$$

where π_t is the productivity adjusted profit of a local monopolist and it is worth recalling that, since an extensive innovation will be displaced by an intensive innovation in the same sector, the expected obsolescence rate takes the same value ϕ_t for extensive *and* intensive innovations.

Differentiating (26), (27) with respect to time and imposing the steady-state restrictions $g_N = n$, $\dot{h}_L = 0, \dot{u}_L = 0, \dot{z}_L = 0, \dot{q} = 0$ we obtain that $\partial v_t / \partial t = \partial v_{N,t} / \partial t = 0$. Thus, (28) and (29) imply the steady state condition:

$$\pi_t / v_t = \pi_t / v_{N,t} \tag{30}$$

which can be written $v = v_N$, or, equivalently,.

$$(1 + \delta) \lambda u_{L,}^{\theta - 1} m^{\theta} = \beta z_L^{\varepsilon - 1} m^{\varepsilon} \tag{31}$$

Using (20) and (22) we obtain:

$$g_A = \lambda\delta u_{L,}^{\theta} m^{\theta} = \lambda\delta (u_{L,} / z_{L,})^{\theta} (n/\beta)^{\theta / \varepsilon} \tag{32}$$

From (20) and (31):

$$g_A = [\delta / (1+ \delta)] n (u_{L,} / z_{L,}) \tag{33}$$

This yields:

$$(u_{L,} / z_{L,}) = [(1 + \delta) \lambda n^{(\theta - \varepsilon) / \varepsilon} \beta^{- \theta / \varepsilon}]^{1 / (1 - \theta)} \tag{34}$$

$$g_A = \delta [(1 + \delta)^{\theta} \lambda n^{1 - \varepsilon} \beta^{- \theta / \varepsilon}]^{1 / (1 - \theta)} \tag{35}$$

In the special, but convenient case $\theta = \varepsilon$ (34) and (35) simplify to:

$$(u_{L,} / z_{L,}) = [(1 + \delta) \lambda \beta^{-1}]^{1 / (1 - \theta)} \tag{34'}$$

$$g_A = \delta n [(1 + \delta)^{\theta} \lambda \beta^{-1}]^{1 / (1 - \theta)} \tag{35'}$$

Thus we reach the striking conclusion that in the endogenous model as specified above, an identical marginal innovation cost for intensive and extensive R&D makes $(u_{L,} / z_{L,})$ and g_A depend only on technological parameters. Instead, the steady-state shares $u_{L,}$, $z_{L,}$ and h_L depend also on the preference parameters ρ and σ. In particular, for $\theta = \varepsilon$ we have :

$$z_{L,} = \{ [(\rho+\alpha n) / \alpha n] + [(1 + \delta)\lambda / \beta]^{1 / (1 - \theta)} [1 + (\sigma\delta + 1) / \alpha(1 + \delta)]\}^{-1} \tag{36}$$

The reason why the model is still qualified to be called endogenous is that a policy variable such as an innovation subsidy (see Aghion and Howitt (1998), p. 419) would affect the rate of growth, if it exerts an asymmetric influence on the cost from one additional unit of labour effort in extensive and intensive R&D. To understand this point, it is worth considering the case below, where the cost asymmetry does not arise from a policy variable, but from a slight generalisation of the innovation technology considered above.

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[10] Cf Aghion and Howitt (1998), p. 109-110.

3.1.2 Asymmetric innovation cost

Suppose that every unit of labour invested in R&D at time t is combined with a quantity of capital $A_{t,\,\mathrm{Max}}\, T_A$, in the case of intensive R&D and $A_t\, T_N$ in the case of extensive R&D. In this section we assume $T_N \neq T_A$. In other words, labour and capital are perfectly complementary inputs to innovation activities, intensive and extensive, but the ratio between the two inputs is different in the two set of activities, even after adjustment is made for the productivity levels $A_{t,\,\mathrm{Max}}$ and A_t. The case $T_N = T_A$ yields conditions identical to those obtained in the previous section, with the understanding that the terms K and q must be everywhere replaced with $h_K K$ and $h_K q$, where h_K is the fraction of total capital employed in the output sector (to produce intermediate goods). u_K and z_K are the fractions of total capital employed in intensive and extensive R&D, respectively. With this notation:

$w_t = (1-\alpha)\, h_{L,\,t}^{\;-\alpha}\, h_{K,\,t}^{\;\alpha} q_t^{\alpha} \mathrm{A}_{\,t}$

$r_t = \alpha^2\, h_{L,\,t}^{\;1-\alpha}\, h_{K,\,t}^{\;\alpha-1} q_t^{\alpha-1}$

$u_{K,\,t} = (1+\delta)\, u_{L,\,t}\, q_{\,t}^{\,-1} T_A$

$z_{K,\,t} = z_{L,\,t}\, q_{\,t}^{\,-1} T_N$

$h_{K,\,t} = 1 - u_{K,\,t} - z_{K,\,t}$

Condition (25) is now replaced by:

$$\lambda\,(u_{L,,\,t}\, L_{\,t} / N_{\,t})^{\theta-1}\, V_{\,t} = w_t + r_t\, A_{t,\,\mathrm{Max}}\, T_A \qquad (37)$$

$$\beta\,(z_{L,\,t}\, L_{\,t} / N_t)^{\varepsilon-1}\, V_{N,\,t} = w_t + r_t\, A_t\, T_N \qquad (38)$$

(26) and (27) are replaced by :

$$v_{\,t} = [1/\lambda(1+\delta)]\;\; (u_{L,,\,t}\, L_{\,t} / N_{\,t})^{1-\theta}\, h_{L,\,t}^{\;-\alpha}\, h_{K,\,t}^{\;\alpha}\, q_{\,t}^{\;\alpha}[(1-\alpha) + \alpha^2\, h_{L,\,t}\, h_{K,\,t}^{\;-1} q_{\,t}^{\;-1} T_A\,] \qquad (39)$$

$$v_{N,\,t} = [1/\beta]\,(z_{L,\,t}\, L_{\,t} / N_t)^{1-\varepsilon}\, h_{L,\,t}^{\;-\alpha}\, h_{K,\,t}^{\;\alpha}\, q_{\,t}^{\;\alpha}[(1-\alpha) + \alpha^2\, h_{L,\,t}\, h_{K,\,t}^{\;-1}\, q_{\,t}^{\;-1} T_N\,] \qquad (40)$$

Recalling that in steady state $v = v_N$, and assuming for simplicity $\theta = \varepsilon$, we obtain:

$$u_L / z_L = \{\lambda(1+\delta)\,[(1-\alpha) + \alpha^2\,(r/\alpha^2)^{1/(1-\alpha)}\, T_N\,] / \beta\,[(1-\alpha) + \alpha^2 (r/\alpha^2)^{1/(1-\alpha)} T_A\,]\}^{1/(1-\theta)}$$

It turns out that u_L / z_L is related to the steady-state rate of interest, which depends on the preference parameters ρ and σ. In particular, it can be easily checked that the sign of $\partial(u_L / z_L) / \partial r$ is positive if $T_N - T_A > 0$ and is negative if $T_N - T_A < 0$. Moreover, using the fact that (32) holds also in the present case, we can see how similar considerations apply to the relation between g_A and the rate of interest. In fact, substituting for u_L / z_L from (41) into (32), the resulting expression of g_A is the function $f(r, \lambda, \delta, \beta, \alpha, \theta, T_N, T_A)$. We can write:

$$g_A = (r-\rho) / \sigma = f\,(r, \lambda, \delta, \beta, \alpha, \theta, T_N, T_A)$$

If $T_N - T_A \neq 0$, then r is a non-redundant argument of *f()* and, given n, g_A and r are simultaneously determined by technology and preferences. If $T_N = T_A$ the simultaneity collapses and g_A is determined by (35').

3.2 Non endogenous R&D growth

Referring back again to system (19), the crucial assumption of the non-endogenous R&D growth models is Det $[I - \Gamma] \neq 0$. In particular, referring to the case $[I - \Gamma]^{-1} > 0$, standard results of linear algebra lead to the following proposition which extends to the economy with expanding varieties and technology levels a result, similar in spirit, of Eicher and Turnovsky (1999).

Proposition 3.2.1: Assume $\Gamma \geq 0$. Assume also that, for each row, the row sum of the elements of Γ is positive and lower than 1. Then, for every $n > 0$, there exist positive values g_A, g_N, g_K that are solutions to (14)-(15)-(16) and such that $g_l = n - g_N > 0$.

Recalling that $0 < \alpha < 1$, a quick look at equation (18) will suffice to see that the following holds:

Proposition 3.2.2: If, in addition to the assumptions of proposition 2.1, we have $(\gamma + 1 - \alpha) \geq 0$, then $g_K > n$ (positive per-capita-output growth).

Remark 3.2.1: The *if* condition of proposition 3.2.2 amounts to the existence of increasing returns to scale in the output sector. The assumption of Proposition 3.2.1 implies, but is not equivalent to, aggregate decreasing returns to scale in extensive and intensive search.

Thus, where the equations of system (19) are not linearly dependent (notably, a condition of full measure in the relevant parameter space) the steady-state growth rates of output, technology levels and varieties are completely determined by population growth and the technological parameters. These rates are therefore independent of preferences, and of savings rates in particular.

The above propositions extend to a three-sector environment the formal characterisation of the class of two-sector non-endogenous growth models first laid down by Eicher and Turnovsky

(1999). From a formal view point the seminal paper of Arrow (1962), where technology accumulation is driven by learning rather than deliberate R&D investment, belongs to the same class. Within the family of R&D growth models, the best-known non-endogenous example is probably Jones (1995) (see also Jones (1998) and (2000)), where the author abstracts from the expansion of varieties, so that $g_N = 0$ and $g_l = n > 0$. In particular, Jones (1995) assumes $\xi = 0$ (no physical capital input in R&D) and $0 < -\chi < 1$, so that his two-sector version of system (19) boils down to

$$\begin{bmatrix} -\chi & 0 \\ -(1-\alpha) & (1-\alpha) \end{bmatrix} \begin{bmatrix} g_A \\ g_K \end{bmatrix} = \begin{bmatrix} \theta n \\ (1-\alpha)n \end{bmatrix}$$

and the conditions of propositions 2.1, 2.2 are trivially satisfied.

It may be worth observing how the steady-state relation $g_c = g_A = (r - \rho) / \sigma$ continues to hold, but the direction of causality at work here is such that, given n, technology determines g_A and r is then determined by g_A and preferences. Instead, in the endogenous model with asymmetric cost of innovation effort between extensive and intensive R&D we have that technology and preferences simultaneously determine g_A and r.

4. Is n an upper bound for g_N?

As it turns out, the available examples of endogenous and non-endogenous R&D growth models share the prediction that, *in steady state*, the expansion of varieties proceeds at a pace which is *not faster* than the pace of population growth. In particular, $g_N^* = n$ in the endogenous and $g_N^* < n$ in the non endogenous models considered above. On a closer examination, however, these predictions are the by-product of quite special assumptions. Both the endogenous and the non-endogenous model admit extensions such that g_N^* may be larger than n.

To see this, consider again system (19) under the simplifying restriction $\gamma = \alpha - 1$. In this case, the third equation in (19) yields $g_K^* = g_A^* + n$. Since the matrix $[I - \Gamma]$ reduces to

$$\begin{bmatrix} -\chi & 0 & -\xi \\ -\nu & 1-\tau-\varepsilon-\psi & -\psi \\ -(1-\alpha) & 0 & 1-\alpha \end{bmatrix}$$

we have $\text{Det}([I - \Gamma]) = -(\xi + \chi)(1 - \tau - \varepsilon - \psi)(1 - \alpha)$.

We may consider a version of the endogenous model with $\chi = 0$, $\xi > 0$, $\upsilon > 0$, where the crucial restriction $\mathrm{Det}([I - \Gamma]) = 0$ is now fulfilled by $\tau + \varepsilon + \psi = 1$. In this case

$$\dot{N} / N = \beta z_L^{\varepsilon} (L / N)^{\varepsilon} A^{\upsilon}$$

which in steady state requires $\varepsilon (n - g_N) + \upsilon g_A = 0$. If $0 < \upsilon < \varepsilon$, this yields $g_K = g_A + n > g_N$. Since from (16) $g_K = g_N - (\theta / \xi)(n - g_N)$ we conclude that $g_N > n$ and $g_A > 0$ are consistent with a steady state path.

In the non endogenous model with the matrix $[I - \Gamma]$ as above, simple calculations reveal:

$$g_N - n = n[(\tau+\varepsilon+\psi-1)(\xi+\chi)-\upsilon(\varepsilon+\xi)]/[(1-\tau)(\xi+\chi)-(\xi+\theta)(\upsilon+\psi)]$$

$$g_A = n[(\tau+\varepsilon+\psi-1)(\xi+\theta)]/[(1-\tau)(\xi+\chi)-(\xi+\theta)(\upsilon+\psi)]$$

Thus, a sufficient condition for a steady state with $g_N > n$ and $g_A > 0$ is: $\tau < 1$, $\tau + \varepsilon + \psi > 1$; $\xi + \chi > 0$, υ and ψ sufficiently close to zero.

5. Research employment and productivity

A second and deeper problem is posed to the R&D growth models by the dramatic long-term rise of the researchers/employment ratio observed in the U.S.A. (and the advanced countries more generally), compared to the relatively constant performance of the U.S. per-capita GDP (and productivity) growth in the period 1950 – 1993 (see Jones (2000)). A reason why in this respect the U.S. experience may be more revealing is that it is less influenced by the transient component of productivity growth in 1950-1970 which is generally associated to technological catching-up.

These stylised facts are not only at variance with the scale effect on the growth rate displayed by the first generation of endogenous R&D growth models and criticised by Jones (1995). The evidence is more generally at variance with the possibility to approximate (if at a very aggregate level) the long term evolution of innovation activity and productivity growth in the U.S. (but also in the advanced countries) through the hypothesis that this economy has been growing in the neighbourhood of a single steady-state path. More specifically, endogenous and non-endogenous models alike are faced with the problem of

(i) explaining how the rising researchers/employment ratio $(1 - h_L)$ can be reconciled with the behaviour of productivity growth;

(ii) identifying the causes of the rising researchers/employment ratio.

A first way of answering these questions is to suppose that the rise in $(1 - h_L)$ corresponds to a transition between different steady states with constant growth rate g_A induced by *exogenous* changes in one or more technological parameters.

A second and more ambitious way is much in the spirit of Pasinetti (1981) and searches for *rules of structural change* that may get closer to explain the observed phenomena and above all the finding that growth trajectories are not well approximated by a steady state path. In the remainder of this paper we shall expand on these two lines of investigation.

To this end, we shall refer to the simplified versions of system (19) that feature in 'standard examples' of endogenous and non-endogenous R&D growth models. In particular, physical capital is not an input to innovation activity, intensive and extensive, hence $\xi = 0$, $\psi = 0$; the productivity of the extensive innovation effort does not depend on the technology level A, that is, $\upsilon = 0$; the aggregate production function does not depend on the number of varieties N, thus $\gamma = \alpha - 1$.

5.1 Looking for appropriate parameter changes

Referring to the U. S. experience in the second half of the twentieth century, we may observe how the rate of interest, the capital output ratio, and the growth rate of per capita GDP have been 'relatively constant'[11] over the period. Since the model structure implies $\sigma g_A + \rho = r = \alpha^2 K/Y$, we derive the restriction that α has been constant; we are also led to formulate the 'working hypothesis' that the preference parameters σ and ρ were unchanged throughout. With this situation in mind we consider what, if any, changes of the technological parameters of the non endogenous and endogenous models can answer the issues posed under *(i)* and *(ii)* above.

5.1.1 Non-endogenous model

With the assumptions of proposition 3.2.1 in place, in particular $0 < -\chi < 1$, $\varepsilon + \tau < 1$ the non endogenous model yields the steady-state predictions:

$$g_Y = g_A + n$$
$$g_N = \varepsilon\, n / (1 - \tau)$$
$$g_A = \theta\, (1 - \tau - \varepsilon)\, n / [- \chi\, (1 - \tau)]$$

Notice that $\partial g_A / \partial \tau < 0$; $\partial g_A / \partial \chi > 0$. Moreover, the growth rate of per capita output is independent of δ, the proportional productivity effect of quality innovations; it is also independent

[11] At least in the sense specified in the introduction to this paper.

of λ and β, the parameters that, for any given innovation effort, regulate the arrival rates of intensive and extensive innovations, respectively. Using the condition that, in equilibrium the agent is indifferent between investing one extra unit of labour effort in intensive research, extensive research or output production, we derive the steady state value of $(u_L + z_L)$.

$$(u_L + z_L)^{-1} = 1 + \frac{\theta n(1-\tau-\varepsilon)(1+\sigma\delta) - \chi\delta[(\rho-n)(1-\tau-\varepsilon)+\rho\varepsilon]}{(1+\delta)(1-\tau-\varepsilon)\alpha\theta n - \chi\delta\varepsilon\alpha n}$$

We may observe how λ and β do not affect the steady-state researchers/employment ratio. Moreover, simple but tedious calculations reveal:

$\partial(u_L + z_L)/\partial\delta < 0$ if $\sigma \geq 1$

$\partial(u_L + z_L)/\partial\chi > 0$ if $\sigma \leq 1$

$\partial(u_L + z_L)/\partial\tau > 0$ if $\sigma \leq 1$

Depending on the preference parameters, the model produces two candidate explanations for the observed long term rise of the researchers/employment ratio: either a fall of δ, leaving g_A unaffected, and/or concomitant increases of χ and τ, both raising the share $(u_L + z_L)$, while exerting mutually compensating effects on g_A. The two types of parameter changes would affect the composition of research employment in opposite directions.

$$z_L / u_L = -\chi\varepsilon\delta / [(1-\tau-\varepsilon)\theta(1+\delta)]$$

$$z_L = \frac{-\chi\varepsilon\delta\alpha n}{(1+\delta)(1-\tau-\varepsilon)\alpha\theta n - \chi\varepsilon\delta\alpha n + \alpha n(1+\sigma\delta)(1-\tau-\varepsilon) - \chi\delta[(\rho-n)(1-\tau-\varepsilon)+\rho\varepsilon]}$$

It can be easily checked that $\partial(z_L / u_L) / \partial\delta > 0$ and $\partial z_L / \partial\delta > 0$. A parametric fall of δ would unambiguously produce an absolute decline in the steady-state share of the extensive-research employment. Instead, if g_A is to remain constant in the face of a ceteris-paribus rise of χ and τ, the term $(1-\tau-\varepsilon) / -\chi$ is bound to fall. Thus, in this case the model predicts a rise of z_L / u_L, hence a rise of the extensive-research employment share.

To gain some understanding of the problems raised by this line of reasoning, it is worth considering the case $\sigma \geq 1$, which is usually considered more realistic than its counterpart $\sigma < 1$. On a growth path with constant g_A, falling δ, and the remaining parameters held constant, we have:

$$g_A = [\frac{\dot{\delta}_t}{\delta_t} + \theta(\frac{\dot{u}_{L,t}}{u_{L,t}} + n - g_{N,t})\frac{1}{-\chi}]$$

On the assumption that transition paths are monotonic, the growth in the number of varieties $g_{N,\ t}$ is expected to fall along the growth path, as a result of the falling share of the extensive-research employment. Unless θ is close to zero, the implication is that the rate of decline in δ must have the same order of magnitude of the growth rate of u_L. In turn, this is predicted to be strictly higher than the growth rate of the research-employment share $u_L + z_L$. Since the latter is known to be, on average, large in the period 1950 – 1993, our conclusion here is that the rate of decline in δ which is required by this line of reasoning may be unplausibly high..

5.1.2 Endogenous model

In addition to the simplifying assumptions stated at the outset of section 5.1, the endogenous model we are considering assumes $\chi = 0$, $\varepsilon + \tau = 1$ and $\theta = \varepsilon$. The innovation technology is that considered in section 3.1.1 generating a symmetric cost from one additional unit of labour effort across extensive and intensive innovations. The fact that with this technology the steady state growth rate does not depend upon preferences is unconsequential here, because the present exercise is conducted under the 'working hypothesis' that the preference parameters ρ and σ and the technological parameter α are unchanged. In steady-state equilibrium, the growth rate of per capita output is:

$g_A = \delta n\,[(1+\delta)^{\theta}\ \lambda\ \beta^{-1}]^{1/(1-\theta)}$

The research employment shares are:

$$u_L = \frac{[(1+\delta)\lambda/\beta]^{1/(1-\varepsilon)}}{1+(\rho/\alpha n)+[1+\frac{\delta\sigma+1}{(1+\delta)\alpha}][(1+\delta)\lambda/\beta]^{1/(1-\varepsilon)}}$$

$$z_L = \frac{1}{1+(\rho/\alpha n)+[1+\frac{\delta\sigma+1}{(1+\delta)\alpha}][(1+\delta)\lambda/\beta]^{1/(1-\varepsilon)}}$$

It can be easily checked that

$\partial u_L / \partial (\lambda/\beta) > 0$

$\partial z_L / \partial (\lambda/\beta) < 0$

$\partial (u_L + z_L) / \partial (\lambda/\beta) < 0$ and $\partial (u_L + z_L) / \partial \delta < 0$ if $\sigma > [(1 + \delta)\rho - n] / \delta n$

$\partial (u_L + z_L) / \partial (\lambda/\beta) > 0$ and $\partial (u_L + z_L) / \partial \delta > 0$ if $\sigma \leq 1$.

This shows how the endogenous model can not easily reconcile the drastic rise of $(u_L + z_L)$ with the simultaneous approximate constancy of g_A. The reason is that for a wide range of the preference parameters, the technological parameters δ and λ/β affect not only $(u_L + z_L)$, but also g_A in the same direction. In this range, the concomitant changes in δ and λ/β that leave g_A unaffected, exert (at least to some extent) mutually offsetting effects on the research-employment share $(u_L + z_L)$. In the remaining range $1 < \sigma \leq [(1 + \delta)\rho - n] / \delta n$ the results are ambiguous, in that they depend on further restrictions on parameters.

5.2 Growth and structural change

In section 5.1 we argued that the convenient and widespread (at least in standard applications of growth theory) interpretation of growth paths as trajectories in the neighbourhood of, or at least converging to, a steady state, may become a strait-jacket when it comes to interpret phenomena such as the long-term rise of $(u_L + z_L)$. In this respect, R&D models have paid mostly lip service to the lesson of eminent scholars on economic development, such as Adam Smith, Allyn Young, Joseph Schumpeter and Simon Kuznets. Their idea that there are deep reasons why growth is systematically associated with structural change is not easily reconciled with a model structure which is deliberately designed in order to obtain the steady-state property.

A recent change in this state of affairs is a paper by Sergio Rebelo and co-authors (Kongsamut, Rebelo and Xie (2000)) showing that the new growth theory has eventually placed structural change on its research agenda. In the focus of that paper is the long term employment shift away from agriculture in favour of services , which is so typically associated with the process of economic growth. Clearly, these changes have at least in part to do with changes in the composition of consumers’ expenditure associated with the long term rise of per-capita income. A tradition in economic theory, from Kuznets (1957) to Pasinetti (1981) had already emphasized this order of phenomena.

One may ask whether the observed long term rise of the research-employment share may be similarly associated with the long-term rise of per capita income through its effects on the composition of consumer’s expenditure. A possible line of explanation is the following. Consider a non-endogenous R&D model of economic growth with differentiated consumer goods, and only one intermediate good, which is used, together with labour, to produce every consumption good i with the production function:

$$Y_t = A_t\, L_{y(i),t}^{\;1-\alpha}\, x_t^{\alpha}$$

A_t is the productivity index associated with the best-practice quality of the intermediate good at t.

Suppose that the rise of per-capita income makes consumers *increasingly* inclined to pay attention to the birth date of a good, even when its service characteristics are close to those of the old goods. That is, the rise of per-capita income produces a form of satiation with respect to the 'old' goods which loose market shares in favour of the 'new' goods. The expected pay-off from the research effort to create a new consumption good would be influenced by the rising per-capita income in at least two ways. Ceteris paribus, the new good would enjoy a larger market share in the period immediately following its first introduction. At the same time, the rising per capita income would produce a faster economic obsolescence, that is, a more rapidly declining market share, during the economic life of this good. Assume conditions such that the first effect prevails and the outcome is a long-term rise of the expected pay-off from the creation of a new consumption good, relative to creation of a quality innovation. In equilibrium, the ratio z_L / u_L between the extensive and the intensive R&D employment would increase with per capita income to make the return of one additional unit of labour effort identical in the two activities. Thus, equilibrium paths with a long term rise of the research-employment share may well be consistent with a constant growth of the productivity index A.

In the remainder of this section we would like to sketch a second line of explanation which is more easily related to the formal structure outlined in the previous sections. The explanation rests upon the problem of complementarity between intermediate goods. In the new-growth literature, the problem of complementarity between intermediate goods has been introduced in relation to the idea of a sequence of general-purpose technologies (GPTs). The adoption of a GPT requires the previous creation of a set of intermediate goods that are specific to it. When the GPT s first appears a labour share is shifted from manufacturing to R&D (phase 1); next, after the intermediate goods required by s have been invented all employment is shifted to manufacturing until the GPT (s + 1) arrives (phase 2). The idea is exploited by Helpman and Trajtenberg (1994) and Aghion and Howitt (1998) to study the relation between growth and cycles. The notion of a steady state is correspondingly extended by these authors to the effect that in an economy with a constant population "a steady-state equilibrium is one in which people choose to do the same amount of research each time the economy is in phase 1 …" (Aghion and Howitt (1998), p. 248).

We suggest that a similar set of ideas can be conducive to phenomena of structural change within a framework which is borrowed, with some important variations or qualifications, from the R&D growth models considered in this paper.

For the sake of simplicity, let us assume away the problem of extensive R&D by assuming that at every date there is an unchanging continuum of intermediate-good varieties ordered on $\Re_+$; to employ these varieties in production, their appropriate technology level must developed. $[0, \Lambda_A]$ is the set of complementary intermediate-good inputs *necessary* to implement the technology level A in the production of final output. N_t is the number of intermediate goods *used* at t. There is only one final good Y. Its production function is:

$$Y_t = N_t^{\alpha - 1} L_{Y,t}^{1 - \alpha} \left[\int_{v=0}^{\infty} P(A_{v,t}) \, x_{v,t}^{\alpha} \, \partial v \right] \tag{41}$$

$P(A_{v,t})$ is the productivity index associated to the technology level $A_{v,t}$ of variety v. with:

$P(A_{v,t}) = A$, if $0 \le v \le \Lambda_A$ and $A_{v,t} = A_{j,t} = A$ for all $v, j \in [0, \Lambda_A]$;

$P(A_{v,t}) = 0$ otherwise.

The above assumption formalises a strong form of incompatibility between intermediate goods of a different technology level. We say that technology level A has been implemented if $A_{v,t} = A_{j,t} = A$ for all $v, j \in [0, \Lambda_A]$. Variety v is *necessary* to the implementation of A if and only if $v \in [0, \Lambda_A]$.

If technology level $A(t)$ is implemented at time t, there is an instantaneous knowledge spillover such that $A_{v,t} = A(t)$ for every $v \in [0, \infty]$. The implementation of a *higher* technology level is instead costly, because it requires the higher level is independently developed for every necessary variety as the result of a deliberate R&D effort. The number $\phi_{v,t}$ of intensive innovations in sector v at t evolves according to the *deterministic* process:

$$\phi_{v,t} = \lambda \, (u_{L,v,t} L_t)^{\theta} A_{v,t}^{\chi} \tag{42}$$

If every innovation has a proportional effect δ on the technology level $A_{v,t}$, we obtain:

$$\dot{A}_{v,t} = \delta \, \lambda \, (u_{L,t} L_t / N_t)^{\theta} A_{v,t}^{\chi + 1} \tag{43}$$

Higher technology level are of higher complexity and their implementation requires a larger number of necessary intermediate inputs. Assume that the number of necessary varieties evolves according to:

$$\Lambda_{A(t)} = A_t^{\eta} \qquad \eta > 0 \tag{44}$$

This implies that, if $g_\Lambda(t)$ is the proportional growth rate of $\Lambda_{A(t)}$ at time t, then:

$$g_{\Lambda}(t) = \eta\, g_A(t) \tag{45}$$

It goes without saying that the strong complementarities of the form we have described imply that the market implementation of a higher technology level will face a host of co-ordination problems. Here we are not concerned with this feature, however important it may be. Our aim is simply to show that equilibrium paths on which the productivity index A_t grows at a constant rate $g_A > 0$ are *not* steady state paths and have a rising share u_L of R&D employment.

In the equilibrium at time t we have $N_t = \Lambda_{A(t)}$. With g_A constant, from (43) and (45) we obtain:

$$-\chi\; g_A = \theta\,(n + \frac{\dot{u}_{L,t}}{u_{L,t}} - g_{\Lambda}(t)) \tag{46}$$

hence:

$$[\eta - \chi]\, g_A = \theta\,(n + \frac{\dot{u}_{L,t}}{u_{L,t}}\,) \tag{47}$$

Recalling that the 'congestion effect' in R&D implies $\theta < 1$, and that our considerations suggest $\chi < 0$, it is easy to see how, given n, the higher η , the higher the growth rate $\frac{\dot{u}_{L,t}}{u_{L,t}}$ required to elicit a given productivity growth g_A . Thus, with η sufficiently large, the value $g_A \approx 0.02$ prevailing in the period 1950-1993 would not have been possible in the presence of a constant labour share in R&D. Indeed, a growth rate g_A of the observed dimension can not be a steady-state growth rate and can not be sustained 'for ever'.

If the argument above offers a tentative explanation of how the long-term rise of the researchers/employment ratio can be reconciled with a constant growth rate of productivity, what is yet to be explained is the source of the rising researchers/employment ratio.

Here we offer as a working hypothesis, to be explored by future work, that the preference structure with constant inter-temporal elasticity of substitution is replaced by a preference structure such that the rising per-capita consumption causes a slowly rising inter-temporal elasticity of substitution. It is worth observing how the required change of σ has not to be large, because a very small, apparently negligible, shift away from employment in manufacturing, in favour of research is sufficient to explain that:

(i) $\frac{\dot{h}_{L,t}}{h_{L,t}}$ is negative but very close to zero, as in the data;

(ii) $\frac{\dot{u}_{L,t}}{u_{L,t}}$ is positive and significantly large, as in the data.

Of course, the consistency between *(i)* and *(ii)* has to do with the fact that h_L is quite close to 1 and u_L is close to zero[12].

[12] The U.S. researchers/employment ratio was 0.008 in 1993. See Jones (2000), p. 16.

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