QUADERNI

Università degli Studi di Siena
DIPARTIMENTO DI ECONOMIA POLITICA

LUIGI M. TOMASINI

Technology Levels and the World Distribution of income: A Model

n. 351 – Maggio 2002

Abstract - In the model it is suggested that gains from trade depend on the stock of wealth possessed by each country and on its level of technology. In a system of N countries which exchange goods and services the results of an interaction is determined by the wealth of each country and by its level of technology. National production and consumption are two other ways for a country to lose or gain wealth. Production depends on each country's technology level and its existing wealth; consumption only on each country existing stock of wealth. We show how wealth depends on the technology level of each country and obtain this last as a function of wealth. This allows us to find the distribution function of wealth, and since in a steady state the wealth of a country does not vary with time, from the distribution of wealth we can compute the distribution of income among countries. Assuming that the technology levels are normally distributed, the distribution of wealth and income among countries which depends on those levels is highly skewed: As a result very few countries will be wealthy and very many poor.

1. A relevant role in determining the pattern of world trade and the world distribution of income is undoubtedly played by technological progress. However , for a number of reasons, the role of technology in international trade has received relatively little attention. Paul Krugman has formulated a highly styled model of international trade in which an innovating North develops and produces **new** products which after a time lag become **old** and can be produced (via technology transfers) also in the South .Trade takes place due to lags in the adoption of new technology by the South.

Assuming labor to be the only factor of production , the demand for new and old goods determines relative wages : higher in the North and lower in the South. This differential reflects the quasi rent that the North earns on its monopoly for the production of new goods . Innovation – an increase in the number of new goods – will increase the wage differential , while technology transfer – a transformation of some new goods into old goods – will lower the wage differential . In his model, developed countries (the North) to maintain their real income must continually innovate. The world distribution of income will henceforth depend on the rates of innovation and technology transfers.

Assuming that for each country the level of technology possessed is a strategic variable, the gains from trade will depend – in the model I suggest – on two variables : the stock of wealth and the level of technology each country has. As a result it will be shown that the world distribution of income will depend on the world distribution of technological progress, favouring those countries which experiment a continuous high level of technological growth.

2. Consider a system of N , $N \geq 2$ countries which exchange goods among them. Each country can be described by two variables : the stock of wealth (w) and the level of technology (t). Countries interact in pairs at equal interval of time and , as all interactions take place, by assumption, in the same interval of time, each country has the same number of exchanges per unit of time. In such a process a country will lose while the other will gain wealth and this according to how much wealth the gainer has and to the difference in technology levels,

Let w_1, w_2 and t_1, t_2 be the stock of wealth and the technology level, respectively , of countries 1 and 2 , before exchange takes place and let α, ($\alpha>0$) be an exchange coefficient in the interaction.

Consider the case where $w_1 < w_2$ and $t_1 \leq t_2$. After exchange takes place, country 1 will have a stock of wealth

(1) $w_1 = w_1 + \alpha\,(t_1 - t_2)\, w_1$

while country 2 will have a stock of wealth

(2) $w_2 = w_2 - \alpha\,(t_1 - t_2)\, w_1$

For values of $t_1 > t_2$,country 1 gains while country 2 loses. The opposite is true for $t_1 < t_2$. This implies that the country that gains in the exchange process is the one that has a higher technology level. From eqs (1) and (2) we see that gains and losses are, by assumption , proportional to the least wealthy country.

3 . Let us assume , for mathematical adhockery , to have t vary from $-\infty$ to $+\infty$ with mean zero . If $t_i < 0$ (i = 1, ..., N), as in (1) and in (2) , then the i.th country has a propensity to lose in the exchange process . The opposite is true for $t_i > 0$

Let $p(t)\,dt$ be the fraction of countries whose t is between t and t + dt, so that

(3) $$\int_{-\infty}^{+\infty} p(t)dt = 1$$

where p(t) is assumed to be normally distributed over the N countries and is symmetric with respect to t = 0 , i.e. ,

(4) $$p(t) = (1/\sqrt{2\pi}) \cdot e^{-t^2/2}$$

which implies that very few countries have a high level of technology and that very few have a zero one .

If p(t) is defined as in (4) and given that

(5) $$\int_{t}^{\infty} t \cdot e^{-t^2/2} dt = 1/2 \int_{t^2}^{\infty} e^{-t^2/2} d(t^2) = e^{-t^2/2}$$

then

(6) $$\int_{t}^{\infty} t.p(t)dt = p(t).$$

It should be pointed out that (1) and (2) make mathematically possible for a country to lose more than the stock of wealth it has. (The possibility of a country indebtedness makes such a case plausible). This requires, however large absolute values of t 's which are , because of (3) very unlikely since $p(t) \to 0$ when $| t | \to \infty$

If all the interactions have the same length γ per unit of time the i.th country with a level t_i of technology interacts $1/\gamma$ other countries . Of those , $(1/\gamma).p(t^t)\, dt^t$ have a level of technology between t^t and $t^t + dt^t$. The i.th country will lose to all countries with $t^t > t$.

Given (1) the total loss per unit of time will be

(7) $$\frac{\alpha}{\gamma} \int_{t}^{\infty} w(t)(t' - t)p(p')dt'$$

while the total gain from countries with $t^t < t$ will be

(8) $$-\frac{\alpha}{\gamma} \int_{-\infty}^{t} w(t')(t' - t)p(pt')dt'$$

Two other ways give rise to the possibility of a country losing or gaining wealth : national production and consumption. The first is defined as a country gain in wealth without taking away from other countries ; the second as a drawing on single countries stock of wealth . Both take place without any interaction with other countries . Production depends on each country's technology level and its existing wealth ; consumption only on each country existing stock of wealth. Both production and consumption are assumed to be continuous processes.

Let the rate of production of wealth by each country be a linear function of w and t . The production p per unit of time is then :

(9) $p = A + B\,w(t) + D\,t,$

where A , B and D are constants .

Similarly , the consumption c per unit of time is

(10) $c = G + E\,w(t)$

where G and E are coefficients .

Three processes are therefore available to each country for gaining or losing wealth : (i) interaction with other countries ; (ii) production ; (iii) consumption.

Combining (7) , (8) , (9) and (10) , we find

(11) $$-\frac{\alpha}{\gamma}\int_{-\infty}^{t} w(t')(t'-t)p(t')dt' - \frac{\alpha}{\gamma}\int_{t}^{\infty} w(t)(t'-t)p(t')dt' + A + (B-E)w(t) + Dt - G = 0$$

which says that in a steady state the algebraic sum of all gains per unit time through exchange plus production minus consumption must equate zero for each country.

4 . Now let us define

(12) $\beta = \dfrac{\alpha}{\gamma}; \quad \delta = \gamma(B-E)/\alpha; \qquad s(t) = \int_t^{\infty} p(t')dt'$

Differentiating (11) with respect to t , given (6) we have :

(13) $\int_{-\infty}^{t} w(t')p(t')dt' + w(t)s(t) + (dw(t)/dt)[t.s(t) - p(t) - \alpha] - D\beta = 0$

Differentiating this expression with respect to t , we obtain

(14) $\alpha + [p(t) - t \cdot s(t)]\ (d^2 w(t)/dt^2) - 2s(t)[dw(t)/dt] = 0$

Define now d w (t) / dt = v (t) and let

(15) $w(t) = \int_0^t v(t)dt + P$

where P is an integration constant .

Rearranging (14) we have now :

(16) $[dv(t)/dt] - [2s(t)/(\alpha + p(t) - t \cdot s(t))] \cdot v(t) = 0$

For the integral of eq. (16) we find

$$(17)\quad v(t) = Q\int_0^t \exp\left\{\int_0^\psi (2s(x)dx\,/(\alpha + p(x) - sx(x))\right\}d\psi$$

where Q is an integration constant .

Introducing this into (15) we have

$$(18)\quad w(t) = P + Q\int_0^t \exp\left\{\int_0^\psi (\psi s(x)dx)\,/(\alpha + (p(x) - xs(x))\right\}d\psi$$

which is a solution of (14) .

To determine P and Q , let w' (t) = dw (t) / dt , then from (18) we note that

P = w (0) ; Q = w' (0) .

Now let Q F (t) indicate the right-hand side of (17) and let F' (t) = dF (t) / dt , then

F (0) = 0 ; F' (0) = 1 .

Therefore (18) can be written

w (t) = w (0) + w' (0) F (t)

which is a solution of (14) .

5. Solving (18) for t we obtain the level of technology as a function of w . Introducing this into (4) , we find the distribution function of wealth : the number of countries whose stock of wealth is between w and w + dw .

As in a steady state the wealth of a country is invariant with respect to time , its income is equal to his consumption (eq. 10) . Hence from the distribution of wealth we can find the distribution of national incomes .

Let n (y) be an income distribution , i.e. n (y) dy gives us the number of countries with income between y and y + dy .Therefore , we have

(19) $$N = \int_0^\infty n(y)dy$$

where N is the total number of countries and

(20) $$Y = \int_0^\infty yn(y)dy$$

where Y is the world total income .

The fraction of all countries that have an income less than or equal to Y is given by

(21) $$(1/N) \int_0^y n/y)dy$$

while

(22) $$(1/Y) \int_0^y n/y)dy$$

represents the fraction of the world total income obtained by this fraction of countries.

As both fractions are functions of y , eliminating it we obtain a relation which gives us the fraction of all countries who receive a given fraction of the world total income .

Therefore , assuming – as we do – that technology levels are normally distributed , the countries distribution of wealth and income , which depends on those levels , is highly skewed . In fact ,as a result of interactions formalized in (1) and (2) very few countries are wealthy and very many poor .

References

1. Krugman, P. (1979) , A Model of Innovation , Technology Transfer and the World Distribution of Income , Journal of Political Economy ,vol. 87.

2 .Landahl , H.D. , (1950) , Note on a Mechanism of the Distribution of Wealth , Bulletin of Mathematical Biophysics ,vol. 12 .

3. Tomasini , L., (1969), A Model of Distribution of Wealth, Cowles Foundation Paper.