SOME CRITICAL REMARKS ON THE PRINCIPLE OF PROPORTIONAL TRANSFERS

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Abstract

Fleurbaey and Michel (FM, (2001)) propose a class of income transfers proportional to the benefits of the agents involved. We critically discuss this approach.
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1 Introduction

The Pigou-Dalton Principle of Transfers (hereafter PDP) requires that any mean-preserving progressive transfer lowers the value of a function evaluating inequality of a given income distribution. It has an essential role in the theory of inequality measurement and gave rise to a lot of research. The PDP was already discussed by Muirhead (1903), who proved that if y_k is the income of individual k, $k = 1, ..., n$, if $y_i < y_j$, and if an amount Δ of income is transferred from individual j to i, the income inequality is diminished provided $\Delta \leq y_j - y_i$. It is a basis for inequality comparisons even if it does not allow to compare distributions defined over population of different sizes or with different means. For these and other reasons, the PDP have been criticized by many scholars as e.g. Shorrocks and Foster (1987), Moyes (1994) and Chateauneuf (1996), who point out there is room for alternative inequality criteria. More recently, Fleurbaey and Michel (FM) (2001) have shown that the Pigou-Dalton Principle is a too weak criterion since it serves mainly theoretical purposes and does not take into account that redistribution policies are costly and entail some loss in the aggregate amount of benefits to be shared. In few words, the PDP is silent on the so-called "*problem of the pierced bucket*": namely taxing rich people to transfer income to poors entails a net loss of money. Then, they focus on a subclass of transfers of the Pigou-Dalton type and consider transfers which are proportional to the benefits of the agents involved. They argue in favor of percentage transfers which cost $\delta\%$ of income to the rich and increase the income of the poor by $\delta\%$. Moreover, Fleaurbaey and Michel show that an additive separable (utilitarian) social welfare function $W(x) = \sum_i u(x_i)$ with a strong concavity of u evaluates a proportional transfer as a good for the welfare of the society.

Our aim is to criticize this approach.

2 Notation and Definitions

We consider a given finite population $N = \{1, \ldots, i, \ldots, n\}$ of individuals. A ranked *income distribution*

$$x = (x_1, ..., x_i, ..., x_n)$$

is a finite collection of positive real numbers such that $0 < x_1 \leq \ldots \leq x_i \leq \ldots \leq x_n$. Let x_i be interpreted as the income of individual i in the population N. The set of all ranked income distributions for the population

N is denoted by

$$\aleph = \left\{x \in \mathbb{R}^n_{++} : x_1 \leq \ldots \leq x_i \leq \ldots \leq x_n\right\}$$

An *inequality criterion* $\preceq$ is an ordering on $\aleph$ and can be identified with a subset $\preceq$ of $\aleph \times \aleph$. When $x \preceq y$, we shall say that y is (weakly) more unequal than x.

Definition 1 (Majorization) *A vector y majorizes a vector x, denoted $x \preceq y$, if x can be derived from y by a finite number of Pigou-Dalton transfers (each satisfying the restriction $\Delta \leq y_j - y_i$).*

Let us denote by Υ_{PD} the set of PD transfers, i.e. $(y, x) \in \Upsilon_{PD}$ if and only if x can be derived from y through transfers of Pigou-Dalton type. Suppose function $\varphi : \mathbb{R}^n \to \mathbb{R}$ is to be used as a measure of inequality. Then:

Definition 2 *$x \preceq y$ and x is not a permutation of y implies $\varphi(x) \leq \varphi(y)$.*

A theorem (due essentially to Schur (1923), see Marshall and Olkin (1979) chapter 1 for details), establishes that φ should be strictly Schur-concave.

All this entails that the notion of the Pigou-Dalton Principle by Fleurbaey and Michel (FM, 2001 pg.4) is not well defined.

They argue that the PDP is silent about transfers where the receiver gets less than the donor gives away. It is true. Nevertheless, the critique is not robust. They are asking to the criterion much more content than it has.

Dalton (1920, pg.351) describes the "principle of transfers" as follows:

> *"If there are only two income-receivers and a transfer of incomes takes place from the richer to the poorer, inequality is diminished. [...] The transfer must not be so large as to more than the reverse the relative positions of the two income receivers."*

Then, it necessarily is mute when the attempt to make distribution x "more nearly equal" than distribution y entails a loss of money.

A principle of transfers that should fit well the idea of loss in the transfers of Fleurbaey and Michel is the following:

Definition 3 (Proportional Transfer Principle (PTP)) *For x, $y \in \aleph$, if*

$$x_i = y_i (1 + \Delta) \leq (1 - \Delta) y_j = x_j \tag{1}$$

and $y_k = x_k$ for all $k \neq i$, j and $\Delta \in \left[0, \frac{y_j - y_i}{y_j + y_i}\right]$, then $W(x) > W(y)$.

Such a criterion introduces an upper bound to the loss. In the original definition PTP (FM, (2001) pg.4), Δ is a (generic) positive number, while, of course, inequality 1 does not hold for any positive number.

3 Some critical remarks

Suppose to apply the PTP to a distribution $x = (10, 800, 1000)$, which costs to the richest $\delta = 10\%$ of their income and increases the poorest's income by $\delta = 10\%$. The distribution we obtain is the following $x' = (11, 800, 900)$. The net loss in the aggregate amount of income is huge: for increasing income of less favorite people of one unit we "spent" 99 units of income. It is straightforward to guess that no ethical (i.e. concerned for equity) decision-maker could impose a so-costly redistribution policy.

As the loss increases with the gap between donor and receiver, the PTP seems to suggest that income must be transferred from a quintile (ith) of the distribution to that situated next to it (the i-1th). Such a prescription, for $\delta = 10\%$, embodies $x' = (10, 880, 900)$, whose redistribution only costs 20 units of income. People could agree (specially if they are relatively richer), on such a policy, but an ethical decision maker strictly promotes a redistribution such that the worst-offs receive an increasing in their benefits. Then, according to the constraint on the loss, we should enforce a sequence of PTP from a quantile to the next in order to minimize the cost of any single transfer. Nevertheless, such a sequence of transfers highly increases the total expenditure for redistribution. On the contrary, when transfers are costly, to minimize their number should be a goal of any policy-maker. But, according to PTP, it is strictly better to sharing out the percentage δ of money to be redistributed, such that $\delta = \sum_{i=1}^{n} \delta_i$. As n approaches to infinite and δ is close to be zero $y_i\delta \cong \delta$. In other words, if we want to reduce losses of any PTP, we have to divide it as much as possible and admit transfer smaller and smaller, such that, to the limit, a Proportional Transfer is indistinguishable from a transfer of PD's type.

The normative counterpart of the PTP seems to suggest "*to transfer less to people who have less*", because this "practice" is costly. If we suppose there exist people with no-income or whose income approaches (i.e. can be consider close) to $x = (0, 0, 100)$, we cannot apply PTP anymore or we can only enforce transfers close to be null. This means that the principle of proportional transfers is not a suitable inequality criterion.

Finally, let us denote the class of all PTP by Υ_{PTP}. Every pair $(y, x) \in \Upsilon_{PTP}$ if and only if Definition 3 holds. Consider the following two class of additive welfare functions:

$$W_1(z) = \sum_i U(z_i) \text{ such that } U(x_i) = \begin{cases} A + B\frac{z_i^r}{r}, & r < 1, \ r \neq o \\ A + B\log z_i, & r = 0 \\ \text{where } B > 0 \text{ and } A \text{ are constants} \end{cases}$$

and

$$W_2(z) = \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n} \min(x_i, x_j)$$

It is possible to show that W_1 and W_2 do not satisfy the PTP as

$$W_s(x) \leq W_s(y) \quad \text{whenever} \quad (y, x) \in \Upsilon_{PTP} \text{ with } s = 1, 2.$$

As W_1 and W_2 represent the class of social evaluation functions corresponding to the class of Atkinson-Kolm-Sen and Gini inequality indices and these two families of indices are the most applied in empirical works, the aim of making "discussion about inequality aversion more concrete" (FM, 2001 pg.2) is vain.

4 Conclusion

We have shown as the principle of proportional transfers is far to be a suitable inequality criterion under a positive and normative point of view. It is not a nice "restriction" of the PDP of transfers, which cannot, in our opinion, be criticized on the field of efficiency.

We are not concerned with the growth problem which FM consider in their work.

Finally, we do not review the paper of Aboudi and Thon (2001) as it characterizes the two preorders covered in FM's paper by simply considering a change of variables in the distributions and reproducing the Hardy-Littlewood-Polya (1934) well-known lemma (see Marshall and Olkin (1979) chapter 1, for more details).

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