QUADERNI

Università degli Studi di Siena
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Financial Fragility and Economic Fluctuations: Numerical Simulations and Policy Implications

n. 407 – Ottobre 2003

Abstract: This paper proposes a simple prototype model that describes the complex dynamics of a sophisticated monetary economy. The interaction between the current and intertemporal financial constraints of economic units brings about irregular fluctuations at the micro and macro levels. By means of qualitative dynamic analysis and numerical simulations, we reformulate in more operational terms, and extend in a number of new directions, the model suggested recently by one of the authors (Vercelli, 2000) to study the interaction between financial fragility, modelled in terms of structural instability, and dynamically unstable financial fluctuations.

JEL Classification: B41; E12; E32; E52

Keywords: Complex dynamics, Structural instability; Financial fragility; Economic fluctuations; Numerical simulations

Acknowledgements: We would like to thank Roberto Dieci and the participants in the Session on "Experimental and Simulation Approaches" at the Conference on "Complex Behavior in Economics" (Aix en Provence, Marseilles, France, May 8-12, 2003) for their useful comments and suggestions. All remaining errors and shortcomings are our responsibility. Financial support from the Ministry of Education, University and Research (National Research Group 2000 on "Interazioni fra agenti eterogenei, mercati imperfetti, fragilità finanziaria e politica monetaria") is gratefully acknowledged.

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1 Introduction

In a sophisticated monetary economy the interaction between current and intertemporal financial constraints generates cyclical fluctuations characterised by dynamic instability. This has been pointed out by a prestigious and diversified tradition of thought including, among others, Wicksell (1898), Irving Fisher (1933), Keynes (1936), and Minsky (1982). What in our opinion has not been thoroughly analysed is the role of structural instability in this process. In this paper we intend to contribute to this analysis by interpreting the financial fragility of economic units not as an index of dynamic instability, as is usual in the literature mentioned above, but as an index of structural instability. In fact, the higher the degree of financial fragility the smaller the size of the shock that is sufficient to bring about a structural change in the behaviour of the unit.[1] For well known reasons (well analysed by Minsky, 1982), the fragility of financial units fluctuates pro-cyclically with the financial fluctuations, increasing progressively in the boom phase the danger of a severe financial crisis. This raises an awkward dilemma for policy: when and how is it necessary to thwart the boom in order to avoid over-investment and an ensuing financial crisis? In order to answer this question it is crucial to understand the nature of the feedback between dynamic and structural financial instability.

To this end, we present a model that tries to capture in the simplest possible way the nexus between the financial fragility of economic units, conceived in terms of structural instability, and dynamically unstable financial fluctuations.

The approach here adopted is based on that suggested recently by one of the authors (Vercelli, 2000). The original model was formalised in such a way as to render it intuitively understandable in economic terms, while the model elaborated here is expressed in more operational terms in order to facilitate the application of standard mathematical techniques such as qualitative dynamic analysis and numerical methods. We hope that in this version the approach may better reveal its constructive potential and act as a starting point for more complex and detailed analyses. To this end we have retained as far as possible the simplicity of the original model, aiming at capturing what we believe to be the essential mechanism of financial fluctuations and crisis: the interaction between current and intertemporal financial conditions. If this building block

[1] We define a system as structurally unstable whenever in consequence of a small shock ε it changes the qualitative properties of its dynamic behaviour. We have introduced elsewhere (Vercelli 1991 and 2001) the distinction between structural instability in the strict mathematical sense when the shock ε is infinitesimal, and ε-instability when the shock is small but finite. In the latter case ε measures the minimum size of a shock that brings about a qualitative change in the dynamic behaviour of the system. The concept of structural instability here utilised is that of ε-instability.

can withstand critical scrutiny, other important features of financial fluctuations and crises may be added in the future.

The structure of the paper is as follows. First, in Section 2, Vercelli's original heuristic model of the dynamic behaviour of a financial unit is restated and its (local) stability properties are analysed in some detail. The dynamic behaviour of the model is based on the feedback between the current financial flows of a financial unit and its financial fragility. Such feedback is shown to bring about dynamically unstable fluctuations in the behaviour of the unit and, as a consequence, the possibility of bankruptcy. In Section 3 a simpler variant of the original model is suggested in order to facilitate the aggregation of the behaviour of the single units. In Section 4 the modified model is taken as a basis for deriving, through trivial but rigorous aggregation procedures, its macroeconomic counterpart. The macro model so obtained is a *piecewise linear system* such that the explosive fluctuations in the behaviour of single financial units generally translate into persistent fluctuations of the aggregate financial variables. In Section 5 the main implications of the dynamic behaviour of the aggregate model are shown and interpreted by means of numerical simulations. In Section 6 a few policy implications of the aggregate model are discussed in some detail. In Section 7 some concluding remarks on the potential and the limits of the approach here outlined are briefly sketched.

2 Dynamic behaviour of the financial units

In this paper all decision makers are modelled as financial units. The rationale of this approach rests on the observation that the financial constraints and objectives of economic agents have assumed a crucial role in shaping their behaviour. This is a long-run process already pointed out, among others, by Wicksell (1898) and Keynes (1936) that has accelerated in the last two decades. The Financial Intensity Ratio of Goldsmith (FIR), i.e. the ratio between financial and real wealth, doubled in this period in the USA, UK, and France (where the FIR reached values between 2.1 and 2.9 %) and increased almost as much in Italy and Germany (where the FIR reached values between 1.3 and 1.4%) (Nardozzi, 2002, p. 15). This trend has been observed not only in corporations but also in households. For example, the ratio of stocks to household wealth exceeded 55% in the U.K., 40% in the USA, 30% in France and Italy (ibidem, p. 17). The analysis of the financial determinants of economic behaviour is therefore becoming a general issue that should not be confined to financial corporations.

In this paper we seek to focus in particular on the interaction between the decisions of financial units and their current and intertemporal financial constraints, without examining the details of their decision-making processes.

Each financial unit i is characterised in each period t by a certain amount of financial outflows e_{it} in consequence of its purchases of goods and services, and financial inflows y_{it} from the sale of its goods and services. The *current financial ratio* k_{it} is then defined as the ratio between total financial outflows and total financial inflows realised by the financial unit i in a certain period:

$$k_{it} = \frac{e_{it}}{y_{it}}$$

When this ratio is greater than one, the financial unit reduces the current outflows or increases debt (i.e. the commitment to future outflows) or a mix of the two. This, in turn, affects the financial constraints faced by the unit in the future.

To take account of this, we consider, as the crucial variable that defines the financial robustness of the financial unit i, its *intertemporal financial ratio*, i.e., the ratio between the sum of discounted expected outflows and the sum of discounted expected inflows.

If r stands for the interest rate, k_{it}^* for the intertemporal financial ratio, and e_{it+n}^* and y_{it+n}^* for the expected outflows and inflows at time $t + n$ respectively, then:

$$k_{it}^* = \frac{\sum_{n=0}^{m} \left[e_{it+n}^* / (1+r)^n \right]}{\sum_{n=0}^{m} \left[y_{it+n}^* / (1+r)^n \right]}$$

where, for $n = 0$, $e_{it}^* = e_{it}$ and $y_{it}^* = y_{it}$ (Vercelli 2000, pp. 144-145).

Given this definition it follows that the *condition for financial sustainability* of the financial unit i is:

$$k_{it}^* \leq 1$$

We may now define the financial fragility of a unit as its structural sensitivity to shocks (in the sense of ε-structural instability defined in footnote 1), whose degree is measured by the minimum shock that would jeopardize its solvency. In our case, therefore, the degree of financial fragility is measured by $1 - k_{it}^*$. In this definition financial fragility defines the structural instability of the financial unit, i.e. the distance from the threshold that by definition changes the qualitative characteristics of its behaviour.

If we suppose that each unit defines a desired value of financial fragility ($1-\mu_i$) beyond which it is reluctant to go, taking account of the risk of its financial position and its degree of risk aversion, we have that whenever $k_{it}^* \geq 1-\mu_i$, the financial unit reacts by reducing k_{it} in order to reduce k_{it}^*. On the other hand, whenever $k_{it}^* < 1-\mu_i$, the financial unit tends to increase the size of its investment which implies an increase in k_{it}. Of course the larger is μ_i the lower the probability of a shock of larger size jeopardizing its solvency. The choice of μ_i is related to the degree of risk aversion. Risk lovers would choose a negative μ_i (Ponzi finance) hoping in a positive shock, but we can assume that in general most financial units choose a sufficiently large μ_i to be fairly safe.

The simple feedback mechanism between k_{it} and k_{it}^* we have just described is captured by the following dynamic system:

$$\frac{k_{it+1}^* - k_{it}^*}{k_{it}^*} = \beta\left(k_{it} - 1\right) \tag{1}$$

$$\frac{k_{it+1} - k_{it}}{k_{it}} = -\alpha\left[k_{it}^* - \left(1-\mu_i\right)\right] \tag{2}$$

where α, $\beta > 0$ for all i.[2]

Equations (1) and (2) define the dynamics of the monetary economy under consideration. The equilibrium points (or steady states) of the model economy – obtained by setting $k_{it+1}^* = k_{it}^* = \bar{k}_i^*$ and $k_{it+1} = k_{it} = \bar{k}_i$ in (1) and (2) – are the origin of the axes and $P_i = (1 - \mu_i, 1)$. In order to determine their nature, we consider the Jacobian matrix of the system, defined as:

$$\mathbf{J}\left(\bar{k}_i^*, \bar{k}_i\right) = \begin{bmatrix} 1+\beta\left(\bar{k}_i - 1\right) & \beta\bar{k}_i^* \\ -\alpha\bar{k}_i & 1-\alpha\left(\bar{k}_i^* - 1 + \mu_i\right) \end{bmatrix}$$

Computing $\mathbf{J}\left(\bar{k}_i^*, \bar{k}_i\right)$ at the origin, we obtain:

[2] Equations (1) and (2) are the same as in the original formulation of the model (Vercelli 2000, p. 147), apart from the fact that we have left out the exogenous shocks which appear in both right-hand sides. Though from the very beginning of this paper we have assumed the existence of shocks that intervene in the crucial definition of financial fragility, we do not need at this stage to make explicit their existence in the equations of the model.

$$\mathbf{J}(0,0)=\begin{bmatrix}1-\beta & 0\\ 0 & 1+\alpha(1-\mu_i)\end{bmatrix}$$

so that the eigenvalues of the Jacobian matrix in this case are *always real and distinct* and such that:

$$\lambda_1 = 1-\beta$$
$$\lambda_2 = 1+\alpha(1-\mu_i) > 1, \text{ always}$$

Thus, we can conclude that, for $0 < \beta < 2$, the origin is a *saddle point,* whereas for $\beta > 2$ the origin is an *unstable node*. In Fig. 1(*a*), for example, where $\alpha = \beta = 0.1 < 2$, the origin of the axes proves to be a saddle point.[3]

On the other hand, computing $\mathbf{J}\left(\bar{k}_i^*, \bar{k}_i\right)$ at P_i, we obtain:

$$\mathbf{J}(1-\mu_i,1)=\begin{bmatrix}1 & \beta(1-\mu_i)\\ -\alpha & 1\end{bmatrix}$$

Thus, in this case, the characteristic equation is:

$$\lambda^2 - 2\lambda + 1 + \alpha\beta(1-\mu_i) = 0$$

such that the eigenvalues are *two complex conjugate numbers*, greater than one in modulus:

$$\lambda_{1,2} = 1 \pm i\sqrt{\alpha\beta(1-\mu_i)}$$

We can then conclude that P_i is an *unstable focus* (see Fig. 1(*a*),(*b*)). As a consequence, in the region of validity of the linear approximation, the financial fluctuations resulting at the microeconomic level from the interaction between the current and intertemporal financial ratios are characterised by dynamic instability.

[3] All figures and simulations presented in the paper can be easily reproduced with simple MATLAB codes written by S. Sordi that are available upon request.

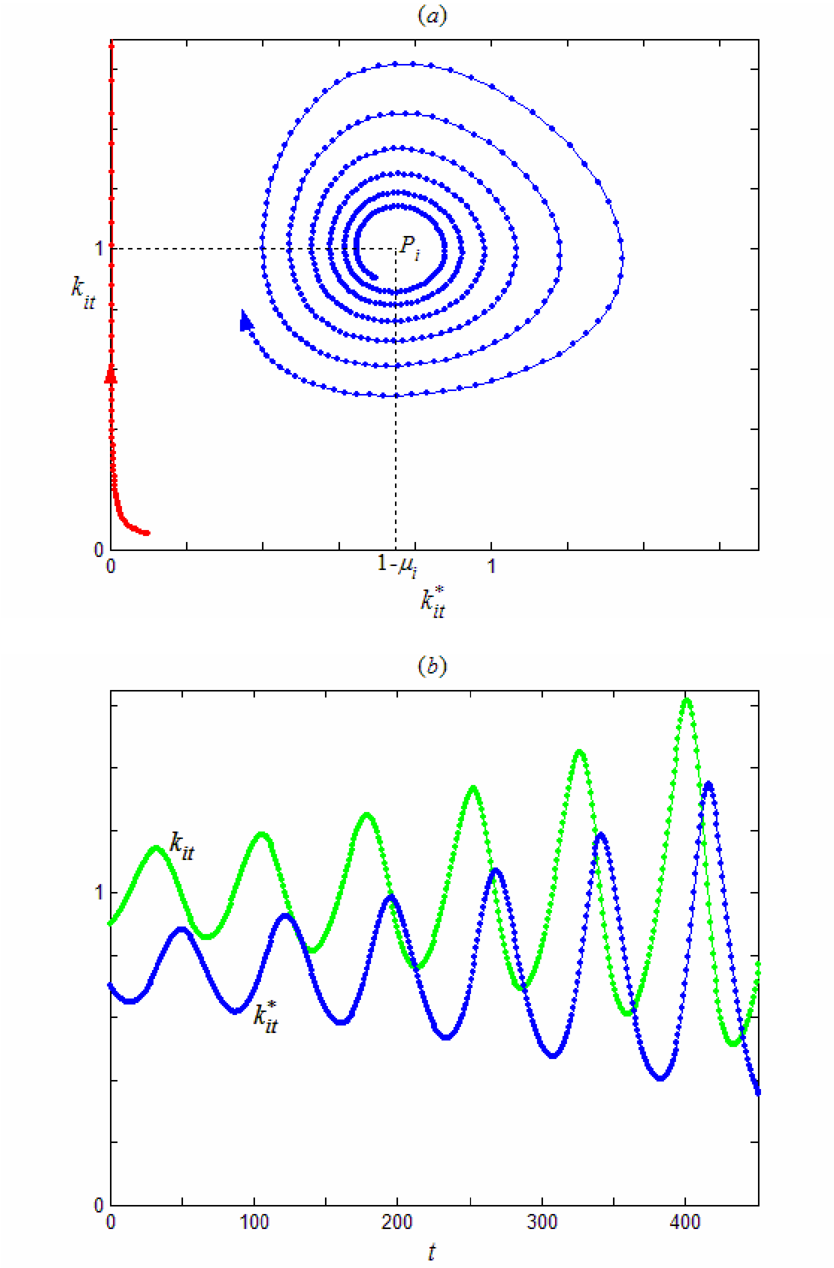

Fig. 1. (*a*) Phase diagram for the dynamic system (1)-(2) of the original model with $\alpha = 0.1$, $\beta = 0.1$, $\mu_i = 0.25$ and initial conditions (0.1, 0.05), (0.7, 0.9) and (*b*) k^*_{it} and k_{it} versus time with initial condition (0.7, 0.9)

3 The dynamics of the financial unit: an alternative formulation

Before proceeding with our investigation, it is convenient to simplify the dynamic system (1)-(2) by expressing the feedback between the two variables k_{it} and k^*_{it} in terms of *first differences* rather than *growth rates*. While preserving its basic economic contents, this simpler formulation will allow us to concentrate on the aggregate features of the model since, as will become apparent in the next section, it will simplify the derivation of the corresponding macroeconomic model.

Writing first differences rather than growth rates in the left-hand sides of both (1) and (2), the dynamic system becomes:

$$k^*_{it+1} = k^*_{it} + \beta\left(k_{it} - 1\right) \tag{3}$$

$$k_{it+1} = k_{it} - \alpha\left[k^*_{it} - \left(1-\mu_i\right)\right] \tag{4}$$

What we have obtained is a *linear* dynamic system which has, as a unique steady state, the positive equilibrium P_i of the original model. Moreover, as in the original model, the latter proves to be an *unstable focus*.[4]

Thanks to its simplicity, this alternative formulation of the feedback between the current and the intertemporal financial ratio is more suitable than the original one for investigating the consequences of further modifications and extensions of the model. To introduce one of these, let us notice that in (4), it is assumed that the financial unit adjusts its current financial ratio depending on the deviation of the intertemporal financial ratio from the desired level. As in the original model, moreover, it is assumed that the adjustment always takes place at a constant velocity, *independently of how far the intertemporal financial ratio is from the desired level* $\left(1-\mu_i\right)$. A possible, more plausible, alternative is to assume that the adjustment of the current financial ratio takes place at a speed that *does* positively depend on the absolute value of the deviation of the intertemporal financial ratio from $\left(1-\mu_i\right)$:

$$\alpha = \alpha\left(\left|k^*_{it} - \left(1-\mu_i\right)\right|\right)$$

where $\alpha'\left(\cdot\right) \geq 0$, or, by taking a linear approximation:

$$\alpha\left(\left|k^*_{it} - \left(1-\mu_i\right)\right|\right) \approx \alpha_0 + \alpha_1\left|k^*_{it} - \left(1-\mu_i\right)\right| \tag{5}$$

where $\alpha_0 > 0$ and $\alpha_1 \geq 0$.

Inserting (5) in (4) we obtain the following modified version of the second equation of the dynamic system of our model:

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[4] Indeed, if we write the homogeneous system in matrix form, and we indicate the coefficient matrix with **A**, then we find that tr(**A**) = 2 and det(**A**) = 1 + $\alpha\beta$ > 1. Thus, the discriminant of the characteristic equation is always negative and the two conjugate complex roots are always greater than one in modulus. See, for example, Lorenz (1993, p. 117).

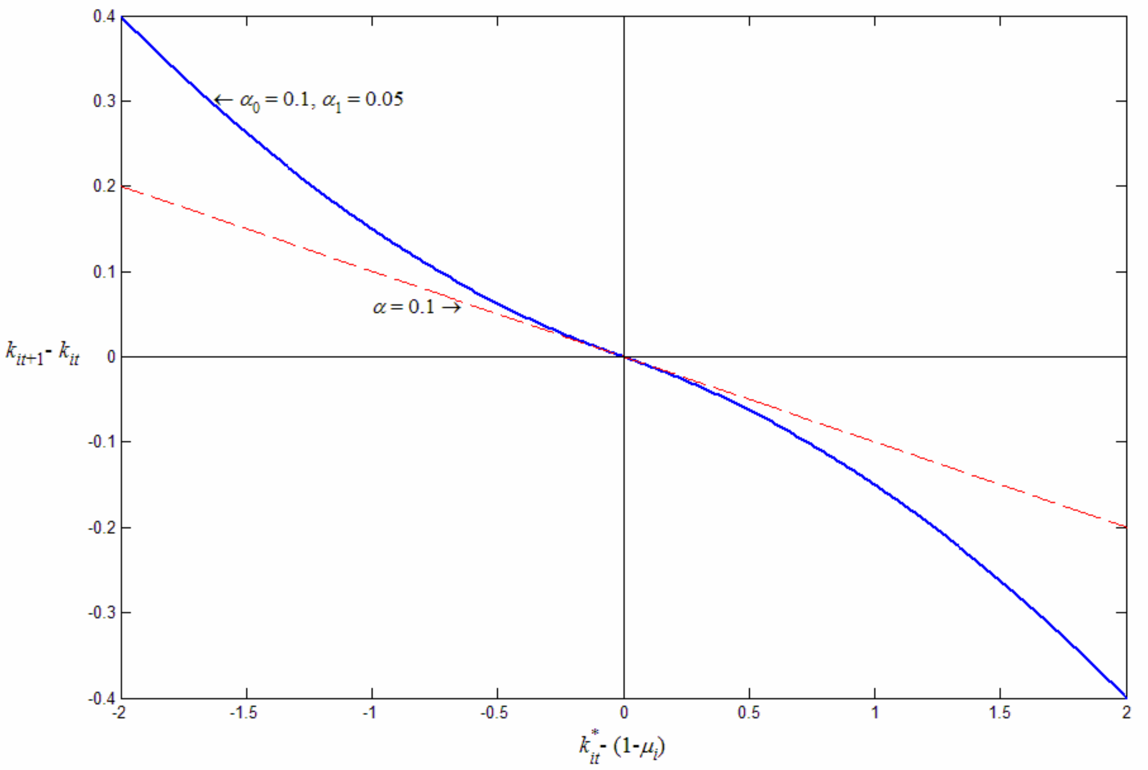


Fig. 2. The non-linear adjustment of the current financial rate to the difference between the intertemporal financial ratio and its desired level

$$\begin{aligned} k_{it+1} &= k_{it} - \alpha_0\left[k^*_{it} - (1-\mu_i)\right] - \alpha_1\left|k^*_{it} - (1-\mu_i)\right|\left[k^*_{it} - (1-\mu_i)\right] \\ &= k_{it} - \alpha_0\left[k^*_{it} - (1-\mu_i)\right] - \alpha_1 \operatorname{sign}\left(k^*_{it} - (1-\mu_i)\right)\left[k^*_{it} - (1-\mu_i)\right]^2 \end{aligned} \quad (4')$$

As shown in Fig. 2, this modification simply implies that, unlike in the previous case that is also drawn in the figure for comparison, the relation between the adjustment $k_{it+1} - k_{it}$ and the difference $k^*_{it} - (1-\mu_i)$ is now non-linear: the more (the less) in absolute value the intertemporal financial ratio is away from its desired level, the more (the less), in proportion, the financial unit is willing to vary its current financial ratio.[5]

An important feature of this non-linear specification is that, although it contains the modulus $\left|k^*_{it} - (1-\mu_i)\right|$, P_i is still a steady state of the modified dynamic system (3)-(4′). Moreover, the right-hand side of (4′) is differentiable at P_i and such that:

$$\left.\frac{\mathrm{d}k_{t+1}}{\mathrm{d}k^*_{it}}\right|_{P_i} = -\alpha_0 - 2\alpha_1 \operatorname{sign}\left(k^*_{it} - (1-\mu_i)\right)\left[k^*_{it} - (1-\mu_i)\right]\Big|_{P_i} = -\alpha_0$$

[5] Of course, the non-linear specification (4′) reduces to the original linear case when $\alpha_0 = \alpha$ and $\alpha_1 = 0$.

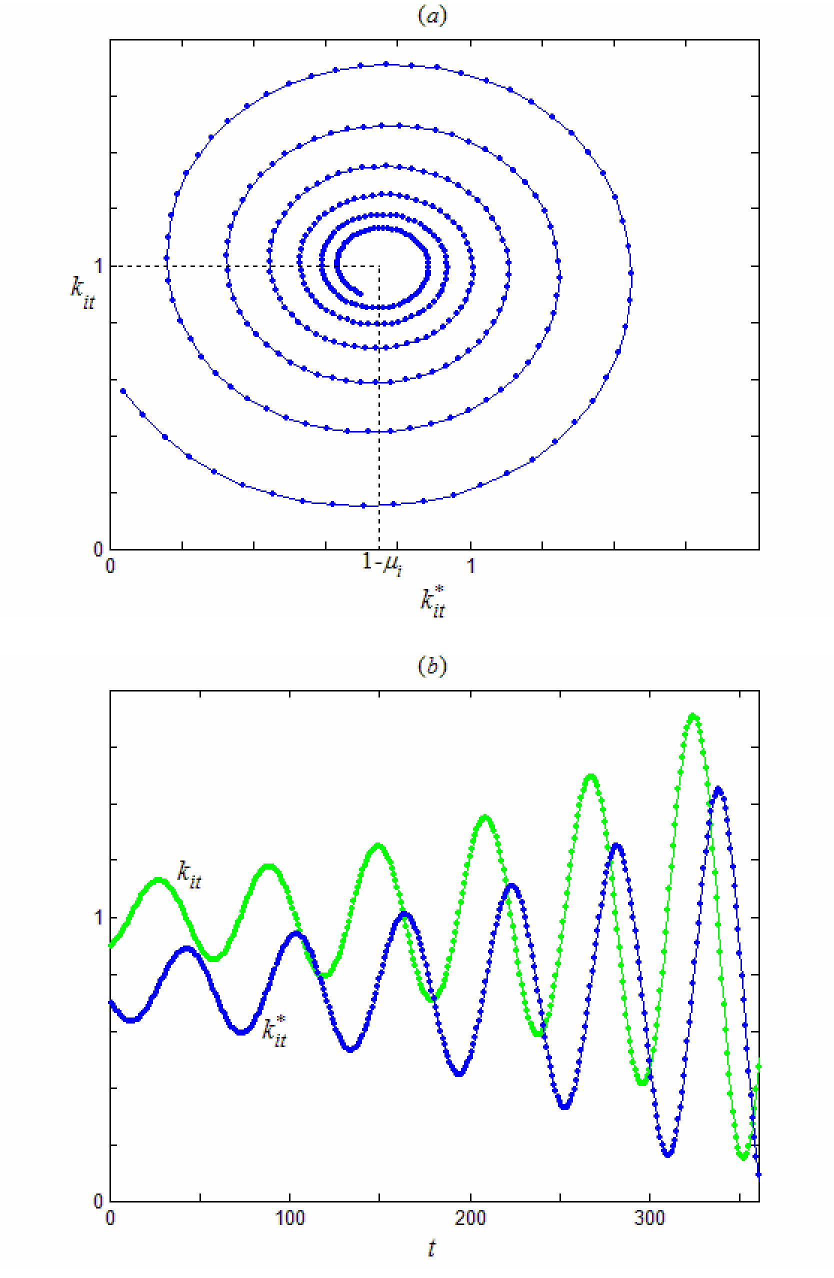


Fig. 3. (a) Phase diagram for the dynamic system (3)-(4′) with $\alpha_0 = 0.1$, $\alpha_1 = 0.05$, $\beta = 0.1$, $\mu_i = 0.25$ and initial condition (0.7, 0.9) and (b) k^*_{it} and k_{it} versus time

Thus, apart from the change in notation from α to α_0, the non-linear specification of the adjustment mechanism we have introduced has no effect on the Jacobian of the dynamic system at the steady state P_i. As a consequence, the stability properties of the linear dynamic system (3)-(4) remain valid for the modified non-linear dynamic system (3)-(4′). The results of the numerical simulations for the modified model are shown in Figs. 3(a),(b).

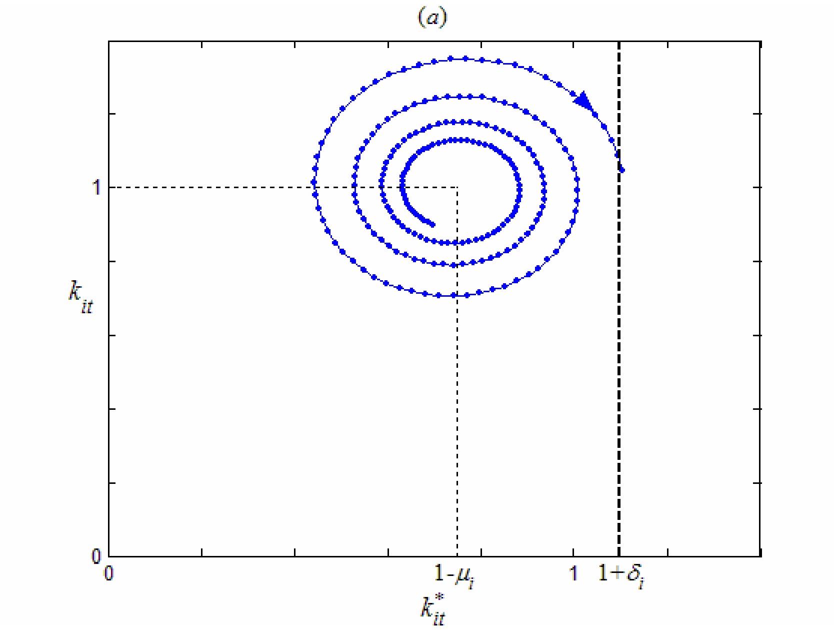
To sum up, it is worth stressing that all versions of the model we have considered seem to confirm the basic dynamic instability of the fluctuations generated by the interaction between the current and intertemporal financial constraints faced by financial units in a sophisticated monetary economy. It seems remarkable, in particular, that such instability characterises the model *even in the last case we have analysed*, i.e., under the hypothesis of more reasonable behaviour on the part of financial units and neglecting the "irrational exuberance" that characterises the expectations of financial units in the boom.

An important consequence of this intrinsic dynamic instability of the system is that, in the absence of constraints or interventions, sooner or later the financial unit will reach the zone characterised by financial unsustainability (i.e., where $k_{it}^* > 1$). When this happens, if the unit does not succeed in coming back very quickly to the region of financial sustainability, it is bound to become virtually insolvent and to go bankrupt unless it obtains further credit from other units.

Since in the present framework the existence of stocks is not explicitly considered we can represent what happens when a financial unit goes bankrupt simply by assuming that the dynamic system (3)-(4′) only holds until the intertemporal financial ratio of the unit is less than or equal to a certain value $\left(1+\delta_i\right)$, which we take to represent the *maximum financial exposure of a certain unit accepted by its creditors*. This refinement of the model redefines the financial fragility of the economic unit as $\left(1+\delta_i\right)-k_{it}^*$, where the parameter δ_i may be positive or negative depending on its financial position and reliability. In what follows, for simplicity, we assume that the distribution of firms' financial positions is such that, on average, δ_i is equal to zero.

The consequences of such a change for the dynamics of the current and intertemporal financial ratios may be appreciated by examining Figs. 4(*a*),(*b*). Of course, a thorough modelling of the bankruptcy process would require a more sophisticated model. However, the figures illustrate the basic idea of bankruptcy that occurs when a financial unit has reached the maximum financial exposure accepted by its creditors, which is represented in the figures by the vertical straight lines at $\left(1+\delta_i\right)$ and is forced to converge rapidly towards economic irrelevance (what is often called the "death" of the financial unit).[6]

[6] Of course, while a few financial units "die", others "are born". The "demography" of financial units is believed to be very important to account for the actual dynamics of the economic system (Delli Gatti *et al.*, 2003). However, for the sake of simplicity, in this early stage of implementation of the model it is going to be completely neglected.



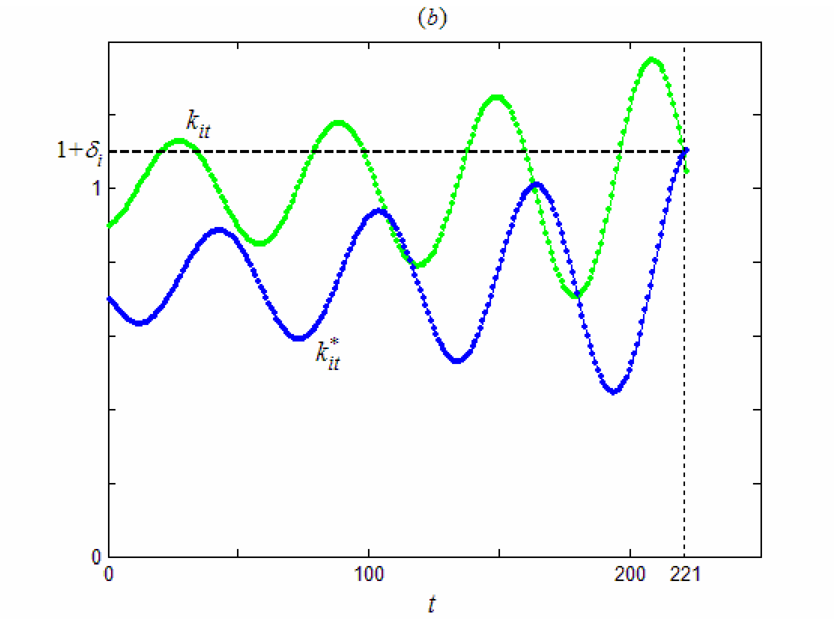


Fig. 4. (*a*) Phase diagram using the modified version of the model with bankruptcy with $\alpha_0 = 0.1$, $\alpha_1 = 0.05$, $\beta = 0.1$, $\mu = 0.25$, $\delta_i = 0.1$ and initial condition (0.7, 0.9); (*b*) k^*_{it} and k_{it} versus time

4 The dynamic system for the entire economy

As we have already stressed, the formulation of the model with first differences of the variables in the left-hand side rather than growth rates, makes it much easier to aggregate the micro behaviour of financial units in order to represent the financial behaviour of the economy as a whole. Thus, it provides rigorous, though trivial, micro-foundations to the analysis of macroeconomic behaviour and policy.

In fact the micro system represented by equations (3) and (4) is linear and therefore makes it straightforward to calculate the average values of the variables. In order to do so, let s denote the number of financial units operating in the economy. Then, summing over units, from (3)-(4), we obtain:

$$\sum_{i=1}^{s} k_{it+1}^{*} = \sum_{i=1}^{s} k_{it}^{*} + \beta\left(\sum_{i=1}^{s} k_{it} - s\right) \tag{6}$$

$$\sum_{i=1}^{s} k_{it+1} = \sum_{i=1}^{s} k_{it} - \alpha\left[\sum_{i=1}^{s} k_{it}^{*} - \left(s - \sum_{i=1}^{s} \mu_i\right)\right] \tag{7}$$

Finally, from (6) and (7), using K_t and K_t^* to denote the average current and intertemporal financial ratios respectively, such that

$$K_t = \frac{\sum_{i=1}^{s} k_{it}}{s},\ K_t^* = \frac{\sum_{i=1}^{s} k_{it}^{*}}{s},$$

we obtain for the entire economy:

$$K_{t+1}^{*} = K_t^{*} + \beta\left(K_t - 1\right) \tag{8}$$

$$K_{t+1} = K_t - \alpha\left[K_t^{*} - \left(1 - \mu\right)\right] \tag{9}$$

where $\mu = \sum_{i=1}^{s} \mu_i$.

We may show that the dynamic system (8)-(9) is a good approximation of the dynamic system for the entire economy even when the financial units follow a more sophisticated adjustment mechanism as in (3)-(4′). Indeed, the only difference is that, from (4′), summing over the s financial units, one obtains:

$$\sum_{i=1}^{s} k_{it+1} = \sum_{i=1}^{s} k_{it} - \alpha_0\left[\sum_{i=1}^{s} k_{it}^{*} - \left(s - \sum_{i=1}^{s} \mu_i\right)\right] - \alpha_1 \sum_{i=1}^{s} \operatorname{sign}\left(k_{it}^{*} - (1-\mu_i)\right)\left[k_{it}^{*} - (1-\mu_i)\right]^2$$

from which, on average:

$$K_{t+1} = K_t - \alpha_0 \left[K_t^* - (1-\mu)\right] - \frac{\sum_{i=1}^{s} \alpha_1 \operatorname{sign}\left(k_{it}^* - (1-\mu_i)\right)\left[k_{it}^* - (1-\mu_i)\right]^2}{s} \tag{9'}$$

Clearly, the more financial units operate close to their desired value of financial fragility and/or – of course – the smaller is α_1, the closer the right-hand side of (9′) is to the right-hand side of (9). In what follows we assume that this is the case and we use equations (8) and (9) in order to study the dynamics of the entire economy.

Microeconomic financial fragility may now be defined as the size of the minimal shock that induces a financial unit to go bankrupt $\left(1+\delta_i - k_{it}^*\right)$, while *macroeconomic financial fragility* may be defined as the size of the minimal shock that makes the economic dynamics unsustainable, demanding immediate and drastic policy interventions in order to push the economy back into the sustainable area.[7]

In our aggregate model economy the system as a whole never enters the region with an intertemporal financial ratio greater than one since we have assumed that the financial units go bankrupt as soon as they hit their crucial barrier 1 + δ_i and that the mean value of δ_i is zero. This assumption forces us to reformulate the first of the two equations of the aggregate dynamic system. Indeed, the right-hand side of (8) is valid only if it is less than or equal to one. Otherwise, it is forced to be equal to one. To formalise this discontinuous dynamics of the average intertemporal financial ratio, we re-write equation (8) as:

$$K_{t+1}^* = \begin{cases} K_t^* + \beta\left(K_t - 1\right) & \text{if } K_t^* + \beta\left(K_t - 1\right) \le 1 \\ 1 & \text{otherwise} \end{cases}$$

or, in shorter notation:

$$K_{t+1}^* = \min\left\{K_t^* + \beta\left(K_t - 1\right), 1\right\} \tag{8'}$$

which is a *piecewise linear equation*.

It is not difficult to understand that such a change, which is tantamount to considering a "*ceiling*" for the aggregate intertemporal financial ratio, has very impor-

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[7] Under the pressure of competition and imitation, financial units are pushed to increase their financial exposure in order to maximise consumption and/or investment to the point where the marginal expected returns of further exposure equals the marginal risk premium. The risk is related to the probability that a shock may increase the intertemporal financial ratio over the threshold 1 + δ_i . The larger is μ_i the lower the probability of a shock of larger size jeopardizing its solvency.

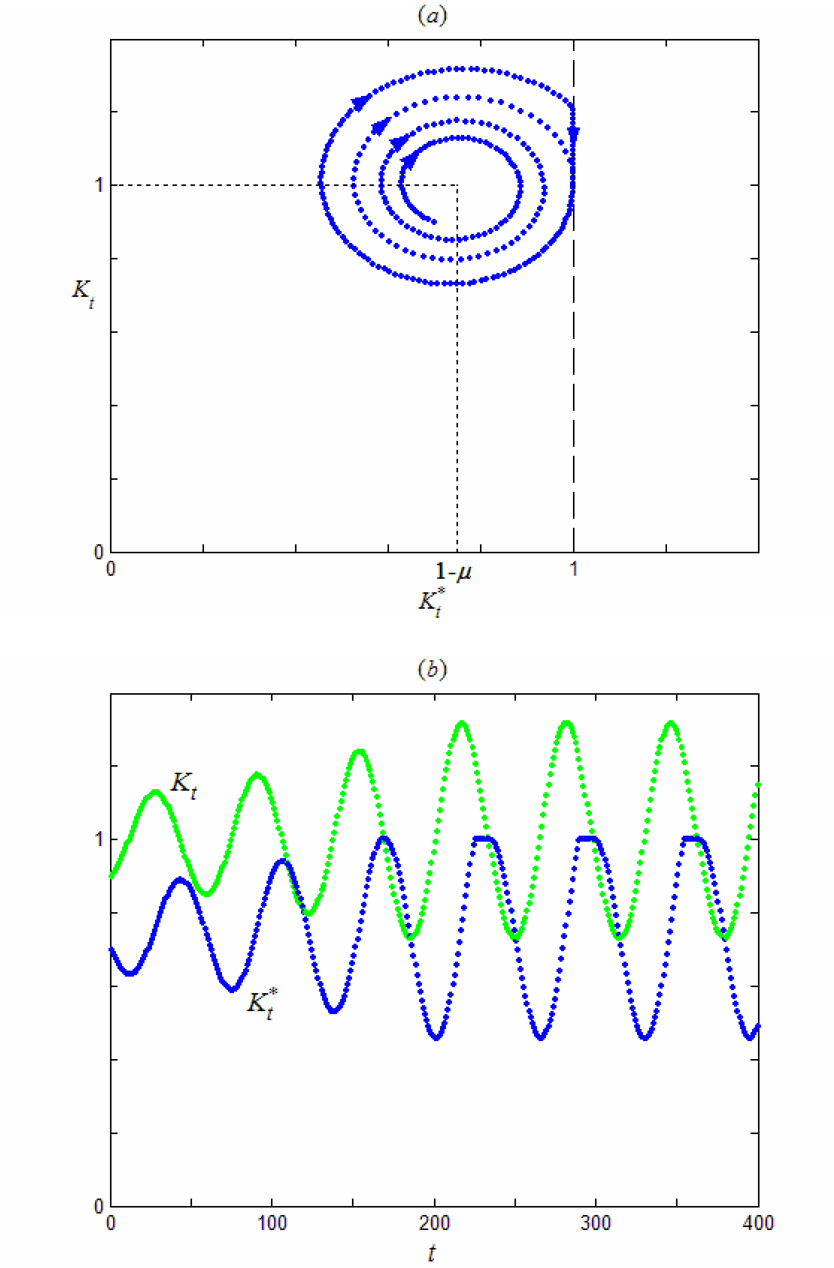

Fig 5. (*a*) Phase diagram for the entire economy using the modified (piecewise linear) version of the model with $\alpha = 0.1$, $\beta = 0.1$, $\mu = 0.25$ and (*b*) K_t and K_t^* versus time

tant consequences for the dynamics of the system.[8] As shown in Figs. 5(*a*),(*b*), when we simulate the dynamic system (8′)-(9), the dynamically unstable fluctuations become persistent.

[8] Unstable linear models of this kind, with "ceilings" and/or "floors" for certain variables, have been recently extensively examined in the literature, for example, by Hommes in the context of Hicks' 1950

5 Implications for the overall financial aggregates of the economy

We intend now to analyse the implications of the dynamics of the current and intertemporal financial ratios for the financial aggregates of the entire economy. In order to do so, we note that the aggregate outflows of the private sector of the economy at time *t,* which we denote by E_t, are the sum of an endogenous component, private aggregate outflows, E_t^{pr}, and an exogenous component: public net outflows E_t^{pu} that correspond to the sum of deficit spending ($D_t = G_t - T_t$, i.e. public expenditure minus revenues from taxation and other inflows from the private sector, etc.)[9] and the change in the quantity of money in circulation ΔM_t. If the net public outflows are positive they translate into an equal additional amount of private inflows and vice versa. This defines two different policy regimes that are crucial in determining the qualitative characteristics of the dynamic behaviour of the macro system: an "inflationary regime", when $E_t^{pu} = D_t + \Delta M_t > 0$, and a "deflationary regime", when $E_t^{pu} < 0$.

The aggregate outflows E_t translate into contemporaneous aggregate inflows Y_t since, from the accounting point of view, the cash outflow of the buyer is identically equal to the cash inflow of the seller. On the contrary we may assume that there is a lag of one period between the realised inflows and the realised outflows of the financial units and therefore of the corresponding aggregates (Vercelli 2000, p. 149). Thus, the dynamic behaviour of cash flow aggregates is determined by the following system:

$$Y_t = E_t \tag{10}$$

$$E_t = K_t Y_{t-1} + E_t^{pu} = f\left(Y_{t-1}; K_t\right) \tag{11}$$

where the dynamics of K_t is determined by equations (8′) and (9).

This model may be interpreted as the financial counterpart of the simple Keynesian income-expenditure model underlying the multiplier (see Vercelli, 1991 and 2000) (Fig. 6). The main difference, at the same time formal and substantial, is that in the simple Keynesian model, the slope of the aggregate demand line – given by the marginal propensity to consume – is fairly stable along the business cycle and it is very unlikely to be or become larger than one. As a consequence, the equilibrium of the model is dynamically stable. On the contrary, in the model we are now considering, the financial counterpart of such a slope, given by the average current financial ratio, tends to

trade cycle theory (Hommes 1993, 1994, 1995). See also Sordi (2003), where piecewise linear dynamical systems are analysed in the context of Goodwin's (1951) non-linear accelerator model.

[9] Taxation in turn may be considered as the product of the aggregate inflows of the private sector multiplied by the average tax rate *t*: $T_t = tY_t$ (see Fig. 6).

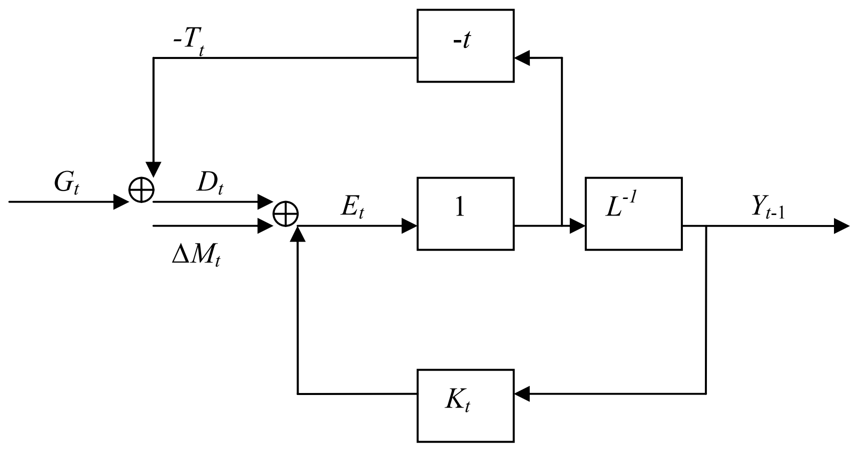

Fig. 6. The aggregate inflows-outflows interaction

oscillate pro-cyclically with the financial cycle and is very likely to become greater than one in the boom, thus making the system dynamically unstable in this phase. In addition, generally speaking, the shocks that impinge on the financial side of the economy are usually more frequent and larger than the real shocks. This comparison clarifies why the study of the financial side of the economy is so important if the purpose is to detect and control its intrinsic instability.

As we have already stressed, the dynamics of the model depends crucially on the policy regime. This may be clearly seen if we analyse and compare the dynamic behaviour of the system under the two above-mentioned regimes. First of all, by means of numerical simulations (represented graphically with the help of *cobweb-type diagrams*), we may show that in the inflationary regime, when the initial conditions are above equilibrium and $K_t < 1$, we have a depression characterised by dynamic stability. However the depression endogenously transforms itself in an expansion characterised by dynamic instability as a consequence of the progressive increase in K_t triggered by the feedback mechanism between K_t and K_t^* described by equation (8′) and (9). When the initial conditions are below equilibrium and initially $K_t < 1$ we have an expansion characterised by dynamic stability which endogenously transforms itself into an expansion characterised by dynamic instability, with an increasing K_t, until K_t diminishes under the unity triggering a new depression.

The first iterations for the two cases with initial values of aggregate inflows respectively greater and smaller than the *initial* equilibrium level – and an initial average current financial ratio less than one, are shown in Fig. 7.

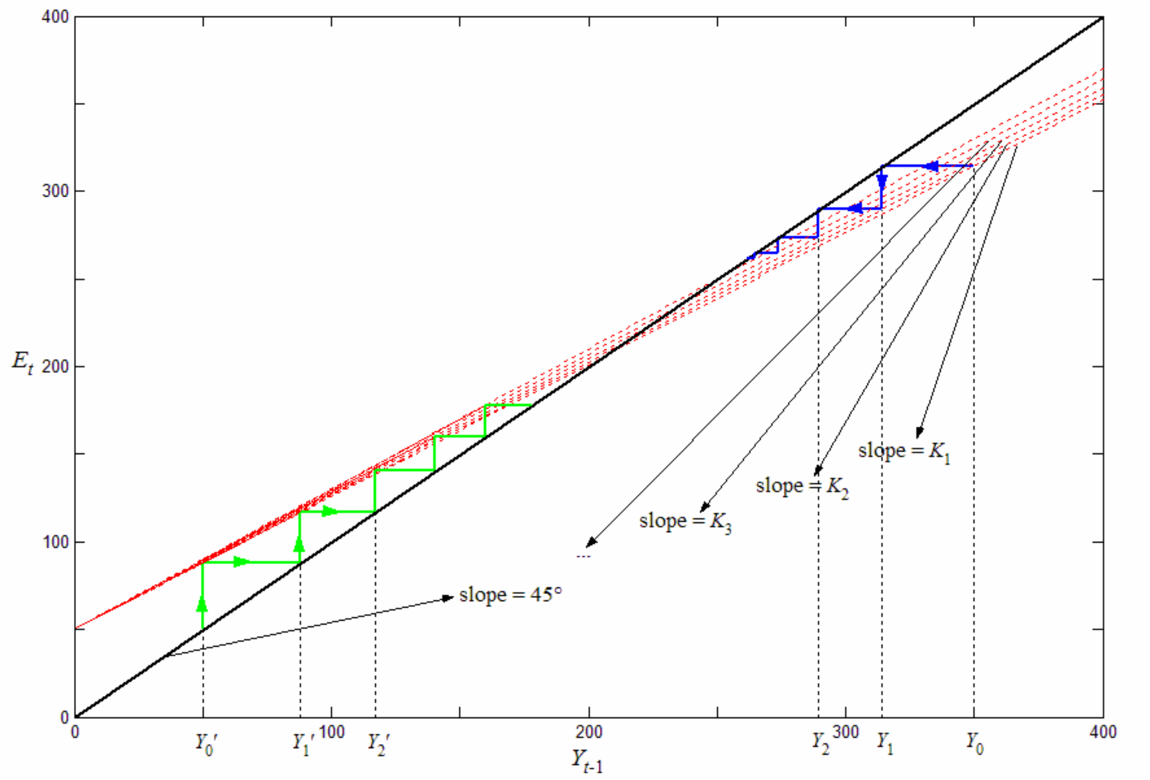

Fig. 7. First iterations (n = 5) of equations (10)-(11) with $E^{pu} = 50 > 0$ and initial conditions $Y_0 = 350$, $Y_0' = 50$ and $K_1 = 0.75 < 1$

However, this is not the end of the story as is clearly shown in the Figs. 8, 9, 10, and 11 where more iterations are considered.

In the case depicted in Fig. 8 the dynamically stable expansion endogenously transforms itself into a boom characterised by dynamic instability. This happens also in the case of a dynamically stable recession that transforms itself into a recovery characterised by *dynamic stability* (see Fig. 9) and, then, as soon as K_t becomes greater than one, into a boom characterised by *dynamic instability*. An aspect of the dynamics of the model economy worth stressing is that the transition from the recovery phase to the boom that we have just described is characterised by a radical *structural change* which modifies the dynamic characteristics of the system. As soon as K reaches a value greater than one, the equilibrium suddenly shifts to the South-West quadrant becoming unfeasible and at the same time dynamically unstable. On the basis of the feedback mechanisms considered in (8′) and (9), however, we know that in the boom both K and K^* increase until K^* reaches the "desired" level $(1 - \mu)$ after which K starts to decrease.

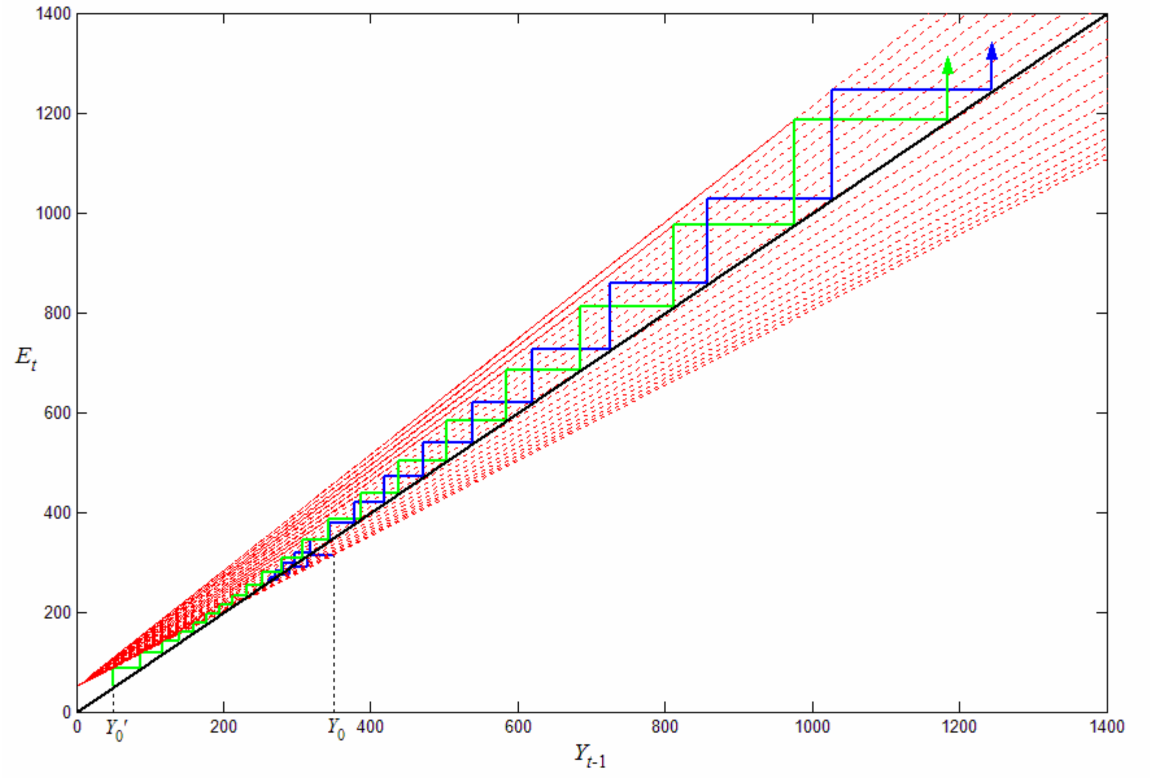

Fig. 8. More iterations (n = 20) of equations (10)-(11) with $E^{pu} = 50 > 0$ and initial conditions $Y_0 = 350$, $Y_0' = 50$ and $K_1 = 0.75 < 1$

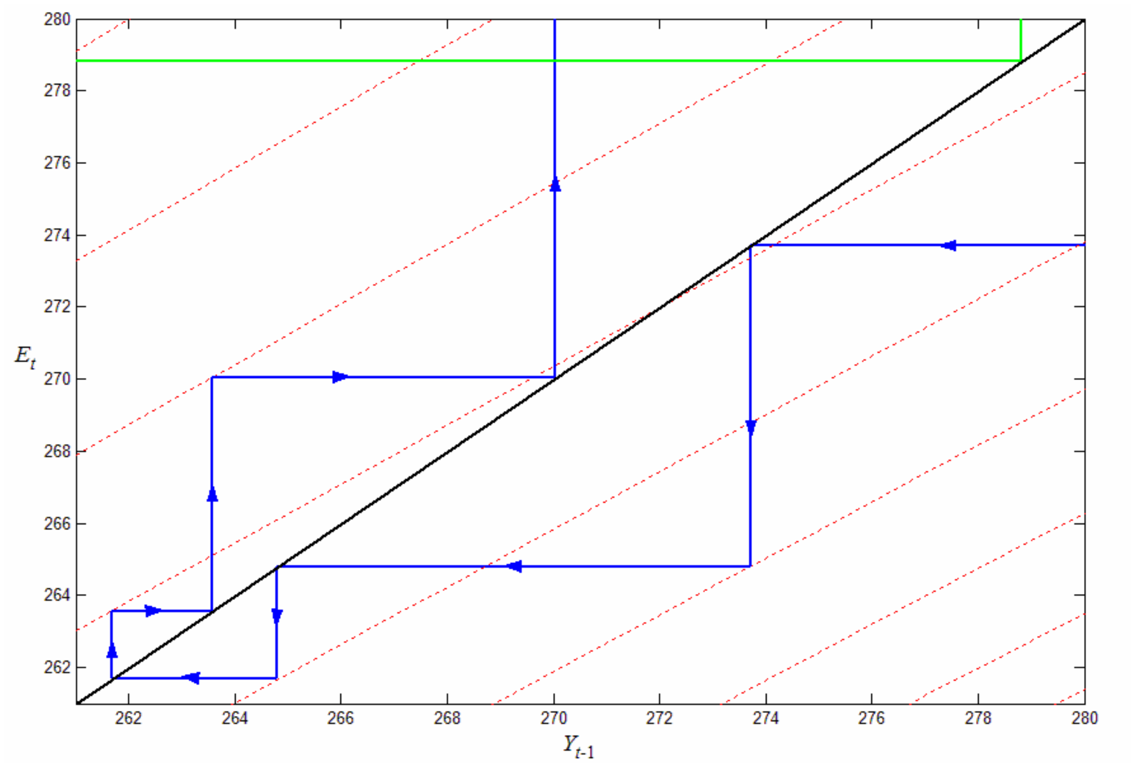

Fig. 9. Enlargement of a portion of Fig. 8

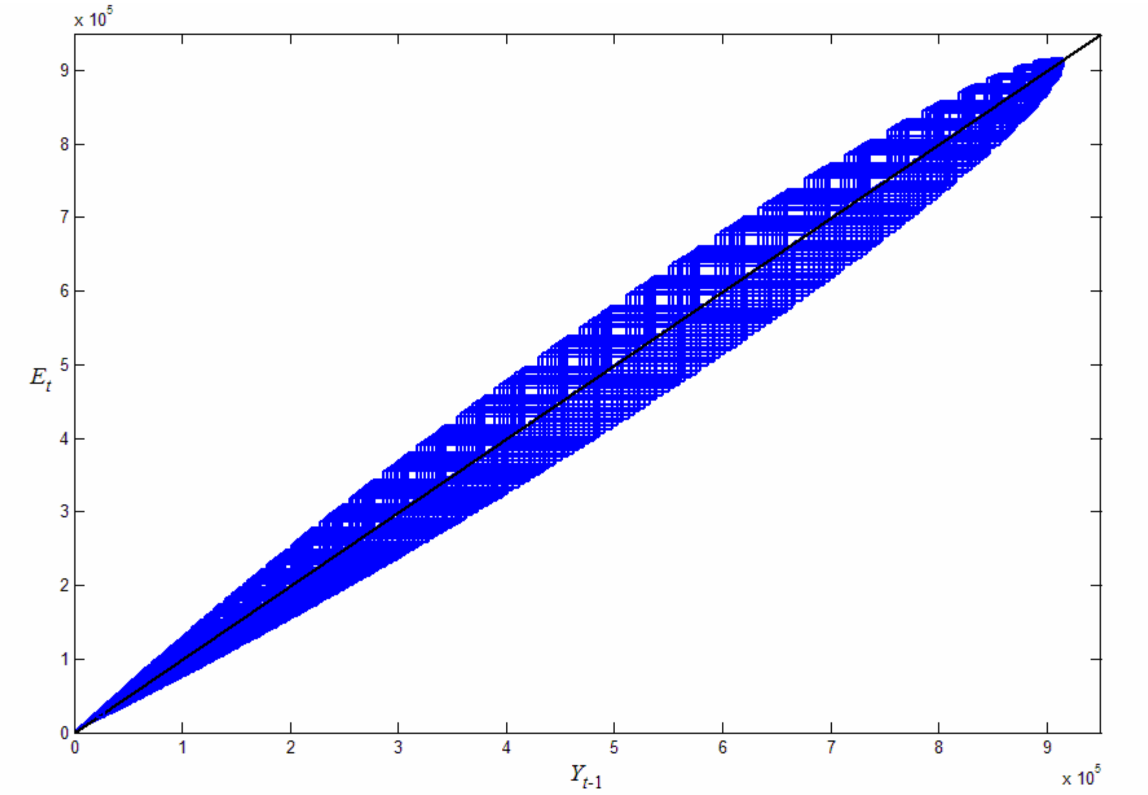

Fig. 10. The dynamic behaviour of the cash flow aggregates (n = 2000), neglecting the first 500 iterations, with initial conditions Y_0 = 350 and K_1 = 0.75

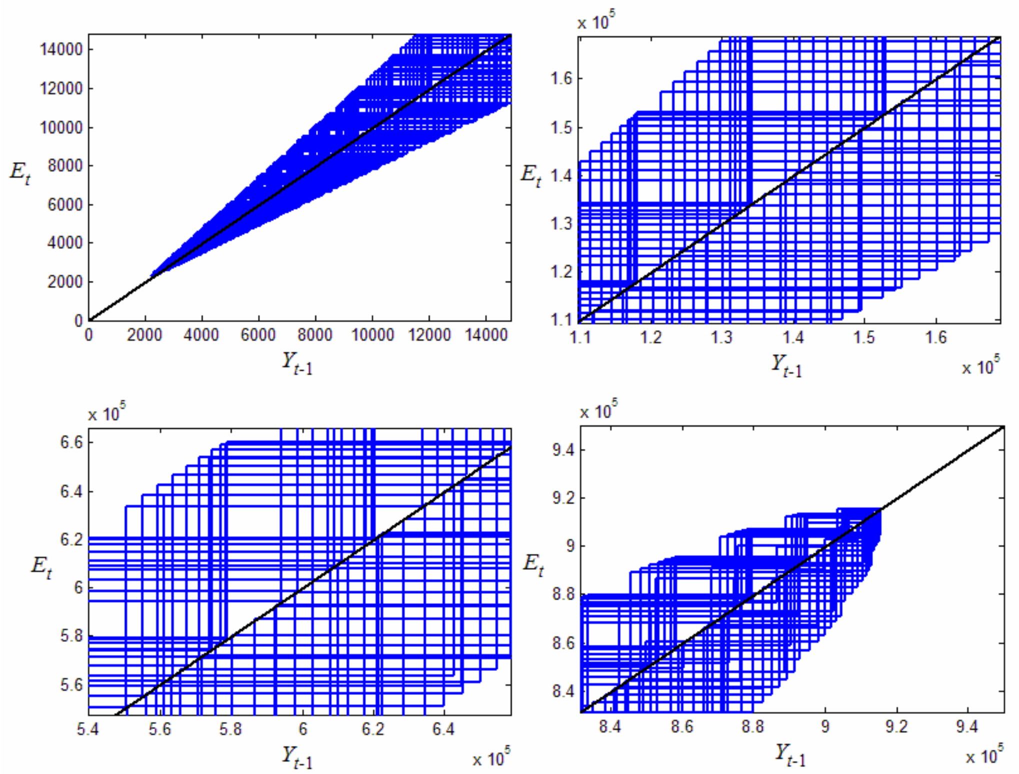

Fig. 11. Sample enlargements of portions of Figure 10

As soon as K again becomes smaller than unity, the recovery may begin and the transition is once more characterised by an endogenous structural change such that the equilibrium again moves to the North-East quadrant and becomes dynamically stable. Given our representation in terms of a piecewise linear system, we know that the cyclical fluctuations of the current and intertemporal aggregate financial ratios start again along lines similar to those of the preceding cycle with the consequences for the overall financial aggregates of the economy shown in Figs. 10 and 11. In short, the persistence of the fluctuations in the average current and intertemporal financial ratios leads to very irregular fluctuations of the overall financial aggregates of the economy.

As for the "deflationary regime", it is important to stress that there is a crucial difference with respect to the case of the "inflationary regime" so far analysed. The main difference is that in the case of a "deflationary regime" there is a serious risk that the system becomes *trapped* in a path of downward dynamic instability. In this case, as shown in Fig. 12 and, on a larger scale, in Fig. 13, it may happen that – notwithstanding the progressive increase in K_t, which, sooner or later, becomes greater than one – the economic system continues to deflate.

This "deflationary trap" may be interpreted as a simple formal representation of the "debt-deflation" mechanism that, according to Irving Fisher (1933), characterised the Great Depression of the 1930s and, according to Minsky (1982), is typical of the most severe financial crises. In fact, in a deflationary regime a reduction of inflows of a financial unit is bound to induce a reduction of its outflows and so on. Only an injection of extra inflows in the form of more expenditure or more money from outside the private sector may reverse the vicious circle.

The unstable dynamic behaviour of the economic system under the two policy regimes has policy implications that we intend to discuss in the next section.

6 Policy implications

The economic fluctuations described in the model elaborated in the preceding sections tend to push at the end of the boom many financial units, and consequently the entire economy, into the area of pronounced financial fragility, i.e. very near the barrier of pathological structural change. In this situation a very small negative shock may be enough to bankrupt the most fragile financial units and to trigger a generalised financial crisis that may lead to a severe recession or even a depression. This situation typically occurs for each financial cycle of sizeable length and magnitude. The boom nurtures overoptimistic expectations and, at the same time, also generates further opportunities

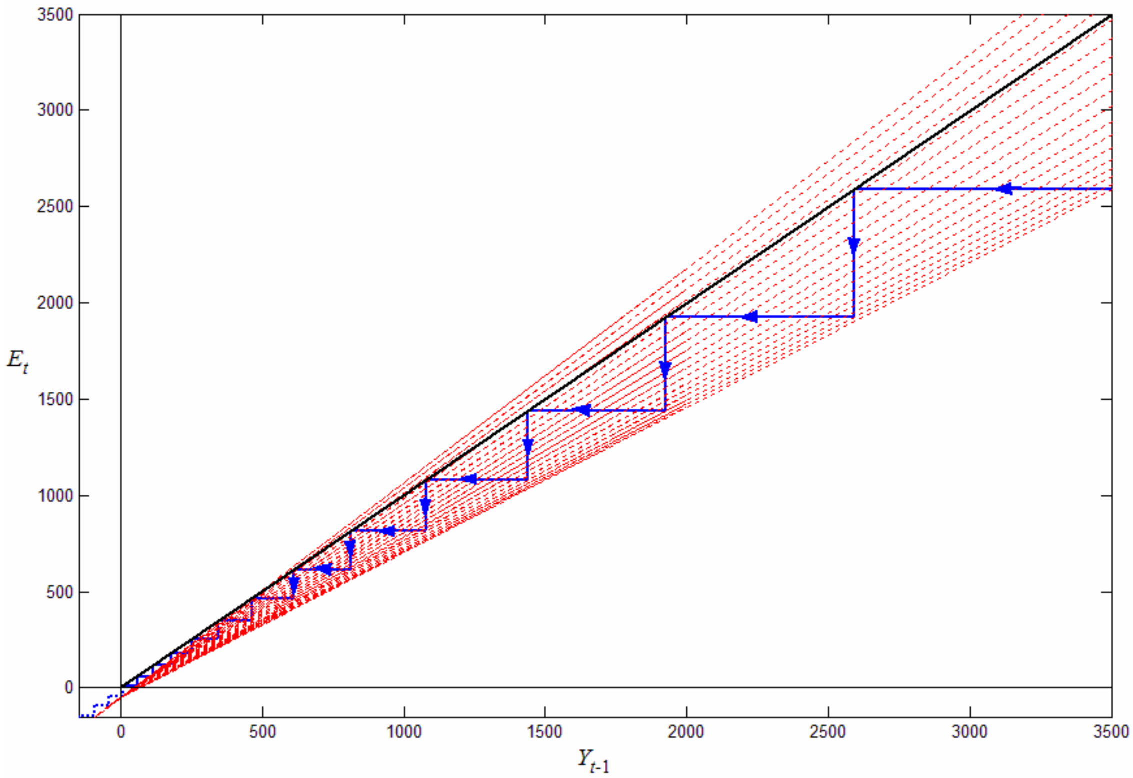

Fig. 12. First iterations (n = 18) of equations (10)-(11) with $E^{pu} = -50 < 0$, with initial conditions Y_0 = 3500 and $K_1 = 0.75 < 1$

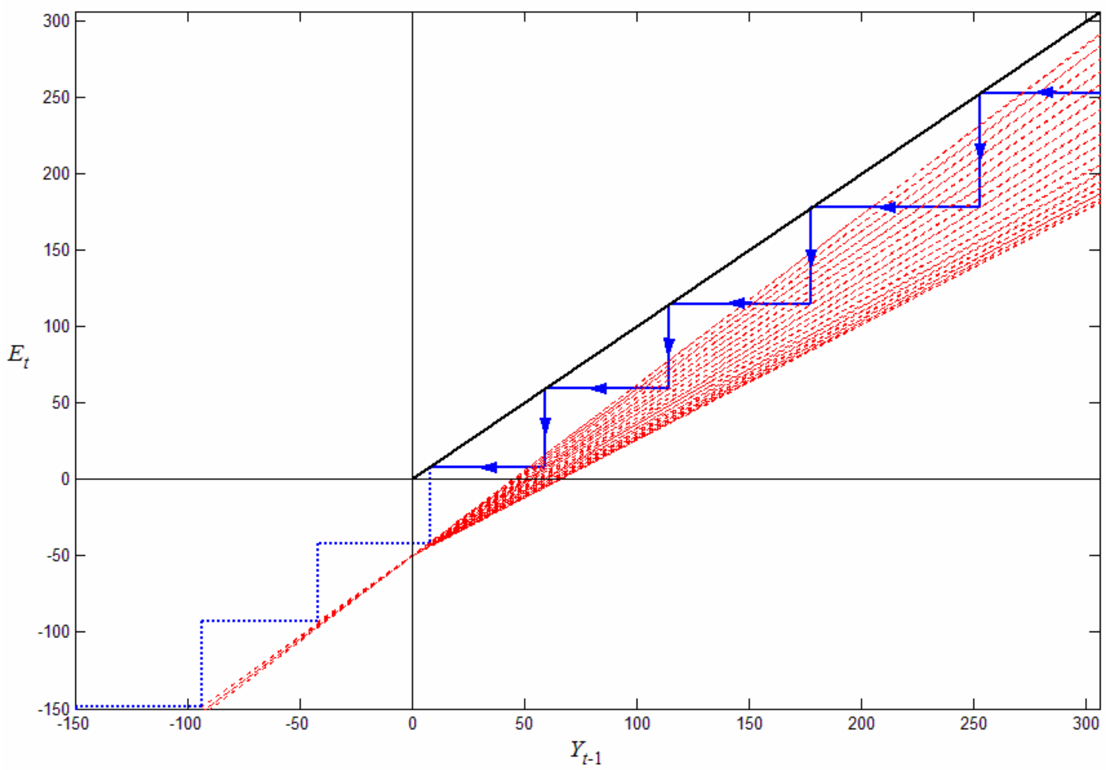

Fig. 13. Enlargement of a portion of Fig. 12

for expenditure, investment and growth.

In recent times pathological structural change was triggered, especially in the case of relatively weak units or sectors or economies, even when they were still relatively far from the crucial barrier of financial unsustainability. In fact an increasing propensity to herd behaviour favoured by the globalisation of capital markets and the ensuing massive and precipitous shifts of hot money from one firm or sector or country to another may rapidly amplify a very small shock, eventually producing a shock of such magnitude as to exceed the system's threshold of structural stability.

In the model presented in this paper both dynamic instability and structural instability play a crucial role. The financial side of the market is seen as liable to dynamic instability and requires (dynamic) stabilisation interventions on the part of policy makers. Observed fluctuations, generally speaking, are unstable but not explosive because the fluctuations are constrained by a financial ceiling[10] beyond which the most fragile units go bankrupt. Fortunately often the inversion occurs before that limit precisely because policy, though it does not succeed in achieving the full dynamic stability of the system and may even help to provoke unintended fluctuations, is fairly successful in ensuring that the economy does not breach reasonable safety thresholds. However what makes the control of the economy particularly difficult is the growing role assumed by structural instability in an increasingly sophisticated financial economy. As we have argued above, very small shocks, rapidly amplified by herd behaviour, may induce the bankruptcy of many financial units as well as recession and depression for the economy as a whole.

These setbacks may be avoided through structural policy interventions that integrate the traditional counter-cyclical policies. The latter should intervene early in the boom as soon as a safe level of financial fragility is exceeded. These interventions should be cautiously "disinflationary" in the sense that they should reduce the positive value of E^{pu} that characterizes the inflationary regime in order to check the progressive increase in financial fragility without being deflationary, i.e. without shifting the economy into a "deflationary regime". In addition this policy should, as far as possible, avoid increasing the rate of interest as this would augment the intertemporal financial ratio and therefore also the financial fragility of the units, particularly those more financially exposed. The space of *manoeuvre* for productive counter-cyclical policies is therefore extremely limited. Thus, it is crucial to intervene with structural measures that prevent an excessive increase in the financial fragility of economic units. This is done, e.g., in the

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[10] Our model defines a sort of limit cycle beyond which the financial side of the economy cannot go.

financial sector by imposing a limit on the maximum ratio between liabilities and assets of banks. The recent agreement between supervision authorities of industrialised countries called "Basel 2" aims to impose more rigour, although the crucial importance of the relationship between financial fragility and financial fluctuations is dangerously played down in its current version.

In particular we should keep in mind that we have to prevent an economy falling into a "deflationary trap". In this case, as we have seen in the previous section only an injection of extra inflows in the form of more expenditure or more money from outside the private sector may reverse the vicious circle, provided that this injection is sufficient to shift the economy into the "inflationary" regime. In the 1930s the deflationary trap was triggered by the inadequate policy response to the Wall Street crash, a response that consisted in ill-judged deflationary measures. The industrial economies have learned from this experience and the teachings of Keynes how to react to financial crises characterised by insufficient aggregate demand. They have thus avoided falling into a deflationary regime or switching from it to an inflationary regime in order to avoid or abort a "deflationary trap". However in recent times these acquisitions have been apparently questioned by approaches that give priority to monetary stability and balanced budgets and have a strong belief in the self-regulating ability of the market.[11] Our model supports the idea that an economy which falls into a "deflationary trap" may be rescued only by public policy interventions aiming to shift the economy into an "inflationary" regime. However we have to stress that in logical terms a very moderate deficit spending and/or creation of money is sufficient to open the escape hatch from the deflation trap. Their optimal amount depends on the cyclical and structural characteristics of the depression to be faced.

7 Concluding remarks

The model herein presented and discussed on the basis of a few simple numerical simulations of its dynamic behaviour is conceived as a first elementary illustration of an approach meant to analyse the interaction between dynamic and structural instability in a sophisticated financial economy. Financial fluctuations produced by the intrinsic dynamic instability of a globalised financial economy endogenously produce a change in the dynamic regime of the financial units that eventually brings about the bankruptcy of many of them and a generalised financial crisis often followed by a sharp downturn

[11] Stiglitz (2002) gives a list of examples with particular reference to the deflationary policies forced by the IMF on economies characterised by a lack of aggregate demand.

of the economy. Attempts by the managers of the financial units, and by policy makers, to avoid or abort a severe financial crisis may be frustrated by the high degree of financial fragility of the units and the entire economy after a prolonged boom. In such a situation a very small shock, suddenly amplified by herd behaviour, may lead to the bankruptcy of many financial units and a financial crisis of the entire economy. Usual counter-cyclical policies are insufficient to ensure that such an outcome is avoided, as they have to be supplemented by, and coordinated with, structural policies aimed at preventing an excessive increase in financial fragility. The capital requirements and financial regulation performed by central banks are a case in point. However, after a prolonged boom, the financial fragility of the economy tends to increase to dangerous levels and a *disinflationary* policy becomes unavoidable. In the light of our model, however, it is crucial that such a policy should not become *deflationary* since such a policy regime could lock the financial side of the economy into a "deflationary trap" from which it would be very difficult to escape.

Among the first steps necessary to develop this approach we should mention: the introduction of financial stocks in the balance sheets of the financial units; a more satisfactory process of aggregation based on the parameters that characterise the distribution of financial units rather than on the linear approximation herein adopted; a more accurate modelling of the bankruptcy process; analysis of the crucial role of the interest rate in affecting both the dynamic and structural instability and their nexus, and explicit analysis of the real consequences of the financial fluctuations.

References

Delli Gatti, D., Gallegati, M., Giuliani, G., Palestrini, A., 2003, Financial fragility, patterns of firms' entry and exit and aggregate dynamics. Journal of Economic Behavior and Organization 51, 79-97.

Fisher, I., 1933, The debt-deflation theory of great depressions. Econometrica 1, 337-57.

Goodwin, R.M., 1951, The nonlinear accelerator and the persistence of business cycles. Econometrica 19, 1-17.

Hicks, J.R., 1950, A Contribution to the Theory of the Trade Cycle (Clarendon Press, Oxford).

Hommes, C.H., 1993, Periodic, almost periodic and chaotic behaviour in Hicks' nonlinear trade cycle model. Economic Letters 41, 391-397.

Hommes, C.H, 1994, Quasi-periodic and strange, chaotic attractors in Hicks' nonlinear trade cycle model, In: J. Grasman and G. van Straten (Eds.), Predictability and Nonlinear Modelling in Natural Sciences and Economics, Kluwer Academic Publishers, Dordrecht, pp. 591-598.

Hommes, C.H., 1995, A reconsideration of Hicks' non-linear trade cycle model. Structural Change and Economic Dynamics 6, 435-459.

Keynes, J.M., 1936, The General Theory of Employment, Interest, and Money (Macmillan, London).
Lorenz, H.-W., 1993, Nonlinear Dynamical Economics and Chaotic Motion, 2nd ed. (Springer Verlag, Berlin).
Minsky, H.P., 1982, Can "It" Happen Again? Essays on Instability and Finance (Sharpe, New York).
Nardozzi, G., ed., 2002, I rapporti tra finanza e distribuzione del reddito: un'interpretazione dell'economia di fine secolo (Roma, Luiss Edizioni).
Sordi, S., 2003, Asymmetric investment functions and limit cycles: the multiplier-accelerator interaction 'continuously' revisited. Paper presented at the Eight Viennese Workshop on "Optimal Control, Dynamic Games and Nonlinear Dynamics: Theory and Applications in Economics and OR/MS", Vienna, May 14-16, 2003.
Stiglitz, J.E., 2002, Globalization and Its Discontents (Allen Lane, London).
Vercelli, A., 1991, Methodological Foundations of Macroeconomics. Keynes and Lucas (Cambridge University Press, Cambridge).
Vercelli, A., 2000, Structural financial instability and cyclical fluctuations. Structural Change and Economic Dynamics 11, 139-156.
Vercelli, A., 2001, Minsky, Keynes and the structural instability of a sophisticated monetary economy, In: R. Bellofiore and P. Ferri (Eds.), Financial Fragility and Investment in the Capitalist Economy. The Economic Legacy of Hyman Minsky, Vol. 2, Edward Elgar, Cheltenham, U.K., pp. 33-52.
Wicksell, K., 1898, Geldzins und Guterpreise (Fischer, Jena). English translation: 1936, Interest and Prices (Macmillan, London).