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The Competitive Outcome of the Entry-Capacity and Pricing Game in a Large Market

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Abstract - We model long-run price competition as a two-stage entry-capacity and pricing game among many potential entrants. Each solution of the game is found to reproduce a long-run competitive equilibrium provided the latter is characterized by a sufficiently large market.This result extends to the long run the oligopolistic foundation of perfect competition provided for the short run by Allen and Hellwig (1986a, 1986b) and Vives (1986a).

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1 Introduction[1]

Since Bertrand critique to Cournot the relation between perfectly competitive equilibria and equilibria with price-setting sellers of an homogeneous commodity has attracted considerable interest among economists. In contrast with the popular view that "two is enough for competition", the consideration of capacity constraints has clarified that the outcome of price competition depends on the degree of industry concentration: short-run analyses of a single market have shown that it is in a large economy - where the size of each firm is sufficiently small relative to the whole market - that the equilibrium of the pricing game tends to reproduce the competitive outcome (see Allen and Hellwig, 1986a, 1986b, and Vives, 1986a). The present paper pushes this oligopolistic foundation of perfect competition a step further. We provide a stylized model of price competition under free entry, which extends the above result to the long run where the number and the capacity of active firms are endogenously determined.

Some remarks are in order to clarify how our analysis departs from recent models also addressing the entry and pricing decisions of firms in a single market. Models have been proposed with potential entrants facing a simultaneous entry-pricing decision (Sharkey and Sibley, 1993; Marquez, 1997) as well as two-stage models of perfect information where entry takes place first and price setting subsequently (Elberfeld and Wolfstetter, 1999, Thomas, 2002). Both types of models have focussed on the impact of a fixed (and sunk) cost of entry while neglecting capacity constraints - active firms are taken to produce any quantity that is demanded at constant marginal costs. Leaving aside the case where production does not require any capital equipment, the notion of no capacity constraints can be rationalized if the firms are "in the long run" when setting prices. Assume long-run constant returns and short-run constant returns up to full capacity utilization. If the firms first choose prices and then, based on resulting buyers' decisions, capacities, lowest-priced firms would clearly undertake the capacity decisions enabling them to meet the entire demand. Consequently, higher-priced firms would have no residual demand left, as in classic Bertrand models.

Unlike the aforementioned works, the present paper takes the firms as capacity constrained when setting prices while neglecting any fixed cost of

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entry. We analyse a dynamic game of perfect information where potential entrants make simultaneous entry-capacity decisions in the first stage and active firms set prices in the second stage. A distinctive feature of the model is capital indivisibility, which makes the firm capacity choice set a discrete one.

Section 2 analyzes a two-stage entry-capacity and quantity game among firms acting as price takers at the stage of quantity decisions. The "long-run competitive equilibria" thus determined will serve as a benchmark to evaluate the outcome of the entry-capacity and pricing game. Then Section 3 analyses the pricing subgame and Section 4 solves the entire entry-capacity and pricing game. Each solution of the entire game is found to yield a long-run competitive equilibrium so long as, at the latter, the firm size is sufficiently small relative to the whole market.

2 Long-run competitive equilibria

We consider a single market of an homogeneous product. Let $D(p)$ and $P(Q) \equiv D^{-1}(Q)$ denote market demand and the inverse demand, respectively, p being the price and Q the total quantity. For the sake of simplicity, a linear demand curve will be assumed throughout, i.e., $P(Q) \equiv a - bQ$ (where $a, b > 0$). $\mathcal{Z} = \{1, ..., i, ..., z\} \subset \mathcal{I}$ denotes the set of potential entrants, where $\mathcal{I} \equiv \{1, 2, ...\}$ is the set of positive integers.

A single technique, employing capital goods and other inputs, is assumed to be available at the time of investment decisions. Though extreme in many cases, this assumption permits to capture capacity indivisibility, a fact arising from the indivisibility of capital equipment and the absence of a continuum of techniques. The decision problem faced by any $i \in \mathcal{Z}$ in the first stage is whether or not to enter and, if entering, the size of capital equipment, which in turn determines the capacity $\overline{q}_i$ at its disposal in the second stage. To incorporate capital indivisibility, the capacity choice set is taken to be $\mathcal{I}$, where we have normalized to 1 the minimum feasible capacity. Given $\overline{q}_i$, the firm produces at cost $c_i(q_i) = r\overline{q}_i$ any quantity $q_i \leq \overline{q}_i$, r being the (sunk) unit cost of capacity. (We might as well have assumed a (constant) positive marginal cost for any $q_i \leq \overline{q}$). Long-run average cost is thus constant at r under full capacity utilization while decreasing over any range of output $(f,\ f+1]$ (where $f \in \mathcal{I}$) between two adjoining capacities. Incidentally, 1 is the minimum efficient output, i.e., the minimum output attaining minimum average cost. Each potential entrant aims at maximizing its profit $\pi_i = p_i q_i - r\overline{q}_i$, where p_i is the price of i's output.

Let $\overline{q}^o \equiv (\overline{q}_1^o, ..., \overline{q}_i^o, ..., \overline{q}_z^o)$ and $\overline{Q}^o \equiv \sum_{i=1}^{z} \overline{q}_i^o$ denote, respectively, the vector of capacities (hereafter, the "industry configuration") and the total capacity resulting from the entry-capacity decisions made in the first stage. At any $\overline{q}^o$, $\overline{q}_{-i}^o$ stands for the vector of capacities by firm i's rivals, g for any of the firms with the largest capacity, $A^\circ \equiv \{i : \overline{q}_i^o > 0\}$ and $n^o \equiv \# A^o$ for the set and the number of active firms, respectively. In the second stage everyone is informed on $\overline{q}^o$.

We first characterize "long run" competitive equilibria (LRCE). They are defined as subgame perfect equilibria of an entry-capacity game in which active firms are price takers and the price is at the market-clearing level.[2] Let $p^w(\overline{q}^o)$ and $Q^w(\overline{q}^o)$ denote, respectively, price and aggregate output at the market-clearing equilibrium corresponding to any $\overline{q}^o$. It is immediately checked that $p^w(\overline{q}^o) = P(\overline{Q}^o)$ and $Q^w(\overline{q}^o) = \overline{Q}^o$ if $\overline{Q}^o \leq D(0)$, whereas $p^w(\overline{q}^o) = 0$ and $Q^w(\overline{q}^o) = D(0)$ if $\overline{Q}^o \geq D(0)$. Denote $\overline{Q}^c$ the largest $\overline{Q}$ yielding nonnegative profits under market clearing and $p^c \equiv P(\overline{Q}^c)$ the corresponding market-clearing price; clearly, $\overline{Q}^c \in \mathcal{I} : D(r) - 1 < \overline{Q}^c \leq D(r)$. Further, denote $\{\overline{q}^*\}$ the set of the least concentrated industry configurations consistent with a total capacity of $\overline{Q}^c$. At any $\overline{q}^*$, $\# A^* \equiv n^* = \overline{Q}^* = \overline{Q}^c$. To guarantee feasibility of any $\overline{q}^*$, a large number of potential entrants will be assumed throughout ($z \geq n^*$).

In the following, $\pi_i^w(\overline{q}^o) \equiv (p^w(\overline{q}^o) - r)\overline{q}_i^o$ will denote firm i's profit at the market-clearing equilibrium corresponding to any $\overline{q}^o$; also, for any given $\overline{q}_{-i}^o$, $\pi_i^w(\overline{q}_i, \overline{q}_{-i}^o) \equiv (p^w(\overline{q}_i, \overline{q}_{-i}^o) - r)\overline{q}_i$ will denote firm i's market-clearing profit as a function of $\overline{q}_i$, with $\overline{q}_i$ viewed as a continuous variable. Concavity of $\pi_i^w(\overline{q}_i, \overline{q}_{-i}^o)$ follows straightforwardly from $D''(p) \leq 0$; under our assumption of a linear demand curve, $\partial^2 \pi_i^w(\overline{q}_i, \overline{q}_{-i}^o)/\partial \overline{q}_i^2 = -2b$ so long as $\overline{q}_i + \sum_{j \neq i} \overline{q}_j^o < D(r)$.

The following proposition establishes that the set of competitive industry configurations coincides with $\{\overline{q}^*\}$.

Proposition 1 *(i) At a LRCE it can be neither* $\overline{q}^o : \overline{Q}^o \neq \overline{Q}^c$ *nor (ii)* $\overline{q}^o : \overline{Q}^o = \overline{Q}^c; n^o < n^*$.

(iii) Any $\overline{q}^*$, *involving* $p^w(\overline{q}^*) = p^c$ *and* $Q^w(\overline{q}^*) = \overline{Q}^c$, *is part of a LRCE.*

[2] Thus, similarly as in Vives (1986b), price taking occurs at the stage of quantity decisions whereas, in the first stage, each firm recognizes the impact of its capacity decision on the market-clearing price.

Proof. (i) At $\overline{q}^o : \overline{Q}^o > \overline{Q}^c$, $\pi_i^w(\overline{q}^o) < 0$ for any $i \in A^o$. At $\overline{q}^o : \overline{Q}^o < \overline{Q}^c - 1$, any firm $u \notin A^o$ would gain from entering with $\overline{q}_u = 1$ since $P(\overline{Q}^o + 1) > r$. This holds at $\overline{Q}^o = \overline{Q}^c - 1$ too so long as $D(r) \notin \mathcal{I}$, since then $P(\overline{Q}^c) > r$. In the special cases where $D(r) \in \mathcal{I}$, implying $P(\overline{Q}^c) = r$, the event of $\overline{q}^o : \overline{Q}^o = \overline{Q}^c - 1$ at a LRCE is discarded if, at zero profit, entering is slightly preferred to not entering.

(ii) The claim is obvious in the special cases where $D(r) \in \mathcal{I}$. Then $\pi_i^w(\overline{q}^o) = 0$ for any $i \in A^o$, whereas any g would earn $\pi_g^w(\overline{q}_g = \overline{q}_g^o - 1, \overline{q}_{-g}^o) = (P(\overline{Q}^c - 1) - r)\overline{q}_g > 0$ by deviating to $\overline{q}_g = \overline{q}_g^o - 1$. In the more general cases where $D(r) \notin \mathcal{I}$, it still holds true that $\overline{q}_g^o - 1$ is a better response to $\overline{q}_{-g}^o$ than $\overline{q}_g^o$. Let $\overline{q}_i' \equiv argmax_{\overline{q}_i}\ \pi_i^w(\overline{q}_i, \overline{q}_{-i}^o)$. So long as $\sum_{j \neq i} \overline{q}_j^o \leq D(r)$, it turns out that $\overline{q}_i' = 0.5\left(D(r) - \sum_{j \neq i} \overline{q}_j^o\right) = 0.5(\overline{q}_i^o + \alpha)$, where $\alpha \equiv D(r) - \overline{Q}^o$. Expanding $\pi_i^w(\overline{q}_i, \overline{q}_{-i}^o)$ in Taylor series around $\pi_i^w(\overline{q}_i', \overline{q}_{-i}^o)$ yields $\pi_i^w(\overline{q}_i, \overline{q}_{-i}^o) - \pi_i^w(\overline{q}_i', \overline{q}_{-i}^o) = (1/2)\left[\partial^2 \pi_i^w(\overline{q}_i, \overline{q}_{-i}^o)/\partial \overline{q}_i^2\right]_{\overline{q}_i = \overline{q}_i'} (\overline{q}_i - \overline{q}_i')^2 = -b(\overline{q}_i - \overline{q}_i')^2$. So what is left is to show that $\overline{q}_g = \overline{q}_g^o - 1$ is closer to $\overline{q}_g'$ than $\overline{q}_g^o$ is. This is immediate if $\overline{q}_g^o > 2$, for then $\overline{q}_g' < \overline{q}_g^o - 1 < \overline{q}_g^o$. If $\overline{q}_g^o = 2$, then $\overline{q}_g' - (\overline{q}_g^o - 1) = 0.5\alpha$; this is less than $\overline{q}_g^o - \overline{q}_g' = 1 - 0.5\alpha$ since $\alpha < 1$ when $\overline{Q}^o = \overline{Q}^c$.

(iii) At any $\overline{q}^*$, a unilateral deviation to $\overline{q}_i > \overline{q}_i^*$ by any $i \in A^*$ or $u \notin A^*$ results in a loss. ■

3 The pricing subgame

In the second stage of the entry-capacity and pricing game, each $i \in A^o$ sets p_i to maximize π_i, and hence quasi rent $p_i q_i$, given the strategy profile p_{-i} expected on the part of rivals. Denote $d_i(p_i, p_{-i})$ the demand facing i at (p_i, p_{-i}). With firms producing on demand $q_i = \min\{d_i(p_i, p_{-i}), \overline{q}_i^o\}$. Whenever relevant, the efficient rationing rule is assumed, hence $d_i(p_i, p_{-i}) = \max\left\{0, D(p_i) - \sum_{j \neq i} \overline{q}_j^o\right\}$ when i alone sets the highest price.

For any $i \in \mathcal{Z} : \sum_{j \neq i} \overline{q}_j^o \leq D(0)$, let $\widetilde{p}_i^o = \widetilde{p}_i(\sum_{j \neq i} \overline{q}_j^o) \equiv \arg\max_{p_i} p_i(D(p_i) - \sum_{j \neq i} \overline{q}_j^o)$. It is $0 < \widetilde{p}_i^o \leq P(\sum_{j \neq i} \overline{q}_j^o)$; also, $\max_i \widetilde{p}_i^o = \widetilde{p}_g^o$ as $\widetilde{p}_i'(\sum_{j \neq i} \overline{q}_j) < 0$ for $\sum_{j \neq i} \overline{q}_j < D(0)$. Next, let $\widetilde{q}_i^o = \widetilde{q}_i(\sum_{j \neq i} \overline{q}_j^o) \equiv \arg\max_{q_i} P(q_i + \sum_{j \neq i} \overline{q}_j^o) q_i$. With $\widetilde{q}_i^o \leq \overline{q}_i^o$, $\widetilde{q}_i^o$ represents i's Cournot (short-run) best response to an aggregate output of $\sum_{j \neq i} \overline{q}_j^o$.[3] Clearly, $\widetilde{p}_i^o = P(\widetilde{q}_i^o + \sum_{j \neq i} \overline{q}_j^o)$.

[3] $\widetilde{q}_i^o$ can also be interpreted as Cournot long-run best response under costless capacity

It must preliminarily be emphasized that any $\overline{q}^o : \overline{Q}^o \neq D(c)$ involving market clearing at an equilibrium of the pricing subgame (PS) is characterized by a large market. More specifically, what is required is a sufficiently small level of $\overline{q}_g^o/\overline{Q}^o$, the one-firm concentration ratio.

Lemma 1 *A small $\overline{q}_g^o/\overline{Q}^o$ is necessary and sufficient (i) for $p_i = p^w(\overline{q}^o) = 0$ $\forall i \in A^o$ to be an equilibrium of the PS at $\overline{Q}^o > D(0)$, and (ii) for $p_i = p^w(\overline{q}^o) = P(\overline{Q}^o)$ $\forall i \in A^o$ to be the equilibrium of the PS at $\overline{Q}^o < D(0)$.*

Proof. (i) Each firm charging a zero price is an equilibrium if and only if $\sum_{j\neq i} \overline{q}_j^o \geq D(0)$ $\forall i \in A^o$,[4] i.e., if and only if $\sum_{j\neq g} \overline{q}_j^o \geq D(0)$. The latter inequality, also written $\overline{q}_g^o/\overline{Q}^o \leq 1 - D(0)/\overline{Q}^o$, is implied by $\overline{Q}^o > D(0)$ if $\overline{q}_g^o/\overline{Q}^o$ is small enough.

(ii) From concavity of $p_i(D(p_i) - \sum_{j\neq i} \overline{q}_j^o)$, each firm charging $P(\overline{Q}^o)$ is an equilibrium if and only if

$$\left[\frac{\partial(p_i(D(p_i) - \sum_{j\neq i} \overline{q}_j^o)}{\partial p_i}\right]_{p_i=P(\overline{Q}^o)} = \overline{q}_i^o + P(\overline{Q}^o)\,[D'(p)]_{p=P(\overline{Q}^o)} \leq 0 \qquad \forall i \in A^o, \tag{1}$$

i.e., if and only if $\overline{q}_g^o \leq -P(\overline{Q}^o)\,[D'(p)]_{p=P(\overline{Q}^o)}$. This in turn can be written

$$\overline{q}_g^o/\overline{Q}^o \leq \eta_{p=P(\overline{Q}^o)}, \tag{2}$$

where η_p denotes absolute elasticity of $D(p)$ at p. Uniqueness of equilibrium can be established similarly as in Kreps and Sheinkman (1983). ■

On reflection, (1) (and hence (2)) is equivalent to $P(\overline{Q}^o) \geq \widetilde{p}_g^o$, a fact to be used later.

A pure-strategy equilibrium of the PS does not exist at $\overline{q}^o : \overline{Q}^o \geq D(0); \sum_{j\neq g} \overline{q}_j^o < D(0)$ nor does it at $\overline{q}^o : \overline{Q}^o < D(0); \widetilde{p}_g^o > P(\overline{Q}^o)$. Then existence of a mixed-strategy equilibrium follows from Theorem 5 of Dasgupta and Maskin (1986). In the following we make use of the fact that, at a mixed-strategy equilibrium, the largest firm's payoff equals the Stackelberg profit.

building ($r = 0$).

[4] Remaining equilibria have sufficiently many firms charging a zero price so that $\sum_{j\neq i:p_j=0} \overline{q}_j^o \geq D(0)$ for any $i : p_i = 0$.

Lemma 2 *At any $\overline{q}^o$ for which no pure-strategy equilibrium exists, denote $\overline{p}_i^o$ the supremum of the support of i's equilibrium strategy and let $\overline{p}^o \equiv \max_{i\in A^o}\{\overline{p}_i^o\}$. It is $\overline{p}^o = \widetilde{p}_g^o$ and g's equilibrium expected profit is $\pi_g(\overline{q}^o) = \widetilde{p}_g^o\widetilde{q}_g^o - r\overline{q}_g^o$.*

Remark. For the proof of this property, see Kreps and Scheinkman (1983) for duopoly, Vives (1986a) for symmetric oligopoly, Boccard and Wauthy (2000) and De Francesco (2003) for asymmetric oligopoly.[5] Incidentally, in view of $\widetilde{p}_i^o = P(\widetilde{q}_i^o + \sum_{j\neq i}\overline{q}_i^o)$ and $(P(\widetilde{q}_i^o + \sum_{j\neq i}\overline{q}_i^o) - r)\overline{q}_i^o = \pi_i^w(\widetilde{q}_i^o, \overline{q}_{-i}^o)$, one can write g's equilibrium expected profit as $\pi_g(\overline{q}^o) = \widetilde{p}_g^o\widetilde{q}_g^o - r\overline{q}_g^o = (\widetilde{p}_g^o - r)\widetilde{q}_g^o - r(\overline{q}_g^o - \widetilde{q}_g^o) = \pi_g^w(\overline{q}_g = \widetilde{q}_g^o, \overline{q}_{-g}^o) - r(\overline{q}_g^o - \widetilde{q}_g^o)$. □

At a later stage use will be made of the following property.

Lemma 3 *At any $\overline{q}^o : \overline{Q}^o = \overline{Q}^c$, $\left[\partial\pi_i^w(\overline{q}_i, \overline{q}_{-i}^o)/\partial\overline{q}_i\right]_{\overline{q}_i=\overline{q}_i^o} < 0$ for any $i \in A^o$.*

Proof. At $\overline{q}^o : \overline{Q}^o = \overline{Q}^c$, $\left[\partial\pi_i^w(\overline{q}_i, \overline{q}_{-i}^o)/\partial\overline{q}_i\right]_{\overline{q}_i=\overline{q}_i^o} = -b\overline{q}_i^o + p^c - r$. Expanding p^c in Taylor series around $P(D(r)) = r$ yields $p^c = r + [P'(Q)]_{Q=D(r)}(\overline{Q}^c - D(r)) = r - b(\overline{Q}^c - D(r))$. Thus $\left[\partial\pi_i^w(\overline{q}_i, \overline{q}_{-i}^o)/\partial\overline{q}_i\right]_{\overline{q}_i=\overline{q}_i^o} = b\left[D(r) - \overline{Q}^c - \overline{q}_i^o\right]$; this is less than zero if and only if $\overline{q}_i^o > D(r) - \overline{Q}^c$, which is actually the case given that $D(r) - \overline{Q}^c < 1$. ■

4 The entry-capacity and pricing game

We have just seen that in a sufficiently unconcentrated industry price competition yields the short-run competitive equilibrium. A similar result will be achieved in this section for the long run where the industry configuration is endogenously determined by the entry-capacity decisions of the potential entrants: the long-run competitive outcome obtains in so far as it is characterized by a sufficiently large market. More specifically, the nature of the solution of the entry-capacity and pricing game (ECPG) will be found to depend on whether or not the following condition holds.

[5] De Francesco (2003) provides an alternative proof for oligopoly in the face of a non-trivial error in the proof by Boccard and Wauthy.

Condition 1 *(a) Equilibrium prices are* $p^w(\overline{q}^*) \equiv p^c$ *in the PS associated to any* $\overline{q}^*$. *(b) Similarly, at any* $\overline{q}^o : n^o = \overline{Q}^o = \overline{Q}^c + 1$ *equilibrium prices are* $p^w(\overline{q}^o) = P(\overline{Q}^o)$ *or, alternatively, expected profit is negative at the mixed-strategy equilibrium.*

Remark. Let us see what is involved into the above condition. As to part (a), applying (2) to any $\overline{q}^*$ yields $1/n^* \leq \eta_{p=p^c}$. By the same token, $1/(n^*+1) \leq \eta_{p=P(\overline{Q}^c+1)}$ under the first variant of (b). When the second variant of (b) holds, at $\overline{q}^o : n^o = \overline{Q}^o = \overline{Q}^c + 1$ the margin of $\widetilde{p}_i^o$ over r is sufficiently small for it to be $\pi_i^o = \widetilde{p}_i^o \widetilde{q}_i^o - r < 0$. Whatever the case may be, Condition 1 is met in so far as the minimum efficient output (i.e., 1) is sufficiently small relative to the long-run competitive aggregate output ($\overline{Q}^c = n^*$).[6] In view of this, an industry which meets Condition 1 is one in which there is a "large market" at a long-run competitive equilibrium. □

Our first task is to solve the ECPG when Condition 1 holds.

Proposition 2 *Let Condition 1 hold. Then: (i) any* $\overline{q}^*$*, which involves firms charging* p^c*, is part of an equilibrium of the ECPG; (ii) at an equilibrium of the ECPG, it can be neither* $\overline{q}^o : \overline{Q}^o \neq \overline{Q}^c$ *nor (iii)* $\overline{q}^o : \overline{Q}^o = \overline{Q}^c; n^o < n^*$.

Proof. (i) By Condition 1(a), at $\overline{q}^*$ each $i \in A^*$ charges p^c and earns $\pi_i^c = \pi_i^w(\overline{q}^*) = p^c - r$ at the equilibrium of the PS. No matter whether $\widetilde{p}_i^* \gtreqless P(\overline{Q}^c + 1)$, any $i \in A^*$ has made a best response to $\overline{q}_{-i}^*$ by choosing the minimum capacity. If $\widetilde{p}_i^* > P(\overline{Q}^c + 1)$, a mixed-strategy equilibrium obtains when any $i \in A^*$ deviates to $\overline{q}_i = 2$, resulting in an expected profit of $\widetilde{p}_i^* \widetilde{q}_i^* - 2r = (\widetilde{p}_i^* - r)\widetilde{q}_i^* - r(2 - \widetilde{q}_i^*) = \pi_i^w(\overline{q}_i = \widetilde{q}_i^*, \overline{q}_{-i}^*) - r(2 - \widetilde{q}_i^*)$. Thus it follows immediately from Lemma 3 that even $(\widetilde{p}_i^* - r)\widetilde{q}_i^* < \pi_i^c$. If $\widetilde{p}_i^* \leq P(\overline{Q}^c + 1)$, a unilateral deviation to $\overline{q}_i = 2$ by any $i \in A^*$ leads to a pure-strategy equilibrium, and hence to the negative profit $2\left(P(\overline{Q}^c + 1) - r\right)$. A fortiori i would suffer a loss if choosing any $\overline{q}_i > 2$ entailing a pure-strategy equilibrium. If choosing $\overline{q}_i > 2$ leading to a mixed strategy equilibrium, i's expected profit $\widetilde{p}_i^* \widetilde{q}_i^* - r\overline{q}_i$ would be negative as $\widetilde{p}_i^* \leq P(\overline{Q}^c + 1) < r$ and $\widetilde{q}_i^* < \overline{q}_i$. Consider now any $u \notin A^*$. By Condition 1(b), with u deviating to $\overline{q}_u = 1$ either a pure-strategy equilibrium obtains - resulting in a loss -

[6] That a large n^* also underlies the second variant of Condition 1(b) follows from $\widetilde{p}_i'(\sum_{j \neq i} \overline{q}_j) < 0$ and the fact that, at $\overline{q}^o : n^o = \overline{Q}^o = \overline{Q}^c + 1$, $\sum_{j \neq i} \overline{q}_j^o = \overline{Q}^c = n^*$ for any $i \in A^o$.

or expected profit is negative at the mixed-strategy equilibrium. A fortiori entering with $\overline{q}_u > 1$ would lead to a loss.

(ii) As regards any $\overline{q}^o : \overline{Q}^o < \overline{Q}^c$, the argument in the proof of Proposition 1(i) still applies since any $u \notin A^o$ might enter with $\overline{q}_u = 1$ and then guarantee itself full capacity utilization by charging $P(\overline{Q}^o + 1)$. At industry configurations $\overline{q}^o : \overline{Q}^o > \overline{Q}^c$ entailing a pure-strategy equilibrium, active firms make losses given that $P(\overline{Q}^o) < r$. It remains to analyze the region of $\overline{q}^o : \overline{Q}^o > \overline{Q}^c$ entailing a mixed-strategy equilibrium. Consider first the subregion where $\overline{Q}^o > \overline{Q}^c + 1$. In this subregion, industry configurations $\overline{q}^o : n^o = \overline{Q}^o$ are immediately dismissed, for active firms earn a negative expected profit as a consequence of Condition 1(b).[7] So we can turn to $\overline{q}^o : \overline{Q}^o > \overline{Q}^c + 1; \overline{q}_g^o > 1$. If $\widetilde{p}_g^o > P(\overline{Q}^o - 1)$, by deviating to $\overline{q}_g = \overline{q}_g^o - 1$ firm g will earn no less than $\widetilde{p}_g^o \widetilde{q}_g^o - r(\overline{q}_g^o - 1)$ at the new mixed-strategy equilibrium,[8] hence more than at $\overline{q}^o$. If $\widetilde{p}_g^o \leq P(\overline{Q}^o - 1)$, $\widetilde{p}_g^o < r$ as $P(\overline{Q}^o - 1) < r$ when $\overline{Q}^o > \overline{Q}^c + 1$. This reveals that g's expected profit $\widetilde{p}_g^o \widetilde{q}_g^o - r\overline{q}_g^o$ at $\overline{q}^o$ is negative. Finally we must analyze the subregion where $\overline{Q}^o = \overline{Q}^c + 1$. If $\overline{q}_g^o = 1$, by the second variant of Condition 1(b) expected profit is negative for each active firm. If $\overline{q}_g^o > 1$, a unilateral deviation to $\overline{q}_g = \overline{q}_g^o - 1$ will raise g's payoff. This is shown by the same argument as above if $\widetilde{p}_g^o > P(\overline{Q}^o - 1)$. If $\widetilde{p}_g^o \leq P(\overline{Q}^o - 1)$ a deviation to $\overline{q}_g = \overline{q}_g^o - 1$ results in a profit of $\pi_g^w(\overline{q}_g = \overline{q}_g^o - 1, \overline{q}_{-g}^o)$ since a pure-strategy equilibrium now obtains. On the other hand, g's expected profit at $\overline{q}^o$ is $\widetilde{p}_g^o \widetilde{q}_g^o - r\overline{q}_g^o = \pi_g^w(\overline{q}_g = \widetilde{q}_g^o, \overline{q}_{-g}^o) - r(\overline{q}_g^o - \widetilde{q}_g^o)$. Then it follows from Lemma 3 that even the term $(\widetilde{p}_g^o - r)\widetilde{q}_g^o$ is less than $\pi_g^w(\overline{q}_g = \overline{q}_g^o - 1, \overline{q}_{-g}^o)$.

(iii) At any such $\overline{q}^o$ a unilateral deviation to $\overline{q}_g = \overline{q}_g^o - 1$ will raise g's profit. The proof is much the same as that of Proposition 1(ii) when equilibrium prices are p^c at $\overline{q}^o$. If not, g's expected profit is $\widetilde{p}_g^o \widetilde{q}_g^o - r\overline{q}_g^o$ at $\overline{q}^o$. Then, if $\widetilde{p}_g^o > P(\overline{Q}^c - 1)$, a deviation to $\overline{q}_g = \overline{q}_g^o - 1$ would raise g's expected profit to no less than $\widetilde{p}_g^o \widetilde{q}_g^o - r(\overline{q}_g^o - 1)$. This applies as well with $\widetilde{p}_g^o = P(\overline{Q}^c - 1)$, in which case a pure-strategy equilibrium obtains at $\overline{q}_g = \overline{q}_g^o - 1$. If $\widetilde{p}_g^o < P(\overline{Q}^c - 1)$, equilibrium prices are $P(\overline{Q}^c - 1)$ at $\overline{q}_g = \overline{q}_g^o - 1$. Then the move from $\overline{q}_g^o$ to

[7] Holding the first variant of Condition 1(b), $\widetilde{p}_i(\sum_{j\neq i} \overline{q}_j) < P(\overline{Q}^c + 1) < r$ at $\sum_{j\neq i} \overline{q}_j = \overline{Q}^c$. Consequently, at any $\overline{q}^o : n^o = \overline{Q}^o > \overline{Q}^c + 1$, a fortiori $\widetilde{p}_i^o < r$ for any $i \in A^o$, given that $\sum_{j\neq i} \overline{q}_j^o \geq \overline{Q}^c + 1$. If the second variant of Condition 1(b) holds, expected profit is negative at any $\overline{q}^o : n^o = \overline{Q}^o > \overline{Q}^c + 1$ because expected quasi rent is the same as at $\overline{q}^o : n^o = \overline{Q}^o = \overline{Q}^c + 1$, whereas capacity cost is higher.

[8] Since rivals produce $\sum_{j\neq g} \overline{q}_j^o$ at most, firm g would sell no less than $\widetilde{q}_g^o = D(\widetilde{p}_g^o) - \sum_{j\neq g} \overline{q}_j^o$ when charging $\widetilde{p}_g^o$.

$\overline{q}_g^o - 1$ can be decomposed in two virtual steps, a reduction of capacity from $\overline{q}_g^o$ to $\widetilde{q}_g^o$ and then from $\widetilde{q}_g^o$ to $\overline{q}_g^o - 1$. The first step would raise g's equilibrium profit to $(\widetilde{p}_g^o - r)\widetilde{q}_g^o$. The second step would add further to g's profit; this follows from $\pi_g^w(\overline{q}_g, \overline{q}_{-g}^o) = \pi_g^w(\overline{q}'_g, \overline{q}_{-g}^o) - b(\overline{q}_g - \overline{q}'_g)^2$ when it is recalled that $\overline{q}'_g < \overline{q}_g^o - 1$ at $\overline{q}^o : \overline{Q}^o = \overline{Q}^c; \overline{q}_g^o \geq 2$. ■

The analysis has so far been concerned with a market that is sufficiently large at a LRCE. It remains to characterize equilibria of the ECPG when Condition 1 is violated.

Proposition 3 *(i) If Condition 1(a) does not hold, then at any equilibrium of the ECPG the industry configuration lies in the set* $\{\overline{q}^*\}$ *and each active firm earns an expected profit of* $\widetilde{p}_i^*\widetilde{q}_i^* - r > 0$.

(ii) If Condition 1(a) holds but Condition 1(b) does not, then at any equilibrium of the ECPG it is $\overline{q}^o : n^o = \overline{Q}^o = \overline{Q}^c + 1$ *and each active firm earns an expected profit of* $\widetilde{p}_i^o\widetilde{q}_i^o - r > 0$.

Proof. (i) Failing Condition 1(a), a mixed equilibrium obtains at $\overline{q}^*$, with each active firm earning $\widetilde{p}_i^*\widetilde{q}_i^* - r > p^c - r \geq 0$. (This follows from $\widetilde{p}_i^*$ being a better response than p^c to $p_{-i} = (p^c, ..., p^c)$.) As regards any $i \in A^*$, deviating to $\overline{q}_i > 1$ results in expected profit falling to $\widetilde{p}_i^*\widetilde{q}_i^* - r\overline{q}_i$. As to any $u \notin A^*$, a loss is expected if deviating to $\overline{q}_u = 1$. In fact, entering with $\overline{q}_u = 1$ yields u the same expected profit $\widetilde{p}_i^o\widetilde{q}_i^o - r = (\widetilde{p}_i^o - r)\widetilde{q}_i^o - r(1 - \widetilde{q}_i^o)$ now earned by any $i \in A^*$. We can prove that even $\widetilde{p}_i^o - r < 0$. By the envelope theorem,

$$\left[\frac{d\pi_i^w(\overline{q}_i = \widetilde{q}_i(\sum_{j\neq i}\overline{q}_j), \overline{q}_{-i})}{d\sum_{j\neq i}\overline{q}_j}\right]_{\overline{q}_{-i}=\overline{q}_{-i}^o} = \left[\frac{\partial\pi_i^w(\overline{q}_i = \widetilde{q}_i^o, \overline{q}_{-i})}{\partial\sum_{j\neq i}\overline{q}_j}\right]_{\overline{q}_{-i}=\overline{q}_{-i}^o} = -b\widetilde{q}_i^o,$$

for any $\overline{q}_{-i}^o$. Since $(\widetilde{p}_i^o - r)\widetilde{q}_i^o = \pi_i^w(\overline{q}_i = \widetilde{q}_i^o, \overline{q}_{-i}^o)$, one can thus write

$$(\widetilde{p}_i^o - r)\widetilde{q}_i^o = \pi_i^w(\overline{q}_i = \widetilde{q}_i^*, \overline{q}_{-i}^*) + \left[\frac{d\pi_i^w(\overline{q}_i = \widetilde{q}_i(\sum_{j\neq i}\overline{q}_j), \overline{q}_{-i})}{d\sum_{j\neq i}\overline{q}_j}\right]_{\overline{q}_{-i}=\overline{q}_{-i}^*} \Delta\sum_{j\neq i}\overline{q}_j = (\widetilde{p}_i^* - r)\widetilde{q}_i^* - b\widetilde{q}_i^*, \qquad (3)$$

where $\Delta\sum_{j\neq i}\overline{q}_j \equiv \sum_{j\neq i}\overline{q}_j^o - \sum_{j\neq i}\overline{q}_j^* = 1$ is the change in the aggregate capacity of rivals faced by any $i \in A^*$ when u deviates to $\overline{q}_u = 1$. Note that

$$\begin{aligned}\widetilde{p}_i^* = P(\widetilde{q}_i^* + \sum_{j \neq i} \overline{q}_j^*) &= r + [P'(Q)]_{Q=D(r)} (\widetilde{q}_i^* + \sum_{j \neq i} \overline{q}_j^* - D(r)) \\ &= r - b(\widetilde{q}_i^* + \sum_{j \neq i} \overline{q}_j^* - D(r)). \end{aligned} \tag{4}$$

Replacing (4) into (3) leads to $(\widetilde{p}_i^o - r)\widetilde{q}_i^o = b\widetilde{q}_i^* \left[D(r) - (\widetilde{q}_i^* + \sum_{j \neq i} \overline{q}_j^*) - 1\right]$, a negative magnitude as $(D(r) - (\widetilde{q}_i^* + \sum_{j \neq i} \overline{q}_j^*) < 1$.

(ii) When Condition 1(b) does not hold, at $\overline{q}^o : n^o = \overline{Q}^o = \overline{Q}^c + 1$ active firms earn a positive expected profit. Arguing much the same as above one can see that $\overline{q}_i^o$ is a best response to $\overline{q}_{-i}^o$ for any $i \in \mathcal{Z}$. ■

5 Conclusion

We have modelled long-run price competition as a two-stage entry-capacity and pricing game among a large number of potential entrants. It turned out that each equilibrium exhibits long-run competitive features - active firms selling at the market-clearing price the minimum efficient output, in a number which is the largest consistent with nonnegative profits -; this, on condition that the minimum efficient output is sufficiently small relative to the long-run competive total output.

It is worth bearing in mind that several complications have been avoided by assuming linearity of the demand curve and the existence of a single technique. Dealing with the more complex case of a strictly concave demand curve and a plurality of techniques is a task we leave for future research.[9]

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