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QUADERNI DELL'ISTITUTO DI ECONOMIA

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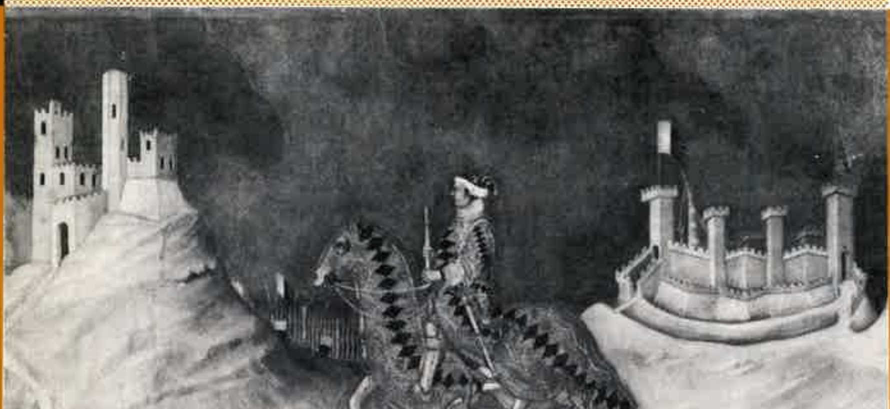
EXPECTATIONS AND THE LIFE SPAN OF THE REGIME



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M. Morishima and T. Sawa

EXPECTATIONS AND THE LIFE SPAN OF THE REGIME

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1. Introduction

One of the methods by means of which expectations are formed, for example expectations concerning the prices of certain goods, is the historical methods whereby the process from the past through to the present is examined and future prices are predicted on the basis of such past experience. This kind of method has been developed in the theory of rational expectation of Muth, Sheffrin and others. Within the historical method there may be two categories, firstly where the regime under consideration is assumed to have been in continuous existence since the infinite past, and secondly where a regime is regarded as having been originated prior to a finite number of periods, prior to which part it did not exist. Much of the theory relating to rational expectation hitherto (e.g. Muth -1961-, Sheffrin -1983-) belong to the first of these two categories; this paper will also concern itself with the second.

There exists in addition another way of forming rational expectations which may be termed the forward looking method. If period t is the present we will assume the price for period t depends on the price for one period into the past, i.e. t-1, *and* on the predicted price for the next period into the future, i.e. t+1. Since any prediction of the price in period t+1 made during period t has to take into account the above relationship which is likely to obatin in period t+1, the price in period t+1 which is predicted during period t must be determined on the basis not only of the price in period t but of the predicted price for t+2 as well. Similarly the predicted price in t+2 is determined in accordance with the predicted prices for both periods t+1 and t+3. Thus any prediction made in the present concerning prices in the following period must ultimately depend on price predictions relating to the distant future. In the case of the horizon of people's perspective being infinitely distant and the life span of the regime being infinite in the direction to the future, one may be interested, among all possible paths of foresight, in the particular one which converges to a stationary state, since it is considered to give the infinitely long sequence of foward looking rational expectations[(1)].

Evidently, this system of the forward looking method is closely related with what Keynes (1936) referred to as a system of shifting equilibrium, "a system in wich changing views about the future are capable of influencing the present situation"[(2)]. Expectations which Keynes deals with are not necessarily those which are formed rationally. Moreover, as it is explicit in Hicks' elaboration[(3)] of Keynes' shifting-equilibrium method, the sequence of expectations which they are concerned with is of a truncated one and, therefore, of a finite length This should be especially so if the remaining life span of the regime is finite. Predictions concerning the near future should mutually be consistent with those made with regard to the more distant future and

it should be possible to reach the near future by following in reverse the predicted path to the more distant future until the point of time when the structure is expected to change. Predicted prices arrived at by this means are said to be predicted by the shifting-equilibrium method. They may also be called quasi-rational expectations, because whilst the prediction of the terminal price (say the oil price on the horizon of the year 2000 providing the structure of the oil market is expected to be unchanged to that year) is given exogenously (say, by an authority which has forecasted it by a crude extrapolation of the trend of prices in the past) and, therefore, is not necessarily rational, the expectations of the price in the interim periods are correctly calculated by utilizing all the available information concerning the structure of the economy. Where people are unable to make a clear prediction concerning the terminal price the Keynes-Hicks shifting-equilibrium model has a degree of freedom of 1, so that there is one path of quasi-rational expectations corresponding to each possible prediction of the terminal price. It will be shown that all these paths will approximate, in most of the periods, the path of the rational expectations according to the 'forward looking' method when the lenght of the paths may be taken as very long. This result shows that there is a formal similarity between the paths of shifting-equilibrium expectations and the paths of efficient growth envisaged by the Turnpike Theorem; any of these paths trace out a 'catenary' motion.

This paper investigates both kinds of method. Section 2 and 3 are concerned with the historical methods, looking at cases both where the regime in question has a long history stretching back to an infinite time in the past, and where this is not the case, respectively. Section 4 gives a remark on Sheffrin's method of obtaining rational expectation. Section 5 deals with the shifting-equilibrium method for the case of the structure having a finite life span. Finally assuming the life span is infinity, Section 6 is concerned with the forward looking method of obtaining rational expectations which has been discussed by Shizuki-Muto (1981), Begg (1982), and others; it, in particular, examines the relation between the rational expectation solutions, by the historical and forward looking methods, and the Turnpike result mentioned above will also be obtained.

2. *Rational expectation: the case of infinite memories*

The argument is based on Muth's market model with inventory adjustment.(4)

$$D_t = -bp_t \qquad \text{(Demand)}$$

$$s_t = c\ {}_{t-1}p_t^e + \eta_t \qquad \text{(Supply)}$$

(1) $$V_t = \alpha\ ({}_tp_{t+1}^e - p_t) \qquad \text{(Inventory)}$$

$$D_t + V_t = s_t + V_{t-1} \qquad \text{(Market)}$$

$$_{t-1}p_t^e = E(p_t \mid I_{t-1}) \qquad \text{(Rational Expectation)}$$

where ${}_tp_{t+1}^e$ is the expectation at t of the price at time t+1, I_t is the information avilable to the supplier at t, and α , b, c are parameters which are assumed to be constant. η_t stands for a serially uncorrelated random disturbance that has mean zero and variance σ_η^2 .

The market equation yields:

(2) $$\alpha\ {}_tp_{t+1}^e - \beta p_t = \gamma\ {}_{t-1}p_t^e - \alpha p_{t-1} + \eta_t$$

where

(3) $$\beta = \alpha + b \quad , \quad \gamma = \alpha + c .$$

Equation (2) has been solved by Muth (1961) and Sheffrin (1983) in two different ways. Relegating the latter to Section 4 below, let us first examine the former in this section. Muth approximates the price p_t by a linear combination of the past history of random variables η_t:

$$(4) \qquad P_t = \sum_{i=0}^{\infty} W_i \eta_{t-i} \quad .$$

He then considers that the expectation of P_t is formed at t-1 as

$$(5) \qquad {}_{t-1}P_t^e = W_0 E\eta_t + \sum_{i=1}^{\infty} W_i \eta_{t-i} = \sum_{i=1}^{\infty} W_i \eta_{t-i}$$

since people do not know, at that time, the value of η_t but only its expected value $E\eta_t = 0$, which gives the rational (minimum mean square error) expectation of η_t. Shifting (4) backward one period and (5) foward one period, we obtain

$$(6) \qquad \begin{aligned} P_{t-1} &= \sum_{i=0}^{\infty} W_i \eta_{t-1-i} \\ {}_tP_{t+1}^e &= \sum_{i=1}^{\infty} W_i \eta_{t+1-i} \end{aligned}$$

respectively. Substituting from all these into (2), Muth obtains a linear equation in terms of $\eta_t, \eta_{t-1}, \eta_{t-2}, \ldots$ which must hold for all possible η's, so that the coefficients of the equation should each be zero. He then obtains a system of equations in terms of W's:

$$(7) \qquad \alpha W_1 - \beta W_0 = 1$$

$$(8) \qquad \alpha W_{i+1} - (\beta+\gamma) W_i + \alpha W_{i-1} = 0 \qquad i = 1, 2, 3$$

Difference equations (8) give

$$(9) \qquad W_i = W_0 \lambda_1^i \qquad i = 1, 2, 3$$

where λ_1 is the smaller root of the characteristic equation,

$$(10) \qquad \alpha\lambda^2 - (\beta+\gamma)\lambda + \alpha = 0$$

(The larger root λ_2 which is found to be greater than 1 is ruled out because $W_i = W_0 \lambda_2^i$ tends to infinity as $i \to \infty$ so that solutions (4), (5), (6) are all explosive. That is, we have meaningless solutions: $P_{t-1} = P_t = {}_{t-1}P_t^e = {}_tP_{t+1}^e = \infty$).
Substituting from (9) into (7), W_0 is determined as

$$(11) \qquad W_0 = 1/(\alpha\lambda_1 - \beta)$$

It then follows from (5) and (6) that

$$(12) \qquad P_t = \lambda_1 P_{t-1} + \varepsilon_t \quad , \quad \varepsilon_t = W_0 \eta_t$$

$$(13)\quad \begin{cases} {}_{t-1}p_t^e = w_0 \left(\sum_{i=1}^{\infty} \lambda_1^i \cdot \eta_{t-i} \right) = \lambda_1 p_{t-1} \\ {}_t p_{t+1}^e = \lambda_1 p_t \end{cases}$$

This means that only the process of p_t which is stochastically stationary (i.e. (12) with $|\lambda_1| < 1$) can be consistent with infinite memories.

3. *Rational expectation: the case of finite memories*

Unless modifications are made, however, the above method of solution devised by Muth (which assumes that the regime (1) has an infinitely long past history of operation) cannot be applicable, where it has operated in the past for a finite number of periods only. Assuming now that (1) has a T-period past history in the current period T, i.e. it started to operate at time 0, we then write

$$(14)\quad \begin{aligned} p_t &= \sum_{i=0}^{t} W_i \eta_{t-i}, \quad p_{t-1} = \sum_{i=0}^{t-1} W_i \eta_{t-1-i} \\ {}_t p_{t+1}^e &= \sum_{i=1}^{t+1} W_i \eta_{t+1-i} \quad {}_{t-1}p_t^e = \sum_{i=1}^{t} W_i \eta_{t-i}, \quad t = 0,1,\dots,T. \end{aligned}$$

In particular, we have, in the first period t=0,

$$(15)\quad p_0 = W_0 \eta_0, \quad {}_0 p_1^e = W_1 \eta_0$$

whilst both p_{-1} and ${}_{-1}p_0^e$ are not available because (1) did not exist at time -1. Substituting from (15), (2) may be written in the form

$$(16)\quad \alpha W_1 \eta_0 - \beta W_0 \eta_0 = \eta_0$$

where we assume $s_0 = 0$ and $v_{-1} = 0$ in (1); η_0 represents the manna at the time of commencement of the regime (1). From (16) obtain

$$(17)\quad \alpha W_1 - \beta W_0 = 1$$

In the second period we have

$$(18)\quad p_1 = W_0 \eta_1 + W_1 \eta_0, \quad {}_1 p_2^e = W_1 \eta_1 + W_2 \eta_0$$

Taking (15) and (18) into account, (2) may be put in the form

$$\alpha(W_1 \eta_1 + W_2 \eta_0) - \beta(W_0 \eta_1 + W_1 \eta_0) - \gamma W_1 \eta_0 + \alpha W_0 \eta_0 = \eta_1$$

so that we obtain

$$\alpha W_1 - \beta W_0 = 1$$

(19)

$$\alpha W_2 - (\beta+\gamma) W_1 + \alpha W_0 = 0 \quad .$$

We have similar equations, for each t = 2, 3, ..., T. Therefore,

$$\text{(20)} \quad \begin{aligned} &\alpha W_1 - \beta W_0 = 1 \\ &\alpha W_{i+1} - (\beta+\gamma) W_i + \alpha W_{i-1} = 0 \quad i = 1, 2, ..., t \quad . \end{aligned}$$

Thus the unknown W's of (14) are determined by solving the difference equations (20). Solutions are

$$\text{(21)} \quad W_i = k_1 \lambda_1^i + k_2 \lambda_2^i \qquad i = 0, 1, ..., T+1$$

where λ_1 and λ_2 are the two roots of the characteristic equation (10) and k_1 and k_2 are constants to be determined as:

$$k_1 = \frac{1}{\lambda_2-\lambda_1} \quad w_0 \lambda_2 - \frac{1+\beta w_0}{\alpha}$$

(22)

$$k_2 = \frac{1}{\lambda_2-\lambda_1} \left(\frac{1+\beta w_0}{\alpha} - w_0 \lambda_1 \right) .$$

Substituting from (21), we may write (14) as

$$\text{(23)} \quad p_t = k_1 \xi_t + k_2 \delta_t ,$$

where

$$\xi_t = \sum_{i=0}^{t} \lambda_1^i \eta_{t-i} \quad \text{and} \quad \delta_t = \sum_{i=0}^{t} \lambda_2^i \eta_{t-i} \quad .$$

(23) can also be written as

$$p_t = k_1 \lambda_1 \xi_{t-1} + k_2 \lambda_2 \delta_{t-1} + (k_1 + k_2) \eta_t \quad .$$

Similarly, this can further be put in the form

$$\text{(24)} \quad p_t = k_1 \lambda_1^2 \xi_{t-2} + k_2 \lambda_2^2 \delta_{t-2} + (k_1+k_2) \eta_t + (k_1 \lambda_1 + k_2 \lambda_2) \eta_{t-1} \quad .$$

In the same way we have

$$\text{(25)} \quad p_{t-1} = k_1 \lambda_1 \xi_{t-2} + k_2 \lambda_2 \delta_{t-2} + (k_1+k_2) \eta_{t-1} ,$$

$$(26)\quad p_{t-2} = k_1 \xi_{t-2} + k_2 \delta_{t-2} .$$

In view of the fact that λ_1 and λ_2 are the roots of the characteristic equation (10) we obtain from (24), (25) and (26) the following auto-regressive model concerning p_t:

$$(27)\quad \alpha p_t - (\beta+\gamma)p_{t-1} + \alpha p_{t-2} = \alpha(k_1+k_2)\eta_t + $$
$$+ \alpha\left[(k_1\lambda_1+k_2\lambda_2) - \frac{\beta+\gamma}{\alpha}(k_1+k_2)\right]\eta_{t-1}$$

In view of (22), this can be rewritten as

$$(27')\quad \phi(B)\, p_t = \Theta(B)\,\eta_t ,$$

where

$$\phi(B) = 1 - \frac{\beta+\gamma}{\alpha} B + B^2$$

$$\Theta(B) = W_0 + \left(\frac{1+\beta W_0}{\alpha} - \frac{\beta+\gamma}{\alpha} W_0\right) B = W_0 + \frac{1-\gamma W_0}{\alpha} B$$

with B being a backward operator, i.e. $B p_t = p_{t-1}$, $B^2 p_t = p_{t-2}$, etc. and $W_0 = k_1+k_2$.

Note that

$$\phi(B) = (1-\lambda_1 B)(1-\lambda_2 B) .$$

Since $|\lambda_1| < 1$ and $|\lambda_2| > 1$, the stochastic process of p_t, (27) or (27'), is non-stationary in general circumstances. Only in a special case where W_0, that is p_0 / η_0 from (15), statisfies the condition,

$$(28)\quad -\lambda_2 W_0 = \frac{1-\gamma W_0}{\alpha} ,$$

(27') is reduced to the Muth process (12), or

$$(1-\lambda_1 B)\, p_t = W_0\, \eta_t .$$

Otherwise, (27') can be written

$$(29)\quad (1-\lambda_1 B)\, p_t = \frac{W_0 - \Theta B}{1-\lambda_2 B}\, \eta_t$$

where

$$\Theta = \frac{\gamma W_0 - 1}{\alpha} .$$

Because the history of the regime is finite we have

$$B^k \eta_t = \eta_{t-k} = 0$$

for any k>t; hence from (29) we get

$$(1-\lambda_1 B) p_t = W_0 \Big[1 + (\lambda_2 - \frac{\Theta}{W_0}) B + \lambda_2 (\lambda_2 - \frac{\Theta}{W_0}) B^2 + \dots + \lambda_2^{t-1} (\lambda_2 - \frac{\Theta}{W_0}) B^t \Big] \eta_t .$$

Comparing this with the Muth process (12), we find that there is a difference between them of the amount

$$(W_0\lambda_2 - \Theta)\eta_{t-1} + \lambda_2 (W_0\lambda_2 - \Theta)\eta_{t-2} + \dots + \lambda_2^{t-1} (W_0\lambda_2 - \Theta)\eta_0 .$$

Since $\eta_{t-1}, \eta_{t-2}, \dots, \eta_0$ are known, this means that the Muth formula always produces systematically biased expectations if it applied to a market of a finite past history.

4. *A remark on Sheffrin*

Let us now turn to Sheffrin(5). By a different method he has arrived at tha same conclusion as Muth obtained. Assuming that the price p_t follows

a first-order stationary autoregressive scheme of the form (12) with some λ_1 whose absolute value is less than 1 and random variables ε_t (with zero mean and variance σ_ε^2) which are serially uncorrelated, Sheffrin calculates ${}_{t-1}p_t^e$ and ${}_tp_{t+1}^e$ at $E(p_t \,/\, p_{t-k}, k \geqq 1)$ and $E(p_{t+1} \,/\, p_{t+1-k}, k \geqq 1)$, respectively, so that

$$(30) \qquad {}_{t-1}p_t^e = \lambda_1 p_{t-1} \quad , \quad {}_tp_{t+1}^e = \lambda_1 p_t \quad .$$

He then substitutes from (12) and (30) into (2); thus,

$$[(\alpha\lambda_1^2 - (\beta+\gamma)\lambda_1 + \alpha] \, p_{t-1} = \eta_t - (\alpha\lambda_1 - \beta)\, \varepsilon_t \quad .$$

This enables him to find the values of λ_1 and W_0 of (12) which are yet undetermined. That is, λ_1 must be the smaller (one) of the roots of the characteristic equation, (10), and W_0 the reciprocal of $(\alpha\lambda_1 - \beta)$. In this way he confirms the same solutions as Muth has obtained.

Unlike Muth's, this method devised by Sheffrin may look to be applicable to the case of regime (1) having only a finite number of period of operation in the past, as well as to the original case where it may be traced back endlessly into the past. It must be noted, however, that if it is applied to the former case Sheffrin has implicitly to assume that $k_2 = 0$, i.e.

$$p_{t-1} = \left(\sum_{i=0}^{T-1} \lambda_1 \ \eta_{T-1-i}\right) / (\alpha \lambda_1 - \beta) .$$

This assumption is *ad hoc* and no rationale can be given to it. Once it is removed, his method fails to be applicable, where the length of the past history of the regime is finite.

5. *Shifting equilibrium expectation*

In this section we try to formulate Keynes' and Hicks' view of expectations. Assuming that prices and price-expectations of the products and the factors of production are given exogenously, Keynes and, particularly, Hicks are concerned with drawing up supply and demand curves of outputs and inputs for the current period and for each particular future period. They are also concerned with examining the effects on these outputs and inputs of a change in the expected prices as well as of a change in current prices. In these analyses, they regard the expected prices as changing independently from each other. However, they are, as a matter of fact, not given in an arbitrary way but are interrelated with each other, forming a system of consistent expectations, in the same way as current prices prevailing in a state of general equilibrium are consistent with each other.

In the context of the Muth model we may observe this view since prices and price-expectations are interrelated in the following manner: first,

in the basic equation (2), once ${}_{t-1}p_t^e$, and p_{t-1} and η_t are given at the commencement of period t, the current price p_t is determined in the period, depending on the expectation ${}_tp_{t+1}^e$ made for p_{t+1}. Thus the current price in period t depends on the expectation concerning the immediate future period t+1 as well as the past. And if we may believe that regime (1) will continue to prevail and there will be no structural change in the foreseeable future, say, period t+T, the expectation concerning period t+1 will depend on the expectation concerning period t+2 and the current price in period t; and so on for each of the subsequent periods until t+T. This link of expectations leads us to say that the current price ultimately depends on our view of the price in the remotest future t+T, the terminal date of the regime. T is referred to as the remaining life span.

This view of interrelated expectations may, more precisely, be formulated by assuming that expectations, ${}_tp_{t+1}^e$, ${}_tp_{t+2}^e$, ... , ${}_tp_{t+t}^e$, which are simultaneously formed at time t for the price in the immediate future, p_{t+1} and for those in the subsequent periods, p_{t+2} , p_{t+3} , ... , p_{t+T} , are consistent with each other. We also assume that people have a clear idea of the value of the expected price at the terminal date, so that in the sequence of expectations consistently formed the final term ${}_tp_{t+T}^e$ takes on an exogenously determined specific value p. This is obviously a strict, unrealistic assumption which we shall relax later. But, for the sake of simplicity, we will proceed by basing our analysis on this rather undesirable assumption as far as we can.

Let us begin with defining *consistent* expectations. Expected prices are said to be *consistent* with each other if they satisfy the basic equation (2) on the assumption that the disturbances η's take on the average value which is zero in each period, that is to say

$$\begin{aligned}
&\alpha\, {}_tp^e_{t+2} - \beta\, {}_tp^e_{t+1} = \gamma\, {}_tp^e_{t+1} - \alpha\, p_t\,, \\
(31)\quad &\alpha\, {}_tp^e_{t+3} - \beta\, {}_tp^e_{t+2} = \gamma\, {}_tp^e_{t+2} - \alpha\, {}_tp^e_{t+1}\,, \\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
&\alpha\, {}_tp^e_{t+T} - \beta\, {}_tp^e_{t+T-1} = \gamma\, {}_tp^e_{t+T-1} - \alpha\, {}_tp^e_{t+T-2}\,.
\end{aligned}$$

(Consistent expectations are said to be in a state of shifting equilibrium.)

Then, to obtain a consistent corrent -and expected- price set, p_t, ${}_tp^e_{t+1}, \dots, {}_tp^e_{t+T}$, we may solve equations (2) and (31) with respect to these prices; so we have

$$(32)\quad {}_tp^e_{t+k} = m_1 \lambda_1^k + m_2 \lambda_2^k \qquad k = 0, 1, \dots, T$$

where we write

$$(33)\quad {}_tp^e_t = p_t$$

and λ_1 and λ_2 are the two roots of the characteristic equation (10). Constants m_1 and m_2 are determined in the following way. First, substituting ${}_tp^e_t$ and ${}_tp^e_{t+1}$ from (32) into (2) we get:

$$(34)\quad \alpha(m_1\lambda_1 + m_2\lambda_2) - \beta(m_1 + m_2) = K$$

where K is a constant to be equal to $\gamma\, {}_{t-1}p^e_t - \alpha\, p_{t-1} + \eta_t$. (Note that ${}_{t-1}p^e_t$ and p_{t-1} have already been determined in the previous period t-1 and η_t in period t.) This initial condition, together with the terminal one requiring the ${}_tp^e_{t+T}$ given by (32) for k = T to be equal to the exogenously specified value p, determines the remaining unknowns m_1 and m_2. Since the terminal condition may be written as

$$(35)\quad m_1\lambda_1^T + m_2\lambda_2^T = p$$

we may solve (34) and (35) with respect to m_1 and m_2. Thus we have

$$(36)\quad m_1 = \frac{p\theta_2 - \lambda_2^T k}{\lambda_1^T\theta_2 - \lambda_2^T\theta_1}$$

$$(37)\quad m_2 = \frac{\lambda_1^T k - p\theta_1}{\lambda_1^T\theta_2 - \lambda_2^T\theta_1}$$

where $\Theta_1 = \alpha\lambda_1 - \beta$ and $\Theta_2 = \alpha\lambda_2 - \beta$.

This method for forming consistent expectations may be termed as the shifting equilibrium method which may be compared with the historical method (developed in the previous sections) as well as the forward looking method (discussed below) of determination of rational expectations.

6. *Rational expectation: the forward looking method*

Assuming that the life span of the regime (1) is infinite, the forward looking method of rational expectation solves the following equation:

$$(38) \quad \alpha\, {}_tP^e_{t+k+2} - \beta_t P^e_{t+k+1} = \gamma_t P^e_{t+k+1} - \alpha\, {}_tP^e_{t+k} \qquad k = 0, 1, 2, \ldots, \text{ ad inf.}$$

where ${}_tP^e_t = P_t$. We also have, at the commencement of the sequence $\left\{{}_tP^e_{t+k}\right\}$ $k = 0, 1, \ldots$, ad. inf.,

$$\alpha\, {}_tP^e_{t+1} - \beta\, {}_tP^e_t = K \quad ,$$

where K is defined, as before, as $\gamma\, {}_{t-1}P^e_t + \alpha P_{t-1} + \eta_t$ which is known at t and hence regarded as constant. Since both Shizuki-Muto (1981) and Begg (1982) exclude explosive expexctations, they obtain the unique path of rational expectations

$$(39) \quad {}_tP^e_{t+k} = \lambda_1 (K/\Theta_1) \qquad k = 0, 1, \ldots \text{ ad. inf.}$$

which satisfy (38), where $\Theta_1 = \alpha\lambda_1 - \beta$ as before.

We can now show that in the special case of $m_2 = 0$ the shifting-equilibrium expectations (32) are reduced to the Muth-Sheffrin and the Shizuki-Muto-Begg rational expectations. Because it follows from (32) that $m_2 = 0$ implies ${}_tP^e_t = P_t = m_1$ and ${}_tP^e_{t+k} = \lambda_1^k P_t$, we have in the particular case of k = 1

$${}_tP^e_{t+1} = \lambda_1 P_t = \lambda_1 \quad (K/\Theta_1)$$

which is nothing else but the second equation of (13) (which is the Muth-Sheffrin expectation) and the equation (39) (which is the Shizuki-Muto-Begg expectation). Also we have

$$(40) \quad p = \lambda_1^T P_t = \lambda_1^T \quad (K/\Theta_1)$$

if and only if $m_2 = 0$. Thus rational expectations (13) and (39) are identical with a particular solution to (31) which is obtained when the terminal price p is specified at (40).

Removing the condition, $m_2 = 0$, let us examine the path of the market price P_t. In view of the condition, $\lambda_1 \lambda_2 = 1$, that the two characteristic roots must satisfy we have from (32), (36) and (37)

$$\text{(41)} \quad {}_tp^e_{t+1} = m_1\lambda_1 + m_2\lambda_2 =$$
$$= [1/(\lambda_1^T\Theta_2 - \lambda_2^T\Theta_1)]\,[(\lambda_1^{T-1} - \lambda_2^{T-1})k + (\lambda_2 - \lambda_1)\beta p]$$

Assuming that the value of the terminal price is invariant with respect to an addition of information in period t, i.e.

$${}_tp^e_{t+T} = {}_{t+1}p^e_{t+T} = p$$

we have

$$\text{(42)} \quad {}_{t+1}p^e_{t+2} = [1/(\lambda_1^T\Theta_2 - \lambda_2^T\Theta_1)]\,[(\lambda_1^{T-2} - \lambda_2^{T-2})K + (\lambda_2 - \lambda_1)\beta p] .$$

This assumption may be justified by assuming that the life-span T of the regime (1) is sufficiently remote so that the expectation of p_{t+T} is invariant in spite of the increase in information fron I_t to I_{t+1}. Since

$${}_tp^e_t = p_t = m_1 + m_2$$

from (32) and (33) for k = 0

$$p = m_1\lambda_1^T + m_2\lambda_2^T$$

for k = T .

m_1 and m_2 can be written in terms of p_t and p as

$$m_1 = \frac{\lambda_2^T p_t - p}{\lambda_2^T - \lambda_1^T}$$

$$m_2 = \frac{p - \lambda_1^T p_t}{\lambda_2^T - \lambda_1^T} .$$

Substituting these into the first equation of (41), we obtain

$$\text{(43)} \quad {}_tp^e_{t+1} = g_0p_t + h_0p$$

where

$$g_0 = \frac{\lambda_2^{T-1} - \lambda_1^{T-1}}{\lambda_2^T - \lambda_1^T}$$

(44)

$$h_0 = \frac{\lambda_2 - \lambda_1}{\lambda_2^T - \lambda_1^T}$$

In a similar way, we have

$$(45)\qquad {}_{t+1}p^e_{t+2} = g_1 p_{t+1} + h_1 p$$

where

$$g_1 = \frac{\lambda_2^{T-2} - \lambda_1^{T-2}}{\lambda_2^{T-1} - \lambda_1^{T-1}}$$

(46)

$$h_1 = \frac{\lambda_2 - \lambda_1}{\lambda_2^{T-1} - \lambda_1^{T-1}}$$

Because $\lambda_1 < \lambda_2$, we can easily show

$$(47)\qquad 0 < g_1 < \lambda_1 \qquad \text{for } T > 2 .$$

From equation (2) which holds for period t it is clear that we have for period t+1

$$(48)\qquad \alpha\, {}_{t+1}p^e_{t+2} - \beta p_{t+1} = \gamma\, {}_tp^e_{t+1} - \alpha p_t + \eta_{t+1} .$$

Substituting (43) and (45) into (48) and rearranging the terms, we can write

$$(49)\qquad (\alpha g_1 - \beta)\, p_{t+1} = (\gamma g_0 - \alpha)\, p_t + (\gamma h_0 - \alpha h_1)\, p + \eta_{t+1} .$$

Taking into account the definition of β as α+b with $\alpha > 0$ and b > 0, we can show

$$\lambda_1 < \beta / \alpha < \lambda_2$$

so that $\alpha g_1 - \beta < 0$ from (46). Hence (49) can be written in the form:

$$(50)\qquad p_{t+1} = \frac{\gamma g_0 - \alpha}{\alpha g_1 - \beta}\, p_t + \frac{\gamma h_0 - \alpha h_1}{\alpha g_1 - \beta}\, p + \frac{\eta_{t+1}}{\alpha g_1 - \beta}$$

which can further be rewritten, because of (A.1) and (A.2) in the Appendix, in the form

$$(51)\qquad p_{t+1} = g_0 p_t + h_0 p + \frac{\eta_{t+1}}{\alpha g_1 - \beta} .$$

From (43) and (51) we find that the mean square error of the 'consistent' expectation by the shifting-equilibrium method is given by

$$(52)\qquad E(p_{t+1} - {}_tp^e_{t+1})^2 = \frac{1}{(\alpha g_1 - \beta)^2}\, \sigma^2_\eta$$

where σ_η^2 is the variance of η_{t+1}.

Let us now compare this mean square error with the one resulting when Muth's method of calculating the rationally expected price is adopted. From Muth's formulae (11), (12), (13) it is at once seen that the mean square error of the Muth expectation is given:

$$(53) \quad E(p_{t+1} - {}_tp^{eM}_{t+1})^2 = \frac{1}{(\alpha\lambda_1 - \beta)} \sigma_\eta^2$$

where ${}_tp^{eM}_{t+1}$ is the rational expecation according to the Muth formula. It then follows from (47) that

$$(54) \quad E(p_{t+1} - {}_tp^{eM}_{t+1})^2 > E(p_{t+1} - {}_tp^{e}_{t+1})^2$$

which holds as long as (40) is violated. This implies that unless the price p which people expect at the terminal date of the regime takes on the particular value $\lambda_1^T K/\Theta_1$, the Muth rational expectation is irrational in the sense that there is an expected price (i.e. the one by the shifting equilibrium method), whose mean square error is smaller than the mean square error of the Muth expectation. The same holds for the Begg expectation.

When T is very large, g_0 and h_0 of (44) are approximated by

$$g_0 = \lambda_2^{T-1} \Big/ \lambda_2^T = 1 \Big/ \lambda_2 = \lambda_1 , \qquad h_0 = 0$$

respectively, because $\lambda_2 = 1/\lambda_1$ and the absolute value of λ_1 is less than one. Therefore (43) is approximated by

$${}_tp^e_{t+1} = \lambda_1 p_t .$$

This means that the shifting-equilibrium and the Muth-Sheffrin or the Shizuki-Muto-Begg rational expectations are very near to each other for large T. Thus, from the point of view of the shifting-equilibrium formulation rational expectationists may be interpreted as being always concerned with the case where the life span of the regime is very long. Or we may say that they have supplied a limiting theory which holds true as T tends to infinity. This is the Turnpike Property mentioned in Section 1.

Finally, we have so far assumed that at the point t people can accurately form a precise expectation concerning the value of the price in period t+T. Obviously this is a very strong assumption. An alternative assumption that ${}_tp^e_{t+T}$ is distributed with mean μ_T and variance σ_T^2 is weaker and more satisfactory than the present one. However, the whole argument in sections 5 and 6 holds *mutatis mutandis* when we replace all p's in the formulae (35), (36), etc. by μ_T. Also, it is noted that the above Turnpike result holds true, provided that although μ_T may change as T changes, it remains to be bounded from above for all values of T.

It must be noted that when people have no idea of the value of the terminal prices, p_{t+T}, the current and the expected prices, p_t, ${}_tp^e_{t+1}$, ..., ${}_tp^e_{t+T}$ are all left undetermined with a degree of freedom of one. To fix them we need an additional condition which is decided exogenously; and one may interpret the Muth-Sheffrin or the Shizuki-Muto-Begg equation of rational expectation ${}_tp^e_{t+1} = \lambda_1 p_1$, as such a condition! There is, however, no rational basis for choosing it from among all possible conditions; it is particularly so in the case of the life span of the regime is limited both in the direction towards the past and in the direction towards the future.

Appendix

Bearing (15) and (16) in mind and using $\lambda_1 \ \lambda_2 = 1$ we can verify:

$$\lambda_1 + \lambda_2 = \frac{1}{g_0} + g_1$$

Considering

$$\lambda_1 + \lambda_2 = \frac{\beta + \gamma}{\alpha}$$

this enables us to write

$$\text{(A.1)} \quad g_0 = (\gamma g_0 - \alpha) / (\alpha g_1 - \beta)$$

Next from (15) and (16) we obtain

$$h_0 / h_1 = g_0 \quad .$$

Hence, from (A.1)

$$(\gamma h_0 - \alpha h_1) / (\alpha g_1 - \beta) = [(\gamma g_0 - \alpha) / (\alpha g_1 - \beta)] h_1 =$$

$$= g_0 h_1 =$$

$$= h_0 \ .$$

Thus

$$(A.2) \quad h_0 = \frac{\gamma h_0 - \alpha h_1}{\alpha g_1 - \beta}$$

Footnotes

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(1) See for example Begg (1982), pp. 28-60, and Shizuki-Muto (1981), pp. 39-66.

(2) Keynes (1936), p. 293.

(3) Hicks (1939), pp. 191-201.

(4) Muth (1961), pp. 325-26.

(5) Sheffrin (1983), p. 166.

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