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ERNESTO SCREPANTI

Choice Freedom and Redistribution Policies

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ABSTRACT. Opportunity sets are defined by taking account of budgetary, institutional and legal constraints. Then a cardinal measure of freedom is proposed which is apt to interpersonal comparisons of opportunity sets. Individual choice freedom may expand when a new good is publicly provided free or at a low price. On the other hand it is decreased by taxes. It is argued that progressive taxation redistributes freedom in favour of the poor, all the more so when public revenues are used to finance the public provision of goods, in which case overall freedom may result enlarged.

ERNESTO SCREPANTI, Dipartimento di Economia Politica, Università di Siena

Introduction[1]

Consider a person whose preference mapping and budget constraint are those represented in figure 1. If he is a free utility maximiser he chooses commodity bundle *A*. Compare this situation with that of another person with the same preferences but whose consumption is decided by a benevolent dictator who assigns him precisely bundle *A*. The two individuals get the same satisfaction. Yet there is a substantial difference in their position. The free decisor could choose to buy bundle *B*, which cannot be done by the person subjected to dictatorship. In this case the free individual enjoys less satisfaction but more freedom: he can choose between two opportunities instead of just one - he can choose (Pattanaik and Xu, 1990).
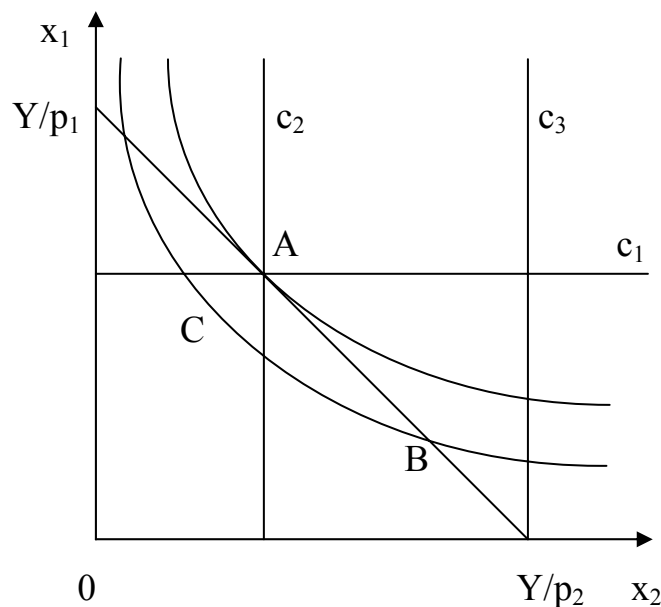

Figure 1

If the free individual is to spend all of his income, his opportunity set consists of the whole budget line. Note however that, from a welfare point of view, he is indifferent between bundles *B* and *C*, so that if he may choose *B* he might choose *C* too. In other words the entire area below and on the budget line represents his opportunity set. When investigating into individual freedom we must abstract from any psychological condition of choice, both from preferences and the degree of rationality. This is what is called an "objectivist" approach to the analysis of freedom (Gravel, Laslier and Trannoy, 1998). In fact it is more than that. It is a non-psychologist, non-welfarist and non-consequentialist approach: non-psychologist, because any psychological characteristic of decisors are overlooked; non-welfarist,

because the utility outcomes of choices are ignored; non-consequentialist, because freedom is valued in itself and not for its economic, social or political consequences. Such an approach attaches an "intrinsic importance" to choice freedom (Gravel, 1994).

In an objectivist approach freedom can be ranked by using a cardinal measure endowed with three basic properties: symmetry, monotonicity to set inclusion and invariance to scaling effects (Xu, 2000). Symmetry means that all choice options are treated equally, in the sense that each one of them offers the individual the same amount of freedom. This property is required because we expect that differences in individual preferences do not affect the amount of freedom. Monotonicity means that any individual's freedom increases when a new option is added to his opportunity sets. Invariance to scaling effects is required because a cardinal measure of freedom is determined up to a dimensionality factor. The values of goods change when their measuring rods vary. Yet the freedom ranking of two opportunity sets must not change. This property is important in that it grants interpersonal comparisons of freedom. Given the measuring rods of all goods available in a given society, we can compare the amounts of freedom of any two citizens. If the measuring rods change in a society, they change in the same way for all citizens. Therefore the relative ranking of their freedoms does not change.

This area of investigation was inaugurated by three seminal works of Kreps (1979), Jones and Sudgen (1982) and Steiner (1983). Then Sen (1985, 1988) clarified the reasons why the freedom of choice must be investigated by abstracting from individual preferences, thus originating the objectivist approach.[2] At the end of the 80s two important papers by Suppes (1987) and Pattanaik and Xu (1990) established a fundamental theorem on cardinality ranking, which simply asserts an individual's quantity of freedom can be measured by counting the number of options contained in his opportunity set. Successively, attempts at generalisation and refinement were done by Klemisch-Alert (1993) Bossert, Pattanaik and Xu (1994), Sudgen (1998), Gravel, Laslier and Trannoy (1998), van Hees (1998), Pattanaik and Xu (1998, 2000), Xu (2000), Basili and Vannucci (2000) and Vannucci (2003).

A cardinal measure is considered quite robust from a theoretical point of view, yet many students believe it has scarce empirical import, for it is impossible to count all the opportunities available to an individual in any practical situation. Therefore it is also considered pretty well useless from a policy point of view. I wish to challenge this opinion by proposing a cardinal measure which is simple to calculate and which can be used to draw some relevant policy implications. In section 1 I expand the definition of opportunity set by introducing several kinds of non-budget constraints to choice freedom. Then I present the cardinal measure in section 2. In section 3 and 4 I show some interesting policy inferences that can be derived from this analysis of freedom.

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[2] There are at least another three different approaches: a diversity, a flexibility and a preference ordering approach. The last two have parted way from a purely "objectivist" view by resorting to some form of indirect utility ranking. See Basili and Vannucci (2000) for a critical survey of the literature.

1. Economic and institutional constraints

Let me first define an opportunity set as bounded by an income constraint. The bundles of goods x_1 and x_2, with prices p_1 and p_2, that can be chosen by an individual with income Y are located in the area of triangle Y/p_1-0-Y/p_2 in figure 1. The budget constraint is $Y \geq x_1p_1 + x_2p_2$. Any point in the triangle bounded by the budget line is an opportunity.[3]

Many choices are constrained by institutions - norms, customs, social habits, conventions etc. This is done through imposing a minimum or a maximum quantity of consumption, that is, through obligations or prohibitions. A case of obligation is compulsory education. One of prohibition is a motorway speed limit. These constraints are represented with vertical or horizontal lines in figure 1. Leftward or downward constraints represent obligations, rightward or upward constraints prohibitions. Thus, if line c_2 defines an obligation, the opportunity set is reduced to the area of the triangle at the right of c_2; if it defines a prohibition, the opportunity set is reduced to the area at its left and below the budget line.

Many goods have no price and are not marketable: joining a party or a church, giving a free speech, enjoying a beautiful landscape, benefiting from free health services etc. Let me call these "social goods".[4] They include public goods, merit goods and any service which is publicly provided free to all citizens. They also include rights, which, in fact, can be likened to public goods. There are no income constraints to the enjoyment of this kind of goods. If x_1 is a social and x_2 a private good, the opportunity set is represented by the entire area leftward the c_3 line in figure 1. Opportunity sets however are not infinite. Either they are rationed, like free public education up to a certain age, or they are bounded by an individual capacity of consumption.[5] If x_1 is a social good rationed to the quantity c_1, the opportunity set reduces to the area below line c_1 and left of line c_3.

Now consider subsistence goods. These pose leftward or downward constraints to opportunity sets. If c_2 is a subsistence consumption of private good x_2, the opportunity set shrinks to the area of the triangle at the right of line c_2. If c_1 too is a subsistence need, the opportunity set shrinks to point A. Subsistence needs reduce freedom, for people are not free to choose to consume less than what is required for survival. Freedom from needs may be attained through raising incomes or transforming private goods into social. Of course subsistence consumption can be both biologically and culturally determined.

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[3] The nature of the constraint varies with the time horizon of choices. So one could refer to a monthly or a yearly income; in a long run intertemporal setting one could refer to a wealth constraint; and so on. Since there is no need to go in deep with this problem here, I will generically talk of "income".

[4] I take this term from Musgrave (1959, pp. 9-14, 43-44), but use it in a different way from him, who calls social goods what are usually named "public goods" and public goods the collection of what he calls social and merit goods.

[5] If social goods were not rationed, opportunity sets would seem to be infinite. However individual consumption of social goods is limited by time availability. In Screpanti (2003) I show that, due to the existence of leisure time constraints, opportunity sets are finite anyway. Moreover the demand for most social goods can hardly be insatiable. For instance, the demand for free health care is not infinite simply because nobody enjoys living in a hospital. In cases like this demand is limited by actual needs.

2. A natural measure of freedom

If only one good exists, and this is a private good with price p_1, the quantity of freedom of a person with income Y is the number of opportunities he is able to choose:

$$L_{(1)} = \frac{Y}{p_1}$$

which is the length of segment $0\text{-}Y/p_1$ in figure 1. $L_{(1)}$ is interpreted not as a quantity of goods but as number of opportunities. Supposing $Y=10,000$ Euro and $p_1=100$ Euro, it is $L_{(1)}=100$. The individual can choose to buy 1 or 2 or 3... or 100 units of good x_1, i.e. he has 100 choice opportunities.

If there are two private goods, x_1 and x_2, his quantity of freedom is:

$$L_{(2)} = L_{(1)} \frac{Y}{2p_2} = \frac{Y^2}{2p_1 p_2}$$

which is the area of triangle $Y/p_1\text{-}0\text{-}Y/p_2$ in figure 1.

If there are three goods, the quantity of freedom is

$$L_{(3)} = L_{(2)} \frac{Y}{3p_3} = \frac{Y^3}{3! \prod_{i=1}^{3} p_i}$$

which is the volume of a tetrahedron with three hedges of length Y/p_1, Y/p_2, Y/p_3.

More generally, the quantity of freedom when n private goods exist can be measured as:

$$(1) \qquad L_{(n)} = \frac{Y^n}{n! \prod_{i=1}^{n} p_i}$$

which is the "volume" of an opportunity set (Xu, 2000). Formula (1) can be interpreted in the following way: $Y^n/n!$ is the volume of a hyper-tetrahedron with n hedges of length Y; it is divided by the volume of a hyper-parallelepiped, $\prod_{i=1}^{n} p_i$, representing one choice opportunity, i.e. a bundle made up of one unit of each commodity; this division yields the number of opportunities available to our individual.

Now, let me introduce social goods. Their price is nil.[6] Therefore the constraints to their consumption are represented through vertical or horizontal lines.

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[6] Spurious social goods with positive tickets (lower than market prices) are likened to private goods.

Also remember that the maximum quantity of an individual's enjoyment of a social good is limited. In other words, social goods do not render opportunity sets infinite. If there are m social goods, the maximum quantities an individual can consume are S_j, with $j=1,...m$. Thus a person's quantity of freedom is:

$$L^* = \frac{Y^n \prod_{j=1}^{m} S_j}{n! \prod_{i=1}^{n} p_i}$$

This measure, though, seems problematic in that its magnitude is very high, which makes it cumbersome to manipulate in empirical research. Therefore it might be useful to modify the formula to get a more manageable measure. I propose the following:

(2) $L = \sqrt[v]{L^*}$

where $v=n+m$.

3. Some quasi obvious implications

A few interesting propositions can be easily proved by looking at the signs of the partial derivatives of equation (2). For instance: freedom is reduced by direct and indirect taxes; is expanded by a reduction in prices (and tickets of spurious social goods) and an increase in the available quantity of a rationed social good.

More interesting is the question: under what conditions does the transformation of a private into a pure social good expand freedom? To answer this question, pose

(3) $\left(\dfrac{Y^{n-1} \prod_{j=1}^{m+1} S_j}{(n-1)! \prod_{i=1}^{n-1} p_i} \right)^{\frac{1}{v}} > \left(\dfrac{Y^n \prod_{j=1}^{m} S_j}{n! \prod_{i=1}^{n} p_i} \right)^{\frac{1}{v}}$

The result is

(4) $S_{m+1} > \dfrac{Y}{n p_n}$

If the maximum quantity of the new social good an individual can consume is higher than his real income (measured in terms of the suppressed good) divided by the number of private goods, freedom is expanded. This kind of policy could easily increase the freedom of the poor. Possibly it would reduce that of the rich; yet not necessarily. In fact the availability of the new social good must be compared, not with the maximum quantity of the suppressed private good the rich could consume

with his income, but with this quantity divided by the number of private goods. Most probably, since *n* is very high, the transformation of a private into a social good would increase the freedom of all. Of course I am here ignoring the effects of the taxation required to finance the new social good production, a problem to which I turn now.

4. Taxation and the redistribution of freedom

The cardinality of the freedom measure enables us to overcome many of the objections traditionally raised against progressive taxation and redistribution policies on account of the impossibility of interpersonal comparisons of utility. An interesting question is: how is freedom redistributed when the government taxes the rich and subsidises the poor?

From now on I introduce the subscripts *h* and *l*, referring to the "rich" (high income) and the "poor" (low income) respectively. Assume the rich is taxed at a rate ρ_h (a negative value) so that his income changes by $dY_h=\rho_h Y_h d\tau<0$. Note that $d\tau$ is the fiscal year. Thus one can write $d\tau=1$ and $dY_h=\rho_h Y_h$. A rich person's freedom will decrease by the amount

$$dL_h = \frac{\frac{n}{v}Y_h^{(\frac{n}{v}-1)}\left(\prod_{j=1}^{m}S_j\right)^{\frac{1}{v}}}{\left(n!\prod_{i=1}^{n}p_i\right)^{\frac{1}{v}}}dY_h$$

Since $dY_h=\rho_h Y_h$,

$$dL_h = \frac{\frac{n}{v}Y_h^{(\frac{n}{v})}\left(\prod_{j=1}^{m}S_j\right)^{\frac{1}{v}}}{\left(n!\prod_{1=1}^{n}p_i\right)^{\frac{1}{v}}}\rho_h$$

and therefore

(5) $\qquad \dfrac{dL_h}{L_h} = \dfrac{n}{v}\rho_h < 0$

where *n/v<1* is a factor transforming an income tax rate into a rate of freedom change.[7]

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[7] If (5) is interpreted as an exercise in ranking, i.e. dL_h/L_h is taken to mean a comparison between two levels of freedom, it is easy to show it is invariant to scaling effects. Let δ_i and δ_j be the measuring rods for goods x_i and x_j, with $i=1,...n$ and $j=1,...m$, and pose $D = \sqrt[v]{\prod_{j=1}^{m}\delta_j / \prod_{i=1}^{n}\delta_i}$. Formula (5) does not change when equation (2) is written $L=(L^*)^{1/v}D$.

With the same procedure we determine the rate of increase in freedom enjoyed by the poor when he obtains an income transfer of $dY_l=\sigma_l Y_l$, with $\sigma_l >0$:

$$\frac{dL_l}{L_l} = \frac{n}{v}\sigma_l > 0$$

Suppose the whole amount of money taxed to the rich is transferred to the poor, that is, $dY_l=-dY_h$, or $\sigma_l Y_l=-\rho_h Y_h$. Since $Y_l<Y_h$, it is $\sigma_l>-\rho_h$, and therefore

$$\frac{dL_l}{L_l} > -\frac{dL_h}{L_h}$$

Taking a given amount of money form the rich and giving it to the poor makes the poor's freedom increase proportionally more than the rich's freedom is reduced.

Now consider a policy of taxing both the rich and the poor, but the former more than the latter. It follows

$$-\frac{dL_l}{L_l} < -\frac{dL_h}{L_h}$$

Progressive taxation *in itself* alters the distribution of freedom in favour of the poor but could reduce overall freedom. At any rate, it may be justified with the possible freedom expansion that can be financed through public revenues. These, for instance, could be used to fund the transformation of a private into a social good. From (4) we know that such a policy expands freedom if $S_{m+1}>Y/np_n$. The overall change in freedom ensuing from the provision is $S_{m+1}-Y/np_n$. This must be added to the decrease in freedom following tax payments, $dL=\rho L(n/v)<0$. The result is $S_{m+1}-Y/np_n+\rho L(n/v)$, which represents the net increase in freedom obtained by a generic citizen. There will be overall freedom expansion if

$$(S_{m+1} - Y_h / np_n + \rho_h L_h n / v) + (S_{m+1} - Y_l / np_n + \rho_l L_l n / v) > 0$$

If both the terms are negative, the policy should not be pursued. If they are both positive, it should be. In practice the first term is presumably negative, for the increase in freedom procured to the rich by the provision of the $(m+1)^{th}$ social good may not make up for the decrease caused by taxation and that provoked by elimination of the nth private good. The sign of the second term should instead be positive. In this case there will be an overall increase in freedom if

$$(S_{m+1} - Y_l / np_n + \rho_l n L_l) > -(S_{m+1} - Y_h / np_n + \rho_h n L_h)$$

Transforming health services or education facilities into social goods could have a tangible consequence on overall freedom. These goods are very expensive and

are hardly accessible to the poor if they are provided privately. Moreover it is a good policy to transform into social goods those commodities that are necessary to conduct a decent social life, e.g. the culturally determined subsistence goods, for the rich does not lose much freedom in this way, whilst the poor could substantially expand his freedom from need. Taxing the rich and the poor to finance a public health service, might reduce the rich citizen's freedom, but does expand the poor's. If the latter effect overcomes the former, a liberal government should pursue the policy. Also note that a way to obtain an overall increase in freedom is to reduce taxation to the poor. The lower is $-\rho_l$, the higher the probability that the first term of the disequation is greater than the second, which is a justification for progressive taxation.

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