QUADERNI

Università degli Studi di Siena
DIPARTIMENTO DI ECONOMIA POLITICA

SIMONE BORGHESI
DAVID ULPH

Sustainability for All? A North-South-East-West Model

n. 348 – Maggio 2002

SUSTAINABILITY FOR ALL? A NORTH-SOUTH-EAST-WEST MODEL

Simone Borghesi*

and

David ULPH†

Abstract

This paper examines whether it is possible for all countries to simultaneously achieve efficient and sustainable allocations of resources even if they do not cooperate in a world with inter-generational and intra-generational externalities. Using a simple model with two governments – one for the north and one for the south – we show that one hemisphere cannot always achieve efficiency and sustainability independently of the other, that is, whatever allocation is chosen by the other hemisphere. However, the north and the south can simultaneously achieve efficiency and sustainability if each government aims separately at these two goals in its own hemisphere.

Keywords: sustainable development, North-South interactions, intra- and inter-generational externalities

JEL classification: D62, F0, O1

* Department of Economics, University of Siena, Piazza San Francesco 7, 53100 Siena, Italy. Tel. +39 0577 289495, fax +39 0577 232661 (Attention: Simone Borghesi), e-mail: borghesi@unisi.it (corresponding author).

† Analysis and Research Office, Inland Revenue, London, and Department of Economics, University College London, England. e-mail: David.Ulph@ir.gsi.gov.uk

Introduction

Sustainability has become widely accepted by policy-makers as an objective of government policy[1] - though by no means the only objective. While there remains some controversy amongst academics and others as to what precisely is meant by this concept, in this paper we adopt the generally held view that the core idea behind sustainability is the requirement of inter-generational equity in the sense that all generations should be equally well off[2].

The simple[3] question on which we wish to focus is whether, in a world made up of many countries with independent governments[4], it is actually possible for all countries to simultaneously achieve sustainability as defined above. That is, when there are many governments acting independently, is it possible to achieve an outcome in which, in each country, every generation is as well off as every other generation in that country - though not necessarily as well off as generations in other countries?[5]

Let us make this question more precise.

In the first place we have to recognise that sustainability is not the only objective of government policy. Another important objective is efficiency. So the question is whether in a world of many countries with independent governments it is possible for them all to simultaneously achieve both efficiency and sustainability.

Secondly, if governments had only a limited range of policy instruments at their disposal then it would hardly be surprising if they found it difficult to simultaneously achieve these two aims. So to make the issue interesting we need to make it clear that in asking this question we are implicitly assuming that each government has complete control over the resource allocation - the time-paths of consumption and production - within its own country. Amongst other things this means that we are assuming that each government can carry out whatever inter-generational transfers it wishes within its country.

Thirdly we have to clarify what we mean by independent governments. By this we mean two things: (i) when a government chooses the resource allocation within its own country it does so non-cooperatively - i.e. taking as given the resource allocation decisions of all other governments[6]; (ii) there are no transfers between countries - i.e. no intra-generational transfers.

Now if the countries were also completely independent and were not linked in any way, then the problem of achieving efficiency and sustainability in such a multi-country world would be precisely

---

[1] See, for example, the report of the World Commission on Environment and Development (WCED, 1987) known as the Brundtland Report).

[2] This is the view that underpins the widely discussed prescription of Hartwick (1977) for achieving intergenerational equity by re-investing resource rents. This is widely held to be a prescription for achieving sustainability.

[3] Though a simple and rather obvious question, to our knowledge this has not previously been fully analysed. The closest is a paper by Bhaskar (1995). The main difference between his paper and ours is that (i) he treats production and hence damage as given; (ii) he allows for inter-generational altruism, and determines the equilibrium transfers between the current and the future generations.

[4] If we had a multi-country world, but if either there were some single over-arching government or else if governments acted cooperatively then, provided governments had enough policy instruments, there is no obvious reason why the simultaneous achievement of sustainability could not be achieved.

[5] Some writers - e.g. Pezzey (1989) and Turner (1991) - strengthen the requirement of sustainability to also include intra-generational equity. In a world of asymmetric countries this stronger equity condition will require intra-generational transfers and so certainly could not be obtained by independent governments.

[6] This means that when we consider the overall resource allocation produced by the simultaneous decisions of all governments, then we are considering the Non-Cooperative Nash Equilibrium.

the same as that in a single-country world, and we know that in a wide class of cases it is indeed possible to achieve both efficiency and inter-generational equity. But of course countries are linked in a number of ways - the most obvious being through trade and environmental externalities. In this paper we are going to ignore trade links and focus purely on environmental externalities. What we have in mind are externalities like CO_2 emissions which are a prime source of global warming. These externalities have two features: (i) they are global externalities - it is the total emissions by all countries which generates the damage; (ii) they are stock externalities - it is the cumulative stock of CO_2 in the atmosphere which does the damage, and this stock decays very slowly. Put differently, what we have in mind are externalities which are both *inter-generational* and *intra-generational.*

So the question is whether, in a world with inter-generational and intra-generational externalities between countries and where countries are governed by very powerful but independent governments, it is possible for all countries to simultaneously achieve efficient and sustainable allocations of resources.

For simplicity, consider the case where there are just two countries/governments. We want to understand how, in the presence of these externalities, the ability of one government to achieve both efficiency and sustainability might be affected by the decisions taken by the other government. It turns out to be useful to break this issue down into three separate, but related, questions.

1) Can each government always achieve both equity and efficiency *whatever* (feasible) resource allocation is chosen by the other?

If the answer to this is yes then of course we have answered the question with which we are concerned. But the answer to this could be negative, for, given the externalities, if one government behaves in a very perverse way it may be impossible for the other government to take corrective action to restore inter-generational equity.

It is then natural to ask whether either government could achieve both efficiency and sustainability if the other government is acting in a more rational fashion. One obvious question to ask here is:

2) Can each government always achieve both equity and efficiency whenever the other at least chooses an *efficient* allocation - even though it may not share the same commitment to equity?

A positive answer to this will mean that both governments can in principle achieve both equity and efficiency. However, a negative answer to our second question could come about if one government, though committed to efficiency, puts a great deal of weight on the current generation. This may lead it to take actions that create so many problems in the future that, because of the externalities, the first government is unable to achieve inter-generational equity.

So this brings us to the question with which we started:

3) Can each government achieve both efficiency and equity when the other government is also trying to achieve *both efficiency and equity*?

To answer these three questions we set up a very simple model. In this model there are in fact four countries, but they are organised into two hemispheres - the north and the south. The are two countries in each hemisphere, one in the east and one in the west. There are just two governments - one for the north and one for the south. There are two goods: a produced good and leisure. The produced good is produced by labour alone. Each country is endowed with labour alone and this labour is immobile and so cannot be transferred between countries. There is a common technology in the north which is more productive than the common technology in the south.

Production generates emissions which cause damage. The damage in the west is caused by the combined output in the north-west and south-west - our analogue of the intra-generational externality. There is a prevailing wind that causes pollution in the west to be carried east, so pollution in each eastern country depends on the entire output of all four countries. This provides us with a geographical analogue of the stock/inter-generational externality, in which the west stands for the current generation and the east for the future.

The government in the north can control the total amount of labour supply, and hence production, in each northern country, and can make whatever transfers of this output it wishes between the consumers in the two northern countries. The government in the south has similar powers. There are no transfers between the north and the south.

Using this model we can give the following answers to our three questions.

1) If utility is sufficiently sensitive to the good that can be transferred between countries/generations, then it is indeed possible for each government to achieve equity and efficiency whatever allocation is chosen by the other government. In this case the answers to the remaining two questions are also positive. However, when utility is not very sensitive to transfers, then there may be allocations chosen by one government which make it impossible for the other to achieve both equity and efficiency. In particular this arises if one country concentrates a sufficiently large amount of its production in the east (future).
2) Even in situations where the answer to question 1 is negative it is possible that the answer to question 2 is positive, so each country can achieve both equity and efficiency as long as the other is acting efficiently. This is particularly true when productivity in each country is fairly low, since then output and damage are fairly low, and transfers can compensate for damage. However, when productivity in one country is high, and when, though acting efficiently, it gives a lot of weight to the west (current generation), then the other government may find it impossible to achieve both equity and efficiency.
3) Even when question 2 has a negative answer, question 3 may still have a positive answer, namely, both countries can simultaneously achieve both equity and efficiency. Calibrations of the model for different values of the parameters show that this result holds not only when countries are equally productive, but also when one is markedly more productive than the other. The same result applies when we allow for technological progress to introduce large differences between west and east, so that the future generation has higher productivity or lower environmental damages per unit of output than the current generation.

The plan of the paper is as follows. Section 1 sets out the model and formalises the three questions posed above. Section 2 shows that in one country world it is always possible to achieve both equity and efficiency. The next three sections deal with a two country model and address in each turn each of our three questions. Section 6 concludes.

Section 1 The Model

There are four countries: north-west, north-east, south-west and south-east.

There are two commodities: a produced good, *X*, and leisure, *L*. In each country there is a single representative consumer, endowed solely with 1 unit of time that can be used for work or leisure. These consumers, and hence their labour, are immobile.

The utility function of each consumer is the same. Thus if a consumer consumes *x* units of *X*, and *l* units of *L*, the utility obtained is

$$u(x,l) = \phi(x) + \psi(l),$$

where

$$\phi(x) = \begin{cases} \frac{1}{1-\beta} x^{1-\beta}, \beta > 0, \beta \neq 1 \\ \log(x), \beta = 1 \end{cases}$$

$$\psi(l) = \log(l) + (1-l)$$

$\phi(.)$ is just a conventional iso-elastic utility function where, if *X* were the only commodity, the parameter β could be interpreted as a measure of risk aversion. It is important to notice that

$$x \to 0 \Rightarrow \phi(x) \to \begin{cases} -\infty, \beta \geq 1 \\ 0, 0 < \beta < 1 \end{cases} \qquad (1)$$

$\psi(.)$ has the following properties that will be important later on:

$$\psi(1) = 0;\ \psi(l) \to -\infty \text{ as } l \to 0;\ \psi'(l) = \frac{1-l}{l}, \text{ so } \psi'(1) = 0;\ \psi'(l) \to +\infty \text{ as } l \to 0$$

The assumption that the marginal utility of leisure is zero when the consumer does no work is made to avoid the possibility later on of certain boundary solutions where it might be optimal to have no work done in particular country. Ruling out these boundary cases is not at all essential to the arguments we wish to make, and simply avoids a messy proliferation of sub-cases which detracts from the main insights.

X is produced by labour alone. We wish to distinguish production of *X* from consumption of *X*, so in all that follows *y* will denote units of output of *X*.

We assume that there is a common constant returns to scale technology in the two southern countries, whereby to produce *y* units of *X* in any southern country requires $c_s y$ units of labour from that country. The two northern countries also have a common constant returns to scale technology, and we allow the possibility that this is more productive than the southern technology. So to produce *y* units of *X* in any northern country requires $c_n y$ units of labour from that country where $0 < c_n \leq c_s$.

Production of *X* generates emissions of some pollutant, which causes damage to consumers. We assume that the northern technology is just as polluting as the southern technology, and that emissions per unit of output are constant. Thus we can measure units such that 1 unit of output of X generates 1 unit of emissions.

Emissions cross boundaries. There is uniform mixing of pollutants between the north and south. However, there is a prevailing westerly wind which carries all the pollution from the west to the east. Total emissions in each western country are therefore given by $E_w = y_{nw} + y_{sw}$, while total emissions in each eastern country are $E_e = y_{nw} + y_{sw} + y_{ne} + y_{se}$.

All consumers face exactly the same damage function, whereby exposure to E units of emissions causes damage equivalent to a loss of $\frac{D}{2}.E^2$ units of utility. The important point here is that, given the quadratic nature of the damage function, marginal damage in one hemisphere depends on decisions taken in the other hemisphere, so decisions are fundamentally inter-connected.

There is a single government in the north which can determine the level of production of X in each of the two northern countries. It can transfer X, but not leisure, between these two countries, so it can control consumption of X in the two countries, subject only to the constraint that total consumption equals total production. Similarly, there is a single government in the south with exactly the same powers. However there are no transfers between the north and the south**.**

Let $z_h \equiv (x_{hw}, y_{hw}, x_{he}, y_{he}),\ h = n, s$ denote an allocation of resources chosen by the government in hemisphere h. This allocation is *feasible* iff

(i) $z_h \geq 0$;

(ii) $y_{hw} \leq \frac{1}{c_h}; y_{he} \leq \frac{1}{c_h}$;

(iii) $x_{hw} + x_{he} \leq y_{hw} + y_{he}$.

Let F_h be the set of feasible allocations for hemisphere h.

Given the global nature of the pollutant welfare in each country depends on the allocations chosen by each of the two governments. So let $z = (z_n, z_s)$. Then welfare in each of the four countries is given by

$$V_{hw}(z) = \varphi(x_{hw}) + \psi(1 - c_h y_{hw}) - \frac{D}{2}(y_{nw} + y_{sw})^2;\ h = n, s$$

$$V_{he}(z) = \varphi(x_{he}) + \psi(1 - c_h y_{he}) - \frac{D}{2}(y_{nw} + y_{sw} + y_{ne} + y_{se})^2;\ h = n, s$$

Each government takes as given the allocation chosen by the other government, and chooses a feasible allocation between the two countries in its own hemisphere which satisfies two criteria.

The first is *Pareto Efficiency*.

Now the locus of allocations which are *Pareto Efficient* for the north (given the allocation in the south) is just the locus of all allocations which solve the problem

$$\underset{z_n \in F_n}{MAX} \delta_n V_{nw}(z_n, z_s) + (1-\delta_n) V_{ne}(z_n, z_s) \qquad (2)$$

as the parameter δ_n varies between 0 and 1. Let $z_n = \zeta_n(z_s; \delta_n)$ denote the allocation that solves (2).

Similarly the locus of allocations which are Pareto Efficient for the south (given the allocation in the north) is just the locus of all allocations which solve the problem

$$\underset{z_s \in F_s}{MAX} \delta_s V_{sw}(z_n, z_s) + (1-\delta_s) V_{se}(z_n, z_s) \qquad (3)$$

as the parameter δ_s varies between 0 and 1. Let $z_s = \zeta_s(z_n, \delta_s)$ denote the allocation that solves (3).

We are interested in allocations which are *simultaneously Pareto Efficient* in both the north and south. Let $z^e(\delta_n, \delta_s)$ be the Nash equilibrium allocation[7] when the government in the north operates according to (2) and the government in the south behaves according to (3). This solves the equations

$$z_n^e = \zeta_n(z_s^e, \delta_n); \; z_s^e = \zeta_s(z_n^e, \delta_s).$$

This allocation is simultaneously Pareto efficient. The set of all simultaneously Pareto Efficient allocations is generated by allowing δ_n and δ_s to vary independently between 0 and 1.

The second objective of governments is assumed to be *fairness*. That is, they would like the welfare of the country in the west of their hemisphere to equal the welfare of the country in the east.

The issue to which this paper is addressed is how the ability of one government to achieve an allocation which is both *fair and efficient* might be affected by the decisions taken by the other country.

More specifically we are interested in the following three questions:

- Can one government achieve a fair and efficient allocation whatever (feasible) action is chosen by the other?
- Can one government achieve a fair and efficient allocation when the other government is at least acting efficiently?
- Can one government achieve a fair and efficient allocation when the other government is trying to achieve both fairness and efficiency? Put differently, can both countries simultaneously achieve fairness and efficiency?

---

[7] Given the fact that each hemisphere is maximising a twice differentiable strictly concave objective function on a compact set of actions, standard results guarantee that such a Nash equilibrium exists.

These questions can be posed more formally as follows.

1 (a) $\forall z_s,\ z_s \in F_s$ does there exist $\overline{\delta}_n,\ 0 \le \overline{\delta}_n \le 1$ such that

$$V_{ne}\left[\zeta_n\left(z_s,\overline{\delta}_n\right),z_s\right] = V_{nw}\left[\zeta_n\left(z_s,\overline{\delta}_n\right),z_s\right]?$$

If it exists, denote this by $\overline{\delta}_n = \Delta_n\left(z_s\right)$.

(b) $\forall z_n,\ z_n \in F_n$ does there exist $\overline{\delta}_s,\ 0 \le \overline{\delta}_s \le 1$ such that

$$V_{se}\left[z_n,\zeta_s\left(z_n,\overline{\delta}_s\right)\right] = V_{sw}\left[z_n,\zeta_s\left(z_n,\overline{\delta}_s\right)\right]?$$

If it exists, denote this by $\overline{\delta}_s = \Delta_s\left(z_n\right)$.

2 (a) $\forall \delta_s,\ 0 \le \delta_s \le 1$ does there exist $\overline{\delta}_n,\ 0 \le \overline{\delta}_n \le 1$ such that

$$V_{ne}\left[z^e\left(\overline{\delta}_n,\delta_s\right)\right] = V_{nw}\left[z^e\left(\overline{\delta}_n,\delta_s\right)\right]?$$

If it exists, denote this by $\overline{\delta}_n = \overline{\Delta}_n\left(\delta_s\right)$.

(b) $\forall \delta_n,\ 0 \le \delta_n \le 1$ does there exist $\overline{\delta}_s,\ 0 \le \overline{\delta}_s \le 1$ such that

$$V_{se}\left[z^e\left(\delta_n,\overline{\delta}_s\right)\right] = V_{sw}\left[z^e\left(\delta_n,\overline{\delta}_s\right)\right]?$$

If it exists, denote this by $\overline{\delta}_s = \overline{\Delta}_s\left(\delta_n\right)$.

3 Does there exist a pair $\left(\overline{\delta}_n,\overline{\delta}_s\right)$, $0 \le \overline{\delta}_n \le 1,\ 0 \le \overline{\delta}_s \le 1$ such that

a) $V_{ne}\left[z^e\left(\overline{\delta}_n,\overline{\delta}_s\right)\right] = V_{nw}\left[z^e\left(\overline{\delta}_n,\overline{\delta}_s\right)\right]$
and
b) $V_{se}\left[z^e\left(\overline{\delta}_n,\overline{\delta}_s\right)\right] = V_{sw}\left[z^e\left(\overline{\delta}_n,\overline{\delta}_s\right)\right]$?

Before proceeding to answer these questions notice the following points.

1. Intuitively it seems obvious that positive answers to question1 imply positive answers to question 2 which in turn implies a positive answer to question 3.
2. More formally, we can claim that if all the various reaction functions defined above are continuous then indeed
   (i) a positive answer to question 1(a) (respectively 1(b)) implies an affirmative answer to question 2 (a) (respectively 2(b));
   (ii) positive answers to questions 2(a) and 2(b) imply an affirmative answer to question 3.

3. However, negative answers to 1(a) and/or 1(b) may still be consistent with a positive answer to question 2(a) and/or 2(b). While negative answers to questions 2(a) and/or 2(b) can be consistent with positive answers to question 3.
4. However, a negative answer to question 3 implies negative answers to questions 1 and 2.

To elaborate on point 2(i) above, notice that the function $\overline{\Delta}_n(\delta_s)$ is implicitly defined as a solution to the equations

$$\begin{aligned} \overline{\delta}_n &= \Delta_n(z_s) \\ z_s &= \zeta_s\left[\zeta_n\left(z_s, \overline{\delta}_n\right), \delta_s\right] \end{aligned}$$

These equations just map the compact set $[0,1]\times F_s$ onto itself, and, if the functions are continuous, a solution is guaranteed by Brouwer's Fixed Point Theorem.

In order to explore the answers to our three questions it is useful to begin by first briefly understanding why, if there were just a single hemisphere, it would always be possible to achieve both equity and efficiency.

Section 2 The Single Hemisphere Case

Suppose that there were just a single hemisphere with one western country and one eastern country. To simplify notation, drop all hemisphere subscripts.

To see whether there is an efficient allocation which is also fair, consider the two allocations at the extreme ends of the locus of Pareto Efficient allocations.

Suppose first that $\delta = 1$. The problem is to choose $z \geq 0$ so as to

$$MAX\,\phi(x_w)+\psi(1-cy_w)-\frac{D}{2}(y_w)^2$$

$$\text{subject to} \quad \begin{aligned} & x_e + x_w \leq y_e + y_w \\ & y_e \leq \frac{1}{c} \end{aligned}$$

Notice that, given the nature of the function $\psi(.)$ the upper bound constraint on production in the west will never bite. It is easy to see that the solution to this is $x_w = \frac{1}{c} + y,\ x_e = 0,\ y_w = y < \frac{1}{c},\ y_e = \frac{1}{c}$, where $y > 0$ is the solution to

$$\left(\frac{1}{c}+y\right)^{-\beta} = \frac{c^2 y}{1-cy} + Dy\,.$$

Obviously, in this allocation, $V_w > V_e$, since the consumer in the east has less consumption, less leisure and more pollution than the consumer in the west.

Consider now the case where $\delta = 0$. Here the problem is to choose $z \geq 0$ so as to

$$MAX \phi(x_e) + \psi(1 - cy_e) - \frac{D}{2}(y_w + y_e)^2$$

$$\text{subject to} \quad \begin{aligned} & x_e + x_w \leq y_e + y_w \\ & y_w \leq \frac{1}{c} \end{aligned}$$

Clearly any solution will involve $x_w = 0$. However, since production in the west damages the east, it may not be optimal to have the consumer in the west working all the time. However, it will only be optimal to have production take place in the east, if the consumer in the west is working all the time. What we can therefore conclude is that

(i) if $c^{1+\beta} \leq D$ then the solution is: $x_w = 0,\ x_e = y,\ y_w = y,\ y_e = 0$, where $y,\ 0 < y \leq \frac{1}{c}$ satisfies the condition $y^{-\beta} = Dy$.

(ii) if $c^{1+\beta} > D$ then the solution is: $x_w = 0,\ x_e = \frac{1}{c} + y,\ y_w = \frac{1}{c},\ y_e = y$, where $y,\ 0 < y < \frac{1}{c}$ satisfies the condition $\left(\frac{1}{c} + y\right)^{-\beta} = \frac{c^2 y}{1 - cy} + D\left(\frac{1}{c} + y\right)$.

In case (i) pollution is the same in the east and in the west, but the consumer in the east has more consumption and more leisure, so $V_e > V_w$. In case (ii) the consumer in the east suffers more pollution than the consumer in the west, but nevertheless is better off since the consumer in the west has zero consumption and zero leisure, and the latter feature gives this consumer infinitely negative utility.

Thus, whatever solution obtains, it is necessarily the case that $V_e > V_w$.

It follows from the construction of the locus of Pareto Efficient allocations that $V_w - V_e$ is monotonically increasing as δ increases from 0 to 1. Consequently there is a unique $\overline{\delta},\ 0 < \overline{\delta} < 1$ such that $V_w = V_e$, and so a single hemisphere can always achieve both equity and efficiency.

In the next section we explore how this argument extends to the case where there are two interdependent hemispheres.

Section 3 The Two Hemisphere Case: Question 1

Consider one of the hemispheres - it does not matter which. To ease notation drop subscript references to the particular hemisphere chosen. Denote the allocation chosen by the other hemisphere by $\tilde{z}$, and consider whether, given this, there is any fair and efficient allocation in the chosen hemisphere. Proceed along the lines of the previous section.

Consider first the case where $\delta = 1$. The problem is to choose $z \geq 0$ so as to

$$MAX\phi(x_w)+\psi(1-cy_w)-\frac{D}{2}(y_w+\tilde{y}_w)^2$$

$$\text{subject to} \quad \begin{aligned} &x_e + x_w \leq y_e + y_w \\ &y_e \leq \frac{1}{c} \end{aligned}$$

It is easy to see that the solution to this is $x_w = \frac{1}{c} + y,\ x_e = 0,\ y_w = y < \frac{1}{c},\ y_e = \frac{1}{c},$ where $y,\ 0 \leq y < \frac{1}{c}$ satisfies the complementary slack inequalities

$$\left(\frac{1}{c}+y\right)^{-\beta} \leq \frac{c^2 y}{1-cy} + D(\tilde{y}_w + y),\ \ y \geq 0$$

The only difference from the case considered in Section 2 is that the damage done by production in the other hemisphere means that we can no longer guarantee that output in the west is positive. Nevertheless it is still true that, in this allocation, $V_w > V_e$, since the consumer in the east has less consumption, less leisure and more pollution than the consumer in the west.

Consider now the case where $\delta = 0$. Here the problem is to choose $z \geq 0$ so as to

$$MAX\phi(x_e)+\psi(1-cy_e)-\frac{D}{2}(y_w+y_e+\tilde{y}_w+\tilde{y}_e)^2 \qquad (3)$$

$$\text{subject to} \quad \begin{aligned} &x_e + x_w \leq y_e + y_w \\ &y_w \leq \frac{1}{c} \end{aligned}$$

Clearly any solution will involve $x_w = 0$. Notice that, given (1), this immediately implies that if $\beta \geq 1,\ V_w = -\infty < V_e$. So we have

Theorem 1 If $\beta \geq 1$, then the answers to questions 1(a) and 1(b) are affirmative.

Corollary If $\beta \geq 1$, then the answers to questions 2(a), 2(b) and 3 are also affirmative.

The intuition behind these results is as follows. If β is large then utility is very sensitive to transfers of *X*. Hence any pattern of damage generated through the allocation of production can be offset through *X* transfers so as to ensure equity.

We therefore need to understand what happens when β is small. So let us assume now - and throughout the rest of the paper - that $0 < \beta < 1$.

To understand whether there is a fair and efficient allocation in this case we need to understand the solution to (3) more fully. As in Section 2, it may not be desirable to push leisure in the west to zero, but it will only be desirable to have production in the east once this has happened. The full solution to (3) therefore takes the following form.

(i) if

$$c^{1+\beta} \leq D\left[1 + c.\left(\tilde{y}_w + \tilde{y}_e\right)\right] \tag{4}$$

then the solution is: $x_w = 0,\ x_e = y,\ y_w = y,\ y_e = 0$ where $y,\ 0 < y \leq \frac{1}{c}$ satisfies the condition

$$y^{-\beta} = D\left[y + \left(\tilde{y}_w + \tilde{y}_e\right)\right] \tag{5}$$

(ii) if

$$c^{1+\beta} > D\left[1 + c.\left(\tilde{y}_w + \tilde{y}_e\right)\right]$$

then the solution is: $x_w = 0,\ x_e = \frac{1}{c} + y,\ y_w = \frac{1}{c},\ y_e = y$, where $y,\ 0 < y < \frac{1}{c}$ satisfies the condition

$$\left(\frac{1}{c} + y\right)^{-\beta} = \frac{c^2 y}{1 - cy} + D\left(\frac{1}{c} + y + \tilde{y}_w + \tilde{y}_e\right)$$

Now if the solution takes form (ii) then, since the consumer in the west gets no leisure, it follows that $V_w = -\infty < V_e$ and hence that a fair and efficient allocation will exist.

When the solution takes the form (i) then

$$V_w = \psi(1 - cy) - \frac{D}{2}\left(y + \tilde{y}_w\right)^2 \ ; \ V_e = \frac{1}{1-\beta} y^{1-\beta} - \frac{D}{2}\left(y + \tilde{y}_w + \tilde{y}_e\right)^2$$

where *y* is determined by (5).

So if

$$\frac{D}{2}\left[\tilde{y}_e^2 + 2\tilde{y}_e\left(\tilde{y}_w + y\right)\right] > \frac{1}{1-\beta} y^{1-\beta} - \left[\log(1 - cy) + cy\right] \tag{6}$$

then $V_w > V_e$ even when $\delta = 0$, and so there is no fair and efficient allocation.

Thus if condition (4) holds, and if, given that, (6) also holds where y is determined by (5), then, for the particular allocation $\tilde{z}$ chosen by the other hemisphere there is no fair and efficient allocation in the chosen hemisphere we are studying.

Notice that if

$$c^{1+\beta} < D \qquad (7)$$

then (4) is satisfied for all feasible $\tilde{z}$. So let us assume that (7) holds for both hemispheres - i.e.

$$c_n^{1+\beta} \le c_s^{1+\beta} < D. \qquad (8)$$

For (6) to hold we require that:

(i) y should be "small" which, from (5), will be true if both D and total output from the other hemisphere are "large";
(ii) a "large" fraction of the output of the hemisphere is produced in the east.

Given this discussion it is easy to construct examples where there is no fair and efficient allocation. For example, if $\beta = 0.5$, $c = 1$, $D = 2$, $\tilde{y}_e = \tilde{y}_w = 0.5$, then y = 0.17965 (from (5)) and (6) holds, therefore in this case no fair and efficient allocation exists.

We thus have:

Theorem 2 If
(2.1) $0 < \beta < 1$;
(2.2) $c_n^{1+\beta} \le c_s^{1+\beta} < D$
then, we can find examples where, for some feasible allocations chosen by one government, the other government finds it impossible to achieve both equity and efficiency.

The question that now arises is whether this negative result depends on the arbitrary nature of the allocation chosen by one government - in particular the fact that a large fraction of the output was produced in the east. Would this result survive if the other government were choosing its output on a more rational basis - i.e. on the principles of efficiency and equity? The problem is that these principles point in different directions. Efficiency considerations argue in favour of shifting production to the east, since this does less damage than production in the west. However, equity considerations point towards shifting production towards the west, so as to try to equalise the damage in the two countries. We therefore need to investigate more carefully the implications of assuming that resource allocations are chosen according to these two principles.

Section 4 The Two Hemisphere Case: Question 2

Theorem 2 showed that, if (2.1) and (2.2) are satisfied then it is possible that if one government chooses certain (feasible) resource allocations, the other government may find it impossible to achieve both equity and efficiency.

In this section we wish to examine whether this result depends on the arbitrariness of the particular resource allocation that was chosen. More precisely we wish to ask whether, if (2.1) and (2.2) continue to hold, it is now possible for one government to always achieve fairness and efficiency if the other government is constrained to at least acting efficiently.

Consider a particular government. Suppose that the other government is acting efficiently with $\tilde{\delta} = 0$. Given our assumption (8) we know from the analysis in the previous section that in any resulting equilibrium allocation it must be the case that $\tilde{y}_e = 0$.

From the analysis we conducted in the previous section, we also know that:

(a) if the government in which we are interested acts efficiently with $\delta = 1$, then, in the resulting simultaneously efficient equilibrium it must be the case that $V_w > V_e$;

(b) if the government in which we are interested acts efficiently with $\delta = 0$, then, given that $\tilde{y}_e = 0$, it must be the case that in the resulting simultaneously efficient equilibrium $V_e > V_w$.

Hence, if $\tilde{\delta} = 0$, there must exist a $\overline{\delta},\ 0 < \overline{\delta} < 1$ such that, in the resulting simultaneously efficient equilibrium, $V_e = V_w$. Thus, perhaps not surprisingly, if one government puts a great deal of weight on future generations (the east) then the other government can always achieve sustainability.

Notice the following points.

(a) From (5), the output chosen by the government acting with $\tilde{\delta} = 0$ does not depend on the level of labour requirements, $\tilde{c}$, in that hemisphere. Hence the critical value $\overline{\delta}$ defined above depends solely on the parameters □, *D* and *c*.

(b) For the two governments the critical values of $\overline{\delta}$ which was defined generically above are just the values $\overline{\Delta}_i(0),\ i = n,s$ of their reaction functions when the other government has chosen to give a zero weight to west.

When the other government acts with $\tilde{\delta} = 0$, then, for given parameter values, and for any given δ chosen by the government in which we are interested it is fairly straightforward to numerically calculate the resulting simultaneously efficient equilibrium. Hence, by evaluating welfare in the east and west, one can numerically calculate[8] the critical value of $\overline{\delta}$ defined above, and hence the $\overline{\Delta}_i(0),\ i = n,s$.

Table 1 below gives the values of $\overline{\Delta}_i(0),\ i = n,s$ for the case where $\beta = 0.5,\ c_n = 0.5,\ c_s = 1$, and where *D* varies over a range of values.

[8] A programme for doing this is available from the authors on request.

Table 1

D	$\overline{\Delta}_n(0)$	$\overline{\Delta}_s(0)$
1.1	0.3506	0.3771
1.5	0.3500	0.3649
2	0.3497	0.3535
3	0.3495	0.3511

As Table 1 shows, the greater the damage caused by pollution the more the government has to weight welfare to the east in order to produce a fair outcome. The east, in fact, suffers more damages from pollution than the west due to the asymmetric nature of the model (production in the west damages the east but not viceversa).

Now consider what happens when $\widetilde{\delta}=1$ i.e. if the other government puts all its weight on the current generation (the west). Once again, if $\delta=1$, then, in the resulting simultaneously efficient equilibrium it must be the case that $V_w > V_e$. Suppose then that $\delta=0$. It is easy to see that the simultaneously efficient allocation associated with these welfare weights used by the two governments is

$$\widetilde{x}_w=\frac{1}{\widetilde{c}}+\widetilde{y},\ \widetilde{y}_w=\widetilde{y},\ \widetilde{x}_e=0,\ \widetilde{y}_e=\frac{1}{\widetilde{c}},\ x_w=0,\ y_w=y,\ x_e=y,\ y_e=0,$$

where $\widetilde{y},\ 0\le\widetilde{y}<\frac{1}{\widetilde{c}}$ and $y,\ 0<y<\frac{1}{c}$ are determined by the conditions:

$$\left(\frac{1}{\widetilde{c}}+\widetilde{y}\right)^{-\beta}\le\frac{\widetilde{c}^{\,2}.\widetilde{y}}{1-\widetilde{c}.\widetilde{y}}+D.(\widetilde{y}+y),\widetilde{y}\ge 0 \qquad (9)$$

and

$$y^{-\beta}=D\left(\frac{1}{\widetilde{c}}+\widetilde{y}+y\right) \qquad (10)$$

where the inequalities in (9) hold with complementary slackness.

The welfare levels in the west and east (for the hemisphere in which the government is operating with $\delta=0$) are then given by:

$$V_w=\psi(1-cy)-\frac{D}{2}(\widetilde{y}+y)^2\ ;\ V_e=\frac{1}{1-\beta}y^{1-\beta}-\frac{D}{2}\left(\frac{1}{\widetilde{c}}+\widetilde{y}+y\right)^2.$$

Notice that if the other hemisphere is quite productive - so $\widetilde{c}$ is small - then, from (10), this will tend to make y small, and more likely that $V_w > V_e$ even though $\delta=0$. Thus the more productive is the other hemisphere the less likely it is that the government we are interested in will be able to achieve both equity and efficiency when $\widetilde{\delta}=1$.

Now let us introduce the following notation. Let $\bar{\bar{\delta}}_s$, $0 < \bar{\bar{\delta}}_s \leq 1$ be the maximum value of δ_s for which the government in the north can achieve a fair and efficient allocation when the government in the south is acting efficiently and giving weight δ_s to the west.

Notice that

(i) if the government in the north can always achieve fairness and efficiency as long as the government in the south is acting efficiently then $\bar{\bar{\delta}}_s = 1$;

(ii) if $\bar{\bar{\delta}}_s < 1$, then $\bar{\Delta}_n\left(\bar{\bar{\delta}}_s\right) = 0$.

We can define $\bar{\bar{\delta}}_n$, $0 < \bar{\bar{\delta}}_n \leq 1$ in analogous fashion.

Once again it is fairly straightforward to calculate these $\bar{\bar{\delta}}_i$, $i = n, s$ numerically[9] and Table 2 presents calculations of these values for the same set of parameters as in Table 1.

Table 2

D	$\bar{\bar{\delta}}_n$	$\bar{\bar{\delta}}_s$
1.1	0.7145	1
1.5	0.6755	0.8823
2	0.6525	0.8077
3	0.6322	0.7346

Thus:

(i) when damage is low (D = 1.1), the north can achieve equity and efficiency whatever efficient allocation is chosen by the south, whereas the opposite is not true;

(ii) for any value of D, the south finds it harder to achieve equity and efficiency than does the north ($\bar{\bar{\delta}}_n < \bar{\bar{\delta}}_S$);

(iii) the greater the damage the more difficult is for each government to achieve equity and efficiency ($\bar{\bar{\delta}}_i$ decreases as D increases).

Thus although it is certainly possible that one government may be able to achieve equity and efficiency whatever efficient allocation is chosen by the other, we can equally find cases where neither given can achieve equity and efficiency when the other, though acting efficiently, gives a very large weight to west (the current generation).

[9] Again, a programme for doing this is available from the authors on request.

Section 5 The Two Hemisphere Case: Question 3

In the previous section we saw that one government may be unable to achieve fairness and efficiency if the other, though acting efficiently, gave a sufficiently high weight to the west (current generation). However, in order to achieve fairness, governments have to give a high weight to the east, so it is still an open question as to whether each government can achieve equity and efficiency when the other is also pursuing these twin goals.

To examine this issue, let us first analyse what happens when both hemispheres are equally productive, so $c_n = c_s = c$. In this case the two hemispheres are identical and, as it can be easily shown, they can then both achieve equity and efficiency. The following is a sketch of the proof.

Suppose first of all that $\delta_i = 1,\ i = N,S$. Then the *symmetric simultaneously efficient equilibrium* is:

$x_{ie} = 0,\ y_{ie} = \frac{1}{c},\ x_{iw} = \frac{1}{c} + y,\ y_{iw} = y$, where $y,\ 0 < y < \frac{1}{c}$ solves the equation

$$\left(\frac{1}{c} + y\right)^{-\beta} = \frac{c^2 y}{1 - cy} + 2Dy.$$

Clearly in this equilibrium $V_{iw} > V_{ie}$.

Now suppose $\delta_i = 0,\ i = N,S$. It is then easy to see that if

$$c^{1+\beta} < 2D \qquad (11)$$

then the *symmetric simultaneously efficient equilibrium* is

$x_{iw} = 0,\ x_{ie} = y,\ y_{ie} = 0,\ y_{iw} = y$, where $y,\ 0 < y < \frac{1}{c}$ solves the equation

$$y^{-\beta} = 2Dy.$$

Since we have already assumed (8) then (11) is satisfied. Clearly in this equilibrium $V_{ie} > V_{iw}$.

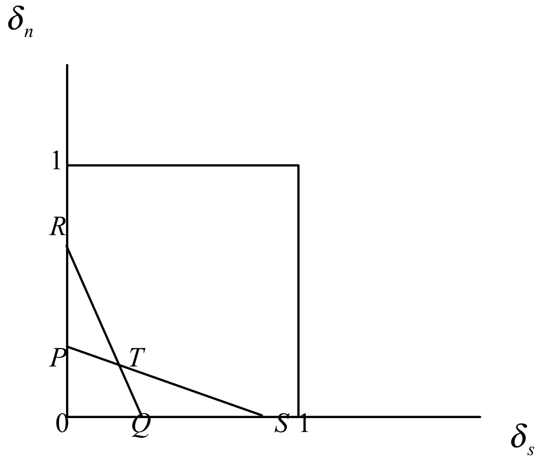
So, by continuity, there must exist a $\overline{\delta}_i,\ 0 < \overline{\delta}_i < 1$ such that if $\delta_i = \overline{\delta}_i$, then, in the corresponding *symmetric simultaneously efficient equilibrium* it will be the case that $V_{iw} = V_{ie}$, (*i*=*N*,*S*).

What happens when the two countries differ in their productivity?

To answer this consider Tables 1 and 2 in the previous section where $\beta = 0.5$, and where $c_n = 0.5,\ c_s = 1$, so the north is twice as productive as the south.

Figure 1 below draws the reaction functions $\overline{\Delta}_i(\delta_j),\ i,j = n,s,\ i \neq j$ of the two governments for a given value of D (D = 1.5). The labelled points are as follows: $P = \overline{\Delta}_n(0), Q = \overline{\Delta}_s(0), R = \overline{\overline{\delta}}_n, S = \overline{\overline{\delta}}_s$.

Figure 1



We have drawn the reaction functions as linear, and, as drawn, they intersect in *T*, showing that there is indeed an outcome where the two governments can simultaneously achieve both equity and efficiency.

Notice the following two points:

(a) The assumption of linearity does not matter since, given the configurations of the points, *P, Q, R, S*, any continuous reaction functions must intersect.

(b) This example shows that even though neither government can achieve equity and efficiency whatever efficient allocation the other chooses (namely, for every δ chosen by the other government), nevertheless as long as both are trying to achieve both equity and efficiency, they can both succeed.

The relative positions of points *P*, *Q*, *R*, *S* are preserved even if we let *D* vary over a range of values. As it emerges from Tables 1 and 2, in fact, it is always:

$$\overline{\Delta}_i(0) < \overline{\overline{\delta}}_i \qquad i = N, S \qquad (12)$$

When $\beta = 0.5$, $c_n = 0.5, c_s = 1$, therefore, there always exists an equilibrium where both countries simultaneously achieve both equity and efficiency.

To examine whether this outcome depends on the specific set of parameters that were used, we analysed how results change as β ranges between 0 and 1. Table 3 presents the correspondent results when $D = 1.5, c_n = 0.5, c_s = 1$. As shown in the Table, $\overline{\Delta}_i(0)$ increases and $\overline{\overline{\delta}}_i$ decreases as β falls, but condition (12) still holds true even at extremely low values of β. Thus, the points *P* and *R*, and the points *Q* and *S* that lie along the same side of the square in Figure 1 get closer and closer as β decreases, but they keep their relative positions, so the curves always intersect.[10]

[10] As it can be easily verified, this result holds for any possible values of *D*, c_n and c_s satisying condition (8). Results are available from the authors upon request.

Table 3

β	$\overline{\Delta}_n(0)$	$\overline{\overline{\delta}}_n$	$\overline{\Delta}_s(0)$	$\overline{\overline{\delta}}_s$
.025	0.4835	0.5348	0.4809	0.6297
.2	0.4201	0.5604	0.4166	0.6827
.5	0.35	0.6755	0.3649	0.8823
.7	0.3164	0.8359	0.3517	1

The results presented in Table 3 seem intuitively appealing. A decrease in β, in fact, implies lower aversion to inequality between west and east within each hemisphere. This will induce each government to give less weight to the east in its objective function, namely, to the country that is more damaged by pollution, so that $\overline{\Delta}_i(0)$ increases. A lower aversion to inequality, moreover, reduces $\overline{\overline{\delta}}_i$, indicating that each government finds it more difficult to achieve equity and efficiency.

We then examined several extensions of the model allowing for further differences between west and east in terms of cost and damage parameters. In particular, we repeated the analysis described above for the following scenarios:

1) the whole environmental damage is suffered by the east (the future generation): $D_w = 0$ and $D_e > 0$.
2) technological progress reduces the environmental damages per unit of output in the east: D_w=(1+g) D_e.
3) technological progress reduces the unit cost of production in the east, that is, it increases future productivity: $c_w = (1+g)c_e$.

This analysis aimed at enhancing the gap between west and east within each hemisphere to examine whether under such conditions it is still possible for each government to restore equity and thus ensure both equity and efficiency in its own hemisphere. Simple calculations show that in all these cases condition (12) always holds true, no matter how large the difference between west and east in terms of production costs and damages. In general, we could not find any single counterexample where the two reaction functions do not cross. This seems to suggest that although production may generate considerable damage, the government can use the ability to transfer output between countries to offset the damage and so produce an equitable outcome.[11] Thus, even if North and South do not cooperate, it is still possible for both hemispheres to achieve an allocation that is both efficient and equitable provided each hemisphere aims at these twin goals.

Finally, it is important to point out that this result applies not only to the notion of sustainability adopted in this paper (i.e. all generations should be equally well off), but also to the notion of non-declining utility (NDU) that has been proposed in the literature (e.g. Pearce et al., 1989). If we adopt the latter criterion, an allocation is considered as sustainable as long as the future generations are not worse-off than the current one, namely, as long as $V_{iw} \leq V_{ie}$ (i = N,S) in the present model. Since $V_{iw} - V_{ie}$ is strictly increasing in δ_i, it follows that the set of the pairs (δ_n, δ_s) that ensure sustainability in hemisphere i according to the NDU criterion is given by all points that lie along and below its reaction function. Thus, the set of points where both hemispheres are simultaneously efficient and sustainable according to the NDU criterion is given by the area *QOPT* in Figure 1 (the intersection of the areas *OPS* and *ORQ*), whereas it is given by point *T* alone if we interpret sustainability as constant intergenerational utility.

[11] Observe that if we account for technological progress (cases 2 and 3 above) $\overline{\overline{\delta}}_i$ increases with g, so that each government finds it easier to achieve equity and efficiency.

Section 6 Conclusions

This paper has considered whether it is possible for all countries to independently achieve both efficiency and sustainability (that is, intergenerational equity) in the presence of phenomena like global warming which produce both inter-generational and intra-generational externalities. To get a deeper understanding of this issue we have broken it down into three separate but related questions:

(1) Can each government always achieve both equity and efficiency *whatever* (feasible) resource allocation is chosen by the other?

(2) Can each government always achieve both equity and efficiency whenever the other at least chooses an *efficient* allocation - even though it may not share the same commitment to equity?

(3) Can each government achieve both equity and efficiency when the other government is also trying to achieve *both equity and efficiency*?

As shown in the paper, the answer to (1) - and hence to (2) and (3) - is positive if utility is sufficiently sensitive to commodity transfers. Otherwise there may be allocations chosen by one government which make it impossible for the other to achieve both equity and efficiency. In particular this can arise if the latter concentrates a sufficiently large amount of its production in the future.

In situations where the answer to (1) is negative, the answer to (2) can be positive, when productivity in each country is fairly low. However, when productivity in one country is high, and when, though acting efficiently, it gives a lot of weight to the current generation, then the other government may find it impossible to achieve both equity and efficiency.

Finally, even when the answer to (2) is negative, question (3) can still have a positive answer: if both hemispheres aim at equity and efficiency, they can both succeed to simultaneously achieve these two goals. It follows that the whole world can achieve equity and efficiency at the intersection point of the two government's reaction functions. Surprisingly enough, this result holds even when we account for marked disparities between hemispheres as well as between the current and the future generations. The existence of an allocation that ensures equity and efficiency in both hemispheres, moreover, does not depend on the specific notion of sustainability adopted in the paper and could be extended to the whole intersection area below the reaction functions if we accepted non-declining utility as a notion of sustainability.

Although the present model is still quite simplified, it may provide some interesting insights on a problem that has been mainly overlooked in the literature so far and it is possible to identify several directions for further work. In particular, the model has no trade, and it would be interesting to explore how this affects the conclusions. Moreover, there is no capital accumulation in the model - i.e. no scope for shifting the production functions between east and west. Again it would be interesting to see how the introduction of this additional source of transfers would affect the main conclusions. Finally, it would be important to try to quantify the model to see whether the possible scenarios described in the paper are likely to arise in a version of the model that best approximates the real world.

References

Bhaskar, A (1995) "Global Warming, Inter-generational and Intra-generational Altruism" in A.Glyn (ed.) *The North the South and the Environment: Ecological Constraints and the Global Economy*, Earthscan Publications, London

Hartwick, J.M. (1977) "Intergenerational equity and the investing of rents from exhaustible resources", *American Economic Review*, 67, 972-974.

Pearce, D., Markandya, A., Barbier, E. (1989) "Blueprint for a Green Economy", Earthscan Publications Ltd., London.

Pezzey, J. (1989), "Economic Analysis of Sustainable Growth and Sustainable Development", Working Paper no 15, World bank Environment Department, Washington.

Turner, R.K. (1991) "Environment, Economics and Ethics", Chapter 11 in D. Pearce (ed) *Blueprint 2: Greening the World Economy*, Earthscan Publications, London.

WCED - The World Commission on Environment and Development (1987) "Our Common Future" (The Brundtland Report), Oxford University Press, Oxford.