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Does less inequality among households mean less inequality among individuals?

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Abstract - Consider an income distribution among households of the same size in which individuals, equally needy from the point of view of an ethical observer, are treated unfairly. Individuals are split into two types, the dominant and the dominated. We look for conditions under which welfare and inequality quasi-orders established at the household level still hold at the individual one. A necessary and sufficient condition for the Generalized Lorenz test is that the income of dominated individuals is a concave function of the household income: individuals of poor households have to stand more together than individuals of rich households. This property also proves to be crucial for the preservation of the Relative and Absolute Lorenz criteria, when the more egalitarian distribution is the poorest. Extensions to individuals heterogeneous in needs and more than two types are also provided.

Key Words: Lorenz dominance, Intra-household inequality, concavity, sharing rule.
JEL Codes: D13, D63, D31.

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1 Introduction

In modern western democracies, there is not much debate about the fact that the ultimate object of concern for economic policy is the well-being of individuals. It has been noticed for a long time that the household forms an informational screen between the government and the individuals. When public authorities target social benefits to some specific group of individuals (e.g. children) or assess the impact of such a policy, their action is limited by asymmetries of information about the allocation of resources within the household. Indeed, in many societies, this allocation is considered as a domain of privacy and as such is protected by the law. A somewhat related informational problem is the fact that the elementary statistical unit remains the household in most data bases that deal with income distribution. There is at least one case where this veil of ignorance would be innocuous for the appraisal or the design of public policies. As pointed out by Bourguignon and Chiappori [3], if the household behavior is such that the intra-household distribution is optimal for the policy maker, then according to a decentralization device, it is sufficient for the state to ensure the resources allocation problem among families in order to achieve a grand optimum among individuals.

Yet, there is some empirical evidence that this rosy picture is out of this world and that the family, like other institutions, may be unfair in the sense that similar individuals from the policy's maker view point may be discriminated in the allocation of resources within a household. The origins of the literature on intra-household inequality are referred to in Sen [21]. He summarizes a number of studies which argue that girls are discriminated relatively to boys. The relevance of gender disparities has been recently recognized in The Human Development Report 1995, which introduces two new measures for ranking countries according to their performance in gender equality (see Anand and Sen [1] for more details). Hence, from the point of view of the decision maker, the most common background might be that some unknown intra-household inequality prevails.

In a bunch of papers, Haddad and Kanbur ([8], [9],[10], [12]) provide a first theoretical demonstration of the relative importance of intra-household inequality with respect to inter-household inequality. In their first paper, Haddad and Kanbur [8] show that the neglect of intra-household inequality is likely to lead to an understatement of the *levels* of inequality. They show that standard measures of inequality in calorie adequacy would be understated by 30 to 40 percent if intra-household inequality was ignored. They also find this problem 'not dramatic' for inequality *comparisons*. More precisely, when intra-household inequality in the two populations is 'sufficiently similar', ignoring intra-household behavior does not reverse the results of inequality comparisons based on a decomposable index of inequality.

Here we deal with the same kind of questions as Haddad and Kanbur and we are concerned with statements about the evolution of inequality at the individual level which can be inferred from the knowledge of the evolution of inequality at the household stage. In other words, by taking into account the fact that intra-household inequality is unobservable but that some general pattern of inequality may be postulated, we exhibit cases where knowing the pattern of inter-household inequality may be sufficient to predict the evolution of overall inequality. A major difference with Haddad and Kanbur analysis is that we are interested in dominance tools like the Lorenz Curve, the Generalized Lorenz curve, with the advantages of robustness of conclusions associated to this approach.

We start by focusing on the simplest possible configuration: given a population of households of the same size, each household is composed of two types of individuals. Suppose that all individuals are homogeneous in the sense that either they are endowed with the same capacity of deriving welfare from income in a utilitarian perspective or their claim to obtain a share of the cake is considered to be identical from an ethical point of view. However, they are distinguished by some characteristics such as sex, age, which do not have to play a role in distribution issues. Despite the fact that the allocation within households ought to be equal, we suppose that the actual distribution of resources within households exhibits some inequality. Why this is so, is not described in the model, but we can imagine that the bargaining power is not equal within the two types of individuals. The precise sharing rule adopted in each household is not known outside the family. Under this veil of ignorance, we simply postulate that all households use the same rule of sharing resources among its members. This assumption can be justified by arguing that some common cultural factor shapes the internal relation within households in a given society.

Hence the problem faced by the ethical observer can be described as follows. He (or she) would like to assess the variation in welfare or inequality at the individual level, but information about the income distribution at this level is not available to him. He is only aware of the distribution of household incomes and of the existence of some discrimination in each household. A given type of individual (not necessarily the same in all households) receives a better treatment. Still he does not know how large the unfairness is. This uncertainty is parallel to the uncertainty concerning the degree of concavity of the utility function in traditional social or stochastic dominance analysis. Taking into account the fact that intra-household decisions are biased, the ethical observer would like to know under which conditions about intra-household behavior the results of welfare and inequality comparisons among household income distributions are preserved at the individual level. It turns out that the 'similarity' of behavior across households is not a sufficient condition.

Our main result shows that welfare gains at the household level according to the General Lorenz test translate into welfare gains at the individual stage with the same criterion if and only if the households share their resources among their members according to a *concave sharing rule*. In other terms, a necessary and sufficient condition to neglect intra-household inequality, when we are interested in rankings of income distributions, is that the poorest households are the more egalitarian. In a dynamic perspective, one should say that the pattern of intra-household inequality must be pro-cyclic, if the sharing rule remains unchanged over time. At this stage, we do not know the plausibility of this condition. Still it has the advantage to be served as a testable restriction in an econometric model of the household.

The next section introduces the basic notations, the tools used in the evaluation of welfare and inequality and the main assumptions on intra-household behavior. The preservation of the General Lorenz dominance criterion is studied in Section 3, while results about the preservation of Absolute and Relative Lorenz rankings are the matter of Section 4. Section 5 shows that the main result of the paper still holds under more general assumptions on household composition or intra-household behavior. The last section concludes with an assessment of the strengths and weaknesses of the paper and by identifying potential ways for further extensions.

2 The setup

2.1 The normative framework

We consider a population composed of n households indexed by $i = 1, ..., n$ (with $n \geq 2$). Let y_i designate the income of the household i. We assume $y_i \in D$ for all $i = 1, ., n$, where $D = [\underline{y}, +\infty)$ is an unbounded interval of $\mathbb{R}_+$. Except when specified, $\underline{y} = 0$. Let $\mathbf{y} = (y_1, y_2, ..., y_n)$ be a generic vector of household incomes with average $\mu_{\mathbf{y}} > 0$ and ordered in an increasing way. The feasible set of households' income distributions is denoted by $\mathbb{D} = \left\{ \mathbf{y} \in \mathbb{R}^n_+ \middle| \ y_i \in D \text{ for all } i = 1, ., n \text{ and } \ y_1 \leq y_2 ... \leq y_n \right\}$. We designate by $\mathbb{D}^*$ the restricted domain with $\underline{y} > 0$. In order to complete the notation, we denote by $\mathbf{e}_n$ the unit vector in $\mathbb{R}^n_+$ and we recall that $\mathbf{y} \geq \mathbf{y}'$ means $y_i \geq y'_i$ for all $i = 1, ., n$. We focus on the Generalized Lorenz (GL) criterion for welfare comparisons and on the Relative (RL) and Absolute Lorenz

(AL) criteria for inequality comparisons (see Shorrocks [22]). For the sake of completeness, we recall the definitions.

Definition 1 *Given* $\mathbf{y},\mathbf{y}'\in\mathbb{D}$,

i) $\mathbf{y}$ *dominates* $\mathbf{y}'$ *according to the Generalized Lorenz criterion, denoted by* $\mathbf{y}\succcurlyeq_{GL}\mathbf{y}'$, *if*

$$\frac{1}{n}\sum_{i=1}^{k} y_i \geq \frac{1}{n}\sum_{i=1}^{k} y_i' \text{ for } k=1,..,n.$$

ii) $\mathbf{y}$ *dominates* $\mathbf{y}'$ *according to the Relative Lorenz criterion, denoted by* $\mathbf{y}\succcurlyeq_{RL}\mathbf{y}'$, *if*

$$\frac{1}{n}\sum_{i=1}^{k}\frac{y_i}{\mu_{\mathbf{y}}} \geq \frac{1}{n}\sum_{i=1}^{k}\frac{y_i'}{\mu_{\mathbf{y}'}}, \text{ for } k=1,.,n.$$

iii) $\mathbf{y}$ *dominates* $\mathbf{y}'$ *according to the Absolute Lorenz criterion, denoted by* $\mathbf{y}\succcurlyeq_{AL}\mathbf{y}'$, *if*

$$\frac{1}{n}\sum_{i=1}^{k}(y_i-\mu_{\mathbf{y}}) \geq \frac{1}{n}\sum_{i=1}^{k}(y_i'-\mu_{\mathbf{y}'}), \text{ for } k=1,..,n.$$

Another criterion plays some role in the analysis. We say that $\mathbf{y}$ dominates $\mathbf{y}'$ according to the *Lorenz* criterion (L), denoted by $\mathbf{y}\succcurlyeq_{L}\mathbf{y}'$, if $\mathbf{y}\succcurlyeq_{GL}\mathbf{y}'$ and $\mu_{\mathbf{y}}=\mu_{\mathbf{y}'}$.

Households are composed of s individuals. Let $\mathbf{x}=(x_1,x_2,...,x_{sn})$ be a generic vector of positive individual incomes ordered in an increasing way. Up to Section 5, we assume that individuals are identical from an ethical point of view. In the dominance approach, this assumption is translated by posing that all individuals have the same utility function u. According to an utilitarian social welfare function, the welfare associated to an individual income distribution $\mathbf{x}$ is larger than the welfare associated to the income distribution $\mathbf{x}'$ if $\sum_{j=1}^{sn}u(x_j)\geq\sum_{j=1}^{sn}u(x_j')$. It is well known (see Shorrocks [22]) that $\mathbf{x}\succcurlyeq_{GL}\mathbf{x}'$ if and only if the above inequality holds for all the class of non-decreasing and concave utility functions u.

It remains to describe how household income is allocated among the family members in our framework.

2.2 Intra-household allocation

We assume that in every household i, labor and non labor incomes of different individuals are pooled to form the household income y_i, which is then shared among the household members. The income devoted to each individual is supposed to be a good proxy for her or his well-being, an assumption which may be accepted in the absence of family public goods. We assume that individuals are treated in an asymmetric way. More precisely, we suppose that each household is composed of two types of individuals, the *dominant* and the *dominated* ones. In our simplest and favorite example, the couple, there is one person of each type, but the framework is sufficiently general to encompass more complex family structures as tribes or even more to figure out the case of large groups like cities, regions or nations. Each dominated individual receives at most an income share equal to the share received by a dominant individual. Thus dominant individuals are the 'rich' within the household and the dominated are the 'poor'. Moreover, it is assumed that each household is composed of the same number d of dominant individuals and δ of dominated ones. Let p_i be the amount received by each dominated individual in household i. We assume that p_i is determined according to a *sharing function* of the household income y_i

$$p_i = f_p^i(y_i),$$

which represents a reduced form of the intra-household decision making. The amount r_i received by each dominant is consequently defined by $r_i = f_r^i(y_i) = \dfrac{y_i - \delta f_p^i(y_i)}{d}$.[1]

Given a vector $\mathbf{y}$ of household incomes, $\mathbf{p}(\mathbf{y}) =(p_1, .., p_j, .., p_{\delta n})$ designates the income vector for dominated individuals, $\mathbf{r}(\mathbf{y}) =(r_1, ..., r_j, ..., r_{dn})$ the income vector for dominant individuals, and $\mathbf{x}(\mathbf{y}) = (\mathbf{p}(\mathbf{y}), \mathbf{r}(\mathbf{y}))$.

We suppose that the sharing functions f_p^i are the same among households, that is, a

[1] The household composition being fixed, we do not introduce δ and d as arguments of the sharing function.

common bias due to a social norm induces a homogeneous intra-household discrimination in the population considered.[2]

Assumption 1 *The functions* $f_p^i : D \to \mathbb{R}_+$ *are identical across households and such that* $f_p(y) \leq \frac{1}{s} y$.

Let us designate by $\mathcal{F}$ the class of functions satisfying Assumption **1** and by $\mathcal{F}^0$ the subset of $\mathcal{F}$ composed of non-decreasing and continuous functions. There are two ways to capture the idea that the situation of the dominated weakens when household income increases: we may think either in *relative* or in *absolute* terms.

To catch the *relative* point of view, let us define $\mathcal{P}$ as the subset of $\mathcal{F}$ of *progressive* sharing functions satisfying: $f_p(\alpha y) \geq \alpha f_p(y)$, $\forall \alpha \in [0,1]$ and $\forall y \in D$. [3] Within this class, the income share of the dominated is decreasing with household wealth, i.e. $\frac{f_p(y)}{y} \downarrow$ with y for $y \neq 0$.[4] If the previous inequality is reversed, the sharing function will be termed *regressive*.

In order to apprehend the *absolute* point of view, we introduce the class $\mathcal{M} \subset \mathcal{F}$, composed of *moving away* functions. Namely, $\forall a, b \in \mathbb{R}_+$ with $a < b$, $f_p(b) - f_p(a) \leq \frac{b-a}{s}$. In order to give an interpretation of this class, let us consider the deviation between the equal split income and the amount devoted to the 'poor guy", $\frac{y}{s} - f_p(y) = \psi(y)$. Requiring ψ to be non-decreasing is tantamount to restrict its attention to the $\mathcal{M}$ class. The division of the family cake is moving away from the equal split as the household income increases along. A *moving closer* sharing function is going on the opposite way. Since for $\underline{y} = 0$ this class

[2] In our framework, the identity of the dominated which corresponds to some exogeneous criterion (sex, age,...) need not be the same in all families.

[3] $-f_p$ is *star-shaped* (see Marshall and Olkin [16] p. 453). The functions of $\mathcal{P}$ are also called *star-shaped at* ∞ in the abstract convexity literature, or simply *star-shaped* by Landsberger and Meilijson [14].

[4] See Marshall-Olkin, Proposition B.9 p. 453. This property is termed *progressivity* in taxation, when f_p is interpreted as the disponible income w.r.t. the gross income (see Lambert [13]).

degenerates into a perfectly egalitarian sharing rule, we will assume $\underline{y} > 0$ in the analysis of absolute variations of income distributions.

The main results of this paper are related to a third set $\mathcal{C} \subset \mathcal{F}$ composed of concave functions. The functions of $\mathcal{C}$ are continuous and non-decreasing (see Moyes [17], Lemma 3.2). On the opposite, classes $\mathcal{P}$ and $\mathcal{M}$ allow for non-monotonic sharing functions. An illustration is provided below for the case of a couple ($d = \delta = 1$).

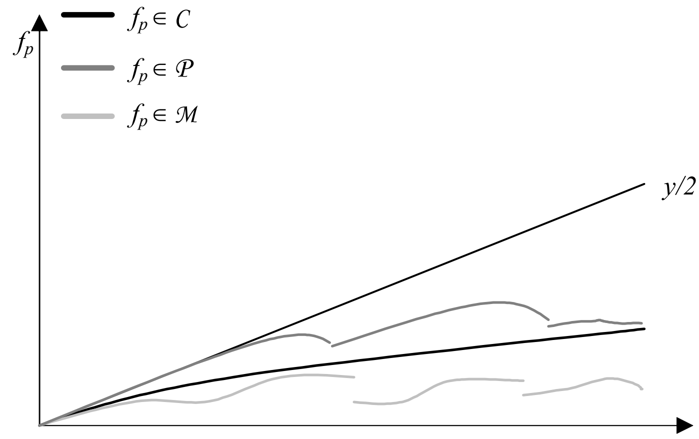


Figure 1: The classes of sharing functions

It is well established that $\mathcal{C}$ is a subset of $\mathcal{P}$ (see Marshall and Olkin, p. 453). We prove that $\mathcal{M}$ contains $\mathcal{P}$

Remark 1 $\mathcal{P} \subset \mathcal{M}$

Proof. Suppose by contradiction that f_p does not belong to $\mathcal{M}$. Then $f_p(b) - f_p(a) > \dfrac{b-a}{s}$, for some $a, b \in \mathbb{R}_+$, with $a < b$, which gives

$$b(\frac{1}{s} - \frac{f_p(b)}{b}) < a(\frac{1}{s} - \frac{f_p(a)}{a}). \tag{1}$$

Both $(\frac{1}{s} - \frac{f_p(b)}{b})$ and $(\frac{1}{s} - \frac{f_p(a)}{a})$ are non negative by Assumption 1. If they are both positive, since $b > a$, then $\frac{1}{s} - \frac{f_p(b)}{b} < \frac{1}{s} - \frac{f_p(a)}{a}$, equivalent to $\frac{f_p(b)}{b} > \frac{f_p(a)}{a}$, which contradicts $f_p \in \mathcal{M}$. If $\frac{1}{s} - \frac{f_p(a)}{a} = 0$, from (1) we get $\frac{b}{s} < f_p(b)$, which is impossible. Finally, if $\frac{1}{s} - \frac{f_p(b)}{b} = 0$ and $f_p \in \mathcal{P}$, then $\frac{1}{s} - \frac{f_p(a)}{a} = 0$ and (1) gives $0 < 0$. ■

We now recall some properties of concave functions which are extensively used in the following.[5]

Claim 1 *Let u be defined on a interval $I \subset \mathbb{R}$. If u is concave on I, then*
$$\frac{u(b_1)-u(a_1)}{b_1-a_1} \geq \frac{u(b_2)-u(a_2)}{b_2-a_2}, \text{ whenever } a_1 < b_1 \leq b_2 \text{ and } a_1 \leq a_2 < b_2.$$

Claim 2 *If u is continuous and not concave on I, then there exists $\bar{x}$ in the interior of I and there exists $\delta > 0$, such that $2u(\bar{x}) < u(\bar{x}+\varepsilon) + u(\bar{x}-\varepsilon)$, $\forall \varepsilon \in (0,\delta)$.*

3 The main result

A starting point is to explore a preservation property restricted to a subgroup of the population composed either of dominated or of dominant types. Such an inquiry might be useful when the type is associated with some definite characteristics such as gender, age, race, nationality.

Remark 2 *Let $f_p \in \mathcal{F}$.*

i) $f_p \in \mathcal{C} \iff [\mathbf{y} \succcurlyeq_{GL} \mathbf{y}' \implies \mathbf{p}(\mathbf{y}) \succcurlyeq_{GL} \mathbf{p}(\mathbf{y}'), \text{for all } \mathbf{y}, \mathbf{y}' \in \mathbb{D}]$

ii) f_r *is concave* $\iff [\mathbf{y} \succcurlyeq_{GL} \mathbf{y}' \implies \mathbf{r}(\mathbf{y}) \succcurlyeq_{GL} \mathbf{r}(\mathbf{y}') \text{ for all } \mathbf{y}, \mathbf{y}' \in \mathbb{D}]$.

Proof. Immediate application of Moyes [17], Theorem 3-1, p. 351 or Marshall-Olkin [16], Theorem A.2 (ii), p. 116. ■

Of course, in the case of a linear sharing rule, the GL dominance among households is inherited both by subpopulation **p** and **r**. The difficulty to extend Remark 2 to the whole population arises from the diverse rankings of individual incomes which are allowed by applying different concave sharing functions to a fixed household income distribution.[6]

[5] For a proof see Marshall and Olkin [16], p. 447 and Yaari [23], p. 1184, respectively.

[6] The number of different rankings among individuals for $n > 1$ couples is given by the following formula:
$$n + \sum_{k=1}^{n-1} (n-k) \prod_{j=2}^{k+1} (n-j).$$

As a first step, we show that a sharing function that saves the GL dominance relation obtained at the household level must be non-decreasing and continuous.

Proposition 1 *Let* $f_p \in \mathcal{F}$.

$[\mathbf{y} \succcurlyeq_{GL} \mathbf{y}' \implies \mathbf{x}(\mathbf{y}) \succcurlyeq_{GL} \mathbf{x}(\mathbf{y}')$ *for all* $\mathbf{y}, \mathbf{y}' \in \mathbb{D}] \implies f_p \in \mathcal{F}^0$.

Proof. f_p must be non-decreasing. Let us suppose by contradiction that there exist some positive scalars a, b such that $a < b$ and $f_p(a) > f_p(b)$. Choosing $\mathbf{y} = (0,..,0,b)$ and $\mathbf{y}' = (0,..,0,a)$ trivially entails $\mathbf{y} \succcurlyeq_{GL} \mathbf{y}'$, but $\sum_{j=1}^{sn-d} x_j < \sum_{j=1}^{sn-d} x'_j$. Hence, $\mathbf{x}(\mathbf{y}) \succcurlyeq_{GL} \mathbf{x}(\mathbf{y}')$ is false.

f_p must be continuous. Let us consider $\mathbf{y} = (0,..,0,a,b)$ and $\mathbf{y}' = (0,..,0,a+b)$. It is easy to see that $\mathbf{y} \succcurlyeq_{L} \mathbf{y}'$ (and *a fortiori* $\mathbf{y} \succcurlyeq_{GL} \mathbf{y}'$) for any positive a and b. In order to secure $\mathbf{x}(\mathbf{y}) \succcurlyeq_{L} \mathbf{x}(\mathbf{y}')$, we need $\sum_{j=1}^{sn-d} x_j \geq \sum_{j=1}^{sn-d} x'_j$. Then f_p must satisfy the following property:

$$\delta\left[f_p(a) + f_p(b)\right] + df_r(a) \geq \delta f_p(a+b) \text{ for all positive } a, b. \quad (2)$$

We can rewrite (2) as: $f_p(b+a) - f_p(b) \leq \frac{1}{\delta}a$, for all positive a, b. Given that f_p is non-decreasing, it is bound to be a Lipschitzian function. ■

An obvious further step is to show that non-decreasingness is sufficient to preserve the GL test when we confine our attention to household income distributions which differ by positive increments.

Claim 3 *Let* $f_p \in \mathcal{F}$.

f_p *is non-decreasing* $\iff$ $[\mathbf{y} \succcurlyeq_{GL} \mathbf{y}' \implies \mathbf{x}(\mathbf{y}) \succcurlyeq_{GL} \mathbf{x}(\mathbf{y}')$, *for all* $\mathbf{y}, \mathbf{y}' \in \mathbb{D}$ *such that* $\mathbf{y} \geq \mathbf{y}']$.

We now proceed to the proof of an important lemma well suited to answer to the preservation question for the unrestricted domain of household income distributions.

Lemma 1 *Given an interval $I \subset \mathbb{R}$, let $u : I \to \mathbb{R}$, $g, h : I \to I$ be continuous non-decreasing functions and w be the composite function defined on I by:*

$$w(y) = \delta u[g(y) - h(y)] + du[g(y) + \frac{\delta}{d} h(y)]$$

with δ and d strictly positive scalars. If u and g are concave and h convex, then w is concave.

Proof. See Appendix. ■

We make use of the lemma in the proof of our main theorem, which identifies the condition on the sharing function which is necessary and sufficient to preserve a GL dominance relation from the household level to the individual one. In the proof, w is nothing else that the welfare computed at the household level.

Theorem 1 *Let $f_p \in \mathcal{F}^0$.*

$$f_p \in \mathcal{C} \iff [\mathbf{y} \succcurlyeq_{GL} \mathbf{y}' \implies \mathbf{x}(\mathbf{y}) \succcurlyeq_{GL} \mathbf{x}(\mathbf{y}'), \text{ for all } \mathbf{y}, \mathbf{y}' \in \mathbb{D}] .$$

Proof. $\implies$ Suppose that f_p is concave and consider $\mathbf{y}, \mathbf{y}' \in \mathbb{D}$ such that $\mathbf{y} \succcurlyeq_{GL} \mathbf{y}'$. We prove that $\sum_{j=1}^{sn} u(x_j) \geq \sum_{j=1}^{sn} u(x'_j)$ for all u non-decreasing and concave, which is equivalent to $\mathbf{x}(\mathbf{y}) \succcurlyeq_{GL} \mathbf{x}(\mathbf{y}')$.

For a given u, let w designate the sum of individual utilities for the household i. Then $\sum_{j=1}^{sn} u(x_j) = \sum_{i=1}^{n} w(y_i)$ and $\sum_{j=1}^{sn} u(x'_j) = \sum_{i=1}^{n} w(y_i)$. Given that $w(y_i) = \delta u(f_p(y_i)) + du(f_p(y_i))$, we can replace $= \frac{y_i}{s} - \psi(y_i)$ instead of $f_p(y_i)$ and $\frac{y_i}{s} + \frac{\delta}{d}\psi(y_i)$ instead of $f_r(y_i)$. We obtain $w(y_i) = \delta u \left[\frac{y_i}{s} - \psi(\frac{y_i}{s})\right] + du \left[\frac{y_i}{s} + \frac{\delta}{d}\psi(\frac{y_i}{s})\right]$. Since $f_p \in \mathcal{C}$, then ψ is convex. Applying Lemma 1, by posing $I = \mathbb{R}_+$, $g(y) = \frac{y}{s}$, $h(y) = \psi(\frac{y_i}{s})$, we get that w is concave. It is easy to see that w is non-decreasing. Since $\mathbf{y} \succcurlyeq_{GL} \mathbf{y}'$, then $\sum_{i=1}^{n} w(y_i) \geq \sum_{i=1}^{n} w(y'_i)$ and therefore $\sum_{j=1}^{sn} u(x_j) \geq \sum_{j=1}^{sn} u(x'_j)$. The reasoning holds for any non-decreasing and concave u.

$\Longleftarrow$ Assume now by contradiction that f_p is not concave. By Claim 2, for some $y^* \in \mathbb{R}_{++}$, there exists $\zeta > 0$ such that, for every ε with $0 < \varepsilon < \zeta$

$$2f_p(y^*) < f_p(y^* - \varepsilon) + f_p(y^* + \varepsilon). \tag{3}$$

Furthermore, (3) combined with $f_p(y^*) \leq \frac{1}{s}y^*$ implies $f_p(y^*) - f_r(y^*) < 0$.[7] Then, by continuity, there exists $\bar{\zeta} > 0$ such that, for every ε satisfying $0 < \varepsilon < \bar{\zeta}$, (3) holds and

$$f_p(y^* + \varepsilon) < f_r(y^* - \varepsilon). \tag{4}$$

We now choose $\mathbf{y} = (0, .., 0, y^*, y^*)$ and $\mathbf{y}' = (0, .., 0, y^* - \varepsilon, y^* + \varepsilon)$. By construction, $\mathbf{y} \succcurlyeq_{GL} \mathbf{y}'$. From (3), we deduce $f_p(y_{n-1}) + f_p(y_n) < f_p(y'_{n-1}) + f_p(y'_n)$ which gives, combined with (4), $\sum_{j=1}^{sn-2d} x_j < \sum_{j=1}^{sn-2d} x'_j$. Hence, $\mathbf{x}(\mathbf{y}) \succcurlyeq_{GL} \mathbf{x}(\mathbf{y}')$ is contradicted. ■

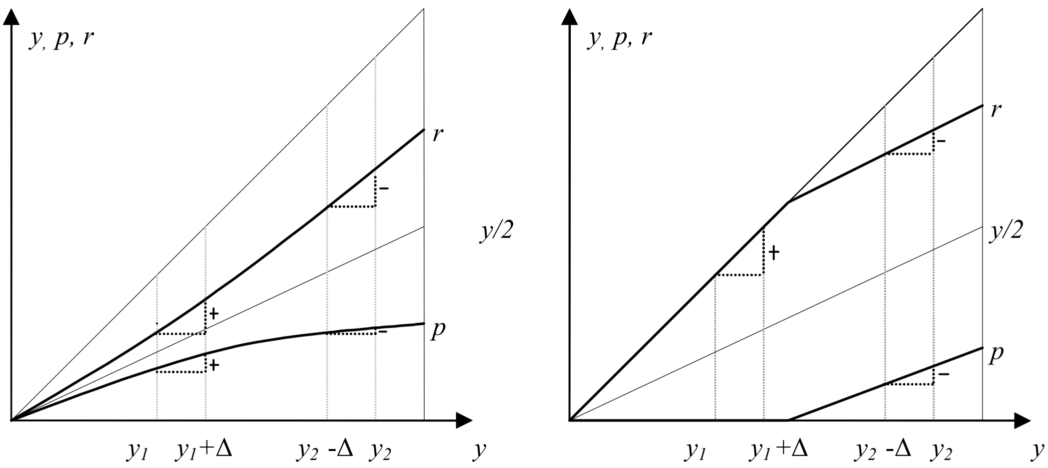
From a purely formal point of view, one may ask whether the opposite is true, namely the concavity of the sharing function is either sufficient or necessary for the GL ranking of distributions observed at the individual level to be preserved at the family one. It is easy to show that concavity is not a sufficient condition.[8]

3.1 Interpretation

The intuition behind Theorem 1 is the following. It is well known that $\mathbf{y} \succcurlyeq_{GL} \mathbf{y}'$ if and only if $\mathbf{y}$ can be obtained from $\mathbf{y}'$ by a finite sequence of *progressive transfers* (also named *Pigou-Dalton transfers*) and *increments* (see Marshall and Olkin, [16], C.6, p. 28 and A.9.a, p. 123). When the sharing function is concave, a progressive transfer between households implies a 'double dividend' on social welfare valued at the individual level. Indeed, a transfer from a richer family to a poorer one becomes a transfer from a less egalitarian household to a more egalitarian one as well. An 'intra-household dividend" supplements the traditional 'inter-household dividend". The Figure 2 illustrates the impact of a transfer of amount Δ from a couple with an initial income y_2 to a couple with an initial income y_1, with $y_1 < y_2$.

[7] Indeed, by assumption $f_p(y^*) \leq f_r(y^*)$. Suppose now that $f_p(y^*) - f_r(y^*) = 0$. This is equivalent to $f_p(y^*) = \frac{1}{s}y^*$. From (3) and $f_p(y) \leq \frac{1}{s}y\ \forall y$, we get $\frac{2}{s}y^* < f_p(y^* - \varepsilon) + f_p(y^* + \varepsilon) \leq \frac{2}{s}y^*$, which is impossible.

[8] A counterexample is given by: $s = 2$, $f_p = \begin{cases} \frac{1}{4}y & y \leq 40 \\ 5 + \frac{1}{8}y & y > 40 \end{cases}$, $\mathbf{y} = (4, 12, 37, 64)$ and $\mathbf{y}' = (4, 10, 40, 48)$.



a- Concave sharing function **b-** Convex sharing function

Figure 2: Effect of a progressive transfer between households

The progressive transfer between households induces three progressive transfers among individuals (see Panel **a**). The dominated of the poor family receives a transfer from the two individuals of the rich family. Moreover, the dominant of the rich family loses at the benefit of the poor household's dominant.

An opposite case with a convex sharing function is represented in Panel **b**: the same progressive transfer among households generates an ambiguous effect. The dominant of the poor family receives a *progressive* transfer $\Delta/2$ from his counterpart of the rich household and a *regressive* one $\Delta/2$ from the dominated of the rich household. Then, the social welfare may improve or get worse, depending on the degree of concavity of the individual utility function.

4 Inequality comparisons

In this section, we focus on inequality criteria which neutralize the differences of the size of the cake. A first remark establishes a rather obvious comparative static property which illustrates the effect of a rise of intra-household inequality in every household.

Remark 3 *Let f_p and $f'_p \in \mathcal{F}$.*

$f_p(y) \geq f'_p(y)\ \forall y \in \mathbb{R}_+ \iff \mathbf{x}(\mathbf{y}) \succcurlyeq_L \mathbf{x}'(\mathbf{y})$ *for all* $\mathbf{y} \in \mathbb{D}$.

Proof. $\Longrightarrow$ If we compare the individual incomes generated by the different sharing functions f_p and f'_p for the household i, then the second distribution can be deduced from the first one through a *mean preserving spread.* Therefore

$$(f_p(y_i)\mathbf{e}_\delta, f_r(y_i)\mathbf{e}_d) \succcurlyeq_L (f'_p(y_i)\mathbf{e}_\delta, f'_r(y_i)\mathbf{e}_d), \tag{5}$$

where $\mathbf{e}_\delta$ and $\mathbf{e}_d$ are unitary vectors belonging to $\mathbb{R}^\delta$ and $\mathbb{R}^d$, respectively. Moreover, given that (5) holds for every household, applying Proposition A.7 (i), p. 121 of Marshall-Olkin [16], we obtain $\mathbf{x}(\mathbf{y}) \succcurlyeq_L \mathbf{x}'(\mathbf{y})$.

$\Longleftarrow$ Suppose by contradiction $f'_p(a) > f_p(a)$ for some $a > 0$. By considering the distribution $\mathbf{y} = (0, ..., 0, a)$, it is easy to see that $\mathbf{x}(\mathbf{y})$ may be obtained from $\mathbf{x}'(\mathbf{y})$ through a mean preserving spread and then $\mathbf{x}(\mathbf{y}) \succcurlyeq_L \mathbf{x}'(\mathbf{y})$ is false. ■

To start from an egalitarian f_p is equivalent to ignore intra-household inequality from a formal point of view. Then, as a particular case, we find again the result obtained by Haddad and Kanbur [8], according to which ignoring intra-household inequality leads to an under-estimation of the income inequality among individuals. We now ask whether concavity of sharing functions is sufficient to secure preservation of L, RL and AL dominance. Inspection of the proof of Theorem 1 allows to state the following remark.

Remark 4 *Theorem 1 holds for the preservation of the Lorenz dominance criterion.*

It turns out that more stringent conditions have to be checked for the two other criteria.

4.1 The relative point of view

A first step is to consider a proportional change in household income distribution. The consequences on individual inequality are investigated in the following lemma which illustrates the relevance of the progressivity property.

Lemma 2 *Let $f_p \in \mathcal{F}$.*

i) *f_p is progressive* $\iff$ $[\mathbf{x}(\alpha\mathbf{y}) \succcurlyeq_{RL} \mathbf{x}(\mathbf{y}), \forall\alpha \in (0,1)$ *and* $\forall\mathbf{y} \in \mathbb{D}_I]$.

ii) *f_p is regressive* $\iff$ $[\mathbf{x}(\alpha\mathbf{y}) \succcurlyeq_{RL} \mathbf{x}(\mathbf{y}), \forall\alpha > 1$ *and* $\forall\mathbf{y} \in \mathbb{D}]$.

Proof. See Appendix. ■

When the sharing function is progressive (respectively regressive), the pattern of inequality among individuals is pro-cyclic (resp. anticyclic), when all household incomes inflate or deflate proportionally. With homothetic changes in household income, concavity of the sharing rule is no more required to preserve the relative Lorenz quasi-order. Unfortunately when we enlarge the comparison to all feasible household distributions, only linear sharing functions guarantee the preservation of the relative Lorenz ranking

Proposition 2 *Let $f_p \in \mathcal{F}$.*

$f_p = \beta y$, with $\beta \in [0, \frac{1}{s}]$ $\iff$ $[\forall\ \mathbf{y}, \mathbf{y}' \in \mathbb{D},\ \mathbf{y} \succcurlyeq_{RL} \mathbf{y}' \implies \mathbf{x}(\mathbf{y}) \succcurlyeq_{RL} \mathbf{x}(\mathbf{y}')]$.

Proof. $\implies$ A direct consequence of Theorem 1.

$\impliedby$ From the necessity part of Lemma 2, it follows that the preservation at the individual level of RL after a scaling up and a scaling down of a household income distribution requires progressivity and regressivity of f_p. ■

Thus, concavity of the sharing function is not sufficient to obtain preservation of the relative Lorenz criterion. It is easy to understand why it is so. Consider two societies where the more egalitarian according to the relative Lorenz criterion is also the richest on average. Hence the more egalitarian may be obtained from the less egalitarian by a finite sequence of Pigou-Dalton transfers and increments. On the one hand, the double dividend generated by progressive transfers which we alluded to in Section 3, is still operating. On the other hand, the increments make the households richer and since a concave sharing function makes rich households more unequal than the poor, increments have a regressive impact on the distribution among individuals which may offset the progressive effect of Pigou-Dalton transfers. Imposing a lower mean for the more egalitarian distribution prevents this outcome to occur.

Proposition 3 *Let* $f_p \in \mathcal{F}$*.*

$$f_p \in \mathcal{C} \iff [\forall\ \mathbf{y}, \mathbf{y}' \in \mathbb{D},\ \textit{with}\ \mu_{\mathbf{y}} \leq \mu_{\mathbf{y}'},\ \mathbf{y} \succcurlyeq_{RL} \mathbf{y}' \implies \mathbf{x}(\mathbf{y}) \succcurlyeq_{RL} \mathbf{x}(\mathbf{y}')].$$

Proof. $\implies$ Let us introduce $\mathbf{y}'' = \frac{\mu_{\mathbf{y}}}{\mu_{\mathbf{y}'}}\mathbf{y}'$. Given that $\mathbf{y} \succcurlyeq_{RL} \mathbf{y}'$ by assumption and $\mathbf{y}'' \sim_{RL} \mathbf{y}'$ by construction, $\mathbf{y} \succcurlyeq_{RL} \mathbf{y}''$. We also have $\mu_{\mathbf{y}''} = \mu_{\mathbf{y}}$, which implies $\mathbf{y} \succcurlyeq_{GL} \mathbf{y}''$. From Theorem 1, a concave f_p implies $\mathbf{x}(\mathbf{y}) \succcurlyeq_{GL} \mathbf{x}(\mathbf{y}'')$. Dividing both individual distributions by $\frac{\mu_{\mathbf{y}}}{s}$, we obtain

$$\mathbf{x}(\mathbf{y}) \succcurlyeq_{RL} \mathbf{x}(\mathbf{y}''). \tag{6}$$

From Proposition 2, we deduce $\mathbf{x}(\mathbf{y}'') \succcurlyeq_{RL} \mathbf{x}(\mathbf{y}')$ which, combined with (6), gives $\mathbf{x}(\mathbf{y}) \succcurlyeq_{RL} \mathbf{x}(\mathbf{y}')$.

$\impliedby$ Similar to the necessity part of Theorem 1. ■

If the 'RL dominant' distribution of household incomes has a higher mean than the 'RL dominated' one, nothing can be immediately concluded about the inequality at the individual level. Nevertheless, in this case, RL dominance implies GL dominance and, via Theorem 1, a higher welfare at the individual level. From a policy point of view, we conclude that, if a concave sharing function remains stable over time, a more and more wealthy society will have to adopt a more and more redistributive policy between households in order to stabilize the level of inequality among individuals.

A further consequence of the previous results concerns the effects of a *progressive taxation* at the *household* level on *individual* inequality. A well-known result, due to Jacobsson [11], states that any after-tax income distribution dominates in the RL sense the before-tax income distribution if and only if the tax system is progressive everywhere. Observe that the mean of the post-tax household income distribution is lower than the mean of the pre-tax income distribution. Then, from Proposition 3, we may state that when the sharing function is concave, *a progressive taxation schedule on household incomes leads to a lower inequality at the individual level* (in the sense of the RL dominance).

To be more precise, following Le Breton et al. [15] we define a taxation scheme G as a mapping from $\mathbb{R}_+$ to $\mathbb{R}_+$ that associates post-tax income $G(y)$ to pre-tax income y such that $G(y) < y$. A taxation scheme G is said to be *rank preserving* over $\mathbb{R}_+$, if $G(y) \leq G(y')$ for all $y, y' \in \mathbb{R}_+$ such that $y < y'$. It is said to be *progressive* if $G(y)/y \leq G(y')/y'$ for all $y, y' \in \mathbb{R}_{++}$, with $y < y'$. Let us call $\mathcal{G}$ the set of rank-preserving progressive taxation schemes on $\mathbb{R}_{++}$.

Corollary 1 *Let* $f_p \in \mathcal{F}$. $f_p \in \mathcal{C} \implies \forall \ \mathbf{y} \in \mathbb{D}, \forall G \in \mathcal{G}, \ \mathbf{x}(G(\mathbf{y})) \succcurlyeq_{RL} \mathbf{x}(\mathbf{y})$.

Proof. A consequence of Proposition 3 and Proposition 3.1 in Le Breton et al. [15]. ■

4.2 The absolute point of view

Now the same kind of questions may be investigated for the absolute Lorenz criterion. By considering a situation when the same amount of money is added to each family's income, we exemplify the interest of *moving away* sharing functions.

Lemma 3 *Let* $f_p \in \mathcal{F}$.

i) f_p *is moving away* $\iff$ $[\mathbf{x}(\mathbf{y}) \succcurlyeq_{AL} \mathbf{x}(\mathbf{y}+\alpha\mathbf{e}_n), \forall \mathbf{y} \in \mathbb{D}$ *and* $\forall \alpha > 0]$

ii) f_p*is moving closer* $\iff$ $[\mathbf{x}(\mathbf{y}+\alpha\mathbf{e}_n) \succcurlyeq_{AL} \mathbf{x}(\mathbf{y}), \forall \mathbf{y} \in \mathbb{D}^*$ *and* $\forall \alpha > 0]$.

ii) $f_p = \frac{y}{s}$ $\iff$ $[\mathbf{x}(\mathbf{y}+\alpha\mathbf{e}_n) \succcurlyeq_{AL} \mathbf{x}(\mathbf{y}), \forall \mathbf{y} \in \mathbb{D}$ *and* $\forall \alpha > 0]$

Proof. See Appendix. ■

Inequality among individuals increases according to the absolute Lorenz criterion when the gains from growth are equally shared among households if and only if the sharing rule is moving away. Now if a same decrease affects all household incomes, the results depends on the domain of the sharing function. If the domain of the sharing function is limited to strictly positive household incomes, then the symmetry of results is kept and the moving closer restriction emerges as the adequate one. If the domain is set to include a null household income, then only egalitarian sharing rules preserve the absolute Lorenz quasi-order.

As a consequence, only *moving constant* sharing functions (that is, functions that are simultaneously moving closer and moving away) preserve the absolute Lorenz criterion, in the full domain.

Proposition 4 *Let* $f_p \in \mathcal{F}$*.*

i) f_p *is moving constant* $\iff [\forall\, \mathbf{y}, \mathbf{y}' \in \mathbb{D},\ \mathbf{y} \succcurlyeq_{AL} \mathbf{y}' \implies \mathbf{x}(\mathbf{y}) \succcurlyeq_{AL} \mathbf{x}(\mathbf{y}')]$.

Proof. $\implies$ A direct consequence of Theorem 1. $\impliedby$ A direct consequence of Lemma 3). ■

When 0 is included in the domain of f_p, *moving constant* sharing functions shrink to pure egalitarian ones and an almost impossibility result is obtained for the preservation of the absolute Lorenz criterion. A more promising result is the analogue of Proposition 3, when we accept to impose that the more egalitarian distribution is also the less wealthy one.

Proposition 5 *Let* $f_p \in \mathcal{F}$*.*

$f_p \in \mathcal{C} \iff [\forall\, \mathbf{y}, \mathbf{y}' \in \mathbb{D},$ *with* $\mu_{\mathbf{y}} \leq \mu_{\mathbf{y}'},\ \mathbf{y} \succcurlyeq_{AL} \mathbf{y}' \implies \mathbf{x}(\mathbf{y}) \succcurlyeq_{AL} \mathbf{x}(\mathbf{y}')]$.

Proof. $\implies$ Let us introduce the distribution $\mathbf{y}'' = \mathbf{y} + (\mu_{\mathbf{y}'} - \mu_{\mathbf{y}})\mathbf{e}_n$. Given the assumption $\mathbf{y} \succcurlyeq_{AL} \mathbf{y}'$ and since $\mathbf{y} \sim_{AL} \mathbf{y}''$ by construction, then $\mathbf{y}'' \succcurlyeq_{AL} \mathbf{y}'$. We also have $\mu_{\mathbf{y}''} = \mu_{\mathbf{y}'}$, which implies $\mathbf{y}'' \succcurlyeq_{GL} \mathbf{y}'$. From Theorem 1, f_p concave implies $\mathbf{x}(\mathbf{y}'') \succcurlyeq_{GL} \mathbf{x}(\mathbf{y}')$. By subtracting the vector $\frac{\mu_{\mathbf{y}'}}{s}\mathbf{e}_{sn}$ from both income distributions, we obtain

$$\mathbf{x}(\mathbf{y}'') \succcurlyeq_{AL} \mathbf{x}(\mathbf{y}'). \tag{7}$$

From Proposition 4, $\mathbf{x}(\mathbf{y}) \succcurlyeq_{AL} \mathbf{x}(\mathbf{y}'')$ which, combined with (7), gives $\mathbf{x}(\mathbf{y}) \succcurlyeq_{AL} \mathbf{x}(\mathbf{y}^{'})$.

$\Longleftarrow$ Similar to the necessity part of Theorem 1. ■

5 Extensions

5.1 More than two types

We analyze the simplest case of three types in households composed of three individuals, although it is possible to generalize the argument to more types, at the price of some additional complexity. A hierarchy prevails among households where the dominated always receives less than the median individual who always gets less than one third, while the share of the dominant always exceeds one third. For notational convenience, f_p (respectively, f_r) is still the dominated (dominant) sharing function and f_m describes the income received by the median individual. f_p satisfies a similar assumption to Assumption 1. $\mathcal{F}^0$ and $\mathcal{C}$ keep their meaning in this context. We now introduce the *group sharing function* f_g, which expresses the income of the group of two first individuals; hence f_g satisfies $f_g = f_p + f_m$. Let $\mathcal{F}^g$ (respectively $\mathcal{C}^g$) designates the set of (resp. concave) group sharing functions. A generalization of Lemma 1 is needed.

Lemma 4 *Given $I \subset \mathbb{R}$, let us define the non-decreasing functions: $u : I \to \mathbb{R}$; h, h_1, $h_2 : I \to I$, such that $h = h_1 + h_2$. Let w be the composite function defined on I by:*

$$w(v) = u[v - h_1(v)] + u[v - h_2(v)] + u[v + h(v)].$$

If u is concave, h and h_1 are convex and $h_2(v) \leq h_1(v)$ $\forall v \in I$, then w is concave.

Proof. See Appendix. ■

For k types, we need $k - 1$ concavity conditions to obtain the aimed preservation result.

Proposition 6 *Let $f_g \in \mathcal{F}^g$, $f_p \in \mathcal{F}^0$ and f_m non-decreasing.*

$$f_p \in \mathcal{C} \text{ and } f_g \in \mathcal{C}^g \iff \left[\mathbf{y} \succcurlyeq_{GL} \mathbf{y}' \implies \mathbf{x}(\mathbf{y}) \succcurlyeq_{GL} \mathbf{x}(\mathbf{y}'), \text{ for all } \mathbf{y}, \mathbf{y}' \in \mathbb{D}\right].$$

Proof. See Appendix. ■

We obtain a 'chain condition", which may easily be extended to more general household structures. If we can rank the s individuals living in a household (or in a tribe) according to their income, and if such a 'hierarchy" is unaffected by the amount of household income, then the concavity of all partial sums of the k poorest individuals for $k = 1, .., s$ is necessary and sufficient to get the preservation of the GL test.

5.2 Differentiation within a type

The second extension copes with the inclusion of some differentiation among individuals of the same type. Up to now, all individuals of the same type are treated on an equal footing. It is worth it to depart from this assumption to test the robustness of our main results to a small change of our framework.

For convenience, let us consider households with two different dominated individuals and a dominant one. The individual sharing functions f_{p^1} and f_{p^2} describe the income received by the two dominated individuals and they satisfy a similar assumption than Assumption 1, namely, they start from 0 and respect $f_{p^1}(y), f_{p^2}(y) \leq \frac{1}{3}y$. As in Section 2, let $\mathcal{C}$ denote the set of concave individual sharing functions.

A complete ranking between the two dominated individuals is not required to obtain a generalization of our main result. For a given household income interval, individual 1 may be the most unfairly treated, while the opposite prevails for some other household income bracket. Let us define the lower contour set of f_{p^j} as $\mathcal{L}_{f_{p^j}} = \left\{(y, x_j) \in \mathbb{R}^2_+ \mid f_{p^j}(y) \geq x_j\right\}$ for $j = 1, 2$. The frontier of the intersection of the two lower contour sets $\mathcal{L}_{f_{p^1}}, \mathcal{L}_{f_{p^2}}$ gives the part of household income received by the poorest individual in any circumstances. Let us denote $\tilde{f}_p(y)$ the sharing function of the *anonymous* dominated. We state without proof a result which is a consequence of Proposition 6.

Corollary 2 *Let $f_g \in \mathcal{F}^g$ and $f_{p1}, f_{p2} \in \mathcal{F}^0$.*

$$\tilde{f}_p \in \mathcal{C} \text{ and } f_g \in \mathcal{C}^g \iff [\mathbf{y} \succcurlyeq_{GL} \mathbf{y}' \implies \mathbf{x}(\mathbf{y}) \succcurlyeq_{GL} \mathbf{x}(\mathbf{y}'), \text{ for all } \mathbf{y}, \mathbf{y}' \in \mathbb{D}] .$$

5.3 Individuals heterogeneous in needs

We now assume that, from the point of view of an ethical observer, there are two types of individuals: the *normal* one (type 1) and the *needy* one (type 2). The analysis is developed for the case of a population of couples comprising one normal and one needy individual. An equivalence scale is here a list of two numbers (e_1, e_2) with $e_1 \leq e_2$ so that the *equivalent incomes* equal to $(\frac{x_1}{e_1}, \frac{x_2}{e_2})$ are assumed to be directly comparable. It is usual to choose a reference type, and w.l.o.g., we choose $e_1 = 1$. Let $\widehat{\mathbf{x}}$ designate the distribution of equivalent incomes of individuals ordered in an increasing way. In this framework, we follow a suggestion made by Ebert (see Ebert [7]) to use the GL test applied to the distribution of *equivalent incomes* across *equivalent individuals* as a criterion for welfare analysis. An equivalent individual is just an individual weighted by its own equivalent scale. Thus, the k^{th} coordinates of this *Equivalent Generalized Lorenz* curve are

$$\frac{\sum_{i=1}^{k}\omega_i}{n(1+e_2)}, \frac{\sum_{i=1}^{k}\omega_i \hat{x}_i}{n(1+e_2)}, \text{ for } k = 1, ...2n, \tag{8}$$

with the weights $\omega_i = 1(e_2)$, if i is of type 1(2). Dominance according to the Equivalent Generalized Lorenz curve, denoted $\widehat{\mathbf{x}}(\mathbf{y}) \succcurlyeq_{EGL} \widehat{\mathbf{x}}(\mathbf{y}')$, has been shown to be equivalent to $\sum_{i=1}^{n} \omega_i u(\widehat{x}_i) \geq \sum_{i=1}^{n} \omega_i u(\widehat{x}'_i)$ for all u non-decreasing and concave by Ebert.

From the 'private' point of view, we assume that there are two types of individuals: the dominated and the dominant ones, as usual denoted by p and r. If the couple is fair according to the observer's ethics, then the share of the household budget devoted to the needy type must be equal to $\frac{e_2}{1+e_2}$. W.l.o.g, we assume that the needier individual is the dominating one.

Let $\mathcal{F}^1$ (respectively $\mathcal{C}^1$) be the class of non-decreasing and continuous (resp. concave) sharing functions such that: $f_p^1(0) = 0$ and $f_p^1(y) \leq \frac{1}{1+e_2} y$.

Corollary 3 *Let $f_p^1 \in \mathcal{F}^1$. Then:*

$$f_p^1 \in \mathcal{C}^1 \iff [\mathbf{y} \succcurlyeq_{GL} \mathbf{y}' \implies \widehat{\mathbf{x}}(\mathbf{y}) \succcurlyeq_{EGL} \widehat{\mathbf{x}}(\mathbf{y}'), \text{ for all } \mathbf{y}, \mathbf{y}' \in \mathbb{D}] .$$

Proof. See Appendix. ■

It is clear that a similar result can be proved in the symmetric case where the dominated individual is the needier one.

6 Concluding remarks

This investigation about the impact of the intra-household inequality on the overall inequality sheds light on the properties of the sharing function. It describes how the part devoted to the more disadvantaged changes as the household income increases along. To study the properties of the sharing function is tantamount to analyze the way intra-household inequality is related to household income. Three properties capture the idea that the more wealthy the household is, the more unequally it behaves. In the *moving away* approach, the deviation with the equal split matters, in the *progressive* approach, the average share counts while the marginal share is relevant for *concavity*.

Although these three approaches have their own merit, the main contribution of this paper is to show that concavity occupies the central stage, even if the other two prove to be useful in the analysis. It allows to preserve the Lorenz criterion when we compare distributions with the same average, the General Lorenz dominance between any couple of distributions, the Relative and Absolute Lorenz dominance when the more egalitarian distribution is the less wealthy one. If we enlarge the comparison for these two criteria to the domain of all distributions, then the class of sharing functions doing the job becomes rather narrow; for instance, we only obtain the linear functions for the relative criterion.

When the domain of comparison shrinks, the admissible class of sharing functions expands. If we restrict the comparison to household income distributions which only differ by proportionate increments (respectively equal increments), then inequality among individuals decreases with growth for an non-decreasing sharing function according to the GL criterion and for a *regressive* (resp. *moving closer*) one according to the RL criterion. Thus, the picture is a little more complex than one would expect. Even if the bottom line is that a positive correlation between within-inequality and household income helps to get the aimed preservation of the inequality relation, we find that when we are looking to the pure effect of growth, it is the opposite which prevails when we are interested in the usual Lorenz criterion.

Deeper extensions will relax basic assumptions of the present model. Four main directions deserve some further investigation. In order to complete the dominance analysis, it would be interesting to consider a population composed of families with different size, for instance couples and singles. A substantial improvement would be equally provided by the introduction of family public goods which represent an important motive for people to live together. In Couprie et al. [5], we address these two questions by resorting to the Sequential Generalized Lorenz test proposed by Atkinson and Bourguignon [2]. More difficult tasks would be to remove the assumption of homogeneous households in terms of the sharing function in use and of the number of types.

In this paper, we resort to a non-structural model of the household: the sharing function is only a reduced form which is compatible with several models of the household behavior.

Another direction of research is to explore the micro-economic foundations of concave sharing functions. A first answer is provided by Peluso and Trannoy [19] who postulate that the sharing function is the outcome of the Samuelson [20] model of household. The families are couples maximizing a 'household welfare function', given by the weighted sum of individual utilities. In future works, Samuelson's model will be extended in the direction of the more appealing 'collective approach', pioneered by Chiappori [4].[9]

All the properties of the sharing functions exposed here may be plainly wrong in trying to describe the behavior of households in any society. Still it is important to stress ultimately that they may be falsified. In Couprie et al. [5], we use non-parametric methods to estimate the shape of the sharing functions on the French household expenditure surveys. The concavity property is not rejected by our data.

Appendix

Proof of Lemma 1. We proceed by *contradiction*. Suppose that w is not concave. Hence, using Claim 2 and the concavity of u the following inequalities hold for some $\alpha, \beta \in I$ with $\alpha < \beta$

$$\begin{aligned}
&2\left[\delta u\left(g(\tfrac{\alpha+\beta}{2}) - h(\tfrac{\alpha+\beta}{2})\right) + du\left(g(\tfrac{\alpha+\beta}{2}) + \tfrac{\delta}{d}h(\tfrac{\alpha+\beta}{2})\right)\right] < \\
&\delta u\left[g(\alpha) - h(\alpha)\right] + du\left[g(\alpha) + \tfrac{\delta}{d}h(\alpha)\right] + \delta u\left[g(\beta) - h(\beta)\right] + du\left[g(\beta) + \tfrac{\delta}{d}h(\beta)\right] \\
&\leq 2\left[\delta u\left(\tfrac{g(\alpha)+g(\beta)}{2} - \tfrac{h(\alpha)+h(\beta)}{2}\right) + du\left(\tfrac{g(\alpha)+g(\beta)}{2} + \tfrac{\delta}{d}\tfrac{h(\alpha)+h(\beta)}{2}\right)\right].
\end{aligned}$$

Concavity of g and non decreasingness of u implies that

$$\begin{aligned}
&\delta u\left(g(\tfrac{\alpha+\beta}{2}) - h(\tfrac{\alpha+\beta}{2})\right) + du\left(g(\tfrac{\alpha+\beta}{2}) + \tfrac{\delta}{d}h(\tfrac{\alpha+\beta}{2})\right) < \\
&\qquad \delta u\left(g(\tfrac{\alpha+\beta}{2}) - \tfrac{h(\alpha)+h(\beta)}{2}\right) + du\left(g(\tfrac{\alpha+\beta}{2}) + \tfrac{\delta}{d}\tfrac{h(\alpha)+h(\beta)}{2}\right). \quad (9)
\end{aligned}$$

By rearranging the terms of (9), we get

$$\delta\left[u\left(b_1\right) - u\left(a_1\right)\right] < d\left[u\left(b_2\right) - u\left(a_2\right)\right]. \quad (10)$$

[9] See Peluso and Trannoy [18] for a first step.

with

$a_1 = g(\frac{\alpha+\beta}{2}) - \frac{h(\alpha)+h(\beta)}{2}$; $b_1 = g(\frac{\alpha+\beta}{2}) - h(\frac{\alpha+\beta}{2})$;

$a_2 = g(\frac{\alpha+\beta}{2}) + \frac{\delta}{d}h(\frac{\alpha+\beta}{2})$; $b_2 = g(\frac{\alpha+\beta}{2}) + \frac{\delta}{d}\frac{h(\alpha)+h(\beta)}{2}$.

Observe that $b_2 - a_2 = \frac{\delta}{d}(b_1 - a_1)$. Moreover, since h is convex, $a_1 \leq b_1 < a_2 \leq b_2$. If $a_1 = b_1$, then $a_2 = b_2$ and (10) is impossible. Thus $a_1 < b_1 < a_2 < b_2$. Dividing both terms of (10) by $(b_1 - a_1)$, we get $\frac{u(b_1)-u(a_1)}{b_1-a_1} < \frac{u(b_2)-u(a_2)}{b_2-a_2}$, which contradicts the concavity of u using Claim 1. ■

Proof of Lemma 2. **i)** $\Longrightarrow$ By assumption $f_p(\alpha y) \geq \alpha f_p(y)$, $\forall \alpha \in [0,1]$ and $\forall y \geq 0$. It implies that for every household i

$$(f_p(\alpha y_i)\mathbf{e}_\delta,\ f_r(\alpha y_i)\mathbf{e}_d) \succcurlyeq_{GL} (\alpha f_p(y_i)\mathbf{e}_\delta,\ f_r(y_i)\mathbf{e}_d)\,. \tag{11}$$

Indeed, suppose by contradiction that, for some integer $1 \leq l \leq d-1$,

$$\delta f_p(\alpha y_i) + l f_r(\alpha y_i) < \delta\alpha f_p(y_i) + l\alpha f_r(y_i). \tag{12}$$

This implies $f_r(\alpha y_i) < \alpha f_r(y_i)$, since $k f_p(\alpha y_i) \geq k\alpha f_p(y_i)$ for $k = 1,..,\delta$. Then, by adding $(d-l)\, f_r(\alpha y_i)$ and $(d-l)\,\alpha f_r(y_i)$ respectively to the LHS and the RHS of (12), we get the contradiction $y_i < y_i$.

Since (11) holds for any household i, by applying Proposition A.7 (iii), p. 121 of Marshall-Olkin [16], we obtain $\mathbf{x}(\alpha\mathbf{y}) \succcurlyeq_{GL} \alpha\mathbf{x}(\mathbf{y})$. Dividing both vectors by $\alpha\frac{\mu_y}{s}$, we get: $\frac{\mathbf{x}(\alpha\mathbf{y})}{\alpha\frac{\mu_y}{s}} \succcurlyeq_{GL} \frac{\mathbf{x}(\mathbf{y})}{\frac{\mu_y}{s}}$, equivalent to $\mathbf{x}(\alpha\mathbf{y}) \succcurlyeq_{RL} \mathbf{x}(\mathbf{y})$.

$\Longleftarrow$ Suppose that $f_p \notin \mathcal{P}$. Then $\exists\alpha \in (0,1)$ and $\exists y^* > 0$, such that: $f_p(\alpha y^*) < \alpha f_p(y^*)$. By picking $\mathbf{y} = (0,..,0,y^*)$ and $\mathbf{y}' = \alpha\mathbf{y}$, we have $\mathbf{y}' \succcurlyeq_{RL} \mathbf{y}$ by construction and $\mu_{\mathbf{y}} = \frac{\mu_{\mathbf{y}'}}{\alpha}$. Since $\frac{f_p(\alpha y^*)}{\frac{\alpha\mathbf{y}^*}{sn}} < \frac{f_p(y^*)}{\frac{\mathbf{y}^*}{sn}}$, then $\sum_{j=1}^{sn-d} x_j < \sum_{j=1}^{sn-d} x'_j$. Hence $\mathbf{x}(\alpha\mathbf{y}) \succcurlyeq_{RL} \mathbf{x}(\mathbf{y})$ does not hold.

ii) can be proved by following a similar argument, since it is easy to see that regressive functions satisfy $f_p(\alpha y) \geq \alpha f_p(y)$, $\forall y \geq 0$ and $\forall \alpha \geq 1$. ■

Proof of Lemma 3. **i)** $\Longrightarrow$ Since $f_p \in \mathcal{M}$, then $f_p(y_i+\alpha)-\mu_{y_i+\alpha} \leq f_p(y_i)-\mu_{y_i}$. By reasoning as in the proof of Proposition 3, we get, for any household i,

$$\left(\left(f_p(y_i)-\mu_{y_i}\right)\mathbf{e}_\delta, \left(\ f_r(y_i)-\mu_{y_i}\right)\mathbf{e}_d\right) \succcurlyeq_{AL} \qquad (13)$$
$$\left(\left(f_p(y_i+\alpha)-\mu_{y_i+\alpha}\right)\mathbf{e}_\delta,\ \ \left(f_r(y_i+\alpha)-\mu_{y_i+\alpha}\right)\mathbf{e}_d\right).$$

Let $\check{\mathbf{x}}(\mathbf{y})$ designate the centered vector of individual incomes. Then, for every household i, (13) is equivalent to $\check{\mathbf{x}}(y_i) \succcurlyeq_L \check{\mathbf{x}}(y_i+\alpha)$. Proposition A.7 (i), p. 121 of Marshall-Olkin [16], gives $\check{\mathbf{x}}(\mathbf{y}) \succcurlyeq_L \check{\mathbf{x}}(\mathbf{y}+\alpha\mathbf{e}_n)$, equivalent to $\mathbf{x}(\mathbf{y}) \succcurlyeq_{AL} \mathbf{x}(\mathbf{y}+\alpha\mathbf{e}_n)$.

$\Longleftarrow$ Suppose that $f_p \notin \mathcal{M}$. Then, $\exists \alpha > 0$ and $\exists y^* > 0$, such that: $f_p(y^*+\alpha) - \frac{y^*+\alpha}{s} > f_p(y^*) - \frac{y^*}{s}$. Let us consider $\mathbf{y} = (y^*, .., y^*, y^*)$ and $\mathbf{y}' = \mathbf{y}+\alpha\mathbf{e}_n$. By construction $\mu_{\mathbf{y}'} = \mu_{\mathbf{y}}+\alpha$ and $\mathbf{y} \succcurlyeq_{AL} \mathbf{y}'$. Since $f_p(y^*+\alpha) - \frac{n(y^*+\alpha)}{ns} > f_p(y^*) - \frac{ny^*}{ns}$, then $\sum_{j=1}^{s\delta}(x_j - \mu_{\mathbf{x}}) < \sum_{j=1}^{s\delta}(x'_j - \mu_{\mathbf{x}'})$ and consequently $\mathbf{x}(\mathbf{y}) \succcurlyeq_{AL} \mathbf{x}(\mathbf{y}+\alpha\mathbf{e}_n)$ does not hold. The proof **ii)** is similar by reversing the appropriate inequalities and **iii)** is due to the fact that *moving closer* sharing functions degenerate into the egalitarian rule if $\underline{\mathrm{y}}= 0$. ■

Proof of Lemma 4. We proceed by contradiction. Suppose that w is not concave: then, for some $\alpha, \beta, \gamma \in I$ with $\alpha < \beta < \gamma$, by Claim 2 and concavity of u, we get (as in the proof of Lemma 1) the following inequality:

$$u\left(\tfrac{\alpha+\beta+\gamma}{3} - h_1(\tfrac{\alpha+\beta+\gamma}{3})\right) + u\left(\tfrac{\alpha+\beta+\gamma}{3} - h_2(\tfrac{\alpha+\beta+\gamma}{3})\right) + u\left(\tfrac{\alpha+\beta+\gamma}{3} + h(\tfrac{\alpha+\beta+\gamma}{3})\right) \qquad (14)$$
$$< u\left(\tfrac{\alpha+\beta+\gamma}{3} - \tfrac{h_1(\alpha)+h_1(\beta)+h_1(\gamma)}{3}\right) + u\left(\tfrac{\alpha+\beta+\gamma}{3} - \tfrac{h_2(\alpha)+h_2(\beta)+h_2(\gamma)}{3}\right) + u\left(\tfrac{\alpha+\beta+\gamma}{3} + \tfrac{h(\alpha)+h(\beta)+h(\gamma)}{3}\right).$$

Now, we pose:

$a_1 = \frac{\alpha+\beta+\gamma}{3} - \frac{h_1(\alpha)+h_1(\beta)+h_1(\gamma)}{3}$; $b_1 = \frac{\alpha+\beta+\gamma}{3} - h_1(\frac{\alpha+\beta+\gamma}{3})$;

$a_2 = \frac{\alpha+\beta+\gamma}{3} - \frac{h_2(\alpha)+h_2(\beta)+h_2(\gamma)}{3}$; $b_2 = \frac{\alpha+\beta+\gamma}{3} - h_2(\frac{\alpha+\beta+\gamma}{3})$;

$a = \frac{\alpha+\beta+\gamma}{3} + h(\frac{\alpha+\beta+\gamma}{3})$; $b = \frac{\alpha+\beta+\gamma}{3} + \frac{h(\alpha)+h(\beta)+h(\gamma)}{3}$.

Since $h_2(v) \leq h_1(v)\ \forall v$, then $a_1 \leq a_2$ and $b_1 \leq b_2$.

Since h and h_1 are convex, then $a \leq b$ and $a_1 \leq b_1$.We remark that

$$b - a = b_1 - a_1 + b_2 - a_2. \tag{15}$$

Moreover, by replacing in (14), we obtain:

$$u(b_1) - u(a_1) + u(b_2) - u(a_2) < u(b) - u(a). \tag{16}$$

We show now that in every possible case a contradiction arises.

Case 1) If $a = b$, then from (15) it follows $b_1 - a_1 = a_2 - b_2$ and from (16):

$$u(b_1) - u(a_1) < u(a_2) - u(b_2). \tag{17}$$

If $a_1 = b_1$,from (15) and (17) we get $0 < 0$.

If $a_1 < b_1$, then $b_2 < a_2$ and given that $b_1 \leq b_2$, if we divide (17) by $b_1 - a_1$, using Claim 1 we contradicts the concavity of u.

Case 2) $a < b$ and $a_1 = b_1$, then we obtain from (15) $b - a = b_2 - a_2$ and from (16)

$$u(b_2) - u(a_2) < u(b) - u(a) \tag{18}$$

Dividing (18) by $a - b$ and applying Claim 1 the concavity of u is contradicted.

Case 3) $a_1 < b_1 < a < b$. Two subcases have to be considered, according to the feature of the function h_2.

3.1) $a_2 < b_2 < a < b$. Introducing $a^* = a + b_2 - a_2$, (16) may be rewritten as follows:

$$u(b_2) - u(a_2) + u(b_1) - u(a_1) < u(b) - u(a^*) + u(a^*) - u(a). \tag{19}$$

From Claim 1 and concavity of u, we get $u(b) - u(a^*) \leq u(b_1) - u(a_1)$ and $u(a^*) - u(a) \leq u(b_2) - u(a_2)$. Then (19) is contradicted.

3.2) $b_2 < a_2 < a < b$ From (15), we get $b_1 - a_1 = b - a + a_2 - b_2$ (observe as in this case $b - a < b_1 - a_1$). By introducing $a^* = a_1 + b - a$, we may rewrite (16) as follows:

$$u(b_1) - u(a^*) + u(a^*) - u(a_1) < u(b) - u(a) + u(a_2) - u(b_2). \tag{20}$$

Given the concavity of u, from Claim 1 we get $u(b_1) - u(a^*) \geq u(a_2) - u(b_2)$ and $u(a^*) - u(a_1) \geq u(b) - u(a)$, which contradict (20).

■

Proof of Proposition 6. $\Longrightarrow$ The sum of individual utilities for the household i is

$$w(y_i) = u[f_r(y_i)] + u[f_p(y_i)] + u[f_m(y_i)]. \tag{21}$$

By replacing $f_r(y_i) = \frac{1}{3}y_i + \psi(\frac{1}{3}y_i)$, $f_p(y_i) = \frac{1}{3}y_i - \psi^1(\frac{1}{3}y_i)$ and $f_m(y_i) = \frac{1}{3}y_i - \psi^2(\frac{1}{3}y_i)$, with ψ and ψ^1 convex and such that $\psi = \psi^1 + \psi^2$ and $\psi^2(y) \leq \psi^1(y)$ $\forall y$, we may rewrite (21) as follows

$$w(y_i) = u\left[\frac{1}{3}y_i + \psi(\frac{1}{3}y_i)\right] + u\left[\frac{1}{3}y_i - \psi^1(\frac{1}{3}y_i)\right] + u\left[\frac{1}{3}y_i - \psi^2(\frac{1}{3}y_i)\right].$$

By posing $v = \frac{1}{3}y_i$, $h = \psi$, $h_1 = \psi^1$ and $h_2 = \psi^2$, Lemma 3-i) guarantees the concavity of w and we may continue with the same reasoning as in the proof of Theorem 1.

$\Longleftarrow$ We proceed by contradiction, by first assuming that f_g is not concave. Then, by Claim 2, there exists $y^* \in \mathbb{R}_+$ and $\xi > 0$ such that $\forall \varepsilon$ satisfying $0 < \varepsilon < \zeta$

$$2[f_p(y^*) + f_m(y^*)] < f_p(y^* - \varepsilon) + f_m(y^* - \varepsilon) + f_p(y^* + \varepsilon) + f_m(y^* + \varepsilon) \tag{22}$$

(22) combined with $f_p(y^*) \leq f_m(y^*) \leq \frac{1}{3}y^*$, implies $f_p(y^*) \leq f_m(y^*) < f_r(y^*)$. Then, by continuity, there exists $\bar{\zeta} > 0$ such that for every ε satisfying $0 < \varepsilon < \bar{\zeta}$, (22) holds and

$$f_p(y^* + \varepsilon) \leq f_m(y^* + \varepsilon) < f_r(y^* - \varepsilon). \tag{23}$$

For some $\varepsilon \in (0, \bar{\zeta})$, we choose two income distributions $\mathbf{y} = (0, .., 0, y^*, y^*)$ and $\mathbf{y}' = (0, .., 0, y^* - \varepsilon, y^* + \varepsilon)$. By construction, $\mathbf{y} \succcurlyeq_{GL} \mathbf{y}'$. From (22) and (23), we get $\sum_{j=1}^{3n-2} x_j < \sum_{j=1}^{3n-2} x'_j$. As a consequence, $\mathbf{x}(\mathbf{y}) \succcurlyeq_{GL} \mathbf{x}(\mathbf{y}')$ is contradicted.

Assume now by contradiction that f_p is not concave. Then, by Claim 2, there exists $y^* \in \mathbb{R}_+$ and $\xi > 0$ such that $\forall \varepsilon$ satisfying $0 < \varepsilon < \xi$

$$2f_p(y^*) < f_p(y^* - \varepsilon) + f_p(y^* + \varepsilon). \tag{24}$$

(24) combined with $f_p(y^*) \leq \frac{1}{3}y^*$, implies $f_p(y^*) - f_r(y^*) < 0$. Then, by continuity, there exists a $\bar{\xi} > 0$ such that, for every ε satisfying $0 < \varepsilon < \bar{\xi}$,

$$f_p(y^* + \varepsilon) < f_r(y^* - \varepsilon). \tag{25}$$

If $f_p(y^*) = f_m(y^*)$, then (24) negates the concavity of f_g which is already necessary. Then $f_p(y^*) < f_m(y^*)$ and for a sufficiently small $\varepsilon \in (0, \bar{\xi})$,

$$f_p(y^* + \varepsilon) < f_m(y^* - \varepsilon). \tag{26}$$

By choosing two income distributions $\mathbf{y} = (0, ..0, y^*, y^*)$ and $\mathbf{y}' = (0, .., 0, y^* - \varepsilon, y^* + \varepsilon)$, we have $\mathbf{y} \succcurlyeq_{GL} \mathbf{y}'$ by construction. Combining (26) with (25), we get $\sum_{j=1}^{3n-4} x_j < \sum_{j=1}^{3n-4} x'_j$, which contradicts $\mathbf{x}(\mathbf{y}) \succcurlyeq_{GL} \mathbf{x}(\mathbf{y}')$. ■

Proof of Corollary 3. $\Longrightarrow$ Let f_p^1 be concave and consider $\mathbf{y}, \mathbf{y}' \in \mathbb{D}$ such that $\mathbf{y} \succcurlyeq_{GL} \mathbf{y}'$. We prove that

$$\sum_{i=1}^{2n} \omega_i u(\widehat{x}_i) \geq \sum_{i=1}^{2n} \omega_i u(\widehat{x}'_i) \ \forall u \text{ non decreasing and concave}, \tag{27}$$

where $\omega_i = \begin{cases} 1 \text{ if } i \text{ is a type 1} \\ e_2 \text{ if } i \text{ is a type 2.} \end{cases}$

It is well-known that (27) is equivalent to $\widehat{\mathbf{x}}(\mathbf{y}) \succcurlyeq_{EGL} \widehat{\mathbf{x}}(\mathbf{y}')$ (see Ebert [6]). For a given u, the sum of utilities of equivalent incomes weighted by the number of equivalent individuals for the household j is given by: $w(y_j) = u\left[f_p^1(y_j)\right] + e_2 u\left[\frac{f_r^2(y_j)}{e_2}\right]$. We may write: $w(y_j) = \delta u\left[g(y_j) - h(y_j)\right] + d\left[g(y_j) + \frac{\delta}{d}h(y_j)\right]$, posing $\delta = 1$, $d = e_2$, $g(y_j) = \frac{y_j}{1+e_2}$, $h(y_j) = \frac{y_j}{1+e_2} - f_p^1(y_j)$ and $h(y_j) = \frac{e_2 yj}{1+e_2} - f_r^2(y_j)$. Since $f_p^1 \in \mathcal{C}^1$, then h is convex. It is easy to see that w is non decreasing. Then, by applying Lemma 1, we get that w is concave. Moreover, $\sum_{i=1}^{2n} \omega_i u(\widehat{x}_i) = \sum_{j=1}^{n} w(y_j)$ and $\sum_{i=1}^{2n} \omega_i u(\widehat{x}'_i) = \sum_{j=1}^{n} w(y_j)$. Since $\mathbf{y} \succcurlyeq_{GL} \mathbf{y}'$, then $\sum_{j=1}^{n} w(y_j) \geq \sum_{j=1}^{n} w(y_j)$, which is equivalent to $\sum_{i=1}^{2n} \omega_i u(\widehat{x}_i) \geq \sum_{i=1}^{2n} \omega_i u(\widehat{x}'_i)$. The reasoning is valid for any non decreasing and concave u.

$\Longleftarrow$ Assume now by contradiction $f_p^1 \in \mathcal{F}^1$ but not concave. By Claim 2, there exist $y^* \in \mathbb{R}_{++}$ and $\zeta > 0$ such that :

$$2f_p^1\,(y^*) < f_p^1(\,y^* - \varepsilon) + f_p^1(y^* + \varepsilon),\ \forall \varepsilon \text{ satisfying } 0 < \varepsilon < \zeta. \tag{28}$$

Moreover, (28) and $f_p^1(y^*) \leq \dfrac{1}{1+e_2}y^*$ imply $f_p^1(y^*) - \dfrac{f_r^2(y^*)}{e_2} < 0$.[10] Then, by continuity, there exists $\bar{\zeta} > 0$ such that, for every ε satisfying $0 < \varepsilon < \bar{\zeta} \leq \zeta$, the following condition is satisfied

$$f_p^1(y^* + \varepsilon) < \frac{1}{e_2} f_r^2(y^* - \varepsilon). \tag{29}$$

For some $\varepsilon \in (0, \bar{\zeta})$, we choose two income distributions $\mathbf{y} = (0, .., 0, y^*, y^*)$ and $\mathbf{y}' = (0, .., 0, y^* - \varepsilon, y^* + \varepsilon)$. By construction, $\mathbf{y} \succcurlyeq_{GL} \mathbf{y}'$. From (28), it follows $f_p^1\,(y_{n-1}) + f_p^1\,(y_n) < f_p^1(\,y'_{n-1}) + f_p^1(y'_n)$. By comparing the curves (**??**) associated to $\widehat{\mathbf{x}}(\mathbf{y})$ and $\widehat{\mathbf{x}}(\mathbf{y}')$, we can conclude that $\widehat{\mathbf{x}}(\mathbf{y}) \succcurlyeq_{EGL} \widehat{\mathbf{x}}(\mathbf{y}')$ is contradicted. ■

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[10] Indeed, by assumption $f_p^1(y^*) \leq \dfrac{1}{1+e_2}y^*$. Suppose now $f_p^1(y^*) - \dfrac{1}{e_2}f_r^2(y^*) = 0$. This is equivalent to $f_p^1(y^*) = \dfrac{1}{1+e_2}y^*$. From (28) and $f_p^1(y) \leq \dfrac{1}{1+e_2}y$, we get:
$\dfrac{2}{1+e_2}y^* < \dfrac{1}{1+e_2}(y^* - \varepsilon) + \dfrac{1}{1+e_2}(y^* + \varepsilon) = \dfrac{2}{1+e_2}y^*$,
which is impossible.

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