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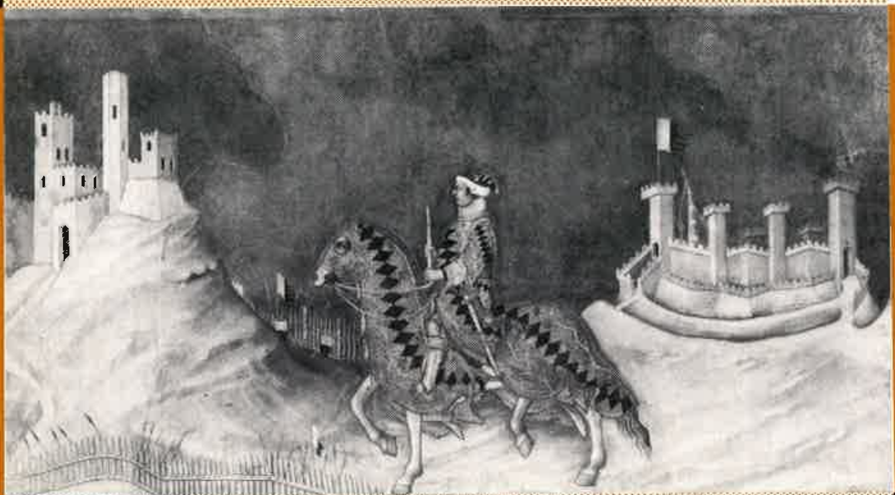
CLASSICAL, NEOCLASSICAL AND KEYNESIAN IN THE LEONTIEF WORLD



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1. Economists are in disarray. They are disagreed with each other in their theoretical models and policy proposals. Some (the classical and neoclassical economists) believe that full employment and full utilization will be realized in labour and capital markets, respectively, as long as the economy works perfectly, while others (the Keynesians) consider that no mechanism is working in the economy for the establishment of full employment in the factor markets. These contrasting perspectives are based on and derived from their antagonistic premises: Say's law for the classical and neoclassical economics and anti- or non-Say's law for the Keynesian economics. Also, on the basis of the wage-fund theory the classical economists advocate that the real-wage rate should be reduced, if Say's law does not work perfectly so that there remains some positive amount of unemployment to be removed by the government's policy. Neoclassical and Keynesian economists, on the other hand, might direct our attention to the fact that labour and capital are complementary, rather than substitutive, so that a decline in the real-wage rate would give rise to a decrease in the demand for the consumption goods but does not create any significant substitution effect between labour and capital in favour of the former. These contradictory views still coexist in the general field of economics. Economists are divided into their respective camps.

However, in the specialised field of the input-output analysis, the situation is completely different. The research workers all seem satisfied by applying the Leontief inverse, $(I - A)^{-1}$, to the final demand vectors. They all seem to be Keynesian; they are homogeneous in reasoning and have no substantial quarrel among them. However, to form a peaceful family is not necessarily a good thing. It is a world which lacks in incentive to think in a different way, so that there is a devastating tendency that analysis becomes mechanical. To avoid this impasse we introduce, in this paper, classical elements into the Leontief framework and show that classical view and policy proposals may be consistent with Leontief economics. Since the classical wage-fund theoretic approach is very suitable for the analysis of agriculture, one may conclude from this paper that input-output models for less developed countries should be the one which is proposed in the final part of the paper, that is the one of the 'classical' Leontief family rather than the present conventional one of the Leontief-Keynes type.

2. Let us consider an economy consisting of two classes (working and capitalist classes) and two industries (consumption goods and capital goods industries). Price, quantity and production coefficients referring to the consumption goods industry are represented by Greek letters: π is the price of the consumption goods, ξ the output of the consumption goods, χ the capital coefficient and λ the labour-input coefficient of the consumption goods industry. p, x, k, ι are those for the capital goods industry, respectively. We also assume that one unit of capital goods provides one unit of capital services and represent the price of the latter by q.

In the following price variables are all expressed in terms of the consumption goods (say, wheat); π is, therefore, always unity; w expresses the real wage rate and p the amount of wheat by which one can buy a unit of the capital goods. Since the 'classical school' assumes that it takes one period (say, a year) to produce the consumption good (wheat), the capitalists of that industry must have enough capital that enables them to by χ units of capital goods (machine) and employ λ units of labour, per unit of output, one year before output is actually produced. On the other hand, in the capital goods industry production is made instantaneously, so that there is no need for paying wages and buying capital goods in advance. Assuming that no capital goods depreciate in both industries, the total cost (wages and interest) per output amounts to:

$\lambda w + \chi pr$	for consumption goods,
$\iota w + kpr$	for capital goods.

where r denotes the rate of profit (or interest). These should be equal to the prices of the respective outputs. Taking account of the fact that the price of

the consumption goods is discounted because of the production lag, we obtain

(1) $\lambda w + \chi pr = \pi/(1 + r) = 1/(1 + r)$

(2) $\iota w + kpr = p$

respectively. In the case of instantaneous production being assumed for the consumption goods industry as the neoclassical and Keynesian economists do, the price-cost equation (1) is of course replaced by

(1') $\lambda w + \chi pr = 1.$

In equations (1), or (1'), and (2), we may write pr as q; the profits or the price to be paid for capital services used.

The demand-supply (or input-output) equation for the consumption goods is written in the following form. First, in the case of the classical school which is based on the wage fund theory, let $\bar{\xi}$ be the output of the consumption goods from the activity in the previous period. In the current period the two industries start production at the levels, ξ and x, respectively. Total wages amount to $w(\lambda\xi + \iota x)$. Assuming that workers do not save and taking the consumption goods as the numeraire ($\pi = 1$), the market for the consumption goods is cleared if and only if

(3) $\bar{\xi} = w(\lambda\xi + \iota x).$

In the case of no production lag, as is assumed by neoclassical and Keynesian economists, this is reduced to

(3') $\xi = w(\lambda\xi + \iota x).$

As for the saving-investment equation, we have already assumed in (3) or (3') that capitalists do not consume and workers do not save, so that the total savings (or the total capitalist income) amount to

$$r\left[p(\chi\xi + kx) + w\lambda\xi + pr\chi\xi\right].$$

This is equal to investment which is $\Delta\xi + px$ in the case of the classical school (where $\Delta\xi$ represents the increment of the wage funds) and px in the neoclassical case of no production lag. Hence the saving-investment equation is written in the form:

(4) $\Delta\xi + px = r\left[p(\chi\xi + kx) + w\lambda\xi + pr\chi\xi\right]$

or

(4') $x = r(\chi\xi + kx)$.

In these cases it must be noted that Say's law prevails in Keynes' sense, because there is no independent investment function. Where investment decisions are made, say by entrepreneurs, independently of savings, investment x should be fixed at a particular level i determined by entrepreneurs. Therefore, we must have, in place of (4'),

(4'') $i = x = r(\chi\xi + kx)$.

The final set of equations are the full employment equation for labour and the full utilization equation of capital, both of which can be shown to hold when Say's law holds:

(5) $\lambda\xi + \iota x = L$

(6) $\chi\xi + kx = K$

where L stands for the total working population and K for the total stock of

capital. Where Say's law does not prevail, the full-employment-full-utilization equilibrium would not necessarily be realized. In particular, if i is set too low, we would have an underemployment-undercapacity production. Hence

(5') $\lambda\xi + \iota x \leq L$

(6') $\chi\xi + kx \leq K$.

Thus we have the following three systems. The first is the classical system based on the wage-fund theory and Say's law:

(1)	$1/(1+r) = \lambda w + \chi pr$	the price-cost equations
(2)	$p = \iota w + kpr$	
(3)	$\bar{\xi} = w(\lambda\xi + \iota x)$,	the wage-fund theory,
(4)	$\Delta\xi + px = r\left[p(\chi\bar{\xi} + kx) + w\lambda\bar{\xi} + pr\chi\xi\right]$	Say's law
(5)	$\lambda\xi + \iota x = L$	the full employment,
(6)	$\chi\xi + kx = K$,	the full utilization

The second is the neoclassical system which assumes no production lag and Say's law:

(1')	$1 = \lambda w + \chi pr$	the price-cost equations,
(2)	$p = \iota w + kpr$	
(3')	$\xi = w(\lambda\xi + \iota x)$,	the multiplier theory
(4')	$x = r(\chi\xi + kx)$,	Say's law,
(5)	$\lambda\xi + \iota x = L$,	the full employment,
(6)	$\chi\xi + kx = K$,	the full utilization.

It is noted that the demand-supply equation for the consumption goods (3') is referred to as the multiplier theory, because it gives rise to an increase in the output of the consumption goods ξ wherever the output of the capital goods is increased. This result is diagonally in contrast with the one obtained under the wage-fund theory (3), that is, that ξ decreases when x increases. Finally, by negating both premises of the classical school, i.e. the wage-fund theory and Say's law, we obtain the Keyneslan system:

$$\left.\begin{array}{lll}(1') & 1 = \lambda w + \chi pr \\ (2) & p = \iota w + kpr\end{array}\right\} \quad \text{the price-cost equations,}$$

$$(3') \quad \xi = w(\lambda\xi + \iota x), \qquad \text{the multiplier theory,}$$

$$(4'') \quad i = x = r(\chi\xi + kx), \qquad \text{the independent investment function,}$$

$$(5') \quad \lambda\xi + \iota x \leq L, \qquad \text{underemployment,}$$

$$(6') \quad \chi\xi + kx \leq K, \qquad \text{undercapacity.}$$

Strict inequality would prevail in either (5') or (6'), or both, if the level of investment i is sufficiently low.

3. Assuming that the price-cost equations - (1) and (2), or (1') and (2) - and the market-clearing equations - (3) and (4), or (3') and (4'), or (3') and (4'') - always hold true, let us concentrate our attention upon the factor markets for labour and capital. First, under the classical wage-fund theory, the excess demand for labour can be written, in view of (3), as

$$(7) \quad ED_L = (\lambda\xi + \iota x) - L = (\bar{w} - w)L/w$$

where $\bar{w} = \xi/L$, which obviously stands for the availability of the consumption goods per worker. If the actual wage rate w equals $\bar{w}$, the excess demand for labour ED_L is zero; so $\bar{w}$ may be referred to the full employment level of the wage rate. If w is lower than it, there is an excess demand for labour, but a wage rate above $\bar{w}$ gives rise to an excess supply of labour. It is thus seen from this point of view that unemployment is a consequence of a too high wage rate, whilst a shortage of labour results from a too low wage rate. On a plane measu-

ring w along the vertical axis and x/K along the horizontal axis, the curve of the excess demand for labour being zero, $ED_L = 0$, is expressed, in view of (7), as a straight horizontal line through $\bar{w}$ i.e., nn' in Figure 1. The upper (lower) half of the plane divided by the line is the region where excess supply of (or demand for) labour prevails.

The excess demand for capital is zero if and only if

$$(8) \quad ED_K = [\chi(\xi/K) + k\,(x/K) - 1]\,K$$

vanishes. In view of (3) as well as the definition of $\bar{w}$ we have

$$\xi/K = (1/\lambda)\,(\bar{w}/w)(L/K) - \iota(x/K)\,.$$

Substituting from this into (8) we obtain

$$ED_K = 1/\lambda[(\bar{w}/w)(L/K)\chi + (k\lambda - \chi\iota)\,(x/K) - 1]\,K$$

Assuming that the consumption goods industry is more capital intensive than the capital goods industry (i.e. $\chi/\lambda > k/\iota$, so that the part in the parentheses on the right-hand side of the above expression is negative), we find that the equilibrium curve of capital utilization, $ED_K = 0$, traces out a downward sloping curve which starts from the wage rate

$$w = \bar{w}\,(\chi/\lambda)\,(\iota/K)$$

at $x/K = 0$, and eventually approaches $w = 0$ when x/K tends to infinity. Keeping x/K constant at a certain level and decreasing (or increasing) w to a level which is lower (or higher) than the equilibrium level on the mm' curve we obviously have an excess demand (or supply) of capital.

Let us now denote the region where excess supply prevails for both labour and capital by A, the region where excess supply prevails for labour and excess demand for capital by B, the region where we have excess demand for both labour and capital by C, and finally the region where we have excess demand for labour and excess supply for capital by D. These are divided by the two curves mm' and nn' as Figure 1 illustrates. It is noted that A, B, C, D are located anticlockwise around the intersection W of the mm' and nn' curves. W is obviously a general equilibrium point where equilibrium is established for both labour and capital.

Let us now turn to the neoclassical and the Keynesian regime, which are identical except that in the latter investment is not flexible but regulated according to a certain independently decided investment function. In these regimes the output of the consumption goods is proportional to the output of the capital goods because of the multiplier theory (3'); substituting this relationship into (5) or (5'), we have

$$ED_L = \lambda \; x w\iota/(1-w\lambda) + \iota x - L$$

which is simplified in the form:

$$(9) \qquad ED_L = x\iota/(1-w\lambda) - L.$$

It is immediately found that the real wage rate w should be fixed according to the formula,

$$w = (1/\lambda)(1 - (\iota/L) K (x/K),$$

in order for labour to be fully employed, i.e., in order to have $ED_L = 0$. Therefore we have a straight full-employment curve which starts at $w = 1/\lambda$ when x/K is set at zero and ends at $w = 0$ when x/K takes on the value, $(1/\iota)(L/K)$. Keeping x constant and increasing (or decreasing) w, it is seen from (9) that we have

an excess demand for (or supply of) labour, so that above (or below) the full-employment line there is the region of excess demand for (or supply of) labour.

To obtain the full-utilization-of-capital curve we substitute the multiplier formula into (6) or (6'): then

$$(10) \qquad ED_K = [\chi \; w\iota/(1-w\lambda) + k] \; x - K$$

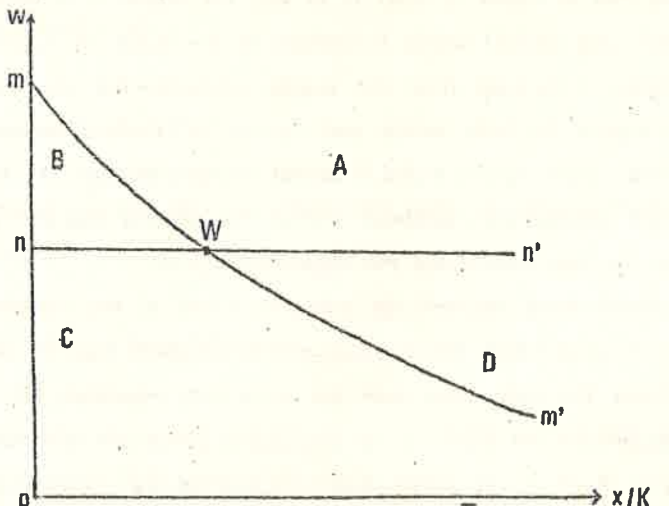
from which we obtain, where capital goods are fully utilized,

$$w = [1 - kx/\bar{K}] \,/\, [\lambda + (\chi\iota - k\lambda) \; x/\bar{K}]$$

This implies that w is zero when x/K is $1/k$ and $w = 1/\lambda$ at $x/K = 0$. (Note that the part within the parentheses in the denominator of the expression is positive). This, together with the full-employment-of-labour line obtained in the above, produces Figure 2. As is in Figure 1 the entire plane is divided into four sections: A, B, C, D. Their arrangement, however, is in complete opposition to the one we had in Figure 1. That is to say, the region C of excess demand for both labour and capital which is nearest to the origin in Figure 1 is now located in Figure 2 farthest from the origin; similarly, the position of region A (of excess supply for both labour and capital) is reversed between the two figures. However, since regions B and D do not change positions, the four regions, A to D, locate themselves clockwise around the general equilibrium point W in Figure 2 for the neoclassical and the Keynesian regime.

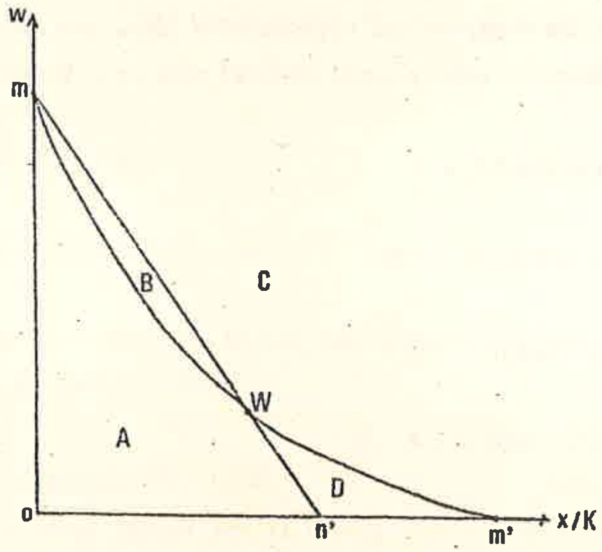
Under these three regimes the economy works in the following manner. First we are concerned with the classical and neoclassical regimes which satisfy Say's law. Given the real wage rate the price-cost equations (1) and (2) (or (1') and 2)) determine the price p of the capital goods and the rate of profits r. Then, with given x, the equilibrium output ξ of the consumption goods is determined by (3) or (3'). Once (1), (2), (3) (or (1'), (2), (3')) are satisfied, the saving-investment equation (4) or (4') is shown to hold identically. This is because of the lack of independent investment function that is imposed by Say's law. Therefore, in the factor markets for labour and capital we have two variables, w and x - note that ξ is a function of w and x - which adjust themselves such that an equilibrium is established in each factor market. There is no obstacle to realizing the full-employment-full-utilization equilibrium, (5) and (6); the temporary equilibrium W will be established.

Where we have an independent investment function i, the saving-investment equation (4'') is no longer an identity; x must be adjusted to satisfy it. Hence, in two factor markets we only have a single variable w, and the regime is a system of overdeterminancy. Either one of the two inequalities, (5') and (6'), or both of them, will not be fulfilled with equality. In particular, if x is fixed too low, both (5') and (6') will hold with strict inequality. Thus, in the Keynesian regime the Walrasian temporary equilibrium is generally impossible regardless the assumption we make concerning the flexibleness or rigidness of the wage rate w. It was Keynes who first emphasized the significance of the role played by Say's law in the theory of unemployment.



$$m = \bar{w} \cdot \frac{L}{\lambda} \Big/ \frac{K}{\chi}, \qquad n = \bar{w} \frac{\bar{\xi}}{L}$$

Fig. 1



$$m = 1/\chi, \quad m' = 1/k, \quad n' = \frac{1}{\iota} \cdot \frac{L}{K}$$

Fig. 2

4. In the area of dynamics the classical school is concerned with establishing a proposition that the economy eventually approaches a state of stationary equilibrium and the neoclassical school shows that there is a stabilizing tendency towards a state of proportional growth of labour and capital in balance. In contrast Keynesians, such as Harrod in particular, were concerned with characterizing the dynamic process of the economy as a process of divergence from the state of full-employment-full-utilization equilibrium (i.e. Harrod's theorem of centrifugal force). In this section we shall examine these three propositions.

To show the proposition of the classical school we first assume that there is no change in technology, so that χ, λ, k, ι are kept constant. We also assume, for the sake of simplicity, that the rate of growth of the labour force, denoted by ρ and referred to as the natural rate of growth, is independent of the real wage w and constant.

Since Say's law holds, the full employment of labour and the full utilization of capital are realized at every point of time. We have from (5) and (6)

(11) $\lambda(\xi/L) + \iota(x/K)(K/L) = 1$

(12) $\chi(\xi/L) + k(x/K)(K/L) = K/L.$

Eliminating ξ/L we obtain

(13) $g = [1/(\chi\iota - k\lambda)]\,[\chi L/K - \lambda]$

where g represents the rate of growth of the capital stock, x/K. Obviously, g traces out an upwards-sloping straight line: g is zero at $L/K = \lambda/\chi$ and $1/k$ at $L/K = \iota/k$. (See Figure 3). The line crosses the natural-rate-of-growth line at $(L/K)^*$ which gives the long-run labour-capital ratio. Where the actual labour-capital ratio $(L/K)_0$ is lower than the equilibrium ratio, L grows at a higher rate than K, so that L/K increases; that is to say, the labour capital ratio in the next period $(L/K)_1$ is greater than $(L/K)_0$. By the same reason L/K decreases if $(L/K)_0$ is greater than $(L/K)^*$; that is, we have $(L/K)_1 < (L/K)_0$. Thus we obtain the $(L/K)_1(L/K)'_1$ curve in Figure 4 and the point at which it crosses the 45° line gives the long-run equilibrium labour-capital ratio. We can show that the long-run equilibrium point is stable if the $(L/K)_1(L/K)'_1$ curve gently slants downwards from left to right. It is obvious that both labour and capital grow proportionately at the rate ϱ at the long-run equilibrium point.

In the case of the natural rate of growth depending on the real wage rate the stability argument is more complicated, and the stability condition would be more restrictive. In the following we assume that the long-run equilibrium is stable. Since there is one year of production lag for the consumption goods, ξ is only available for consumption at the beginning of the next period, which is shared by $L(1+\varrho)$ workers. The full-employment wage rate in the next period

will be given as $w = \xi/(L(1+\varrho))$ so that the previous equations (11) and (12) can be written as:

(11') $\lambda w(1+\varrho) + \iota(x/K)(K/L) = 1$

(12') $\chi w(1+\varrho) + k(x/K)(K/L) = K/L.$

Since $x/K = g = \varrho$ in the state of long-run equilibrium of balanced growth, we have from (11') and (12')

(14) $w = (1 - kg)/[\lambda + (\chi\iota - k\lambda)g]\,(1+g)$

which is called the long-run consumption-investment frontier because it gives a relationship between consumption per man (w) and investment per capital ($x/K = g$) in the state of long-run equilibrium. It states that $w = 0$ at $g = 1/k$ and $w = 1/\lambda$ at $g = 0$. It also states that consumption per man must decrease when investment per capital increases. This frontier has an intersection with the natural-rate-of-growth curve $\varrho\varrho'$, as is seen in Figure 5. The rate of growth of the working population will be negative if the wage rate is very low; it is zero at the subsistence level of wages and becomes positive for higher wages. Where these two curves intersect each other the natural rate of growth of the work force is equalized with the rate of growth of the capital stock, so that w^* and g^* in Figure 5 give the long-run equilibrium wage rate and growth rate, respectively.

We have so far assumed that technological coefficients χ, λ, k, ι are all constant. This would, however, conflict with the fact that industries, in particular the consumption goods industry, of which agriculture is a main component, will experience diminishing returns sooner or later as the economy continues to grow. In these less favourable circumstances more labour will be needed to produce one unit of the consumption goods, so that λ will be increased. In the following we examine the case of only λ being increased continuously, all the other coefficients being kept constant.

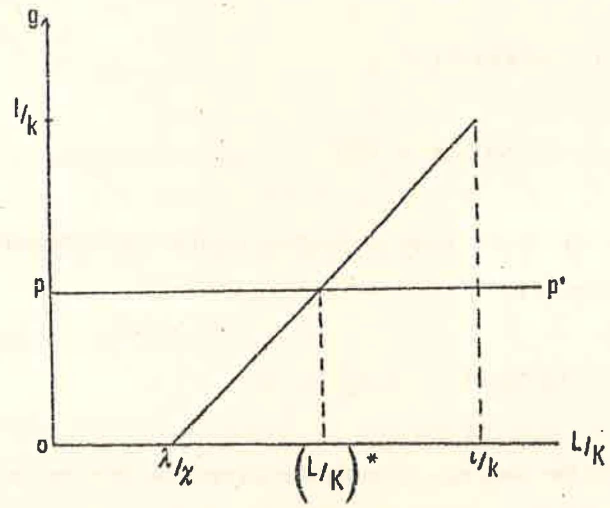


Fig. 3

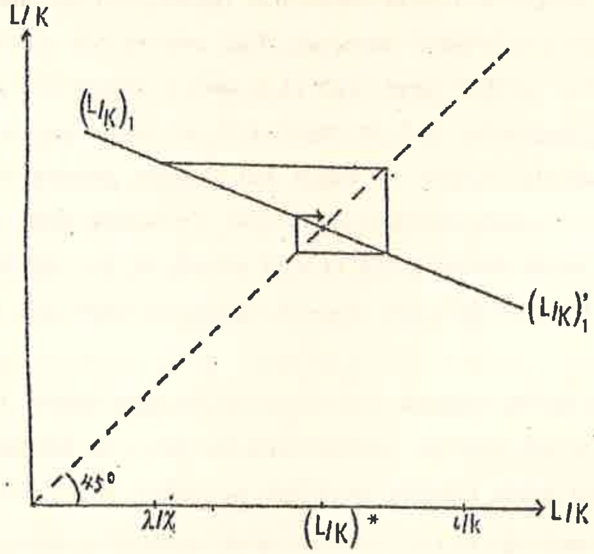


Fig. 4

It can be shown that an increase in λ produces a leftwards shift of the consumption-investment frontier. This is obvious from (14) because, where λ increases, g has to decrease in order for it to fulfil (14) with the same value of w. Then, new long-run equilibrium real wage rate and balanced growth rate will be determinated at smaller values, w** and g**, than before. This process of diminishing wage and growth rates will stop when $1/\lambda$ reaches the subsistence rate w_s. (See Figure 6).

Samuelson has once criticized Marx in saying that his law of wages (which asserts the wage rate to fall as the economy grows) is incompatible with his law of falling rate of profits. It is easy, however, to falsify this criticism. We can derive from the price-cost equations (1) and (2) the factor price frontier in the following form:

(15) $$w = (1 - kr)/[\lambda + (\chi_1 - k\lambda)r] \; (1 + r).$$

Comparing with (14) we find that the factor-price frontier is identifical with the consumption-investment frontier. Therefore, the long-run rate of growth equals the long-run equilibrium rate of interest. We also find that the factor-price frontier shifts leftwards as λ increases. When $1/\lambda$ reaches the subsistence wage rate w_s, the economy is in the stationary state, and ϱ , g, r are all zero. It is thus seen that in the process of diminishing returns prevailing in the consumption goods industry both real wage and profit rates decline together until the latter finally becomes zero.

The neoclassical economy works through time in the following way. Figure 2 is reproduced in Figure 7 below, with addition of the natural rate of growth curve, $\varrho\varrho'$, newly drawn. Note that points m and m' are fixed, but n' moves leftwards or rightwards according as L/K decreases or increases. Because Say's law prevails in the neoclassical economy, the temporary equilibrium W is realized and the capital stock will grow at the rate g^o while the labour force at ϱ^o. As $g^o > \varrho^o$, the labour-capital ratio L/K will decline; so n' moves leftwards and the point of temporary equilibrium (that is the intersection of the full-employment line mn' and the full-utilization curve mm') climbs up the curve. It will finally reach the long-run equilibrium point W* when L/K takes on a particular value (L/K)* such that the line mn' corresponding to this value gets through the point. At W*, the rate of growth of the capital stock equals the rate of growth of the labour force, so that a proportional growth of capital and labour will be obtained. Then diminishing returns will start to appear in the consumption goods industry; λ will increase and point m will be pushed down. The long-run equilibrium point W* slides down along the natural rate curve $\rho\rho'$. Finally, m and W* reach w_s simultaneously, and everything stops. A stationary state will be established. Workers earn the subsistence wages w_s and capitalists earns zero profits.

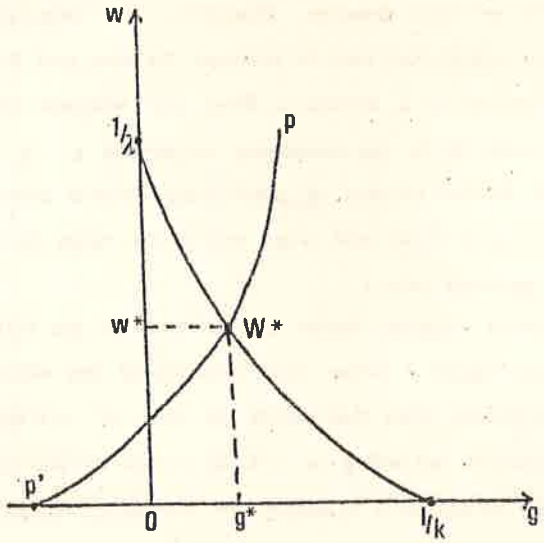


Fig. 5

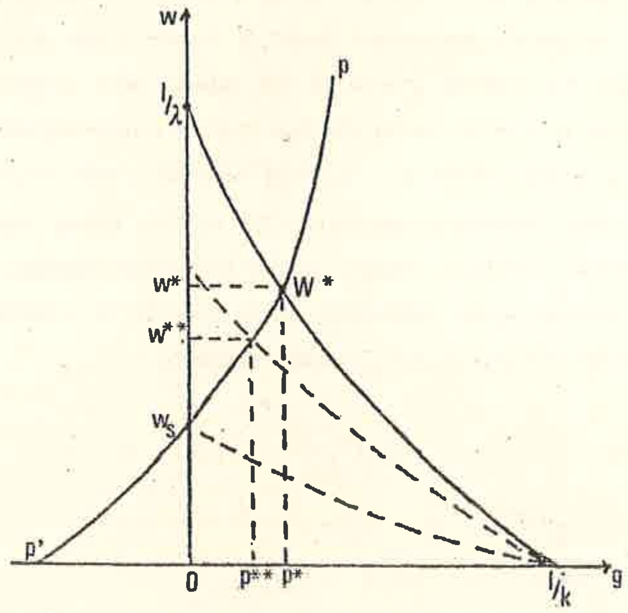
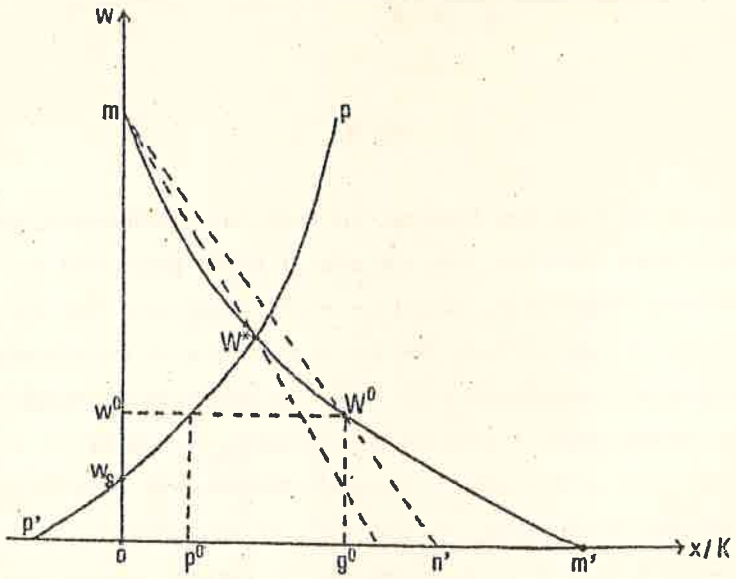


Fig. 6

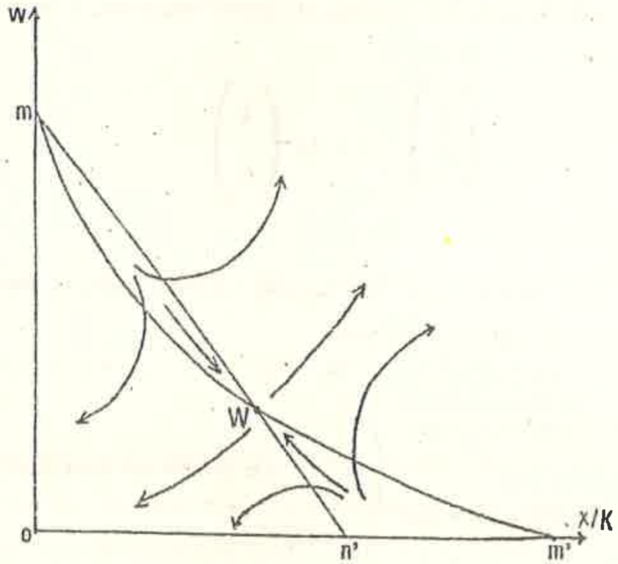
Finally, in the Keynesian economy, the temporary equilibrium is not necessarily realized since Say's law does not hold. It is not guaranteed that the investment decided independently should be at the level such that the existing stock of capital is fully utilized. The actual position a of the economy would be off the temporary equilibrium point W. If a is located in the region A, there would be an excess supply in both markets for labour and capital. If we assume that the wage rate w decreases or increases according as there is an excess supply of or demand for labour, we find that w decreases if a is in A or B and increases if it is in C or D. Investment i may be adjusted such that i is decreased or increased if there is an excess supply of or demand for capital, so that i/K decreases if a is in A or D and increases if it is in B or C. Thus we have a Keynesian phase diagram as Figure 8 illustrates.

It can be easily seen that W is a saddle point. There are two (only two) streams which bring a to W, but at all other points centrifugal forces work. Moreover, the temporary equilibrium point W moves unless the natural rate of growth equals the rate of growth of the capital stock at the actual point a. Therefore, even if a is on one of the two streams converging upon W at some point of time, a is not, at the next point of time, on a new stream which converges on the new temporary equilibrium. It is thus almost impossible that the actual economy eventually settles at the long-run equilibrium, a particular temporary equilibrium which reproduces itself once it is established. This is Harrod's knife-edge property of the Keynesian economy.



$m = 1/\lambda, \quad m' = l/k, \quad n' = \frac{l}{\iota}\frac{L}{K}$

Fig. 7



$m = l/\lambda, \quad m' = l/k, \quad n' = \frac{l}{\iota}\frac{L}{K}$

Fig. 8

5. We have so far neglected Leontief's input-output relationships which we now introduce into the three systems: classical, neoclassical and Keynesian. Let A be the input-coefficient matrix,

$$A = \begin{pmatrix} \alpha & a \\ \alpha' & a' \end{pmatrix}$$

B be its Leontief inverse

$$B := (I - A)^{-1} = \begin{pmatrix} \beta & \beta' \\ b & b' \end{pmatrix}$$

Net outputs of the consumption and capital goods industries $\hat{\xi}$ and $\hat{x}$ are defined as

$$\begin{pmatrix}\hat{\xi} \\ \hat{x}\end{pmatrix} = (I - A)\begin{pmatrix}\xi \\ x\end{pmatrix}$$

Let us first examine the neoclassical Leontief system, which is expressed in terms of the following six equations:

(16)	$1 = \alpha + \alpha' p + \lambda w + \chi pr$	the price-cost equations,
(17)	$p = a + a' p + \iota w + kpr$	
(18)	$\xi = \alpha\xi + ax + w(\lambda\xi + \iota x)$	the multiplier theory,
(19)	$x = \alpha'\xi + a'x + r(\chi\xi + kx)$	Say's law,
(20)	$\lambda\xi + \iota x = L$	the full employment,
(21)	$\chi\xi + kx = K$	the full utilization.

In term of net outputs these are written as

(16') $1 = \dot{\lambda} w + \dot{\chi} pr$

(17') $p = \dot{\iota} w + \dot{k} pr$

(18') $\dot{\xi} = w(\dot{\lambda}\dot{\xi} + \dot{\iota}\dot{x})$

(19') $\dot{x} = r(\dot{\chi}\dot{\xi} + \dot{k}\dot{x})$

(20') $\dot{\lambda}\dot{\xi} + \dot{\iota}\dot{x} = L$

(21') $\dot{\chi}\dot{\xi} + \dot{k}\dot{x} = K$

where

(22) $\dot{\lambda} = \lambda\beta + \iota b \quad , \quad \dot{\iota} = \lambda\beta' + \iota b'$

(23) $\dot{\chi} = \chi\beta + \chi b \quad , \quad \dot{k} = \chi\beta' + kb'$.

Comparing these with the equations of the original neoclassical system (1'), (2), (3'), (4'), (5), (6), we find that their structures are identical, except that variables are net outputs $\dot{\xi}$, $\dot{x}$ and coefficients are defined as in (22) and (23). Hence we obtain a figure similar to Figure 2 and we find that the temporary equilibrium W is established by virtue of Say's law. It is also obvious that other neoclassical properties, such as the stability of the long-run growth equilibrium and the tendency towards the stationary equilibrium, provided that diminishing returns prevails in the consumption goods industry, are equally obtained.

In a similar way, we can show that the Keynesian Leontief system can be written in net terms as

(16'')	$1 = \dot{\lambda} w + \dot{\chi} pr$	the price-cost equations,
(17'')	$p = \dot{\iota} w + \dot{k} pr$	
(18'')	$\dot{\xi} = w(\dot{\lambda}\dot{\xi} + \dot{\iota}\dot{x})$	the multiplier theory,
(19'')	$\dot{x} = i = r(\dot{\chi}\dot{\xi} + \dot{k}\dot{x})$	the saving-investment equation,
(20'')	$\dot{\lambda}\dot{\xi} + \dot{\iota}\dot{x} \leqq L$	underemployment,

$$(21'') \qquad \hat{\chi}\hat{\xi} + \hat{k}\hat{x} \leqq K \qquad \text{undercapacity.}$$

This system is structurally identical with the previous Keynesian system, (1'), (2), (3'), (4''), (5'), (6'). All we have seen concerning the Keynesian system may *mutatis mutandis* obtain for this Keynesian Leontief system. In particular, we observe Harrod's knife-edge property.

In the case of the 'classical' Leontief system, however, the equation of the wage-fund theory,

$$\bar{\xi} = \alpha\xi + ax + w(\lambda\xi + \iota x)$$

cannot be transformed into the original 'classical' form (3), or

$$\bar{\xi} = w(\hat{\lambda}\hat{\xi} + \hat{\iota}\hat{x})$$

in net terms. We still have

$$\bar{\xi} = \hat{\alpha}\hat{\xi} + \hat{a}\hat{x} + w(\hat{\lambda}\hat{\xi} + \hat{\iota}\hat{x}),$$

where $\hat{\alpha} = \alpha\beta + ab$ and $\hat{a} = \alpha\beta' + ab'$. Therefore, the full-employment line 'nn' in Figure 1 is no longer horizontal but an oblique line connecting

$$n = (\hat{\xi}/L) - (\hat{\alpha}/\hat{\lambda}) \quad \text{on the ordinate with}$$

$$n' = h\,(1/\iota)(L/K)$$

on the abscissa, where

$$h = [(\bar{\xi}/L) - (\hat{\alpha}/\hat{\lambda}) / (\hat{a}/\hat{\iota}) - (\hat{\alpha}/\hat{\lambda})].$$

The full-utilization curve is also slightly modified; it takes on the value

$$m = (\bar{\xi}/\hat{\lambda})/(K/\hat{\chi}) - (\alpha/\hat{\lambda}), \quad \text{on the ordinate and another value,}$$

$$m' = [(\hat{\chi}/\hat{\alpha})(\bar{\xi}/K) - 1] / [(\hat{\chi}/\hat{\alpha}) - (\hat{k}/\hat{a})] \quad \hat{a}$$

on the abscissa. In spite of these modifications, the full-employment line and the full-utilization curve divide the plane into four regions, A, B, C, D. Region A where excess supply prevails in both labour and capital markets is farthest from the origin and C where excess demand prevails is the two markets is nearest. Likewise in the classical economy, regions A, B, C, D are located counter-clockwise around the temporary equilibrium point in the classical Leontief economy. This economy works in a similar way as the original classical economy. Therefore, there should be many substantial disagreements and inconsistencies between classical and Keynesian Leontief economists.

It is finally noted that although most of the Leontief economists are Keynesians, the classical group is not entirely empty; in fact, as von Neumann did, we can easily introduce production lags into Leontief models. Then, the classical Leontief economists would become more visible in the Leontief camp; they would start to quarrel with Keynesian Leontiefs. This, however, is a welcome phenomenon, though some might consider it destructive because it would make the Leontief analysis much deeper, as economics, and more appropriate to developing, agricultural countries.

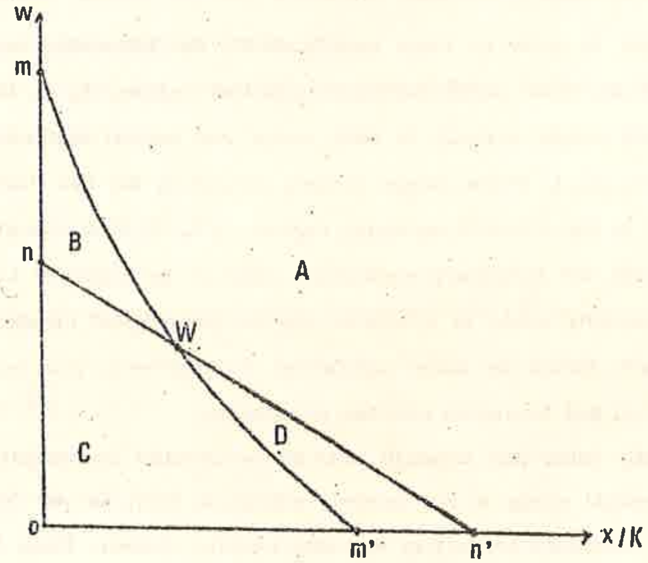


Fig. 9

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