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START UP PROBLEM IN A NETWORK GOOD MARKET: THE USE OF ADVERTISING BY A MONOPOLIST PROVIDER

by

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Abstract: *The start up problem in a network market refers to the necessity for the provider of the good exhibiting positive network externalities to find appropriate tools to make the network start. Rohlfs (1974) was the first to recognise the possible use by the provider of marketing policies in order to attract the critical mass, but a formal treatment of the issue has never been tempted. This essay tries to cover this lack. Through the introduction into the analysis of advertising we examine how this inclusion may modify the received theory.*

Keywords: *Network good, start up problem, advertising.*

Classification J.E.L.: *L19,* M37.

1. Introduction.

Network economics has undoubtedly been one of the most growing field in economics over the last decade. A large number of papers have been produced in the last years trying to analyse the peculiar features that network industries present: the optimal (static and dynamic) pricing policy under different market structures, the choice of compatibility between products and the consequent possibility to adopt a common standard, the timing of the adoption of a new technology presenting network externalities, are only some of the various issues that have been analysed in literature[1].

Notwithstanding this important effort, it is true that there are still several topics which have not been fully explored yet[2]. Surely one of the less investigated ones, in spite of its importance[3], is somehow related to the aggregate access demand and in particular it concerns the way agents formulate their expectations on the size of the network. More precisely, so far there has not been any attempt to study the possible use by the providers of the good of promotional means in order to generate in the population positive expectations about the size that the network will have.

This is quite surprising, since the importance of advertising strategies in network economics is well known. Quoting Shapiro and Varian (1999):

«Marketing strategy designed to influence consumer expectations is critical in network markets.»

The main aim of this paper is to provide a formal analysis of this specific issue.

Following the well-established theory of demand primarily due to Rohlfs (1974) and further developed and completed among the others by Economides and Himmelberg (1995a), we investigate how the inclusion into this basic framework of different assumptions about the advertisement costs faced by a monopolist provider, affects the original analysis. In particular the equilibrium price and the actual size of the network are investigated. A comparison of the results using two alternative hypotheses, will offer us the opportunity to draw some interesting

[1]See Economides (1996) for a complete survey.

[2]Varian (1999) offers some good examples, especially related to the interconnection problem.

[3]Quoting Yang and Barrett (1997): «The very existence of a good and service of a good network depends on firms` expectations of consumer demand, which is in turn a function of individual consumers` expectations of others`

considerations about a possible public intervention which aims at favouring the optimal network size from a social point of view.

The paper will be structured as follows: section [2] briefly describes the received theory on the aggregate access demand for a network good; section [3] develops our own analysis on this framework and finally section [4] is dedicated to some conclusive remarks.

2. The standard theory of the access aggregate demand for a network good.

Rohlfs (1974) was the first paper that provides a detailed analysis of the demand side for a good that exhibits network externalities[4]. Katz and Shapiro (1985), Economides (1993), Economides and Himmelberg (1995a), Economides and Himmelberg (1995b) and Palma and Leruth (1996) developed the concepts introduced by Rohlfs, furnishing a more formal analysis of the issue. A neat exposition of the theory is also provided by Shapiro and Varian (1998) and Shy (2001). We will closely follow all these works in describing how the aggregate demand for a network good is constructed.

Let us consider a network that can potentially serve a number of *N* potential agents, uniformly distributed and indexed by *x* on the unit interval [0,1]. The hypothesis of uniformly distributed agents is not necessary, but it allows us to make the exposition more fluent. What it is necessarily to assume, in general, is a cumulative distribution function of types which is continuous and with positive density everywhere in its support. This more general assumption will originate an access demand curve similar to the one described here, provided that either one of the following conditions holds[5]:

1) the utility of every consumer in a network of zero size is zero;

2) there are immediate and large external benefits to network expansion, when the network is very small;

3) there is a significant density of high-willingness-to pay consumers who are just indifferent on joining a network of approximately zero size.

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demands. The potentially heterogeneous way in which agents form expectations is thus of extraordinary significance in industries characterised by network externalities.»

[4] For network externality is intended the positive change in the utility that a consumer derives from a good, when the number of consumers that purchase the same product increases.

[5] See Economides and Himmelberg (1995a) and Economides and Himmelberg (1995b) .

Each agent will face the binary decision of whether to join the network or not, or in other words whether to buy the good that exhibits network externalities or not.
Agents that present a low value of x are those who have a high willingness to pay for the service, while those indexed by a high value of x are the agents that have a low willingness to pay.
Denoting by n, the total number of consumers that buy the good, with naturally 0 #n #1, and by p the price, we define the utility of a representative agent as:

$U_x = (1 - x)n^e - p;$	if she buys the good	(1)
$U_x = 0$	if she does not buy the good	

where n^e is the expected number of consumers. The utility of each consumer exhibits network externalities: it increases with the expected number of total consumers of the good.
Suppose now that a continuum of potential consumers exists. There will be therefore a particular consumer, indexed by x^*, such that she is indifferent between buying and not buying the good. This consumer is found by:

$$0 = (1 - x^*)n^e - p \qquad (2a)$$

Rearranging (2a):

$$x^* = \frac{n^e - p}{n^e} \qquad (2b)$$

Hence all the consumers that have a higher willingness to pay for the service (x # x^*), will buy the good, while all the agents that have a lower willingness to pay (x ∃ x^*), will not buy it. Thus, the actual number of consumers of the good is $n = N \times x^*$.
The framework is then reduced from a dynamic problem to a static one. It is assumed that the consumers form their expectations identically and they have perfect foresight. At the time of purchase, they can correctly anticipate how many other consumers will buy the good. Formally, $n^e = n = N \times x^*$.

Substituting $n^e = N \times x^*$ into (2b), we obtain the inverse demand function for a good exhibiting positive network externalities:

$$p = (1 - x^*) N x^* \tag{3}$$

Which is drawn in Figure 1, without loss of generality, for $N = 1$.

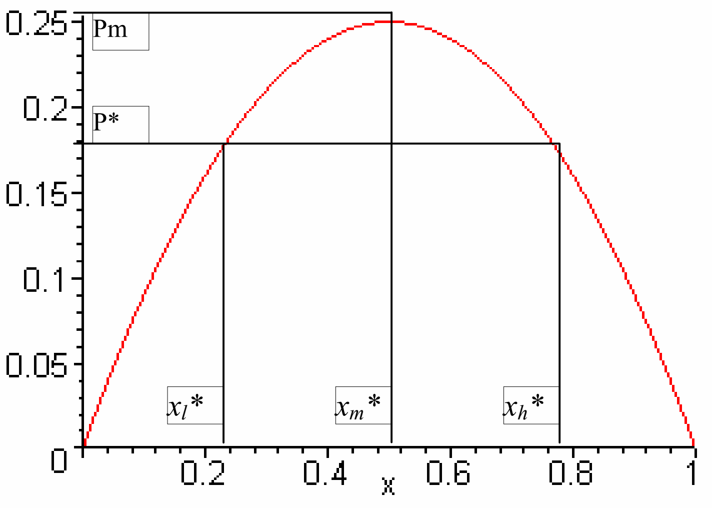

Figure 1.The aggregate access demand curve.

Notice that the aggregate demand is not only formed by the inverted U-shaped curve but it includes even the entire vertical axis. The reason is that for every possible price a null demand is rational as long as very pessimistic expectations dominate the market: no agent is buying the good because everybody is convinced that no one else other than him will eventually buy the good. This is what Economides and Himmelberg (1995b) defines as a sort of chicken-egg paradox: nobody joins the network because the size of the network is zero; but the size of the network is zero because no one has joined it.

For every price such that $0 \leq p < p_m$, we have two possible levels of demand, a low and a high one, with the aggregate demand that is upward sloping at low levels and becomes downward sloping at high levels. This is the result of two different effects: the network and the price effect. On one hand, at small demand levels, every increase in the expected number of consumers makes the good more valuable and attracts those agents that have a relatively high willingness to pay. This is the network effect. On the other hand, at large demand levels, an increase of the same magnitude in the expected number of consumers has a smaller effect on the customers` willingness to pay[6], thus the price has to fall in order to attract those agents that have a relatively low willingness to pay for the good. At the downward-sloping side of the demand curve, the price effect dominates the network effect.

It is evident that this particular shape of the aggregate demand causes the possible existence of multiple equilibria. Consider the case depicted in Figure 1, with the network good sold at price p^*. There are three possible equilibria: one at a network of zero size ($n = 0$), an intermediate one where the network is relatively small ($n_l^* = x_l^*$) and the third one with a large consumers` base ($n_h^*=x_h^*$). Only two of these equilibria are stable. At x_l^*, in fact, every increase in the (expected) number of customers will make the good more valuable causing all the additional agents indexed between x_h^* and x_l^* to be willing to purchase; while every decrease will cause the opposite process, leading to the zero network size equilibrium.

Rohlfs (1974) refers to the point x_l^*, as the *critical mass* of the network good at price p^*. More generally, all the upward-sloping part of the demand represents the critical mass of the service at any given price[7].

Therefore, x_l^* is the minimal amount of consumers which will benefit from joining the network at a given price p^*, any inferior expected number will start a hypothetical process (since the problem is static) that will inevitably end up with a network of zero size, while every expected increase will trigger a "chain reaction" that will lead to the equilibrium constituted by a large number of consumers. The start up problem that usually every provider of a network good, faces, is thus to reach the critical mass for any given price charged. In this contest, the possible role played by the use of promotional means in order to influence the expectations of the agents has never been explored.

The theme is elaborated in the next section.

3. The start up problem.

The first to recognise the importance of marketing policies in order to get beyond the critical mass was Rohlfs (1974):

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[6] Because of the concave functional form of the newtork externality.

[7] This concept of critical mass is essentially taken from nuclear engineering, where it is used to indicate, under radioactive decay conditions, the amount of uranium necessary to start a self-sustaining process of production of neutrons that maintains unchanged its quantity. Any larger amount of uranium will cause an explosive nuclear chain reaction. Any smaller amount of uranium will cause nothing, and it will soon decompose.

«Achieving the static optimal user set may require ruinos (albeit temporary) promotional costs.»

Unfortunately beside this statement he does not propose any formal analysis of the issue.

Then, almost all the subsequent literature seems in a certain way to ignore the problem. It is, in fact, assumed that the price is the only decision variable of the firm given the depicted structure of the demand. This approach is satisfactory under Pareto dominance considerations, since the set of equilibria (provided that they exist) located on the downward-sloping side of the demand curve, Pareto dominates for any given price, all the equilibria characterised by a network of zero size. Although the argument is undoubtedly true, it seems more a useful device to avoid the analysis of the start up problem rather than a way to solve it[8]. It is obvious that a new good has to be publicised in order to be sold: if no one knows that a new product (or service) exists, nobody can then purchase it. Whatever the nature of the good (network or not), the provider has necessarily to publicise its existence. But the scenario is different whether we consider a network good or not. In the classical case, under a pure monopoly regime, the provider only needs to make consumers aware of the existence of the good. Consumers will then decide whether to buy the good or not (regardless of how many other agents have got the same information). Generally, advertising expenditure is not so important, provided that agents already know the existence of the product. In the network good case, the consumer's decision of purchasing is conditional to the expected number of other agents that will buy the product.

It is crucial for the consumer to infer how many agents are expected to buy the item. Thus, it seems natural, in this contest, to allow the monopolist provider to use the promotional policy not only for informing the agents but also for influencing their expectations about the penetration in the market that the product will have. The more a new coming network good is advertised, the higher will be the probability that the good is perceived by the agents as successful (i.e. adopted), the higher will be the size of the expected network (n^e) and hence the size of the actual network (n), allowing the firm to set a higher price. The effect on the strategies of a monopolist provider of two different assumptions regarding the promotional costs, both plausible under different circumstances, is investigated. In either case it is assumed that these promotional costs are

[8]This is clearly stated by Rohlfs (1974):"Viable nonnull equilibrium user sets, are always superior from a static point of view. [....] However this kind of analysis is incomplete and may be misleading without consideration of the start up problem."

necessary to originate an inverted U-shaped aggregate demand curve similar to the one depicted in Figure 1, with the demand curve that does not include the vertical axis anymore. In fact, the role of the advertisement expenditure is essentially to reassure the monopolist provider that after a sufficient amount spent in publicity for any given price, the resulting network size will not be zero. In order to better highlight the effects on the equilibrium of promotional costs, we assume that the firm bears only this possible kind of cost.

Now, consider a monopoly provider that can potentially sell a new good exhibiting network externality to N agents, where without loss of generality, $N = 1$. The agents are continuously distributed and indexed by x in the interval [0,1]. The cumulative distribution function of types has a positive density everywhere in the support. The monopolist necessarily has to bear some promotional costs in order to sell the network good. An increase in the advertisement expenditure produce larger expectations about the size of the network and, under the perfect foresight assumption, a bigger actual network size. The effect on the network size of an increase in advertising expenditure is assumed to be positive at an increasing rate, basically for two classes of reasons. The first one is related to the nature of the good that we are considering: its value rises for any increase in the number of adopters. It seems legitimate to assume that for any given positive size of the network, the provider needs fewer and fewer adverts to induce the agents to purchase the item. The other one is somehow related to the structure of the modern advertising market[9]. Generally, each firm has to bear conspicuous fixed costs in order to set up the campaign (costs for market research, the creation of the advertisements, etc.), and usually the price charged by the agency or by a means of communication for the amount of advertisement space booked is increasing at a decreasing rate.

Moreover, intensive campaigns are more likely to generate in the economy that phenomenon defined in advertising terminology as extra- publicity, making less necessary for the provider to invest in publicity, the more he has already spent on advertisements.

Under the first hypothesis, called "no critical mass effect" (NCME) assumption, the relationship between advertisement costs (A) and network size (x) is defined as follows:

$$A = A(x(p)) \text{ with } A' > 0,\ A'' < 0,\ A'(x) \exists 0 \text{ when } x 6\ 1,\ 0 \# x \# 1. \tag{4}$$

[9] For further reference on this topic, see Brierly (1995).

The advertisement cost function is defined, in this first case, upon the entire population.

The monopolist has to spend an increasing amount of money in adverts to generate in the economy increasing expectations about the size of the network. This approach (intentionally) ignores any possible consideration about the concept of critical mass. We have seen in the previous paragraph, that once the monopolist provider succeeds for a given price p^* to sell the good to x_l^* consumers, since this one is an unstable equilibrium, the probability that he will effectively sell the item to x_h^* is extremely high and can be approximated to one. The second hypothesis assumes therefore that the promotional cost function is defined only on the critical mass values, i.e. $x_l\,(p)$.Under this second case, the monopolist has to provide a positive amount of advertisement expenditure only to make the network start. Once started, because of the network externalities which the good exhibits, the network will grow until reaching a stable equilibrium identified by the downward-sloping part of the demand[10]. Therefore, under the second hypothesis named as "critical mass effect" (CME) assumption, the promotional cost function will be given by the following expression:

$$A_l = A(x_l(p)) \; with \; A_l` > 0, \; A_l `` < 0, \; 0 \# x_l \# x_m. \tag{5}$$

The profit function of the monopolist provider will be:

$$\pi = px(p) - [d_{A_l} A_l(x_l(p)) + d_{Ah} A_h(x_h(p))]; \tag{6}$$

Under the NCME assumption, the monopolist has to decide whether to adopt an intensive promotional campaign (i.e. $d_{A_l} = 0$ and $d_{Ah} = 1$) or on the contrary to spend less in advertising (i.e. $d_{A_l} = 1$ and $d_{Ah} = 0$), for every charged p; while naturally, under the CME assumption, it is always $d_{A_l} = 1$ and $d_{Ah} = 0$.

3a. The first assumption: no critical mass effect (NCME).

The monopolist has to set the price and implicitly the intensity of the promotional campaign in order to maximise profits, in this case given by:

$$\pi = px(p) - A(x(p)) \tag{7}$$

For any given price charged, the provider decides whether to sell the good to a low fraction of the population x_l, with $0 \# x_l \# x_m$, spending an amount of A_l on advertisements ($0 \# A_l \; \# A_m$), or to produce a high quantity of the good x_h, with $x_m < x_h \# 1$, bearing a cost of A_h ($A_h > A_m$).
The first and second order conditions of the monopolist problem are:

$$x(p) + p \cdot \frac{\partial x}{\partial p} = \frac{\partial A}{\partial x} \cdot \frac{\partial x}{\partial p} \tag{8}$$

$$2\frac{\partial x}{\partial p} + p\frac{\partial x^2}{\partial p^2} - \frac{\partial^2 A}{\partial x^2} \cdot \left(\frac{\partial x}{\partial p}\right)^2 - \frac{\partial A}{\partial x} \cdot \frac{\partial^2 x}{\partial p^2} \leq 0 \tag{9}$$

Recalling that x_l ε [0, x_m] and x_h ε] x_m, 1], two possible optimal network sizes can arise:

$$1) \;\; x_l(p) = \frac{\partial x_l}{\partial p} \cdot \left(\frac{\partial A_l}{\partial x_l} - p\right) \tag{10a}$$

$$\text{if} \; \left| \frac{\partial A_l}{\partial x_l} \cdot \frac{\partial^2 x_l}{\partial p^2} \right| \geq 2\frac{\partial x_l}{\partial p} + \frac{\partial^2 A_l}{\partial x_l^2} \cdot \left(\frac{\partial x_l}{\partial p}\right)^2 + p \cdot \frac{\partial x_l^2}{\partial p^2}$$

$$2) \;\; x_h(p) = \frac{\partial x_h}{\partial p} \cdot \left(\frac{\partial A_h}{\partial x_h} - p\right) \tag{11a}$$

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[10]Note again that the problem is set in a static form, so all this process of network growth has to be seen as purely

$$2 \cdot \left|\frac{\partial x_h}{\partial p}\right| + p \cdot \left|\frac{\partial^2 x_h}{\partial p^2}\right| \geq \frac{\partial^2 A_h}{\partial x^2} \cdot (\frac{\partial x_h}{\partial p})^2 + \frac{\partial A_h}{\partial x_h} \cdot \frac{\partial^2 x_h}{\partial p^2}$$

note that $x_l(p)$ can be greater than zero only for $\frac{\partial A_l}{\partial x_l} > p$, and $x_h(p) > 0$ only when $\frac{\partial A_h}{\partial x_h} < p$.

Defining the price elasticity of demand (ε_D) and the price elasticity of advertisement expenditure (ε_A), we can express (10a) and (11a) as follows:

$$1)\ x_l(p) = \frac{A_l}{p} \cdot \frac{\varepsilon_{A_l}}{1+\varepsilon_{D_l}} \tag{10b}$$

$$2)\ x_h(p) = \frac{A_h}{p} \cdot \frac{-\left|\varepsilon_{A_h}\right|}{1-\left|\varepsilon_{D_h}\right|} \tag{11b}$$

where $\varepsilon_{A_l} = \frac{\partial A_l}{\partial x_l} \cdot \frac{\partial x_l}{\partial p} \cdot \frac{p}{A_l} > 0$; $\varepsilon_{D_l} = \frac{\partial x_l}{\partial p} \cdot \frac{p}{x_l} > 0$; $\varepsilon_{A_h} = \frac{\partial A_h}{\partial x_h} \cdot \frac{\partial x_h}{\partial p} \cdot \frac{p}{A_h} < 0$; $\varepsilon_{D_h} = \frac{\partial x_h}{\partial p} \cdot \frac{p}{x_h} < 0$.

A strictly positive solution of (11b) exists as long as $\left|\varepsilon_{D_h}\right| > 1$. For $\left|\varepsilon_{D_h}\right| = 1$, the maximum profit achievable by the monopolist is never positive: to see why, take the first order condition and consider the case where $\varepsilon_{D_h} = \frac{\partial x_h}{\partial p} \cdot \frac{p}{x_h} = -1$. From (10a):

$$p = p - \frac{\partial A_h}{\partial x_h} \quad ; \tag{12}$$

expression (12) can be verified only for $\frac{\partial A_h}{\partial x_h} = 0$ and hence only for $x_h=1$ and $p=0$. So the returns are zero and the monopolist provider will incur a loss represented by the amount of promotional costs (A_h). For $x_l(p)$, the maximum level of achievable profit is represented by:

hypothetical and constituted by a continuous revision of the expectations on the network size.

$$\pi_l = p\frac{A_l}{p}\cdot\left(\frac{\varepsilon_{A_l}}{1+\varepsilon_{D_l}}\right) - A_l \tag{13a}$$

rearranging:

$$\pi_l = A_l\cdot\left[\frac{\varepsilon_{A_l}}{1+\varepsilon_{D_l}} - 1\right] \tag{13b}$$

this expression will be positive only when $\varepsilon_{Al} > 1+\varepsilon_{Dl}$.

For $x_h(p)$, π_h is instead given by:

$$\pi_h = p\frac{A_h}{p}\cdot\left(\frac{-\left|\varepsilon_{A_h}\right|}{1-\left|\varepsilon_{D_h}\right|}\right) - A_h \tag{14a}$$

rearranging:

$$\pi_h = A_h\cdot\left[\frac{-\left|\varepsilon_{A_h}\right|}{1-\left|\varepsilon_{D_h}\right|} - 1\right] \tag{14b}$$

π_h is strictly positive only for $\left|\varepsilon_{D_h}\right| < 1+\left|\varepsilon_{A_h}\right|$.

Summarising, a positive level of $x_l(p)$ can arise only if $\frac{\partial A_l}{\partial x_l} > p$ and $\varepsilon_{A_l} > 1+\varepsilon_{D_l}$; while a positive level of $x_h(p)$ is guaranteed only if $\frac{\partial A_h}{\partial x_h} < p$ and $1 < \left|\varepsilon_{D_h}\right| < 1+\left|\varepsilon_{A_h}\right|$.

Since there is no a-priori contradiction between the second order conditions reported in equations (10a) and (11a), we cannot rule out the case that both values of p are relative maxima at the same time. If this is the case, a simple comparison between π_h and π_l will indicate us which is the global maximum, since $x_l(p)$ and $x_h(p)$ cover all the values of the profit function, and hence even the end points.

So $x_h(p)$ will be the optimal network size to serve, only if :

$$A_h\left[\frac{-\left|\varepsilon_{A_h}\right|}{1-\left|\varepsilon_{D_h}\right|}-1\right] > A_l\left[\frac{\varepsilon_{A_l}}{1+\varepsilon_{D_l}}-1\right] \qquad (15)$$

and naturally $x_l(p)$ will be the optimal quantity to produce when the right-side member of the expression is greater than the left-side one.

First proposition. Either a small or a large network size can arise as the optimal solution to the monopolist provider problem. If $x_l(p)$ is the maximising profit quantity to produce, it has to be that for that particular network size: $\varepsilon_{A_l} > 1+\varepsilon_{D_l}$. *If $x_h(p)$ is the optimum, this has to be in correspondence with the elastic side of the downward-sloping aggregate demand, and such that* $\left|\varepsilon_{D_h}\right| < 1+\left|\varepsilon_{A_h}\right|$.

Hence, the inclusion in the analysis of continuously increasing promotional costs faced by the provider in order to convince the agents to buy the network good, can lead to the negation of the well-accepted proposition[11] which states that the monopoly provider will always operate on the downward-sloping part of the aggregate demand ($x_h(p)$).

Consider now the case of a promotional cost function, given by the following expression:

$$A = \theta A(x(p)) \qquad (16)$$

where θ is a parameter included between 0 and 1.

The parameter scales up or down the advertising cost function. For θ close to zero, this cost is relatively low, for higher values, the cost rises.

Basically the actual value can depend on many factors: the size of the population, the overall economic conditions, the effective goodness of the campaign prepared, are only some possible factors affecting θ. Note that the inclusion of this parameter does not modify the analysis conducted so far.

If $x_l(p)$ is the optimal network size, the effect of a change of the parameter on the optimal price is represented by:

[11] See Economides and Himmelberg (1995b) and Shy (2000).

$$\frac{dp}{d\theta} = \frac{\frac{\partial A_l}{\partial x_l} \cdot \frac{\partial x_l}{\partial p}}{2.\frac{\partial x_l}{\partial p} + p \cdot \frac{\partial^2 x_l}{\partial p^2} - \theta \frac{\partial^2 A_l}{\partial x_l^2} \cdot \left(\frac{\partial x_l}{\partial p}\right)^2 - \theta \frac{\partial A_l}{\partial x_l} \cdot \frac{\partial^2 x_l}{\partial p^2}} < 0 \qquad (17)$$

since $\frac{dx_l}{dp} > 0$, we can infer that $\frac{dx_l}{d\theta} < 0$. If $x_h(p)$ is the optimal network size, the effect of a change of θ on the optimal price is given by:

$$\frac{dp}{d\theta} = \frac{\frac{\partial A_h}{\partial x_h} \cdot \frac{\partial x_h}{\partial p}}{2.\frac{\partial x_h}{\partial p} + p \cdot \frac{\partial^2 x_h}{\partial p^2} - \theta \frac{\partial^2 A_h}{\partial x_h^2} \cdot \left(\frac{\partial x_h}{\partial p}\right)^2 - \theta \frac{\partial A_h}{\partial x_h} \cdot \frac{\partial^2 x_h}{\partial p^2}} > 0 \qquad (18)$$

since $\frac{dx_h}{dp} < 0$; we have that $\frac{dx_h}{d\theta} < 0$.

Second proposition. An increase in the promotional costs leads the monopolist to charge a lower price if the optimal network size is small, a higher price when the optimal network size is big. In both cases the effect is to lower the network size targeted by the monopolist.

3b. *The second assumption: the presence of critical mass effect (CME).*

Under the second assumption, the monopolist profit function will instead be given by:

$$\pi = px_h(p) - A_l(x_l(p)) \qquad (19)$$

The optimal policy for the provider is to fix the price and the level of the promotional campaign in order to match the same quantity of consumers x_l. As "a proof ab absurdo", consider the other two possible cases:

1) the monopolist sets a price that «needs» a greater amount of advertisement expenditure than the level effectively chosen, to attract consumers: p 6 (select) x_l, A^* 6x_l*, with $x_l > x_l^*$. All agents indexed between $(x_l - x_l^*)$ will not subscribe. This will cause that, at that price p, all the agents

indexed between 0 and x_l* will not find the network good valuable. Hence, nobody will join the network and the monopolist will incur a loss given by the amount of money spent on the campaign: $\pi = -A(x_l^*)$.

2) the price is below the critical mass targeted by the promotion: p^* 6 x_l^*, A 6 x_l, with $x_l > x_l^*$.

All agents indexed between $(x_l - x_l^*)$ will subscribe. This will make the good more valuable and all agents indexed between x_2 and x_l will want to subscribe, with $x_2 > x_l$. The subscription of x_2 will in turn raise the value of the good and all agents indexed between x_3 and x_2, with $x_3 > x_2$, will want to join the network. This process will have an end only when $x_n = x_h^*$, since at price p^*, both the x_h^* agents that subscribe and the $(1- x_h^*)$ agents that do not purchase the good are maximising their utilities.

As a result, the monopolist has sold a given quantity of the good at a given price but spending more money than necessary in advertisements. More formally, the provider is not optimising, since:

$$p^* x_h^*(p^*) - A(x_l(p)) < p^* x_h^*(p) - A^*(x_l^*(p^*))$$

given that: $A^*(x_l^*(p^*)) < A(x_l(p))$.

Third proposition. The set of strictly dominant strategies for the provider can only be composed by those {A, p} that individuate the same x_l.

Given this proposition, we can formulate the monopolist problem as follows:

$$\max_p \pi = p x_h(p) - A(x_l(p)) \tag{20}$$

the first and second order conditions are now given by:

$$x_h(p) + p \cdot \frac{\partial x_h}{\partial p} = \frac{\partial A_l}{\partial x_l} \cdot \frac{\partial x_l}{\partial p} \tag{21a}$$

$$2\frac{\partial x_h}{\partial p} + p\frac{\partial {x_h}^2}{\partial p^2} - \frac{\partial^2 A_l}{\partial {x_l}^2} \cdot (\frac{\partial x_l}{\partial p})^2 - \frac{\partial A_l}{\partial x_l} \cdot \frac{\partial^2 x_l}{\partial p^2} \leq 0 \tag{22}$$

where the second order condition is verified only if:

$$2\frac{\partial x_h}{\partial p} + p\frac{\partial x_h{}^2}{\partial p^2}\frac{\partial A_l}{\partial x_l}\cdot\frac{\partial^2 x_l}{\partial p^2} \geq \frac{\partial^2 A_l}{\partial x_l{}^2}\cdot\left(\frac{\partial x_l}{\partial p}\right)^2 \tag{23}$$

Provided that a solution exists, we can express the optimal quantity in terms of the elasticities as follows:

$$x_h(p) = \frac{A_l}{p}\cdot\frac{\varepsilon_{A_l}}{1-\left|\varepsilon_{D_h}\right|} \tag{21b}$$

$x_h(p)$ will be greater than zero only for $\left|\varepsilon_{D_h}\right| < 1$. The monopolist will never produce along the part of the aggregate demand curve that has an elasticity equal to one. To see why, consider again, the first order condition when $\varepsilon_{D_h} = \frac{\partial x_h}{\partial p}\cdot\frac{p}{x_h} = -1$.

$$x_h(p) - x_h(p) = \frac{\partial A_l}{\partial x_l}\cdot\frac{\partial x_l}{\partial p} \tag{24}$$

the expression can only be true when $\frac{\partial A_l}{\partial x_l\cdot}\cdot\frac{\partial x_l}{\partial p} = 0$, but since both $\frac{\partial A_l}{\partial x_l}$ and $\frac{\partial x_l}{\partial p}$ are always greater than zero, this can never be the case.

A positive profit level will arise as long as:

$$A_l\left[\frac{\varepsilon_{A_l}}{1-\left|\varepsilon_{D_h}\right|} - 1\right] > 0 \tag{25}$$

that can be verified only if $\varepsilon_{A_l} > 1-\left|\varepsilon_{D_h}\right|$.

Fourth proposition. Under the hypothesis that the monopolist needs to provide a positive level of advertisement expenses only to get beyond the critical mass, he will always operate on the inelastic part of the downward-sloping side of the aggregate demand. At the optimum it also has to be that $\left|\varepsilon_{D_h}\right| > 1-\varepsilon_{A_l}$.

Consider now again the case of a multiplicative shock to the cost function: $A_l = \theta A(x_l(p))$.

A change in the parameter θ will affect the variables of our interest, computed at the optimum, in this way:

$$\frac{dp}{d\theta} = \frac{\dfrac{\partial A_l}{\partial x_l} \cdot \dfrac{\partial x_l}{\partial p}}{2.\dfrac{\partial x_h}{\partial p} + p \cdot \dfrac{\partial^2 x_h}{\partial p^2} - \theta \dfrac{\partial^2 A_l}{\partial x_l^2} \cdot \left(\dfrac{\partial x_l}{\partial p}\right)^2 - \theta \dfrac{\partial A_l}{\partial x_l} \cdot \dfrac{\partial^2 x_l}{\partial p^2}} < 0 \tag{26}$$

since $\dfrac{dx_h}{dp} < 0$, $\dfrac{dx_h}{d\theta}$ will be greater than zero.

Fifth proposition. Under the second assumption (CME), an increase in the promotional costs needed to induce the agents to buy the network good, leads the monopolist to charge a lower price and this in turn will cause a higher rate of penetration of the good among the population.

At first glance, the result may seem counterintuitive: the more the monopolist perceives as difficult and costly to persuade the economy to purchase the product, the more the network good will be then adopted among the agents. The rationale behind the proposition is that for saving money on a particularly expensive promotion, the provider will find optimal to set a low price in order to target a small critical mass. But given the characteristic of network externalities that the good exhibits, the low price will at the end attract a greater number of consumers. This proposition could even be viewed as a possible claim in support of a taxation scheme on the promotional campaign sustained by the monopolist in a network good market. A progressive tax imposed by the government on the amount spent by the provider on publicity will in fact have the same effect of the multiplicative shock that we have analysed, increasing the rate of adoption of the good.

3c. An interpretation of the two assumptions.

The use of the first rather than the second assumption about the promotional cost function leads to substantial different results. Before attempting a comparison between them, it is necessary to dwell on the nature of the two hypotheses. It has already been claimed that they both seem plausible under different circumstances. More precisely the NCME assumption seems to fit better a scenario where agents are initially not aware of the existence of the good. The monopolist provider needs increasing advertisement expenses not just to convince the agents to purchase, but first of all to inform them of the existence of the good. High-willingness-to-pay consumers are more easily reachable and convincible than the low-willingness-to-pay consumers. In this scenario any "chain reaction" effect is simply impossible to occur: once the provider targets x_l* agents by his promotional campaign, the only maximum feasible network size is just x_l*, since the remaining $(1- x_l*)$ agents do not even know that the good exists.

The best fitting example of a possible network good generally sold in this environment is a party. The party's organiser has to provide publicity as to inform individuals as to attract them to the event. Naturally a relatively low level of advertisements will be enough to inform (and to convince) the high-willingness-to-pay consumers, since they are keener on parties and night life in general and hence ready to get any chance of enjoyment. A much higher level of publicity is instead needed to get the low-willingness-to-pay agents aware of the party, since they usually do not attend parties or meet people who regularly attend parties. A large amount of money has hence to be invested to inform and to persuade this kind of individuals. Ruling out any possible consideration about congestion, the organiser will face the same problem as the monopolist provider does in the first case analysed.

The CME assumption implies a substantial difference respect to the NCME one: it is implicitly assumed that every agent in the economy is *ex-ante* aware of the existence of the good or they become aware of it, after any minimum positive amount spent by the provider on advertisements. In this case the promotional policy has only the scope of persuading agents to buy and no longer of informing them. In particular, it is assumed that every agent in the economy is aware of the strength of the campaign set up by the provider. Each agent expects that the more the network good is publicised, the more it will be successful (adopted) and hence valuable. Once the provider bears a A_l* level of advertisement expenditure in order to sell x_l* units of the good, any infinitesimal shift from this unstable equilibrium will assure him to sell the good to x_h* consumers, because of the network externalities. The probability of a shift will be approximately one, since it can also be caused by an infinitesimal increase in the level of the promotional expenditure ($A_l*+\varepsilon$, with $\varepsilon \approx 0$), for the mechanism illustrated in the second part of the demonstration of the third proposition.

Clearly there is a wide range of network goods for which the second assumption is plausible: telephone, fax, e-mail, are all good examples. In general, every item that as soon as invented is known by everyone in the community can confidently be treated under this hypothesis.
As already said, these two assumptions entail rather different implications for the optimal choices of the monopolist provider. The next section investigates this issue.

3d. A comparison of the results.

Under the first assumption (NCME), the maximum profit level achievable by the monopolist provider has been indicated by equations (13b) and (14b) for respectively x_l and x_h as optimal quantities to produce. The maximum profit level under the second assumption (CME) is instead given by (25). Evidently, this last expression will always be greater than equation (14b) given the nature of the two hypotheses, but equation (25) is always even greater than equation (13b). In fact for every particular optimal p^*, the maximum level of profit given by the (13b) is:

$$\pi_l = A_l * \cdot \left[\frac{\varepsilon_{A_l} *}{1 + \varepsilon_{D_l}} - 1 \right] \tag{27}$$

the level of profit for the same price, will be instead represented under the second assumption, by:

$$\pi_h = A_l * \cdot \left[\frac{\varepsilon_{A_l} *}{1 - \left| \varepsilon_{D_h} \right|} - 1 \right] \tag{28}$$

The level of profit given by (28) is always greater than (27), since $\left| \varepsilon_{D_h} \right| > -\varepsilon_{D_l}$ is always verified.
Hence, as easily predictable, the maximum monopolist provider profit is always higher when we consider the CME rather than the NCME assumption.
Considering now the size of the network, we can obviously state that the rate of penetration of the network good under the CME assumption will always be greater than the rate under the NCME assumption when a low quantity is the optimal choice. More interestingly, it is legitimate even to say that also in the other case (x_h optimal), the quantity produced by the monopolist will

be greater under the second assumption. Both the solutions can in fact coexist only if the demand presents an elasticity greater than one in some parts and less than one in some others. More precisely, for $x_h^{\,1}$ (first assumption: NCME) has to be $\left|\varepsilon_{D_h}\right| > 1$, while for $x_h^{\,2}$ (second assumption: CME) has to be $\left|\varepsilon_{D_h}\right| < 1$. The shape of the downward-sloping side of the inverted U aggregate demand allows us to assert that the price elasticity is decreasing in absolute value in respect of the network size. This can also be seen more formally, differentiating the demand elasticity with respect to price:

$$\frac{d\left(\frac{\partial x_h(p)}{\partial p}\cdot\frac{p}{x_h(p)}\right)}{dp} = \frac{\partial^2 x_h(p)}{\partial p^2}\cdot\frac{p}{x_h(p)} + \frac{\partial x_h(p)}{\partial p}\cdot\left[\frac{1}{x_h(p)} - \frac{p}{x_h(p)^2}\cdot\frac{\partial x_h(p)}{\partial p}\right] < 0 \qquad (29)$$

Recalling that the demand elasticity for x_h is negative, equation (29) implies that $\left|\varepsilon_{D_h}\right|$ always decreases for a reduction in price. So if both solutions exist it has necessarily to be that $x_h^{1} < x_h^{2}$.

Sixth proposition. The CME assumption always leads to a higher profit level for the monopolist provider and to a larger network size.

The proposition implies that there is no conflict between social welfare and the monopolist own interest. In a scenario on which every agent knows the existence of the good and the monopolist uses the advertisement expenditure only to convince people to purchase, the level of profit achievable by the provider is greater and the rate of penetration is higher than in an uninformed environment.

Thus, in case the knowledge of the good was not naturally spread over the community, a public authority could intervene to diffuse the information, advantaging the provider and aiding the adoption of the good among the population. The interesting point is that, by the fifth proposition, though the information process could be costly, the authority could partly or totally recover the money spent, imposing a progressive tax on the provider advertisements. This will naturally lower the monopolist profit but will even favour the goal of the universal service, where for universal service is usually intended a network which everyone has the opportunity to subscribe to a reasonable cost.

4.Conclusion

The inclusion of the hypothesis of advertisement expenses faced by the provider in order to sell the good responds to the need of adding a realistic feature to the analysis of networks.

The monopolist problem in a network market sensibly changes once these promotional costs are introduced. Moreover, different hypotheses about the form of the advertisement expenditure that the provider has to bear, affect in different ways his strategies and the resulting network size. In particular, a scenario characterised by perfect information of the agents regarding the existence of the product will lead to a superior equilibrium from both the monopolist and the social point of view. As already said, this fact can allow a public authority to intervene in the market in the opposite case where agents are initially not aware of the good. Once effectuated the process of spreading the information over the population, the authority can always positively influence the size of the network imposing a progressive taxation scheme on the advertisement expenditure.

That `s the story so far but, naturally, further research is needed. Our study of promotional policy is placed in a static setting, while the formulation of the problem in dynamic terms could give the opportunity to better delineate the use of marketing means in a network market. Moreover, we have restricted our treatment to the case of monopoly, but it would be even more interesting to investigate which role advertisement could play in a more competitive market structure and how the resulting network size would be affected. These are all totally unexplored themes. So far, advertisement policy has not attracted the attention of network economists.

Finally, we would like to conclude the present work reporting the old and popular quote: "advertising is the very soul of the commerce". If this is true for every market is even true (and maybe even more plausible) for a market of a network good.

Therefore, the study of advertising in a network market is extremely important and should not be under valuated.

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