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QUADERNI DELL'ISTITUTO DI ECONOMIA

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THEORETICAL FOUNDATIONS OF A NEW MONEY SUPPLY HYPOTHESIS FOR THE FRG

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CONTENT

Introduction

This paper is a report on an attempt to derive a special money supply hypothesis for the FRG.

Such an attempt proved to be necessary given that the Brunner-Meltzer approach (BM-approach), which is dominating money supply theory also in Germany, is lacking microfoundations(1). In that approach, behavioural functions of the banks and the nonbank public are not derived from optimization but are postulated ad hoc. Occasional references to the general portfolio theory are no substitute because the standard results of portfolio theory have been derived for utility maximizing, riskaversive investors. The general portfolio theory is useful to derive the behavioural functions of the non-bank public but inappropriate for the explanation of the behaviour of riskneutral, profitmaximizing banks.

With microfoundations missing the BM-approach is lacking a basis for integrating institutional particularities of the monetary system of the FRG. Consequently, in the past some important institutional details of the German monetary system have not been integrated into the money supply theory as developed by adherents to the BM-approach. Examples for neglected institutions are the regulation of the money market by the Bundesbank and quotas for central bank credit to commercial banks. The neglect of quotas for rediscount and lombard credit is sometimes defended on the ground that quotas normally are not exhausted and therefore do not form a binding restriction for bank behaviour. However, it can be shown that this argument is a fallacy and that unexhausted quotas are in fact binding, too.

The adherents of the BM-approach have compensated for the lack of microfoundations by political propaganda. Some institutions, not fitting the ad hoc framework of the BM-approach, were successfully propagated away. The abolishment of "liquid assets" from the Bundesbank regulation of the money market (i.e. the elimination of price fixing open market policy) is one example, the disappearance of the concept of "free liquid reserves" from the monthly reports of the Bundesbank is a symptom of another one.

However, if available institutions seem inappropriate from the perspective of the BM-approach then this is not necessarily a defect of these institutions but may actually be a signal of impotence of the underlying theoretical approach.

Depending on the theoretical approach initially chosen existing institutions may have no chance of proving to be reasonable. In this sense it is true that theoretical approaches contain institutional prejudices and the BM-approach is no exception.

In the following we shall present a theoretical framework that provides such a chance mainly to those institutions of monetary policy of the FRG that were designed for longer-run adjustments (Grobsteuerung). For the same purpose "liquid assets" which have disappeared from the stage of monetary policy have not been excluded from the analysis a priori.

The paper is divided into two parts. In the first part behavioural functions of a representative bank are derived from an optimizing approach.

Based on this microfoundation a macro money supply hypothesis is developed in the second part.

I. Micro part

1. The model of a representative banking firm

To overcome the deficiencies of the BM-approach we shall consider the maximization problem of a representative bank under the following assumptions.

a) The representative bank is maximizing expected profits, is a price taker, has only demand deposits and is subject to various restrictions. It is planning for one period, is submitted to a special time structure and cannot obtain central bank money from other banking firms both domestically and abroad[(2)].

b) The demand deposit ($\bar{D}$), minimum reserve requirements (r), and all interest rates (i) are predetermined or exogenous to the bank.

The exogeneity of demand deposits follows from three assumptions we assume, firstly, that in the short run and at given rates of interest the bank accepts any volume of deposits. This implies that its supply of demand deposits is infinitely elastic and that the actual volume of deposits is demand determined. Since the bank is assumed, secondly, to be a price taker, interest rates are given by the market. Thirdly, as seen by the bank its volume of credits does not influence its volume of deposits. These assumptions imply that deposits are exogenous (i.e. parametrically given) to the bank.

c) The bank is actively deciding about the desired volume of excess reserves (R^e), the amount of "liquid assets" (L), its volume of customer credits (K) and its borrowings from the central bank via the discount (B_D) and lombard window (B_L).

"Liquid assets" are special money market papers included in the so-called regulation of the money market by the Bundesbank.

"Liquid assets" may be sold to the Bundesbank prior to their maturity at any time on demand of the holding commercial bank. To the extent that commercial banks hold "liquid assets" they can obatin central bank money at any time outside the limits of the rediscount and lombard quotas. (Lombard credit from the central bank is credit against collateral). Since "liquid assets" are earning interest they are a form of liquidity superior to the holding of cash and central bank deposits.

d) Expected profits are the difference between interest earnings and expected costs. Interest earnings derive from customer credits (K) and "liquid assets" holdings (L). The expected costs consist of interest paid for deposits, interest paid for rediscount and lombard credit and expected liquidity costs (E(LK)).

$$(1) \qquad E(Pr) = i_K K + i_L^V L - i_D \left\{ \bar{D} - 1/2\, E[W(\bar{D})] \right\} - [\, i_{BD} D_B + i'_{BL} B_L + 1/2 E(LK)]$$

e) The bank is submitted to a balance sheet constraint, to a minimum reserve requirement, to quotas for rediscount and lombard credit and to non-negativity constraints.

$$(2) \qquad R^e + R^r + L + K = \bar{D} + B_D + B_L \qquad \text{(balance sheet constraint)}$$

$$(3) \qquad R^r = r\bar{D} \qquad \text{(minimum reserve requirement)}$$

$$(4) \qquad \bar{B}_D - B_D \geqslant 0 \qquad \text{(rediscount quota } (\bar{B}_D))$$

$$(5) \qquad \bar{B}_L - B_L \geqslant 0 \qquad \text{(lombard quota } (\bar{B}_L))^{(3)}$$

$$(6) \qquad R^e, L, K, B_D, B_L \geqslant 0 \qquad \text{(non-negativity constraints)}$$

f) The model has a particular time structure. The planning horizon is one period. A period has two subsections of equal length.

Deposits flow to a bank at the beginning of a period. In the middle of a period, i.e. at the beginning of the second half of a period, deposits may be withdrawn. These withdrawls (W) are unknown to the bank at the beginning of a period, not precisely predictable and submitted to a probability distribution. For deposits withdrawn, the bank must not pay interest during the second half of a period. Therefore, half of the expected withdrawals must be deducted in computing the interest payments for deposits. Customer credits, "liquid assets" and credits of the central bank have a duration of one period. By assumption additional customer credits cannot be granted profitably in the middle of the period(4). At that point of time customer credits, if at all, can only be cancelled at considerable expense. In the middle of the period "liquid assets" can be acquired with a remaining term to maturity of half a period, "liquid assets" may be sold to the central bank within its regulation of the money market and credits from the central bank may be obtained for the remaining half of a period.

2. The liquidity problem of the bank

Since deposits may be withdrawn after the first half of a period, while customer credits at this point of time may not or may only be cancelled at extreme costs, the bank has a liquidity problem. In order to meet the deposit withdrawals the bank has to provide for central bank money. This causes interest and liquidity costs (LK). In the beginning of the period, at the time of planning, the withdrawals and therefore the liquidity costs are probability distributed. The expected profits of the bank are reduced by the expected value of the liquidity costs.

The following sources of liquidity are available to the bank:

a) use of excess reserves $(R^e, 0)$;

b) sale of "liquid assets" to the Bundesbank $(L, i_L^{K,e})$;

c) use of unused rediscount quota $(\bar{B} - B_D, i_{BD}^e)$;

d) use of unused lombard quota $(\bar{B}_L - B_L, i_{BL}^e)$;

e) violation of minimum reserve requirements $(r[(1-r)\bar{D}-F], i_S^e)$;

f) premature cancellation of credits $(K - B_D - B_L, i_p^e)$.

This list of liquidity sources is ordered. It starts with the cheapest one and ends with the most expensive one. The first four sources are sometimes added up to form the "free liquid reserves".

$$(7) \qquad F = R^e + L + (\bar{B}_D - B_D) + (\bar{B}_L - B_L)$$

But instead of a positive deposit withdrawal there may be an unforeseen inflow of deposits $(W < 0)$. For additional deposits arriving in the middle of the period the following uses are assumed to exist:

a) repayment of lombard credits to the Bundesbank (B_L, i_{BL}^e);

b) purchase of "liquid assets" from the Bundesbank $(L^P = \infty, i_L^{v,e})$;

c) accumulation of excess reserves $(\infty, 0)$.

This list requires further comment. Firstly, it is an institutional feature of the German monetary system that lombard credits from the central bank can be repaid at any time while rediscount credits from the central bank cannot. Therefore a repayment of rediscount credit does not appear in this list.

Secondly, also this list is ordered. It starts with the most profitable use of liquidity. Liquidity will be acquired or used according to the economic principle.

In acquiring liquidity a more expensive source will not be used before the cheaper sources are exhausted. Inflowing liquidity will be directed to the most profitable use. If that use is exhausted then the next best will follow and so on. For the various sources and uses of liquidity there exist various upper limits and prices which are indicated in brackets.

Thirdly, if we consider an economy that is closed with respect to money and if the bank of our analysis is representing the aggregate of the banking sector then interbank credits may be excluded as an additional source or use for bank liquidity. Fourthly, if the upper limit for additional customer credits in the middle of the period is actually both finite and independent of the variables explicit considered in the model then the present exclusion of additional customer credits in the middle of the period is of no analytical consequence except that it rules out the study of exogenous changes of these upper limits.

Fifthly, from the point of view both of an individual bank and of the banking system as a whole additional "liquid assets" cannot exceed the amount of "liquid assets" that the Bundesbank is able and willing to sell on demand. I simplify matters by assuming that this amount is infinite so that unexpected liquidity inflows in the middle of a period will never be used to build up excess reserves.

3. *Further assumptions*

a) To simplify the analysis we assume that at the beginning of a period no change of quotas or of minimum required reserve ratios is expected to take place within the period, while interest rates may be expected to change in the middle of the period.

b) Minimum required reserves are restricted by

(8) $0 \leq r < 1$

c) At the beginning of a period interest rates are ordered as follows:

(9) $i_p > i_S > i_{BL} > i_{BD} > i_L^K > i_L^V > 0,$

With the exception of i_p interest rates in (9) are determined by the Bundesbank.

Thus, disregarding i_p, (9) may be said to describe Bundesbank behaviour.

d) If the bank expects a change in interest rates then it does not expect a change in the initial ordering as given by (9):

(10) $i_p^e > i_S^e > i_{BL}^e > i_{BD}^e > i_L^{K,e} > i_L^{v,e} > 0$

e) The interest rate policy of the Bundesbank is expected (by the bank) to be stable in the sense that

(11) $i_L^{K,e} < 2/c \; i_L^V$ with $c \leq 1$

where c is the probability of positive withdrawals ($W > 0$).

f) With the exception of excess reserves (R^e) there exists an *interior* solution[5] of the maximization problem such that:

(12) $L, B_D, B_L, \bar{B} - B_D, \bar{B}_L - B_L > 0$

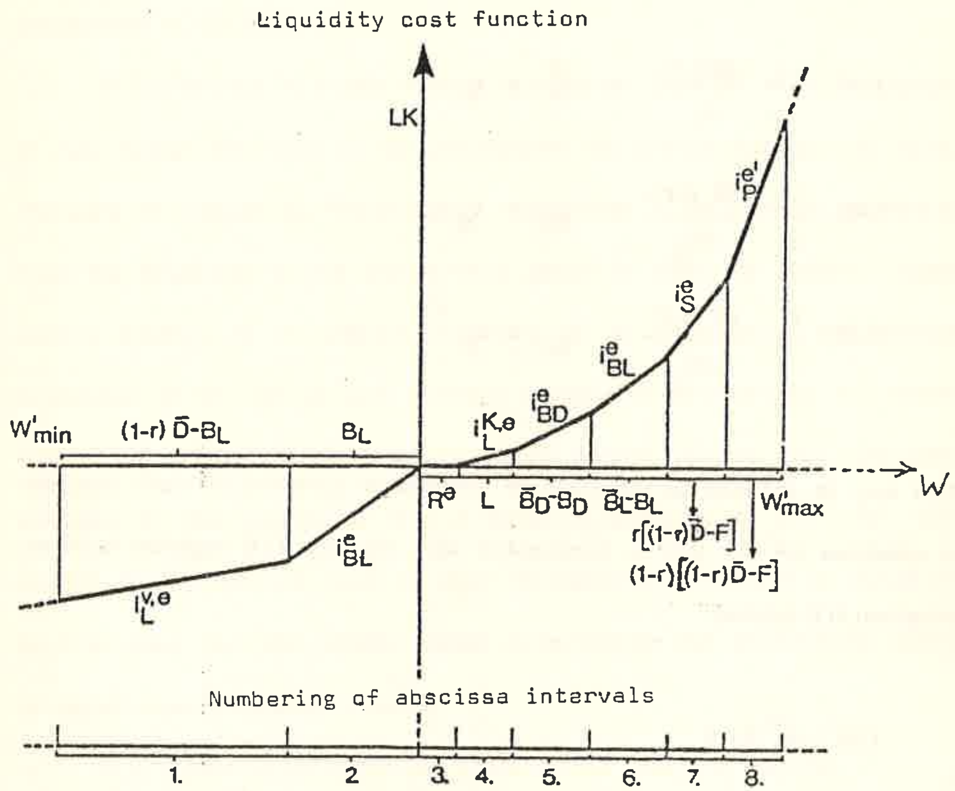
Thus, it is assumed for the optimum that rediscount and lombard credits from the central bank are taken but that the quotas are not exhausted. The assumption of an interior solution also implies that the more expensive lombard credits are used before the rediscount quota has been exhausted. This is both empirically observable and economically plausible for the following reason; firstly, as opposed to rediscount credits, lombard credits may be repaid by the bank at any time. For unexpected deposit inflows repayable lombard credits offer a better investment alternative than "liquid assets".

Secondly, although the lombard rate is higher than the discount rate the possibility of early repayment of lombard credits compensates for the disadvantage of the higher lombard rate and makes lombard credits an attractive source of refinance from the Bundesbank before the rediscount quota has been exhausted.

g) The probability distribution of deposit withdrawals in the middle of a period may be assumed to be arbitrary with a few inessential restrictions[6].

4. The liquidity cost function

The liquidity cost function as presented in the diagram is piecewise



linear[7]. The slopes of the linear pieces are determined by the interest rates which are given exogenously to the bank. The lengths of the linear pieces and therefore the probabilities of the withdrawals to fall into the various intervals are determined by the decisions of the bank.

5. Optimality conditions

Since the objective function is concave, the first order optimality conditions are both necessary and sufficient for a maximum of expected profits.

The following conditions of optimality form part of an equation system

$$(13) \quad -\frac{1}{2}\frac{\partial E(LK)}{\partial L} + i_L^V = i_K$$

$$(14) \quad \frac{1}{2}\frac{\partial E\,(LK)}{\partial B_D} + i_{BD} = i_K$$

$$(15) \quad \frac{1}{2}\frac{\partial E\,(LK)}{\partial B_L} + i_{BL} = i_K$$

that may be analized by the standard methods of comparative static analysis. A condition for R^e that is comparable with equation (13) together with assumption (11) implies

$$(16) \quad R^e \cong 0$$

Equations (2), (3), (13), (14), (15) and (16) form a system of 6 equations to determine the 6 unknowns R^r, R^e, L, K, B_D and B_L.

6. Results

With very few and inessential exceptions all reactions of the bank to changes in the predetermined variables are unequivocal(8). For reasons of scarcity of space we shall discuss only a selection of results and preferably those results that contradict either directly or indirectly the ad hoc assumptions of the BM-theory.

1) The holding of excess reserves is optimally zero ($R^e = 0$). No change of data within the range of our assumptions will change this optimal value. The rate of interest on "liquid assets" would have to be expected to double from the beginning to the middle of a period in order for positive excess reserve holdings to be optimal. According to our assumptions the interest rate policy of the central bank is not as unstable as necessary for this condition to be satisfied. If there are no "liquid assets" because they have been abolished by the Bundesbank then a condition analogous to (11) holds with respect to the discount rate. In order to interpret this result it should be kept in mind that the present model is neglecting the transactions aspect of excess reserve holdings of banks.

2) A change of the *rediscount quota* has the following effects:

$$(17) \quad \frac{\partial K}{\partial \bar{B}_D} = \frac{\partial B_D}{\partial \bar{B}_D} = 1 \quad ,$$

$$\frac{\partial(\bar{B}_D - B_D)}{\partial \bar{B}_D} = \frac{\partial F}{\partial \bar{B}_D} = 0$$

a) Planned customer credits, i.e. the supply of credit will change to the same extent.

b) The change of credit supply is refinanced by an equally sized change in planned rediscount credits from the central bank.

c) All other planned variables do not react. Especially, the amount of unused rediscount quota does not change. But also excess reserves, "liquid assets" and the unused lombard quota remain unchanged. Consequently, the sum of these variables, "free liquid reserves", does not change either.

d) These effects have been obtained in spite of our assumption that the rediscount quota has not been exhausted prior to its change. These results may be made plausible without entering into the technical details of the comparative static analysis by rewriting the balance sheet constraint (2) by means of (3) and (7):

$$(18) \qquad F + K = (1-r)\,\bar{D} + \bar{B}_D + \bar{B}_L$$

On the right hand side of (18) there are only exogenous variables. If the rediscount quota is increased then the left hand side must increase to the same extent. Given a ceteris paribus change of the rediscount quota the volume of deposits and interest rates remain unchanged. Therefore planned "free liquid reserves" which act as precautionary buffer, presumably, will not change either and the adjustment of the left hand side of (18) will necessarily occur via the volume of credits. The mathematical analysis shows

that this presumption is correct for the rediscount quota. It does not hold to the same extent for the lombard quota due to liquidity cost effects which derive from the possibility of an earlier repayment of lombard credits.

3) A change of the *lombard quota* has the following effects:

$$(19) \qquad 0 < \frac{\partial K}{\partial \bar{B}_L} < 1 , \quad 0 < \frac{\partial F}{\partial \bar{B}_L} < 1 , \quad 0 < \frac{\partial B_L}{\partial \bar{B}_L} < 1$$

$$0 < \frac{\partial \bar{B}_L - B_L}{\partial \bar{B}_L} < 1, \quad \frac{\partial B_D}{\partial \bar{B}_L} > 0 , \quad \frac{\partial(\bar{B}_D - B_D)}{\partial \bar{B}_L} < 0$$

a) Planned customer credits, planned "free liquid reserves", planned lombard credits and planned unused lombard credits will all change in the same direction but not to the same extent as the lombard quota.

b) Planned rediscount credits from the central bank will also change in the same direction but not to the same extent as the lombard quota. This implies a reduction of the unused rediscount quota.

c) These effects have been derived under the assumption of an unexhausted lombard quota (prior to its change).

4) Quota policies have been shown to be effective under the assumption of the quotas not being exhausted prior to their change. Thus, the effectiveness of a quota policy is not a corner solution phenomenon.

5) The effectiveness of a change of the rediscount quota is higher than that of a change of the lombard quota. This is due to the liquidity cost effects of the earlier repayment of lombard credits which is not possi-

ble for rediscount credits from the central bank. (Lombard credits do appear on the left hand side of the liquidity cost function while rediscount credits do not (see figure 1).

6) The effects of a variation of *deposits* are as follows:

$$(20) \qquad 1-r > \frac{\partial F}{\partial \bar{D}} > 0, \qquad \varepsilon(F,\bar{D}) < 1, \qquad \frac{\partial B_D}{\partial \bar{D}} < 0, \qquad \frac{\partial B_L}{\partial \bar{D}} < 0$$

a) Planned "free liquid reserves" rise with the volume of deposits.

b) The increase under a) is less than proportional, there are economies of scale(9). (The elasticity ε of F with respect to $\bar{D}$ is smaller than 1). This result contradicts the proportionality assumptions of the BM-approach. It implies that the money -and credit multipliers of the BM-model are directly independent of the volume of deposits. At best the proportionality assumption and the assumption that the multipliers are independent of deposits may be taken as approximations of unknown quality to those relationships that have a microeconomic foundation.

c) If the volume of deposits rises then refinance credits from the central bank decrease. Both rediscount and lombard credits decline. This result contradicts the BM-approach which postulates a positive proportionality between refinance credits (borrowings) from the central bank and the volume of deposits.

d) There is a simple explanation for the result under c). If deposits rise then

the liquidity risk of the bank increases, to which the bank adjusts by increasing the volume of "free liquid reserves". For given interest rates and refinance quotas it is optimal for the bank to extend "free liquid reserves" by increasing unused rediscount and lombard quotas. For given quotas this requires a reduction of refinance credits from the central bank.

7) Further results are stated in table 1 and 2.

8) The bank's demand function for "free liquid reserves" can be written as

$$(21) \qquad \begin{array}{l} F = F\,(\\ \\ \end{array} \begin{array}{cccccccccccccc} i_K, & i_L^V, & i_{BD}, & i_{BL}, & i_L^{V,e}, & i_L^{K,e}, & i_{BD}^e, & i_{BL}^e, & i_S^e, & i_p^e, & \bar{B}_D, & \bar{B}_L, & \bar{D}, & r) \\ - & 0 & + & + & + & 0 & - & + & + & + & 0 & + & + & - \\ & (+) & & & & (+) & & & & & (+) & & & (+) \end{array}$$

The signs below the variables represent the signs of corresponding partial derivatives.

With one exception the signs in brackets hold for the case of a risk aversive bank only, the exception being the plus sign below the minimum required reserves ratio which is indicating plain equivocality in the risk neutral case and a subjective belief of the author that the minus sign is more likely.

9) If we assume that interest rates do not change within a period but only change at the beginning of a period, then the following restriction holds for all interest rates:

Table 1
THE REACTION PATTERN OF DECISION VARIABLES (V_i)
TO PARTIAL CHANGES
OF EXOGENOUS VARIABLES (QUALITATIVE RESULTS)

V_i	i_K	i_L^V	i_{BD}	i_{BL}	$i_L^{V,e}$	$i_L^{K,e}$	i_{BD}^e	i_{BL}^e	i_S^e	i_P^e	$\bar{B}_D$	$\bar{B}_L$	r	$\bar{D}$
B_L	+	0	+	-	-	0	-	?	-	-	0	+	+ (+)	-
R^e	0	0	0	0	0	0	0	0	0	0	0	0	0	0
L	0	+	-	0	0	-	+	0	0	0	0	0	-	+
B_D	+	+	-	+	+	-	+	?	-	-	+	+	+	-
$\bar{B}_D-B_D$	-	-	+	-	-	+	-	?	+	+	0	-	-	+
$\bar{B}_L-B_L$	-	0	-	+	+	0	+	?	+	+	0	+	- (+)	+
F	-	0	+	+	+	0	-	+	+	+	0	+	- (+)	+
K	+	0	-	-	-	0	+	-	-	-	+	+	- (+)	+

Table 2
THE REACTION PATTERN OF DECISION VARIABLES (V_i)
TO COMBINED CHANGES IN INTEREST RATES

V_i	$di_L^V = di_L^{V,e} = di_L^{K,e} > 0$	$di_L^V = di_L^{V,e} > 0$	$di_L^V = di_L^{K,e} > 0$	$di_{BD} = di_{BD}^e > 0$	$di_{BL} = di_{BL}^e > 0$	$di_L^{V,e} = di_L^{K,e} > 0$
B_L		-	0	+	-	-
R^e		0	0	0	0	0
L		+	+	-	0	-
B_D		+	+	-	?	?
$\bar{B}_D-B_D$		-	-	+	?	?
$\bar{B}_L-B_L$		+	0	-	+	+
F		+	0	+	+	+
K		-	0	-	-	-

(22) $di = di^e$

Comparative static analysis with restriction (22) implies a restricted demand function for "free liquid reserves"

$$
\begin{array}{llccccccccccc}
(23) & F = F^r(& i_K, & i_L^V, & i_L^K, & i_{BD}, & i_{BL}, & i_S, & i_p, & \bar{B}_D, & \bar{B}_L, & r, & \bar{D}) \\
 & & - & + & 0 & + & + & + & + & 0 & + & - & + \\
 & & & & (+) & & & & & (+) & & (+) &
\end{array}
$$

where

$$\frac{\partial F^r}{\partial i} = \frac{\partial F}{\partial i} + \frac{\partial F}{\partial i^e}$$

$$\frac{\partial F^r}{\partial \bar{B}_D} = \frac{\partial F}{\partial \bar{B}_D} = \underset{(>0)}{0}$$

$$0 < \frac{\partial F^r}{\partial \bar{B}_L} = \frac{\partial F}{\partial \bar{B}_L} < 1$$

$$0 < \frac{\partial F^r}{\partial \bar{D}} = \frac{\partial F}{\partial \bar{D}} < 1\text{-}r$$

$$0 < \varepsilon(F^r, \bar{D}) = \varepsilon(F, \bar{D}) < 1$$

7. Transition to macro analysis

In the macro part the following vector definitions will be used.

$$I \equiv (i_L^V, i_L^K, i_{BD}, i_{BL}, i_S, i_p)$$

$$G \equiv (i_K, I, \bar{B}_D, \bar{B}_L, r)$$

With these definitions the F^r-function (23) may be restated as

(24) $F = F^r(G, \bar{D})$

The F-function of the macro part corresponds to the F^r-function (24), the index r being dropped for reasons of simplicity.

8. Implications for money supply theory (conclusion)

1) We have found that banks make liquidity provisions not by holding excess reserves but by holding "free liquid reserves". However, "free liquid reserves" are not a subset of the monetary base. Therefore, the monetary base does not include the critical variables of the liquidity provisions of German commercial banks. An alternative concept that avoids this defect is the *potential monetary base* (B^P), which is defined as the actual monetary base (B) plus "free liquid reserves" (F) minus excess reserves (R^e).

(25) $B^P \equiv B + F - R^e$

In the German monetary system the Bundesbank cannot control the monetary base like an instrument (e.g. the rediscount rate). Still, after several institutional changes of the monetary system, the course of the monetary base can only be an intermediary target of monetary policy. This holds true even for a system of flexible exchange rates. This is different for the potential monetary base which can be controlled[10] as if it were an instrument of monetary policy, while commercial banks determine the partitioning of a given potential monetary base into the components actual monetary base and "free liquid reserves" (minus excess reserves).

From this analysis the conclusion emerges that a money supply theory for the FRG must use the potential monetary base as the critical scale variable. Analytically, the actual monetary base has no explanatory power but is just one of several monetary aggregates that require explanation like any other money stock variable. This statement may not be refuted by reference to high correlations between the actual monetary base and money stock aggregates.

2) We have seen that there exist economies of scale in the provision of liquidity by commercial banks. These economies of scale directly contradict the assumption of linear homogeneity as used by the multiplier approach[11] of BM. Since the multiplier approach does not offer any technical advantage in the analysis one is left to defend the approach as a useful approximation, the quality of which is generally unknown.

II. Macro part

1. Macroeconomic supply functions for money and credit

a) Conceptual preliminaries

The treatment of demand deposits as exogenous on the micro-level was justified as follows: firstly, we assumed that commercial banks at given rates of interest, at least in the short run, accept any volume of deposits, i.e. their supply of demand deposits is infinitely elastic[12]. Then the actual volume of deposits is demand determined. Secondly, we assumed that banks take interest rates as given by the market. Thirdly, we assumed that banks do not expect changes in their volume of deposits if they change their volume of credits. These assumptions obviously imply that, at least in the short run, deposits are exogenous (i.e. parametrically given) to the banks.

If these assumptions[13] are maintained for any admissable set of interest rates then the actual volume of deposits of an individual bank is always demand determined and an independent supply function in the usual sense does not exist[14]. Thus, a supply function for the market (of deposits) as a whole cannot be derived by a horizontal addition of individual bank supply curves since the latter simply do not exist.

However, given the currency demand function of the nonbank public, the minimum reserve requirements and the banks' demand function for free liquid reserves, there exists an upper limit for the volume of deposits that is both compatible with these demand functions and with the given stock of potential monetary base. The existence of such an upper limit may be used to define the macroeconomic supply of deposits:

> "The macroeconomic supply of deposits is that quantity of deposits which via the currency demand of the nonbank public, the minimum reserve requirements and the banks' demand for free liquid reserves implies a demand for potential monetary base which is equal to the given stock of potential monetary base".

b) Derivation

According to the preceeding definition the *macroeconomic supply of deposits*, D^S, is that volume of D which for parametrically given k, r, G and $\bar{B}^P$ solves the following stock equilibrium condition for the potential monetary base:

$$(26) \quad (k+r)\, D + F\,(G, D) = \bar{B}^P$$

where

(27)	$C = kD$	(currency demand of nonbank public)
(28)	$R^r = rD$	(minimum reserve requirements)
(29)	$F = F(G, D)$	(banks' demand for free liquid reserves)

We obtain the *demand deposit supply function* by solving (26) for D^S as a function of the parameters $\bar{B}^P$, G and k:

$$(30) \quad D^S = D^S\,(\bar{B}^P, G, k)$$

A deposit supply D^S implies a currency demand kD^S. The sum of the two is called the *macroeconomic supply of money*, M^S.

$$(31) \quad M^S \overset{def}{=} (1+k)\, D^S$$

For parametrically given G, $\bar{B}_D$ and $\bar{B}_L$ the banks' demand for free liquid reserves, the macroeconomic supply of deposits, and the balance sheet constraint (18) imply a volume of credit which will be called the *macroeconomic supply of credit* K^S:

$$(32) \quad K^S \overset{def}{=} (1-r)\, D^S + \bar{B}_D + \bar{B}_L - F\,(G, D^S).$$

Applying (30) to (31) and (32) we obtain the macroeconomic *supply functions* for *money* and *credit*:

$$(33) \quad M^S = M^S\,(\bar{B}^P, G, k)$$

$$(34) \quad K^S = K^S\,(\bar{B}^P, G, k)$$

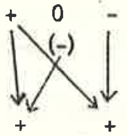
The total differential of (33) and (34) may be derived from the total differential of (26), (31) and (32).

The partial derivatives of the M^S -and K^S- functions are given in table 3.

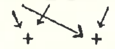
c) Properties

From the sign properties of the demand function for free liquid reserves, as determined in the micro section, the following sign properties of the macroeconomic money and credit supply functions may be determined:

$$(35) \quad M^S = M^S \begin{array}{l}(i_K, \bar{B}^P, \bar{B}_D, \bar{B}_L, i_L^V, i_{BD}, i_{BL}, i_S, i_p, r, k) \\ \;\; + \quad + \quad 0 \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad - \end{array}$$



$$(36) \quad K^S = K^S \begin{array}{l}(i_K, \bar{B}^P, \bar{B}_D, \bar{B}_L, i_L^V, i_{BD}, i_{BL}, i_S, i_p, r, k) \\ \;\; + \quad + \quad + \quad ? \quad - \quad - \quad - \quad - \quad - \quad - \quad - \end{array}$$



For the proper interpretation of these results it is important to note that the signs below (35) and (36) characterise partial derivatives, i.e. the effect of ceteris paribus changes.

However, if either the rediscount or the lombard quota is changed then the potential monetary base is automatically changed to the same extent ($d\bar{B}^P = d\bar{B}_x$, x=D,L). In order to hold the potential monetary base constant, while one of the quotas is changed a balancing open market operation is required. Therefore, the signs below $\bar{B}_D$ and $\bar{B}_L$ represent the effect of a combination of two actions of monetary policy, one action being the quota change, the other one being the open market operation that eliminates

(neutralizes) the implied change in the potential monetary base. The effects of non-neutralized quota variations are specified in table 3 and are indicated in (35) and (36) by means of special arrows.

The effects of neutralized quota changes may be explained by means of (26). According to our micro results for the case of risk-neutral banks, a c.p. change of the rediscount quota leaves desired free liquid reserves unchanged and will not disturb an initial stock equilibrium for the potential monetary base. The volume of deposits and money that is consistent with the stock equilibrium condition for the potential base and therefore the macro supply of deposits and money will remain constant. Since, in the present case, both the supply of deposits and desired free liquid reserves remain unchanged when the rediscount quota changes, it is obvious from the balance sheet constraint (18) that the macro supply of credit will change by the same amount and in the same direction as the rediscount quota:

$$(37) \quad \frac{\partial K^S}{\partial \bar{B}_D} = 1$$

When the lombard quota is increased, c.p., then desired free liquid reserves increase and an initial stock equilibrium for the potential monetary base will be disturbed. To reestablish it, the supply of deposits and therefore the supply of money has to decline. What is the reaction of the supply of credit? Since desired free liquid reserves increase less than the lombard quota the supply of credit declines due to the decline of the supply of depo-

Table 3

THE REACTIONS OF THE MACRO SUPPLY OF MONEY AND CREDIT TO CHANGE IN ITS DETERMINANTS(1), (2)

x	i_K	$\bar{B}^P$	$\bar{B}_D$	$\bar{B}_L$	$i_L^V, i_{BD}, i_{BL}, i_S, i_p$	r	k
$\frac{\partial M^s}{\partial x}$	$-m \frac{\partial F_i}{\partial i_K} > 0$	$m > 1$	$-m \frac{\partial F}{\partial \bar{B}_D} = 0$ (<0)	$-m \frac{\partial F}{\partial \bar{B}_L} < 0$	$-m \frac{\partial F}{\partial x} < 0$	$-m \frac{\partial F}{\partial r} + D^s < 0$	$(1-m)\, D^s < 0$
$\frac{\partial K^s}{\partial x}$	$-(1+a) \frac{\partial F}{\partial i_K} > 0$	$a > 0$	$1-(1+a) \frac{\partial F}{\partial \bar{B}_D} = 1$ $(< 1, \lesseqgtr 0)$	$1-(1+a) \frac{\partial F}{\partial \bar{B}_L} \gtrless 0$	$-(1+a) \frac{\partial F}{\partial x} < 0$	$-(1+a) \frac{\partial F}{\partial r} + D^s < 0$	$-a D^s < 0$

The effects of *non-neutralized* quota changes:

$$m \underset{(>)}{=} \left.\frac{\partial M^s}{\partial \bar{B}_D}\right|_{d\bar{B}^P = d\bar{B}_D} = \frac{\partial M^s}{\partial \bar{B}^P} + \frac{\partial M^s}{\partial \bar{B}_D} = m\left(1 - \frac{\partial F}{\partial \bar{B}_D}\right) \underset{(>0)}{>1}$$

$$m > \left.\frac{\partial M^s}{\partial \bar{B}_L}\right|_{d\bar{B}^P = d\bar{B}_L} = \frac{\partial M^s}{\partial \bar{B}^P} + \frac{\partial M^s}{\partial \bar{B}_L} = m\left(1 - \frac{\partial F}{\partial \bar{B}_L}\right) > 0$$

$$m \underset{(>)}{=} \left.\frac{\partial K^s}{\partial \bar{B}_D}\right|_{d\bar{B}^P = d\bar{B}_D} = \frac{\partial K^s}{\partial \bar{B}^P} + \frac{\partial K^s}{\partial \bar{B}_D} = (1+a)\left(1 - \frac{\partial F}{\partial \bar{B}_D}\right) \underset{(>0)}{>1}$$

$$m > \left.\frac{\partial K^s}{\partial \bar{B}_L}\right|_{d\bar{B}^P = d\bar{B}_L} = \frac{\partial K^s}{\partial \bar{B}^P} + \frac{\partial K^s}{\partial \bar{B}_L} = (1+a)\left(1 - \frac{\partial F}{\partial \bar{B}_L}\right) > 0$$

(1) Inequality signs in brackets hold for the case of risk aversive bank behaviour.

(2) $m = \frac{1+k}{N} > 1,\ N = r \frac{\partial F}{\partial D^s} k,\quad a = \frac{Z}{N} > 0;\ Z = 1 - r - \frac{\partial F}{\partial D^s}$

sits.

Therefore, there are two opposing effects on the supply of credit and it is impossible to judge their relative strength without further information.

If a quota change is not neutralized then it implies an equal change of the potential monetary base. Therefore, the effect of such a change is the sum of two partial effects, one being the effect of the neutralized quota change, the other one being the effect of the implied variation of the potential monetary base. In this case the macro supply of money and credit clearly moves in the same direction as the quota.

It may be shown that the effect on the macro supply of money and credit of a non-neutralized rediscount quota change is larger than the corresponding effects of a change in the lombard quota.

Both the money supply and the credit supply react in the same direction to variations of the rates of interest, of the ratio of required reserves and of the currency ratio. The sign of the reactions may be determined by the kind of disturbance to the stock equilibrium condition for the potential monetary base. Two examples may illustrate this point.

A rise of the ratio of required reserves leads to an excess demand for potential monetary base. The equilibrating volume of deposits, i.e. the macro supply of deposits must fall. This implies a reduction in the macro supply of money and credit. A rise in the interest rate for bank credits, i_k, reduces the demand for free liquid reserves and therefore leads to an excess supply of potential monetary base. The equilibrating volume of deposits, i.e. the macro supply of deposits must rise, implying an increase in the macro supply of money and credit.

2. Equilibrium in the credit market

a) Conceptual preliminaries

We shall now distinguish three sectors, the sector, of commercial banks, the sector of private nonbanks and the state sector which includes the central bank. The aggregate of the sector of private nonbankes and of the state sector is called the nonbank sector.

The *macro supply of credit* is a theoretical aggregate the precise content of which varies with the kind of macro model to which it is applied. If the macro model contains as assets-money, bonds and shares and if banks may buy and hold each of these assets then their supply of bank credit includes their stock demand for bonds and shares, in addition to their loan supply, which either may be treated as part of their demand for bonds or as a separate item.

The *macro demand for credit* is defined as the stock excess supply of non-money assets of the nonbank sector. The excess supply of a non-money asset is defined by the difference between the total stock (either in the portfolio of commerciaL banks or of non-banks), minus the non-bank stock demand for this asset. The demand for loans may either be treated as part of the stock excess supply of bonds or as a separete item.

If by legal or other contraints commercial banks are not permitted to buy and hold certain kinds of assets then the corresponding assets should be excluded from the supply and demand of credit aggregates.

Accordingly, *the* interest rate on bank credits, i_K, is representing a whole spectrum of interest rates, the spectrum depending on the kind of macro model to which the money and credit supply model is applied.

For the present purposes the demand function for bank credit may be specified as follows:

$$(38) \quad K^d = K^d(\underset{-}{i_K}, V)$$

where V is a vector of unspecified variables. The commodity price level and the expected rate of inflation are examples of possible components of V. From the definition of K^d as stock excess supply of non-money assets it is clear that the stock of any non-money asset included may also be treated as a component of V.

b) Equilibrium interest rate

The equilibrium rate of interest for bank credits, i_K, is the solution of the *equilibrium condition* for bank credit for given $\bar{B}^P$, I, k, r, $\bar{B}_D$, $\bar{B}_L$ and V:

$$(39) \quad K^d(\underset{-}{i_k}, V) = K^s(\underset{+}{i_k}, \underset{+}{\bar{B}^P}, \underset{-}{I}, \underset{-}{k}, \underset{-}{r}, \underset{+}{\bar{B}_D}, \underset{?}{\bar{B}_L})$$

The solution function for i_k is:

$$(40) \quad i_k = i_k(\underset{-}{\bar{B}^P}, \underset{-}{\bar{B}_D}, \underset{?}{\bar{B}_L}, \underset{+}{I}, \underset{+}{r}, \underset{+}{k}, V)$$

(?) – – –

The total differential of the i_k-function, (40), may be derived from the total differential of the equilibrium condition for the credit market (39). The partial derivatives of the i_k-function are given in table 4.

c) *Properties*

The signs below the interest function are exactly opposite to the corresponding signs below the credit supply function. This is due to the fact that if V is held constant then we only consider shifts in the credit supply function for a credit demand function that stays put in the i_k, K space. Thus, the explanations given for the signs below the credit supply function immediately carry over to the interest rate function.

If a quota change is not neutralized then the effect of a change in the rediscount quota on the interest rate is stronger than the effect of a comparable change in the lombard quota. This follows from the corresponding difference in the shifts of the credit supply function.

Table 4

REACTIONS OF THE EQUILIBRIUM INTEREST RATE FOR BANK CREDITS TO CHANGE IN ITS DETERMINANTS(1), (2)

x	$\bar{B}^P$	$\bar{B}_D$	$\bar{B}_L$	$i_L^V, i_{BD}, i_{BL}, i_S, i_P$	r	k
$\frac{\partial i_K}{\partial x}$	$\alpha \cdot a < 0$	$-\alpha\left[1-(1+a)\frac{\partial F}{\partial \bar{B}_D}\right] < 0 \; (\lessgtr 0)$	$-\alpha\left[1-(1+a)\frac{\partial F}{\partial \bar{B}_L}\right] \lessgtr 0$	$\alpha(1+a)\frac{\partial F}{\partial x} > 0$	$\alpha(1+a)\frac{\partial F}{\partial r} + D^s) \gtrless 0$	$\alpha a D^s > 0$

The effects of *non-neutralized* quota changes:

$$\left.\frac{\partial i_K}{\partial \bar{B}_D}\right|_{d\bar{B}^P = d\bar{B}_D} = \frac{\partial i_K}{\partial \bar{B}^P} + \frac{\partial i_K}{\partial \bar{B}_D} = -\alpha(1+a)(1 - \frac{\partial F}{\partial \bar{B}_D}) < 0$$

$$\left.\frac{\partial i_K}{\partial \bar{B}_L}\right|_{d\bar{B}^P = d\bar{B}_L} = \frac{\partial i_K}{\partial \bar{B}^P} + \frac{\partial i_K}{\partial \bar{B}_L} = -\alpha(1+a)(1 - \frac{\partial F}{\partial \bar{B}_L}) < 0$$

(1) Inequality signs in brackets hold for the case of risk aversive bank behaviour.

(2) $\alpha = -\left[\frac{\partial K^d}{\partial i_K} + (1+a)\frac{\partial F}{\partial i_K}\right]^{-1} > 0$; m and a are defined in table 3.

3. *Reduced form equations for the macroeconomic supply of money and credit*

a) Derivation

Given the interest rate function we can eliminate the interest rate variable i_k from the supply functions (35) and (36) and obtain the following reduced form equations for money and credit.

$$(41)\quad M^{s,r} = M^{s,r}(\underset{+}{\bar{B}^P}, \underset{-}{\bar{B}_D}, \underset{-}{\bar{B}_L}, \underset{-}{i_L^V}, \underset{-}{i_{BD}}, \underset{-}{i_{BL}}, \underset{-}{i_S}, \underset{-}{i_p}, \underset{-}{r}, \underset{-}{k}, V)$$

$$+ \quad +$$

$$(42)\quad K^{s,r} = K^{s,r}(\underset{+}{\bar{B}^P}, \underset{+}{\bar{B}_D}, \underset{?}{\bar{B}_L}, \underset{-}{i_L^V}, \underset{-}{i_{BD}}, \underset{-}{i_{BL}}, \underset{-}{i_S}, \underset{-}{i_p}, \underset{-}{r}, \underset{-}{k}, V)$$

$$+ \quad +$$

The partial derivatives of (41) and (42) are given in table 5.

b) Properties

By comparing reduced forms with nonreduced forms of the supply of money and credit various observations can be made:

Firstly, the signs below the exogenous determinants of the money and credit supply functions are the same for the two types of functions. In order to explain this observation we compare table 3 and 5 and find that the reaction coefficients in table 5, with few exceptions for the money supply, emerge from those in table 3 via multiplication by T_i-factors which are

Table 5

THE REACTIONS OF THE MACRO SUPPLY OF MONEY AND CREDIT INCLUDING INTEREST RATE EFFECTS(1), (2)

x	$\bar{B}^P$	$\bar{B}_D$	$\bar{B}_L$	$i_L^V, i_{BD}, i_{BL}, i_S, i_p$	r	k
$\frac{\partial M^{s,r}}{\partial x}$	$T_1 m > 0$	$-T_6 m < 0$	$-T_3 m < 0$	$-T_4 m \frac{\partial F}{\partial x} < 0$	$-T_4 m (\frac{\partial F}{\partial r} + D^s) \underset{(>)}{<} 0$	$(1-T_1 m)D^s < 0$
$\frac{\partial K^{s,r}}{\partial x}$	$T_4 a > 0$	$T_4 \left[1-(1+a)\frac{\partial F}{\partial \bar{B}_D}\right] \underset{(\gtrless 0)}{> 0}$	$T_4 \left[1-(1+a)\frac{\partial F}{\partial \bar{B}_L}\right] \gtrless 0$	$-T_4(1+a)\frac{\partial F}{\partial x} < 0$	$-T_4(1+a)(\frac{\partial F}{\partial r} + D^s) \underset{(>)}{<} 0$	$-T_4 a D^s < 0$

The effects of *non-neutralized* quota changes:

$$T_4 m \underset{(>)}{=} \frac{\partial M^{s,r}}{\partial \bar{B}_D}\Bigg|_{d\bar{B}^P = d\bar{B}_D} = \frac{\partial M^{s,r}}{\partial \bar{B}^P} + \frac{\partial M^{s,r}}{\partial \bar{B}_D} = (T_1 - T_6)m = T_4 m (1 - \frac{\partial F}{\partial \bar{B}_D}) > 0$$

$$T_4 m > \frac{\partial M^{s,r}}{\partial \bar{B}_L}\Bigg|_{d\bar{B}^P = d\bar{B}_L} = \frac{\partial M^{s,r}}{\partial \bar{B}^P} + \frac{\partial M^{s,r}}{\partial \bar{B}_L} = (T_1 - T_3)m = T_4 m(1 - \frac{\partial F}{\partial \bar{B}_L}) > 0$$

$$T_4(1+a) \underset{(>)}{=} \frac{\partial K^{s,r}}{\partial \bar{B}_D}\Bigg|_{d\bar{B}^P = d\bar{B}_D} = \frac{\partial K^{s,r}}{\partial \bar{B}^P} + \frac{\partial K^{s,r}}{\partial \bar{B}_D} = T_4(1+a)(1 - \frac{\partial F}{\partial \bar{B}_D}) > 0$$

$$T_4(1+a) > \frac{\partial K^{s,r}}{\partial \bar{B}_L}\Bigg|_{d\bar{B}^P = d\bar{B}_L} = \frac{\partial K^{s,r}}{\partial \bar{B}^P} + \frac{\partial K^{s,r}}{\partial \bar{B}_L} = T_4(1+a)(1 - \frac{\partial F}{\partial \bar{B}_L}) > 0$$

---

(1) Inequality signs in brackets hold for the case of risk aversive bank behaviour.

(2) m and a are defined in table 3, α is defined in table 4 and the T_i are specified in the appendix.

both larger thanthan zero and smaller than one (see appendix). Obviously, the multiplication by T_i-factors implies a reduction of the absolute size of a coefficient, i.e. a reduction of effects. This reduction and the exceptions for money supply will be explained next.

If one of the exogeneous determinants of the supply of credit is changed then in general both the supply of money and credit change directly and the equilibrium of the credit market will be disturbed. The equilibrium interest rate for bank credits will be changed and will feed back to the supply of money and credit. The interest rate effects on the supply of money and credit are indirect, opposed to the direct effects and reduce the direct effects. For logical reasons the indirect effects can never be stronger than the direct effects. If the demand function for credit is never completely interest-inelastic then the indirect effects will with certainty be smaller than the direct effects.

With respect to the exceptions for the money supply the following may be said. If, in the case of risk neutrality, the rediscount quota is changed then there is no direct effect on the supply of money and there is only the indirect effect via the interest rate. If the lombard quota is changed then the rate of interest may rise or fall and the indirect effects via the interest rate are not necessarily opposed to the direct effects on the money supply. A similar statement holds for a change of the rediscount quota in the case of risk aversion.

Secondly, the reduced form equation for the supply of credit is also

an equation for the *equilibrium* supply of credit. This is an implication of the equilibrium interest rate for the bank credit market. The reduced form equation for the money supply is not necessarily an equation for the *equilibrium* quantity of money. Only if the number of assets distinguished in the economy is reduced to two will a credit market equilibrium imply a money market equilibrium and vice versa.

Thirdly, the explanations for the signs below the non-reduced form equations for the macro supply of money and credit may be transferred to the reduced form equations.

Fourthly, even if interest rate effects are included non-neutralized changes of rediscount quotas still have stronger effects on the supply of money and credit than non-neutralized changes (of equal size) of lombard quotas.

4. Completion and extensions

The analysis presented needs to be completed by introducing a price function that links the deposit rate i_D to the bank credit rate i_K (in a manner consistent with our assumptions) and by treating the currency ratio as a function of i_D. The results of this macro section may easily be generalized by allowing for other kinds of deposits besides demand deposits and by using different concepts of the money stock.

Conclusion (Implications for monetary policy)

1) "Free liquid reserves" are an important variable for monetary policy, not as a separate variable but as a component of the potential monetary base which is defined as the sum of the actual monetary base and "free liquid reserves" (minus excess reserves).

2) According to the analysis presented extensions (contractions) of quotas for rediscount and lombard credit have expansionary (contractionary) effects on the macro supply of money and credit even if quotas are not exhausted prior to their change.

3) Not the actual monetary base but the *potential* monetary base is the proper scale variable for monetary policy in Germany. If the Bundesbank is freed from obligations -like those within the European Monetary System- to intervene in the foreign exchange market then it can control the potential monetary base while it cannot control the actual monetary base even if it abstains from interventions in the foreign exchange market.

4) A reintroduction of those "liquid assets" (Liquiditätsanlagen) which were abolished by the Bundesbank in the Seventies would not be an obstacle to perfect control of the potential monetary base. Their former elimination appears to be the consequence of a misguided preoccupation of the Bundesbank with the actual monetary base.

5) Both the actual monetary base and "free liquid reserves" are misleading indicators of monetary policy in Germany. The former attack of the

monetarists on the use of "free liquid reserves" as an indicator of monetary policy was justified but misguided to the extent that it suggested the replacement of "free liquid reserves" by the actual monteray base. According to the analysis presented the two concepts should be synthesized into the potential monetary base.

6) During certain periods of the past the upper limit for Bundesbank lombard credit to an individual bank has not been preannounced explicitly to the individual bank. An upper limit existed nevertheless and in the process of planning was taken into account by the individual bank in a subjective manner. However, in order to obtain an objective value for the potential monetary base the Bundesbank should preannounce the lombard quota to each individual bank as explicitly as it is done for the rediscount quota.

List of symbols

i_k	bank credit interest rate
i_L^V	sales or offer rate (of Bundesbank) for "liquid assets"
i_L^K	buying or bid rate (of Bundesbank) for "liquid assets"
i_D	deposit rate of interest
i_{BD}	discount rate (of Bundesbank)
i_{BL}	lombard rate (of Bundesbank)
i'_{BL}	expected average lombard rate of a period $\equiv \frac{1}{2}(i_{BL} + i^e_{BL})$
i_S	penalty rate for violation of minimum reserve requirements
i_p	penalty rate for premature cancellation of credits
r	rate of required reserves
F	"free liquid reserves" $\equiv R^e + L + (\bar{B}_D - B_D) + (\bar{B}_L - B_L)$
K	bank credits
L	"liquid assets"
$\bar{D}$	deposits (exogenous)
B_D	rediscount credit
$\bar{B}_D$	rediscount quota (exogenous)
B_L	lombard ceredit
$\bar{B}_L$	lombard quota (exogenous)
$\bar{B}_D - B_D$	unused (part of) rediscount quota
$\bar{B}_L - B_L$	unused (part of) lombard quota
R	total reserves

R^e	excess reserves
R^r	minimum required reserves
W	withdrawals
E	expected value operator
e	index for expectations (exception: e = excess in R^e)
W'	$(1 - r)W$ liquidity needed
$\varepsilon(y, x)$	$\frac{\partial y}{\partial x} \cdot \frac{x}{y}$ = elasticity of y with respect to x
Pr	profits
I	$\equiv (i_L^V, i_L^K, i_{BD}, i_{BL}, i_S, i_p)$, vector of interest rates
G	$\equiv (i_K, I, \bar{B}_D, \bar{B}_L, r)$ vector of interest rates and policy parameters
$\bar{B}^p$	potential monetary base
D^s	macro supply of deposits
M^s	macro supply of money
K^s	macro supply of bank credit
$M^{s,r}$	reduced form macro supply of money
$K^{s,r}$	reduced form macro supply of bank credit
V	vector for unspecified determinants of bank credit demand

Appendix

Definitions of the T_i:

(A1)

$$T_1 = 1 + \alpha \, a \frac{\partial F}{\partial i_K} \qquad \alpha = - \left[\frac{\partial K^d}{\partial i_K} + (1+a) \frac{\partial F}{\partial i_K} \right]^{-1} > 0$$

$$T_2 = - \alpha \frac{\partial F}{\partial i_K}$$

$$T_3 = \frac{\partial F}{\partial \bar{B}_L} T_1 + \left(1 - \frac{\partial F}{\partial \bar{B}_L}\right) T_2 = T_2 + T_4 \frac{\partial F}{\partial \bar{B}_L}$$

$$T_4 = 1 + \alpha \cdot (1+a) \frac{\partial F}{\partial i_K} = T_1 - T_2$$

$$T_5 = \left(1 - \frac{\partial F}{\partial \bar{B}_L}\right) T_4 = T_1 - T_3$$

$$T_6 = \frac{\partial F}{\partial \bar{B}_D} T_1 + \left(1 - \frac{\partial F}{\partial \bar{B}_D}\right) T_2 = T_2 + T_4 \frac{\partial F}{\partial \bar{B}_D}$$

Order of magnitude:

$$0 < - \frac{\partial K^d}{\partial i_K} < \infty \Rightarrow$$

$$0 < T_i < 1 \qquad i = 1, 2, ..., 6$$

(A2) $T_1 > T_3 > T_2$

$$T_4 > T_5 \, , \qquad T_6 \underset{(>)}{=} T_2 \quad ; \quad T_3 \qquad \overset{F}{\bar{B}_L}$$

$$
\text{(A3)}\quad
\begin{cases}
\dfrac{\partial K^d}{\partial i_K} \to 0 \Rightarrow &
\begin{cases}
T_4 = T_5 = 0 < T_1 = T_2 = T_3 = T_6 < 1 \\
T_3 \gtrless \dfrac{\partial F}{\partial \bar{B}_L}
\end{cases}
\\[2ex]
-\dfrac{\partial K^d}{\partial i_K} \to \infty \Rightarrow &
\begin{cases}
T_2 = 0 < T_3 \, , \, T_5 < 1 = T_1 = T_4 \\
T_3 = \dfrac{\partial F}{\partial \bar{B}_L} \, , \; T_5 = 1 - T_3 \\
0 \underset{(<)}{=} T_6 = \dfrac{\partial F}{\partial \bar{B}_D} < 1
\end{cases}
\end{cases}
$$

Footnotes

(1) This can be verified by examining the following work: Brunner, Meltzer (1968), Brunner (1973), Burger (1971), Neumann (1977), (1977), Siebke, Willms (1970), (1974), Willms (1971). For the, institutional environment of the US the contributions of Klein (1971), Porter (1961) and others could be considered as a microfoundation. However, typically the representatives of the BM-approach do not refer to this body of literature. For the environment of the FRG such a reference would be entirely misplaced.

(2) In the eyes of the author these restrictions on the sources of central bank money form the strongest limitations of the present analysis. In future work they will be eliminated with priority.

(3) At times the Bundesbank does not announce explicit upper limits for lombard credits. For such cases $\bar{B}_L$ is a symbol for the upper limit of lombard credits as *expected* by our representative bank. Unfortunately, these expected values are generally not true, only approximate certainty equivalents.

(4) This assumption is a technically simple special case of the more general assumption of a fixed upper limit for the additional credits possible. The crucial point of this assumption is not that additional credits cannot be granted but that the amount of additional credits possible is assumed

to be invariant. The explicit introduction of a fixed exogenous nonzero upper limit would not change the following results. The author believes that an endogenous upper limit will not alter the results qualitatively in an essential way.

(5) The assumption of an interior solution is consistent with empirically observable states of the corresponding macroeconomic aggregates. Typically, in the aggregate, quotas are not exhausted.

(6) It is necessary throughout that the density function of withdrawals is not ascending above the fifth and sixth interval (see the numbering of intervals in the diagram for the liquidity cost function).

Further restrictions are only necessary in order to determine the effects of a variation of deposits and can be formulated as alternative sets of sufficient, nonnecessary conditions. In one alternative the density function has to be a negative function of the volume of deposits ($\bar{D}$) for $W \geqslant 0$, independent of the volume of deposits for $W < 0$ and the ratio of probabilities above the seventh and eigth interval has to remain constant if these intervals are maintained while the volume of deposits is changed.

(7) If deposits are withdrawn then required reserves are liberated. Therefore, liquidity is needed only for that part (W') of the outflow of deposits (W) which is not covered by required reserves: $W' = (1-r)\ W$.

(8) If "liquid assets" do not exist then both equation (13) and the unknown L disappear from the model. The results presented under 6.1) - 6.6) will change neither quantitatively nor qualitatively. This follows from the recursive structure of the equations (13) - (15).

(9) The economies of scale of the model are a consequence of the lombard quota. They are not derived by applying the law of large numbers to the stochastic withdrawals occuring in the middle of a period.

(10) If we neglect the interventions in the foreign exchange market to which the Bundesbank is obliged by international treaties, to my knowledge, there is only one feature in the monetary system that stands in the way of perfect control of the potential monetary base. This is the opportunity of commercial banks to sell (via the Privatdiskont-bank-AG), prime bankers' acceptances to the Bundesbank. Within an institutionally fixed but rather low ceiling (4. Mrd. DM = 4 billions of DM) the Bundesbank is acting here entirely passively. Since this ceiling is not allocated by the Bundesbank to individual commercial banks similar to rediscount quotas its analytical treatment in our micro analysis cannot be similar either. Prime bankers' acceptances are usually treated as part of "liquid assets". From the point of view of controlling the potential monetary base the Bundesbank has abolished the wrong part of "liquid assets" when it stopped the money market regulation for treasury bills while maintaining its standby facility for prime bankers'

acceptances.

However, the control problems arising from prime bankers' acceptance are not limited to the control of the potential monetary base but fully extend to the control of the actual monetary base.

(11) Here, the term multiplier approach is applied to a system in which all scale or level variables vary in proportion to the volume of demand deposits. In particular, liquidity provisions and borrowings from the central bank vary in proportion to the volume of demand deposits.

(12) I do not deny that banks may associate a single preferred volume of deposits with a given set of conditions and that in the long run will try to be on a non degenerate supply curve in the usual sense. But in the short run sense, relevant for money supply theory, banks accept any amount of deposits at the given set of conditions.

(13) These assumptions may look stronger than necessary. Alternatively we might have assumed that the bank is a market maker in the following sense. It sets bank credit rates and deposit rates with a long run profit maximizing perspective and then stands by to accept, in the short run, any volume of deposits forthcoming. With uncertainty about customers' demand for deposits, the bank, at least in the short run, would not be in a position to control the *actual* volume of deposits by choosing the level of deposit rates

while it may have control over the expected value of its deposits. Since, in our model, we are dealing with the bank's problem of allocating *actual* deposit inflows deposits would under these assumptions again be exogenous to the bank. However, this weaker set of assumptions would be inconsistent with our intention to derive a price taker's bank credit supply function that together with a credit demand function allows to determine a competitive equilibrium rate of interest on bank credits and, via a price setting function, of the deposit rate. Such an inconsistency could be avoided by assuming that the bank is a price taker on the bank credit market while at the same time acting as a monopolistic price setter on the deposit market. However, it seems difficult to justify such an asymmetry of assumptions and we have chosen to follow the BM-approach with respect to modelling the credit market.

(14) As long as i_K is not determined the deposit supply function consists of the whole price (i_D) - quantity (D) - space and not just of a line or another subset of that space. At the same time i_K cannot be determined on the credit market without knowledge of the volume of deposits that one wants to determine via demand and supply of deposits.

References

Brunner K., Meltzer A.H., Liquidity Traps for Money, Bank Credit and Interest Rates, Journal of Political Economy, vol. 76, 1968, S. 1-37.

Brunner K., Money Supply Process and Monetary Policy in an Open Economy, in: M.B. Conolly, A.K. Swoboda, Hg., International Trade and Money, London 1973.

Burger A.E., The Money Supply Process, Belmont Calif., 1971.

Klein M.A., A theory of the banking firm, Journal of Money, Credit and Banking, vol. 3, 1971, S. 205 - 218.

Neumann M.J.M., A Theoretical and Empirical Analysis of the German Money Supply Process, in: S.F. Frowen, A.S. Courakis, M.H. Miller, Hg., Monetary Policy and Economic Activity in West germany, London 1977, S. 73 - 124.

Neumann M.J.M., Price Expectations and the Interest Rate in an Open Economy, Germany 1960 - 72, Journal of Money, Credit and Banking, Vol. 9, 1977, S. 206 - 227.

Porter R.C., A model of bank portfolio selection, Yale Economic Essays, Vol. 1, 1961, S. 323 - 359.

Siebke J., Willms M., Das Geldangebot in der Bundesrepublik Deutschland: eine empirische Analyse für die Periode von 1958-1968, Zeitschrift für die Gesamte Staatswissenschaft; Bd. 126, 1970, S. 55 - 74.

Siebke J., Willims M., Theorie der Geldpolitik, Berlin, Heidelberg, New York 1974.

Willms M., Controlling Money in an Open Economy: the German Case, Federal Reserve Bank of St. Louis, Review, Vol. 53, 1971, S. 10 - 27.

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