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A Correction to "Bertrand Competition and Cournot Outcomes: Further Results"

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Abstract - Before solving the capacity-pricing game for oligopoly, Boccard and Wauthy (2000) argue that, as under duopoly, at a mixed-strategy equilibrium of the pricing game the largest firm's payoff equals the Stackelberg follower profit. We point to a nontrivial mistake in their argument and see how this important property can be satisfactorily established.

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1 Introduction

In a recent paper Boccard and Wauthy (2000) (hereafter, BW) analyze a two-stage capacity-pricing game. The setting of Kreps and Scheinkman (1983) - constant marginal variable cost up to "capacity", efficient rationing rule, and so on - is extended to oligopoly and by allowing production beyond installed capacity at a constant extra unit cost. Before solving the entire game, BW seek to establish the following property of mixed strategy equilibria of the pricing subgame: for the largest firm(s) expected profit equals the Stackelberg-follower profit when rivals supply their capacity (see Claim 6, pp. 282-3). To our knowledge, this is the first attempt to generalize a property previously established for duopoly (see Kreps and Scheinkman, 1983) and symmetric oligopoly (see Brock and Scheinkman, 1985, and Vives, 1986). Our aim here is to see how this important generalization can satisfactorily be proved, in the face of a nontrivial mistake in the proof supplied by BW.

2 The correction

$D(p)$ and $P(q)$ are demand and the inverse demand, respectively, where p is the price and q the total output; $P(q) > 0$ on a bounded interval $[0, \widehat{q})$ on which $P'(\cdot) < 0$ and $P''(\cdot) \leq 0$. There are $n \geq 2$ firms; x_i is firm i's capacity, x_{-i} the total capacity of i's rivals, and $x = x_i + x_{-i}$. Variable cost is zero up to the firm capacity, hence in the pricing subgame each $i \leq n$ seeks to maximize expected revenue, denoted Π_i. Firm i's Cournot best response to x_{-i} under costless capacity building is denoted $r(x_{-i})$, while $R(x_{-i})$ is the corresponding revenue. Note that $x_i P(x_i + x_{-i})$ is concave in x_i and $-1 < r'(x_{-i}) < 0$ for any $x_{-i} < \widehat{q}$.

In the region of no existence of a pure strategy equilibrium of the pricing subgame, existence of a mixed equilibrium is guaranteed by Theorem 5 of Dasgupta and Maskin (see BW, p. 281). Let $\Sigma_i \equiv \{p_i \mid f_i(p_i) > 0\}$ denote the support of firm i's equilibrium density $f_i(p_i)$, and $\underline{p}_i$ and $\overline{p}_i$, respectively, the infimum (i.e., the greatest lower bound) and the supremum (the lowest upper bound) of Σ_i. Also, let $F_i(p) \equiv \Pr(p_i < p)$, $\overline{H} \equiv \arg\max_{i \leq n} \overline{p}_i$, $\overline{p} \equiv \max_{i \leq n} \overline{p}_i$, and let Π_i^* denote firm i's equilibrium expected revenue.

A few features of mixed equilibria are readily established. First, any $j \in \overline{H}$ sells less than x_j when charging $\overline{p}$: otherwise, it would be $D(\overline{p}) \geq x$, implying that any $i : F_i(\overline{p}) > 0$ has not made a best response since it can sell

x_i by charging $\overline{p}$. Second, $\overline{p}$ is charged with positive probability by one firm at most: indeed, if $\Pr(p_j = \overline{p}) > 0$, charging slightly less than $\overline{p}$ is better than $\overline{p}$ to any $i \neq j$ due to the jump in i's residual demand in the event of j charging $\overline{p}$. Thus, at a mixed equilibrium there exists at least one firm $j \in \overline{H}$ such that no $i \neq j$ charges $\overline{p}$ with positive probability.[1] Arguing similarly as in Kreps and Scheinkman, $\overline{p} = P(r(x_{-j}) + x_{-j})$ and $\Pi_j^* = R(x_{-j}) = \overline{p}r(x_{-j})$ for any such j, with $r(x_{-j}) < x_j$. We can now look at the argument set forth by BW, according to which a contradiction would arise from assuming $x_j < x_1$, where 1 is assumed to be (any of) the largest firm(s).

It must be $R(x_{-1}) = r(x_{-1})P(r(x_{-1}) + x_{-1}) \leq \Pi_1^*$, given that $\overline{p}_1 \leq \overline{p} < P(r(x_{-1}) + x_{-1})$. Further, $x_1 R(x_{-j}) < x_j R(x_{-1})$ (see the proof in the Appendix). In view of all this, BW can write (p. 283)

$$\Pi_j^* = R(x_{-j}) < \frac{x_j}{x_1} R(x_{-1}) \leq \frac{x_j}{x_1} \Pi_1^* < \Pi_1^*. \tag{1}$$

The final stage of their argument relies on writing (p. 283)

$$\Pi_1^* = \underline{p}_1 \left[F_{-1}(\underline{p}_1)(D(\underline{p}_1) - x_{-1}) + (1 - F_{-1}(\underline{p}_1))D(\underline{p}_1) \right], \tag{2}$$

where the expression in square brackets is 1's expected output when charging $\underline{p}_1$. The term $D(\underline{p}_1)$ multiplying $(1 - F_{-1}(\underline{p}_1))$ is mistaken. Notice that $F_{-1}(\underline{p}_1)$ is the probability of the event that $p_i < \underline{p}_1$ for every $i \neq 1$, i.e., $F_{-1}(\underline{p}_1) = \times_{i \neq 1} F_i(\underline{p}_1)$. The complementary event is thus the event that $p_i \geq \underline{p}_1$ *for at least some* $i \neq 1$. Therefore, $(1 - F_{-1}(\underline{p}_1))$ should have been multiplied by $q_1(p_1 = \underline{p}_1 \mid \textit{some } p_i \geq \underline{p}_1)$, i.e., 1's expected output when charging $\underline{p}_1$, conditional on $p_i \geq \underline{p}_1$ for some $i \neq 1$. To understand that $q_1(p_1 = \underline{p}_1 \mid \textit{some } p_i \geq \underline{p}_1) < D(\underline{p}_1)$ it suffices to show that $x_1 < D(\underline{p}_1)$. Notice that $x_{-j} \geq x_1$ (with $x_{-j} = x_1$ if and only if $n = 2$) and recall that $\overline{p} = P(r(x_{-j}) + x_{-j}) < P(x_{-j})$; further, $\underline{p}_1 < \overline{p}$ since it is assumed that $\overline{p}$ is not charged with positive probability by firm 1. Consequently, $D(\underline{p}_1) > r(x_{-j}) + x_{-j} > x_1$.[2]

[1] This means that, for any $i \neq j$, either $\overline{p} \notin \Sigma_i$ or $\overline{p} \in \Sigma_i$ but $\Pr(p_i = \overline{p}) = 0$. As to firm j, $\Pr(p_j = \overline{p}) \geq 0$ depending on the capacity configuration.

[2] Further, if $F_i(\underline{p}_1) > 0$ for some $i \neq 1$, the event that $p_i \geq \underline{p}_1$ for some $i \neq 1$ would also include price vectors by 1's rivals with some $p_i < \underline{p}_1$, resulting in a residual demand for firm 1 less than $D(\underline{p}_1)$ when charging $\underline{p}_1$.

Then, how to achieve the desired contradiction? Write firm 1's equilibrium expected revenue as $\Pi_1^* = \underline{p}_1 q_1(p_1 = \underline{p}_1)$, where $q_1(p_1 = \underline{p}_1)$ is 1's expected output when charging $\underline{p}_1$. Let $(x_j/x_1)\Pi_1^* \equiv x_j \underline{p}_1 k$, where $k \equiv q_1(p_1 = \underline{p}_1)/x_1 \leq 1$. It follows from (1) that $\Pi_j^* < x_j \underline{p}_1 k$. Notice that firm j sells x_j by charging $\underline{p}_1^-$, i.e., slightly less than $\underline{p}_1$: indeed, this results in a residual demand of at least $D(\underline{p}_1) - x_{-j} + x_1 > x_1 > x_j$, since $D(\underline{p}_1) > x_{-j}$ as $\underline{p}_1 < \overline{p} < P(x_{-j})$. Thus $\Pi_j(p_j = \underline{p}_1^-) = \underline{p}_1 x_j \geq x_j \underline{p}_1 k > \Pi_j^*$, a contradiction.

It follows that $\overline{p} = P(r(x_{-1}) + x_{-1})$ and $\Pi_i^* = R(x_{-i})$ for any $i : x_i = x_1$. This is immediate when $x_1 > x_i$ for every $i \neq 1$, since then it must be that $1 \in \overline{H}$ and no $i \neq 1$ charges $\overline{p}$ with positive probability. The case $x_i = x_1$ for some $i \neq 1$ needs further reflection. The above contradiction is avoided by having $x_j = x_1$ for any $j \in \overline{H}$ for which no $i \neq j$ charges $\overline{p}$ with positive probability; further, $\Pi_i^* = R(x_{-i})$ for any $i : x_i = x_1$. Indeed, suppose the latter does not hold, i.e., $1 \in \overline{H}$ but $\overline{p}_i < \overline{p} = P(r(x_{-1}) + x_{-1})$ and $\Pi_i^* > \Pi_1^* = R(x_{-1})$ for some $i : x_i = x_1$.[3] Similarly as above, firm 1 would sell x_1 by charging $\underline{p}_i^-$,[4] hence $\Pi_1(p_1 = \underline{p}_i^-) = \underline{p}_i x_1 \geq \Pi_i^* > \Pi_1^*$, a contradiction.

APPENDIX

Here we report (with a few integrations) the argument by which BW establish that $x_1 R(x_{-j}) < x_j R(x_{-1})$, where, by assumption, $x_1 > x_j$, $j \in \overline{H}$ and no $i \neq j$ charges $\overline{p}$ with positive probability at a mixed equilibrium. Let $\overline{m} \equiv \sum_{i \neq j,1} x_i$, so that $x_{-j} = \overline{m} + x_1$ and $x_{-1} = \overline{m} + x_j$. By letting $\Theta(z) \equiv zR(\overline{m}+z) = zr(\overline{m}+z)P(\overline{m}+z+r(\overline{m}+z))$ our task is to prove that $\Theta(x_1) - \Theta(x_j) < 0$. By the envelope theorem, $\dot{\Theta}(z) = (r(\overline{m}+z) - z)P(\overline{m}+z+r(\overline{m}+z))$. In the following, we repeteadly use the fact that $\partial r(\overline{m}+z)/\partial z < 0$ for $\overline{m}+z < \widehat{q}$. If $r(\overline{m}+x_j) < x_j$, then $\dot{\Theta}(z) < 0$ for any $z \in [x_j, x_1]$, implying $\Theta(x_1) - \Theta(x_j) < 0$. The same implication is drawn if $r(\overline{m} + x_j) = x_j$ since then $\dot{\Theta}(z) < 0$ for $z \in (x_j, x_1]$. The argument is somewhat involved when $r(\overline{m} + x_j) > x_j$. Then, $r(\overline{m} + x) = x$ is solved for $x = x^* > x_j$. Similarly, $r(\overline{m} + x) = x_j$ is solved for $x = y^* > x^*$. Finally, in view of the latter and recalling that $r(\overline{m} + x_1) = r(x_{-j}) < x_j$, it is understood that $y^* < x_1$. This

[3] The case $\Pi_i^* < \Pi_1^* = R(x_{-1})$ is immediately dismissed since firm i earns no less than $R(x_{-1})$ by charging $\overline{p}$. (It earns $R(x_{-1})$ if firm 1 charges $\overline{p}$ with zero probability.)

[4] By charging $\underline{p}_i^-$ firm 1 obtains a residual demand of at least $D(\underline{p}_i) - x_{-1} + x_i > x_i = x_1$ since $\underline{p}_i < \overline{p} < P(x_{-1})$.

leads to[5]

$$\Theta(x_1) - \Theta(x_j) = \int_{x_j}^{x^*} \dot{\Theta}(z)dz + \int_{x^*}^{x_1} \dot{\Theta}(z)dz < \int_{x_j}^{x^*} \dot{\Theta}(z)dz + \int_{x^*}^{y^*} \dot{\Theta}(z)dz$$
$$= \Theta(y^*) - \Theta(x_j) = y^* R(r^{-1}(x_j)) - x_j R(\overline{m} + x_j)$$
$$= y^* x_j P(x_j + r^{-1}(x_j)) - x_j R(\overline{m} + x_j) = x_j \left[y^* P(x_j + \overline{m} + y^*) - R(\overline{m} + x_j) \right].$$

Finally, $y^* P(x_j + \overline{m} + y^*) \leq R(\overline{m} + x_j)$ as $R(\overline{m} + x_j)$ is by definition the maximum payoff in response to an aggregate output of $\overline{m} + x_j$ by rivals.[6] Thus $\Theta(x_1) - \Theta(x_j) < 0$.

[5] Actually, BW (p. 283) write $\int_{x_j}^{x^*} \dot{\Theta}(z)dz + \int_{x^*}^{x_1} \dot{\Theta}(z)dz \leq \int_{x_j}^{x^*} \dot{\Theta}(z)dz + \int_{x^*}^{y^*} \dot{\Theta}(z)dz$. Strict inequality must hold, however, because $\dot{\Theta}(z) < 0$ for $z \geq y^*$ so that $\int_{y^*}^{x_1} \dot{\Theta}(z)dz < 0$.

[6] BW conclude that $y^* P(x_j + \overline{m} + y^*) < R(\overline{m} + x_j)$. Equality cannot be excluded, however (it occurs if, by a fluke, $y^* = r(\overline{m} + x_j)$.)

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