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Asymmetric Attitude Towards Ambiguity and Price Stickiness

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Abstract - The paper address the topic of price stickiness, that is price elasticity below one with respect to nominal shocks. The paper original contribution is also in its approach to modelling: hard uncertainty and macroeconomics are linked within a micro-founded framework. The analysis is carried out in a general equilibrium setting where ambiguity and weak rational expectations are allowed. Ambiguity is introduced in the form of firms lack of knowledge about the relationship between changes in the aggregated stock of money and changes in money distribution across the heterogeneous consumers that populate the economy. It is shown that price stickiness can be generated with rational agents, even in the case a change of the money level does not alter money distribution, by assuming that firms attitude towards ambiguity is asymmetric: ambiguity aversion towards potential positive outcomes and ambiguity seeking towards negative ones. Such asymmetric attitude towards ambiguity is not arbitrary but finds some support in empirical research about economic behaviour.

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Keywords: ambiguity, money neutrality, price stickiness, heterogeneous agents.

The paper has been conceived and written independently from the author's activity at the bank; the views expressed are his own entirely.

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1. Introduction

According to some empirical studies - Greenwald and Stiglitz [1989] for example - the impact of the money stock level changes on both prices and quantities is a "stylised fact". The theoretical "puzzle" (Farmer [1992]) is how to reconcile price "stickiness" - that is price elasticity below one w.r.t. aggregated nominal shocks - with a "standard" model "based on agents rational choices".

The nominal rigidity topic is, of course, an old one, and has already been tackled in many ways: introducing menu costs (Blanchard and Kiyotaky [1987]), near rationality (Akerlof e Yellen [1985]), multiple equilibria (Cooper and John [1988]), staggered contracts (Fisher [1977]), information incompleteness (Lucas [1972]), money social externalities (Farmer [1993]).

This paper represents a contribution towards a solution of the "puzzle" through an application of hard uncertainty theory to microfounded macroeconomics. Hard uncertainty (ambiguity) is introduced in a general equilibrium framework with rational expectations inspired to the Lucas' idea of information incompleteness, allowing for what we call "weak rational expectations". It is shown how with our approach it is possible to overcome the critique that is moved against Lucas on the empirical ground: that because of readily availability of information on nominal aggregates his model is not able to explain the observed price rigidity persistence (Romer [1996]). In our paper information incompleteness is referred to money distribution, that is a much more difficult to observe variable than the money stock aggregated level. Rational firms face an "ambiguous" problem when evaluate the impact of monetary policies on the distribution of nominal endowments across heterogeneous consumers. In this context, non additive probabilities are used to show how firms asymmetric attitude towards ambiguity can *explain a long lasting prices stickiness phenomenon even with respect to monetary policies that do not alter the nominal endowments distribution among agents*.

2. The model

2.1. Studying a uniform monetary policy

Many conditions for prices not to react proportionally to monetary shocks are well known even when agents are rational. Specifically, when a change in the money stock is not (believed to be) once for all, is not immediately implemented, is not fully observed or does not distribute money proportionally to heterogeneous agents' initial nominal balances (Grandmont [1983]). Moreover, creation of money to finance Government spending in real goods (say the construction of a bridge or a highway) changes the economy endowment, therefore is likely to affect relative prices and the real equilibrium.

To avoid these effects, public expenditure is not considered (money is just distributed to consumers directly) and we concentrate on a very specific kind of monetary policy: a once for all, announced, immediately implemented, observed (in its macro features) and proportionally distributed change in the exogenous stock of money level, defined as a *uniform* monetary policy.

2.2. The economic framework

In order to concentrate on the very issue under investigation, our analysis will take place in a very simplified economic framework.

We study a sequence economy, with no future markets, where the per period economy is characterised by standard properties (homogeneity of demand and money neutrality under no information incompleteness and rational expectations).

The stock of money (M), which coincides with the consumers beginning-of-the-period aggregated nominal savings, is exogenous. There are H heterogeneous consumers and J firms engaged in monopolistic competition. Economic agents are subject to a cash-in-advance constraint: they have to use money for transactions.

Let q_j^h be agent h demand for good j and p_j good j price. Assuming a CES utility function, consumer h nominal demand for good j is given by:

$$q_j^h = \frac{p_j^{b-1}}{P^b} n^h \qquad b < 0$$

where n^h represent the per period consumer h nominal expenditure

$$n^h = \sum_j p_j q_j^h$$

and P is an aggregate price level index

$$P = \left(\sum_j \left(p_j \right)^b \right)^{\frac{1}{b}}$$

Production takes place using labour only, of which the offer is proportional to the real wage rate for each consumer ($l^{h,s} = \frac{w}{P}$). Technology is the same for all producers and return to scale are constant; each unit of labour produces a unit of good ($y_j = l_j$).

Timing is as follows. Firms set prices on the base of expected demand, the taken-as-given general price level (we look at a Nash-equilibrium outcome on the ground of common expectations) and the competitively set nominal wage. During this stage firms can observe (virtual) nominal

demand (this is to avoid the problem of over-under capacity and disequilibrium in the goods market). Goods are sold, then wages and profits are paid out.

Since wages are paid at end of the period, consumption is limited to the already accumulated monetary endowment because of the cash-in-advance constraint. Consumers intertemporal consumption/saving decisions are modelled in a very simple way: each consumer is willing to spend for consumption a fraction of his initial monetary holdings (M^h). That fraction is determined, in each period, by a constant consumer-specific parameter ($\phi^h \in [0,1]$)[1]. All ϕ^h s are known by firms. Agent *h* aggregate nominal demand is, therefore

$$n^h = \phi^h M^h$$

where M^h s are neither known by firms nor directly observable.

Then, aggregated nominal demand depends on the money distribution index $\Omega = \sum_h \phi^h M^h$, that is

$$n = \sum_h n^h = \Omega$$

The optimal *j*-price is given by maximisation of expected profits[2] and is given by

$$p_j = \frac{w}{g} \qquad b = \frac{g}{g-1} \quad g \in (0,1)$$

and under the common expectations assumptions prices are thus all equal and $P = J^{\frac{1}{b}} p$.

The temporary equilibrium of the economy is given by

$$\sum_j E[q_j] = \frac{w}{P} H$$

that is

$$\begin{cases} w^* = \dfrac{\Omega^e}{H} \\ p^* = \dfrac{\Omega^e}{gH} \\ q^* = \dfrac{gH\Omega}{J\Omega^e} \end{cases}$$

where *e* refers to expected values.

[1] An assumption that reminds the constant saving rate assumption in the classic Solow growth model, where, as pointed out also by Romer [1996], it is less harmful than it could appear to be (apart form not allowing welfare analysis, which actually is not the focus of our paper).

[2] Since firms are not intertemporally constrained they just maximise current profits.

It is clear that the market outcome depends on how expectations about Ω are formed.

3. Expectations

Before going forward about modelling expectations for Ω we consider Ω^e as given and observe that money neutrality holds only if Ω^e changes by the same proportion the money supply is observed to do. That is to say, for whatever reason, firms expect a non-uniform monetary policy – even if when, in fact, the policy *is* uniform – an equilibrium emerges where prices do not react proportionally to the change in the money supply level.

3.1. Strong rational expectation

It is immediate to show that assuming strong rational expectations for Ω (i.e. $E[\Omega] = \Omega$, an equilibrium can only be characterised by money neutrality. Therefore, by introducing heterogeneous agents it is possible to generate an equilibrium in which firms' expect other firms not to move prices proportionally to the money supply even if the monetary policy implemented is a uniform one. However, such equilibrium is not to be robust w.r.t. strong rational expectations.

3.2. Weak rational expectation

We now proceed arguing that imposing SRE about money distribution is a too stringent condition. *Purely subjective* evaluations are allowed: more precisely, we assume that true ambiguity, and not only risk, is involved when agents evaluate the effects of monetary policies on the distribution of money. Accordingly, Ω is taken as an ambiguous variable.

The attitude towards ambiguity is considered separately w.r.t. that towards risk[3]. One way to model agents choices under ambiguity, and that we shall follow, is the one based on *non additive probabilities* (Gilboa [1987], Schmeidler [1989])[4] (a brief introduction to non additive probabilities is in Appendix A).

[3] The attitude towards risk and that towards ambiguity differ in a fundamental way: while the former is related to the agent's preferences (the utility function), the latter is concerned with his events likelihood perception. Therefore, in principle, the two kind of attitudes are probably unrelated. This insight seems to be verified by some experimental studies. As noticed by Cohen et al. [1985] or Einhorn and Hogarth [1987], people do not display significant correlation between the attitude towards risk and ambiguity. Accordingly, even if it is common to assume risk aversion, this does not implies we necessarily have to work with ambiguity averse agents. In the following we assume risk neutrality when discussing about firms attitude towards ambiguity.

[4] A very brief introduction to this approach is in Appendix B.

4. Ambiguity and price stickiness

4.1. Ambiguity and Choquet expectations

It is assumed that a uniform monetary policy is implemented ($\forall h\ M_1^h = \lambda M_0^h$)[5]. However, only its macroeconomic features ($M_1 = \lambda M_0$) are observed, while its distributive characteristics are to be inferred by firms. Suppose that there are no (fully) "reliable" probabilities describing the relationship between changes in the money supply level and its distribution (or that the model is far too complex for agents to have understood and learnt it). We say that firms face *ambiguity*, not risk, and use the non additive probability approach to model their attitude towards it.

The key element is Ω_1^e of which the true value is $\lambda\Omega_0$, which we assume is also the expected value for Ω_1 in case firms are ambiguity neutral. Proposition 1 highlights that under non-neutrality towards ambiguity it is generally true that $\Omega_1^e \neq \lambda\Omega_0$.

PROPOSITION 1. When agents are ambiguity averse (seeking), the Choquet expected value of Ω is not greater (not lower) than it would be under additive probabilities.

Proof: see Appendix B. □

On one hand, proposition 2 says that under non neutrality towards ambiguity it is generally true that $\Omega_1^e \neq \lambda\Omega_0$, therefore, that non additive probabilities are enough to motivate, even under the assumptions that expectations are formed rationally (weakly), a price elasticity different from one. On the other hand, this result implies an asymmetric price behaviour. Under strict ambiguity aversion it happens that $\Omega_1^e < \lambda\Omega_0$, therefore, when the monetary policy is expansive ($\lambda > 1$) prices react less then proportionally, but, after a restrictive monetary policy ($\lambda < 1$) prices react more than proportionally (an overshooting result).

4.2. A rationale for price stickiness

We first observe that « utility » of firms, i.e. their profits, is, ceteris paribus, increasing in Ω: $\frac{\partial \pi_j}{\partial \Omega} \geq 0$. Therefore, assuming that an expansionary monetary policy does not reduce the nominal

[5] Subscripts refer to time (before and after the uniform monetary policy is implemented).

endowment of any agent[6] ($\frac{\partial M^h}{\partial M} \geq 0$), profits are an increasing function of the aggregate money level ($\frac{\partial \pi_j}{\partial M} \geq 0$) and an increase (reduction) of the money stock represents, ceteris paribus, a good (bad) news for firms.

As a consequence, proposition 2 implies also that *if* firms attitude towards ambiguity is characterised by a *conservative asymmetry*, that is the tendency to show ambiguity aversion for prospective gains and ambiguity seeking for prospective losses, the new expected value of a random variable after some shock tend to remain closer to the former value w.r.t. the case of additive probabilities (and this is the reason for the adjective *conservative*).

In our model this is enough to motivate, from a theoretical point of view, price stickiness in front of nominal *uniform* monetary policies.

Is then asymmetric conservative attitude plausible? The point made is that attitudes towards ambiguity may well be "state dependent". In particular we need to focus on the distinction between "good" and "bad" uncertain outcomes, aiming to argue that attitude towards ambiguity is "asymmetric" according to which of the two cases prevails. The basic idea is that to explain people choices one has to focus on changes in endowments[7]. That may well not make any difference about outcomes ranking, but it seems quite correlated with the attitude towards risk and ambiguity.

Actually, there are only a few empirical investigations about the attitude towards ambiguity.

Cohen et al. [1985] have performed a laboratory-type study, submitting ten experiments involving choices between sure, risky and ambiguous outcomes to a sample of 134 subjects. Experiments are classified according to the positive or negative sign of the "prize". The main finding of interest to us is that the evidence is unambiguously in favour of ambiguity aversion as a typical behaviour when people face a possibly good outcome. In the opposite situation, that of a likely bad outcome, Cohen et al. conclude that the mode is located at ambiguity neutrality, but also that the experimental distribution is asymmetric, with more weigh for ambiguity seeking.

Also Einhorn and Hogarth [1990] use a student-sample laboratory experiment. Like in the Cohen et al. study, they find strong evidence that people are ambiguity averse when uncertain prizes are positive. The result about the typical behaviour when negative prizes are involved is less determinate, but the supported result is at least a lower degree of ambiguity aversion, if not ambiguity seeking.

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[6] And that a contractionary monetary policy does not increase any nominal endowment.

[7] The idea that not only final outcomes but also changes in the state are relevant for agents' choices in not new. See, for example, in a different context, the works under the name of "prospect theory" (Tversky and Kahneman [1992]).

To sum up these results are broadly consistent with the hypothesis of ambiguity aversion for gains and ambiguity neutrality/seeking for losses.

4.3. The duration of price adjustment

In the model developed so far price stickiness is short-living. Firms need only one observation to learn the new value of Θ_j after a monetary shock.

This seems to make the model weak w.r.t. the lack of persistence critique. However this is a false problem. Almost immediate learning is entirely due to the simplicity of the framework specification. With more complex functions or other sources of ignorance (for example ambiguity about the demand functions parameters) slow (not immediate) learning is a normal result.

A slight change in the framework, namely a widely used i.i.d. disturbance, is enough to get that result[8]. To see that suppose ϕ^h are "disturbed" by a common random shock ($\theta \in [0,1]$): $n^h = \theta\phi^h M^h$. Then, the economy equilibrium depend on the "compound" random index $\theta\Omega$

$$\begin{cases} w^* = \dfrac{(\theta\Omega)^e}{H} \\ p^* = \dfrac{(\theta\Omega)^e}{gH} \\ q^* = \dfrac{gH\theta\Omega}{J(\theta\Omega)^e} \end{cases}$$

and learning about Ω cannot be immediate.

5. Conclusions

The paper has explored the possible role of firms attitude towards ambiguity to explain the phenomenon of prices rigidity in a general equilibrium framework with rational maximising agents. In brief, what we find is that firms characterised by a non neutral attitude towards ambiguity may well plan not to alter their prices proportionally w.r.t to the observed change in the stock of money.

Moreover, it turns out that an "asymmetric" attitude towards ambiguity rationalises also the observed behaviour of prices, of which the elasticity seems to be less than one w.r.t. both expansive and restrictive monetary policies. In fact, if firms are ambiguity averse when facing a potential good outcome and ambiguity seeking when the random outcome is bad, sluggish prices are a generic outcome.

This result can be interpreted from two points of view. The first is to consider the asymmetric attitude towards ambiguity as a behavioural axiom of which the empirical implications are not falsified by real word observations and we could content with this achievement. The second is to deal with the problem of axioms "plausibility" on the ground of empirical evidence. We go through this way too, finding some support to the asymmetric attitude hypothesis in the empirical literature. Some studies, in fact, favours it, while others do not reject it.

As a general conclusion, we think we have provided a contribution to support the idea the "puzzle" of nominal price stickiness is not a puzzle at all - even within the realm of rational maximising agents - and that one should not be surprised to see that nominal rigidities characterises the actual working of our economies.

[8] It is worth noting that the same "trick" does not work in the Lucas model, since there the source of short-living real effects of money is the direct observability of the relevant macroeconomic variable(s).

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Appendix A

An additive probability function $\rho : \Omega \to R^+$ is defined by the following properties:

i) $\rho(\varnothing) = 0 \quad \rho(\Omega) = 1$

ii) $A \subset B \Rightarrow \rho(A) \leq \rho(B)$

iii) $\rho(A \cup B) - \rho(A \cap B) = \rho(A) + \rho(B)$

A non additive measure of capacity $\eta : \Omega \to R^+$ has to satisfy conditions *i*) and *ii*) only. As a consequence it is possible, for some partition $\{A_i\}$ of Ω, that $\sum_i \eta(A_i) < \eta(\cup_i A_i)$ or $\sum_i \eta(A_i) > \eta(\cup_i A_i)$.

The first case is the one taken to model ambiguity aversion, and $\eta(\cup_i A_i) - \sum_i \eta(A_i) > 0$ is a measure of ambiguity aversion. Probabilities are then said super-additive, or *convex* since $\forall A, B \subset \Omega$ it is $\eta(A \cup B) - \rho(A \cap B) \geq \eta(A) + \eta(B)$.

With non-additive probabilities it is possible to give an axiomatic foundation to an expected representation of utility which uses the Choquet integral (or summation)[9].

Let $X = \{X_1, X_2, ..., X_n\}$ be a random variable along with the relevant capacities $\{\eta_1, \eta_2, ..., \eta_n\}$. Assume capacities are super-additive: the possible realisations of the random variable are then ordered according to their associated utility levels. Assume it is $U(X_1) > U(X_2) > ... > U(X_n)$ then the Choquet expected utility is

$$CEU(X) = \sum_i W_i U(X_i)$$

where

$$W_i = \left[\eta(X_1 \cup X_2 ... \cup X_{i-1} \cup X_i) - \eta(X_1 \cup X_2 ... \cup X_{i-1})\right]$$

The best outcome X_1 receives a weight equal to the original capacity ($W_1 = \eta_1$) while all the others outcome receive a weight greater or equal to the original capacity ($\forall i > 1 \; W_i \geq \eta_i$): weights are thus biased towards the less favourable outcomes, from which the association between convex (super-additive) probabilities and "pessimism".

[9] See Schmeidler [1989], Gilboa [1987].

Appendix B

We use the relationship between non-additive probabilities and the multiple prior approach to modelling the attitude towards ambiguity.

The question we start from is: what would additive probabilities be like if a pessimistic agent became ambiguity neutral? We postulate that such hypothetical additive probabilities would just represent some upscailing of the sub-additive ones, that is an element of the core of the super-additive probabilities[10].

But, then, we know that the Choquet expectation w.r.t. the non-additive probability is not greater than the standard expected value taken w.r.t. any of the relevant additive probability distribution in the core[11].

In our case, that is to say that[12]

$CE(\pi) \leq E_{core}(\pi)$

or

$(p-w)CE(\Omega) \leq (p-w)E_{core}(\Omega)$

which implies that

$CE(\Omega) \leq E_{core}(\Omega)$

For a optimistic agent the same argument can be used, concluding that the Choquet expectation w.r.t. the sub-additive probability is not inferior than the standard expected value taken w.r.t. any of the additive probability distribution in the relevant core.

[10] Let η a concave capacity on $X = \{x_i\}$. The core of η is defined as the set of additive probability distributions of which the components are not inferior to those of the capacity $Core(\eta) \equiv \left\{ p = \{p_i\} \mid p_i \in [0,1] \; and \; p_i \geq \eta_i \; and \; \sum_i p_i = 1 \right\}$.

[11] Gilboa [1989], for example.

[12] CE stands for *Choquet Expectation*, while E represents the standard expectation operator.