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Dynamic Consistency in Extensive form Decision Problems

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Abstract - In a stimulating paper Piccione and Rubinstein (1997) argued how a decision maker could undertake dynamically inconsistent choices when, in an extensive form decision problem, she exhibits a particular type of imperfect recall named *absentmindedness*. Such imperfection obtains whenever an information set includes histories along the same decision path. Starting from work focusing on the *Absentminded Driver* example, and independently developed by Segal (2000) and Dimitri (1999), the main theorem of this paper provides a general result of dynamically consistent choices, valid for a large class of finite extensive form decision problems without nature.

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1. Introduction

The issue of dynamic consistency in Finite Extensive Form Decision Problems (FEFDP) has recently been at the centre of a very interesting debate. The discussion was stimulated by a paper of Piccione and Rubinstein (1997) (P&R) where, in absence of changes in the decision maker (DM) preferences, imperfect recall was identified as a source for possible inconsistent choices, in particular by the type of limited memory denominated absentmindedness. Such memory limitation is the case when the DM can not distinguish among decision histories along the same path. Piccione and Rubinstein argued that upon reaching an information set characterised by absentmindedness, the DM might be induced, somehow "paradoxically", to revise her original optimal plan. A number of authors Aumann-Hart-Perry (1997), Battigalli (1997), Gilboa (1997), Lipman (1997) have discussed how an approach based on the view that the DM is a collection of different selves could justify these choices, along the decision tree, as a Nash Equilibrium profile of actions taken by the various selves.

Within a one self framework, an alternative route leading to dynamic consistency has been independently undertaken by Segal (2000) and Dimitri (1999). Starting from the celebrated example of the *Absentminded Driver* they argue in support of two main assumptions, which are not part of P&R analysis, that would fully incorporate the DM awareness of possibly being absentminded along the decision tree. Broadly speaking, the first concerns consistency of beliefs with respect to the revised strategy, while the other is a condition of perceived welfare symmetry once at an information set where absentmindedness is exhibited. In short, belief consistency rests on the consideration that, upon reaching an information set with absentmindedness, it would be difficult to make sense of the view that passage from one node to another, within the set, is governed by the behavioural strategy adopted prior to entering that set and not by the (possibly) revised one. The second is based on the observation that, once having reached such an information set, unless further elements would be introduced the DM should associate the same *game value* at all nodes in that set since, otherwise, she would *think* of having the ability to disentangle the nodes that she is finding herself at.

This paper generalises the result to a large class of FEFDP where *nature* is not present, this because we only want to concentrate on the possibility of inconsistent behaviour due to cognitive limitations. The problems we consider incorporate straightforward generalisations of absentmindedness, but do not investigate the role of partial memory limitation. On this point, recent work by Kline (2002) established minimum memory conditions for ex-ante optimal strategies to be dynamically consistent. The paper is organised as follows. In part one of Section 2 we introduce and discuss the above two main cognitive-behavioural assumptions, within the "classical" example of the *Absentminded Driver*. Section 3 specifies the basics of the general model. In Section 4 we define dynamic consistency and the generalised versions of the two main assumptions. Section 5 presents the main result of the work while Section 6 discusses possible extensions and concludes the paper.

2. THE "CONSISTENT" ABSENTMINDED DRIVER[1]

2.1 The Absentminded Driver

We begin this section by recalling the original version of the *Absentminded Driver* problem (AMD henceforth), the motivating example of P&R analysis. The game is as in Fig 1 below.

Insert Figure 1 about here

The payoffs associated to terminal histories of the extensive form decision problem reveal that her goal is to obtain *4*, the highest reward. There is absentmindedness since the driver upon reaching one of the two nodes (highway junctions), *I* and *II*, is unable to distinguish between them. More specifically she has a single information set, indicated by the dotted line connecting nodes *I* and *II*. As a manifestation of imperfect recall, the peculiarity of absentmindedness rests in the fact that information sets contain decision points (two in the case of the AMD problem) along the same path.

[1] This section draws from Dimitri (1999).

Actions available at each intersection are either *c {continue}* or *e {exit}*; hence, the AMD goal is achieved if and only if the sequence of actions *ce* (in this precise order) would be chosen.

In this simplest decision problem P&R detect the possibility of time-inconsistent choices when both pure and (randomised) behavioural strategies are considered. Namely, the plan (on how to choose within the game) formed by AMD before she starts playing may be revised, in spite of unchanged preferences, upon reaching the information set.

This last observation makes dynamic inconsistency a key conceptual issue in the game, in need of further investigation. Indeed, since Strotz (1956), a common explanation for dynamic inconsistency has been given by change in preferences. Hence, on the one hand, since in the problem AMD tastes remain stable along the decision path, here inconsistency would be exclusively due to the type of imperfect memory under consideration; namely, only to a *purely cognitive* aspect. On the other hand, P&R notice, in the model there's an evident tension between the ex-ante and the ex-post individual's choice, in the following sense. As AMD is ex-ante aware of her own ex-post absentmindedness why should she plan an optimal strategy, before entering the game, if she can anticipate that she may have an incentive to deviate from it once at the information set? As a consequence why, for example, isn't AMD considering only either the ex-ante or the ex-post strategy? With the possibility of time inconsistency in extensive form games it is the standard notion of strategy, defined as a contingent (to each information set) plan of actions made at the beginning of the game, that comes to be put under scrutiny. In extensive form decision games with perfect information the distinction between ex-ante and ex-post strategies is irrelevant since, ex-post, players have no incentive to deviate from a previously formed optimal plan of actions. With absentmindedness instead the alleged distinction becomes meaningful and the notion of optimal strategy may be an issue.

Since absentmindedness represents a very specific type of imperfect recall, it is not unreasonable to imagine that different DM may entertain alternative views on how to deal with such a particular cognitive limitation. Then, in principle, we see it as acceptable that inconsistency might emerge in some of the approaches but not necessarily in all of them. Indeed, we very much share the view, made explicit by P&R and Lipman (1997), that there may be more than one plausible way to model a decision maker's mental process in this game.

Let's now discuss, somewhat informally, the P&R argument supporting the possibility of inconsistency in behavioural strategies. In what follows p will be the *ex-ante* (planned before the game starts) probability of choosing action c *{continue}*, $1\text{-}p$ the probability of action e {*exit}* at each node (in the behavioural strategy), q the *ex-post* (at the information set) probability of choosing c, $1\text{-}q$ the probability of choosing e and α the subjective probabilistic belief of AMD to be at node I, conditional to having reached the information set. Moreover $\Pi(p)$ and $\Pi(q)$ indicate, respectively, the ex-ante and ex-post expected payoffs for AMD; finally, $\Pi(q / I)$ and $\Pi(q / II)$ stand for the expected payoffs conditional to, respectively, node *I* and *II*.

i) Ex-ante Before the game starts AMD expected payoff is $\Pi(p)=p(4-3p)$, since she would face a lottery that with probabilities $1-p$, $p(1-p)$ and p^2 provides her with payoffs (respectively) equal to *0*, *4* and *1*. Straightforward maximisation of $p(4-3p)$ leads to $p^*=2/3$ as the optimal behavioural strategy planned to follow, once at the information set.

ii) Ex-post Conditional to being at the information set AMD expected payoff instead becomes $\Pi(q)=\alpha\Pi(q|I)+(1-\alpha)\Pi(q|II)=\alpha[q(4-3q)]+(1-\alpha)[4-3q]$. Hence, maximisation of $\Pi(q)$ entails that the optimal strategy is now $q^*=max\ [0,(7\alpha-3)/6\alpha]$, different from p^* unless $\alpha=1$. Therefore, if $\alpha<1$, any behavioural strategy would be time inconsistent since $p^*\neq q^*$; however, inconsistency would also be the case if $\alpha=1$ too since, believing to be at node *I* with probability one, AMD would presumably choose to continue with certainty and so $q^*=1$.

While point *(i)* is basically uncontroversial, point *(ii)* has been a main source of debate for it is on the *ex-post* reasoning of an individual that alternative approaches can be contemplated. In particular, since AMD mental process is formalised by the expected payoff $\alpha[q(4-3q)]+(1-\alpha)[4-3q]$, we here anticipate that we'll scrutinise the above expression in the following two main directions. First to incorporate an issue of internal coherency in the individual's mental process, reflected in Assumption 3 (*Welfare Symmetry)*, the term $\alpha\Pi(q\,|\,I)$ will change; this in turn implies that AMD game evaluation at the information set will no longer be calculated through the conditional expected payoff $\Pi(q\,|\,I)$. The second direction, made explicit by Assumption 4 *(Belief Consistency)*, will instead propose a definition of α that, to us, represents the most convincing way for the model to embody the fundamental issue of the analysis, namely AMD possible reconsideration of the ex-ante optimal behavioural strategy upon reaching the information set.

We are now ready to discuss in more detail how consistency could emerge once the AMD reasoning would fully incorporate the (explicit and implicit) model assumptions that we now specify.

Assumption 1 (History-independent information partition) *At all non terminal histories AMD information partition is the same.*

In extensive form games and decision problems this is the standard assumption concerning a DM information processing skills. It is however worth stressing it since taken together with the next one (Assumption 2) will entail some important implications.

Being standard, in P&R Assumption 1 is left implicit; what is made explicit instead is the following Assumption 2.

Assumption 2 (Awareness[2] of Absentmindedness*) At all non terminal histories AMD is aware of her own absentmindedness, and takes this knowledge into account when evaluating her welfare.*

Few comments are in order here. Absentmindedness is introduced as AMD inability to distinguish between nodes, at all decision histories in the problem. Consequently, for both Assumptions 1 and 2 to hold it must necessarily be true the further implication that absentmindedness should also relate any *element* that may provide AMD with information concerning the node that she is at the moment. The *element* we are particularly interested in stressing is her mental process at decision nodes.

A possible, informal, argument supporting the point could be the following. Suppose that at node *II* AMD could recall (for example) a consideration that she has previously made, which was not at the beginning of the game. Then, she would immediately infer to be at node *II* and the information set could not be as the one depicted in Fig. 1.

We now come to Assumptions 3 and 4, the main ones for the paper findings. The versions we give below will later be generalised.

Assumption 3 (Welfare Symmetry) *At both decision nodes in the information set AMD thinks that her game value is the same at all histories in the set, and equal to* $\Pi^*(q)$.

[2] The notion of (un)awareness is introduced here only on intuitive grounds. For a formal treatment of the concept see Modica-Rustichini (1994-1999) and Dekel-Lipman-Rustichini (1998).

Assumption 4 (Belief Consistency) *Conditional to having reached the information set, the probabilistic belief of being at a particular node is consistent with the behavioural strategy chosen at the information set (ex-post) and not before the game starts (ex-ante). Formally, if* α *is AMD's belief to be at node I then* $1-\alpha=\alpha q$*. This would clearly imply that*

$$\alpha+\alpha q=1$$

so that $\alpha=1/(1+q)$*.*

The key conceptual finding of the paper is already in the following preliminary proposition.

Proposition *The ex-post evaluation of the game by AMD is* $\Pi^*(q)=\alpha q\Pi^*(q)+(1-\alpha)[4-3q]$*, rather than* $\Pi(q)=\alpha[q(4-3q)]+(1-\alpha)[4-3q]$*, where* $\alpha=1/(1+q)$*. Hence,* $\Pi(p)=p(4-3p)$ *and* $\Pi^*(q)=q(4-3q)$ *have the same functional form, respectively, in p and q so that* $p^*=2/3=q^*$ *and AMD is time consistent.*

Here's the argument supporting welfare symmetry. Following the proposition, interpret $\Pi^*(q)$ as AMD "value of the problem" conditional to being at the information set; we observe that because of Assumptions 1 and 2 she must believe of entertaining the same evaluation of the game at both nodes. Indeed, if this was not so, she would imagine to be able to distinguish between the two junctions. More specifically, if (with belief α) she is at node *I* then her game value is $\Pi^*(q)$; moreover, she is aware that should she then reach junction *II* (with probability *q*) she would have no elements to think that she would not replicate exactly the same reasoning, hence ending up with the same game value $\Pi^*(q)$. This explains the first term on the right hand side of $\Pi^*(q)$. If AMD were instead at node *II* (with belief $1-\alpha$) then she knows that the game will end after the next move; as a consequence, at this node the game evaluation is given, as in P&R, by $(4-3q)$.

It is indeed at this very point that our modelling of AMD mental process differs from P&R proposal; this is not surprising in view of the following conceptual tension. If she were to be at node *I* then she perfectly knows that at *II* she will face a different lottery; P&R are in favour of incorporating such information in her welfare calculations and notice that this leads to time inconsistent choices. However, if we do this we allow AMD to be aware of an element of incoherence in her own reasoning by assuming that she anticipates at *I* the subjective value that she will attach to the game at *II* and yet, in calculating $\Pi^*(q)$, she disregards this information. Still at an interpretative level, we could depict the above situation as AMD facing a trade-off between being *externally* (with respect to the data of the problem) *vs* *internally* (with respect to her own personal evaluation of the game) coherent. This paper will suggest that should AMD opt for the latter then time consistent choices may emerge.

Notice that an analogous approach incorporating future welfare in a DM calculation, rather than mere expected payoff, has recently been advocated and discussed also by Saez-Marti and Weibull (2002) in work concerning time discounting.

We now argue on beliefs α and $1-\alpha$ being consistent with q[3]. Indeed, as AMD has no elements to think that her own reasoning is different at the two nodes (otherwise, again, she would imagine to be able to detect the nodes), she can not but assume that at both junctions she would choose the same ex-post probability q. Hence, q is the transition probability between I to II and consistency between beliefs and behavioural strategies entails $1-\alpha=\alpha q$ so that $\alpha=1/(1+q)$.

In extensive form games (decision problems) without absentmindedness, it is standard to consider beliefs as formed prior to players' actual choices. Namely, conditional to being at an information set, beliefs relative to the nodes in the set are independent of the actions taken at that set. Because of the above symmetric-reasoning considerations, with absentmindedness this may not be so. Conditional beliefs could then be imagined to depend upon behavioural strategies chosen at the set and, due to the belief consistency assumption, come after them. Failure to accept so may entail some drastic consequences; the following is a possible one. Letting $1-\alpha=\alpha p$, would imply imputing to AMD the idea that possible reconsideration of her ex-ante optimal strategy can not occur at node I. Indeed, if this was not so then AMD could not assume that p is the transition probability between the

[3] We stress so since in P&R belief consistency is defined with respect to the probabilistic behavioural strategy p^* chosen ex-ante. Namely, they require $1-\alpha=\alpha p^*=\alpha(2/3)$.

two highway junctions. Consequently, AMD would think of deducing to be at *II* and choose $q^*=0$.

3. THE GENERAL MODEL

We are now ready to pursue the general analysis. We begin so by formally defining the relevant elements of a FEFDP provided by P&R (1987) a specification, for the case of one player and nature, of the definition of a Finite Extensive Form Game given in Osborne and Rubinstein (1994).

Definition 1 *(FEFDP) Is a five tuple* $\Gamma=\langle H,u,C,\rho,\Pi\rangle$ *where*

i) *H is a finite set of sequence of actions of the type* $h=(a_1,\ldots,a_k)$*, where k is a non negative integer. Moreover, H includes the initial history* $\varnothing$ *and if* $h\in H$ *then for all its sub-histories* $h'\trianglelefteq h$ *it is* $h'\in H$*. A sub-history* $h'\trianglelefteq h$ *of the kind* $h'=(a_1,\ldots,a_l)$*, with* $l<k$*, is called prefix. History h is terminal if for no* $h'\in H$ *is* $h\triangleleft h'$*. The set of terminal histories is Z, with generic element* $z\in Z$*. For all non terminal histories* $h\in H\text{-}Z$*, the set A(h) indicates the actions available to the player (whether DM or nature) at h.*

ii) $u: Z\rightarrow R$ *is a VNM utility function.*

iii) $C\subset H\text{-}Z$ *is the set of histories at which chance (nature) moves.*

iv) *for each* $h\in C$ *the probability with which chance chooses* $a\in A(h)$ *is* $\rho(a)$*, where* $\rho(a)>0$ *for all such a.*

v) *D=H-Z-C is the set of histories at which the DM moves.*

vi) *the information processing ability of DM is modelled by a partition P of D. The element of P containing $h \in D$, for all such h, is $P(h)$. At all $h' \in P(h)$ is $A(h)=A(h')$. Henceforth, for simplicity, X will be a generic (element of P and so) information set; the set $A(X)$ indicates the (common) actions available at all histories in X.*

Moreover, we also need to introduce the following

Definition 1a *In $\Gamma=\langle H,u,C,\rho,\Pi\rangle$*

vii) *$\Gamma(h)$ indicates the subgame starting from history h, for all $h \in H\text{-}Z$, while $\Gamma(X)=\cup_{x\in X}\Gamma(x)$ is the set of subgames starting from X.*

viii) *$b^*(h) = bI_{(D-\Gamma(X))}(h)+b'I_{(\Gamma(X)-Z)}(h)$, where I is the standard indicator function, stands for a behavioural strategy adopted by DM that coincides with the behavioural strategy b', at all histories h in $\Gamma(X)$-Z, and with the behavioural strategy b at histories h in D-$\Gamma(X)$. The set of behavioural strategies is B.*

ix) *$V(\Gamma(h)|b)$ stands for the value of the subgame $\Gamma(h)$ and, analogously, $V(X|b)$ stands for the value of the game at the information set X, when in Γ the DM adopts the behavioural strategy b. By $V(\Gamma(\emptyset)|b)=V(\Gamma|b)$ we denote the value of the whole game.*

x) *the probability of reaching history h' from history h, according to the behavioural strategy b, is $p(h'|h,b)$. We denote $p(h'|\emptyset, b)$ as $p(h'|b)$.*

The expected value of $\Gamma(h)$ under b is $E(\Gamma(h)\,|\,b)=\Sigma_{z\in Z}\, p(z\,|\,h,b)u(z)$. Furthermore, $E(\Gamma(\varnothing)\,|\,b)= E(\Gamma|\,b)$

xi) *for all $x\in X$ the conditional (to X) belief of being at history x, when DM adopts strategy b, is $\alpha(x\,|\,b)$.*

Throughout the paper we shall assume the following.

Assumption 5 *$E(\Gamma|\,b)=V(\Gamma|\,b)$*

As we shall see, because of the main behavioural assumptions, the above would not necessarily be true for proper subgames. Finally, we also need the notions below

Definition 2 *In $\Gamma=\langle H,u,C,\rho,\Pi\rangle$ the Successors and Predecessors of history h are defined as follows.*

(i) *(Successors) For each $h\in H\text{-}Z$ the set of its 1-step successors is defined as $S(1,h)=\{h'\in H\ |\,h'= (h,a) \text{ for all } a\in A(h)\}$. The n-step successors of h are defined iteratively as follows $S(n,h)=\{h''\in H\ |\,h''= (h',a) \text{ for all } a\in A(h') \text{ and } h'\in S(n\text{-}1,h)\}$, with n=2,3,…*

(ii) *(Predecessors) For each $h\in H\text{-}\varnothing$, its 1-step predecessor is defined as $P(1,h)=\{h'\in H\,|\,h\in S(1,h')\}$. The n-step predecessor of h is defined iteratively as follows $P(n,h)=\{h''\in H\,|\ h\in S(n,h'')\}$.*

4. DYNAMIC CONSISTENCY

In this section we define the general notion of dynamic consistency and provide generalised versions of Assumptions 3 and 4. We begin with the definition of ex-ante optimality.

Definition 3 *(Ex-Ante Optimality) A behavioural strategy* $b \in B$ *is ex-ante optimal if*

$$V(\Gamma \mid b) \geq V(\Gamma \mid b')$$

for all $b' \in B$.

We now formalise dynamic consistency

Definition 4 *(Dynamic Consistency) A behavioural strategy* $b \in B$ *is dynamically consistent if for every information set X that is reached with positive probability under b is*

$$V(X \mid b) \geq V(X \mid b')$$

for all $b' \in B$.

The definition of dynamic consistency adopted here is an immediate generalisation of the one discussed earlier; it simply stipulates that consistency is the case whenever at each information set reached with positive probability the DM has no incentive to change an ex-ante adopted strategy. Below we generalise Assumptions 3 and 4 ; more specifically, the extensions are given by Assumptions 7 and 8.

Assumption 6 *For all information sets X is* $V(X|b^*)=\Sigma_{x\in X}\alpha(x|b^*)V(\Gamma(x)|b^*)$.

Assumption 7 *At any history x in X the DM thinks that her game value at all histories X is the same and equal to* $V(X|b^*)$. *Moreover,*

$$V(\Gamma(x)|b^*)=\Sigma_{\{h'\in S(1,x)\cap X\}}p(h'|x,b')V(X|b^*)+\Sigma_{\{h'\in S(1,x)-X\}}p(h'|x,b')E(\Gamma(h')|b')$$

At any history in X to calculate $V(\Gamma(x)|b^*)$, for all $x\in X$, the DM reasons as follows. From x, in one step, she can either move to histories within X or to histories outside X. The former possibility can occur with probability $\Sigma_{\{h'\in S(1,x)\cap X\}}p(h'|x,b')$ and, because of Assumption 7, at any such h' would give rise to a game evaluation of $V(X|b^*)$. The latter instead will occur with an overall probability $\Sigma_{\{h'\in S(1,x)-X\}}p(h'|x,b')$ and, at all such h', gives rise to game evaluations equal to the expected values $E(\Gamma(h')|b')$. On this last point we shall come back in the final section.

Assumption 8 $\alpha(x|b^*)=p(x|b^*)/p(X|b^*)$, *where* $p(X|b^*)=\Sigma_{x\in X}p(x|b^*)$.

In words, upon having reached the information set X, the DM has a (conditional belief) to be at a node $x\in X$ that, within the set itself, is consistent with the strategy adopted at X, generically indicated by b'. Clearly, in so far as those nodes in X which are reached from outside the set are concerned, the (conditional) belief must be consistent with the strategy adopted prior to reaching X, generically indicated by b. Notice that in the AMD and GAMD this last point does not appear to be an issue since the information set is reached with probability one. As for the rest, the definition of $\alpha(x\,|\,b^*)$ is fully analogous to one given by P&R.

5. *THE MAIN RESULT*

We are now capable to state the main theorem of the paper establishing dynamic consistency.

Theorem *Assume that in* $\Gamma=\langle H,u,C=\varnothing,\Pi\rangle$ *assumptions 1-2 and 6-8 hold. Then if* $b\in B$ *is ex-ante optimal is dynamically consistent.*

Proof Assume that, when implementing b, X can be reached with positive probability. Then

$$V(\Gamma\,|\,b)=E(\Gamma\,|\,b) = \Sigma_{z\in Z}\,p(z\,|\,b)u(z) = \Sigma_{z\in Z\text{-}Z(X)}\,p(z\,|\,b)u(z)+ \Sigma_{z\in Z(X)}\,p(z\,|\,b)u(z)$$

where $Z(X)\subseteq Z$ is made of those terminal histories having at least one prefix ending in X. Moreover,

$$V(\Gamma\,|\,b)=\Sigma_{z\in Z\text{-}Z(X)}\,p(z\,|\,b)u(z)+ \Sigma_{h\in \underline{X}}\,p(h\,|\,b)E(\Gamma(h)\,|\,b) \qquad (1)$$

where $\underline{X}=\{h\in X \mid P(n,h)\notin X \text{ for all } n\in N\}$.

It is however more convenient to write $V(\Gamma \mid b)$ as follows

$$V(\Gamma \mid b)=\Sigma_{z\in Z\text{-}Z(X)}\, p(z \mid b)u(z)+ p(\underline{X} \mid b)\Sigma_{h\in \underline{X}}\, p(h \mid b)E(\Gamma(h) \mid b)/p(\underline{X} \mid b) \quad (2)$$

where $p(\underline{X} \mid b)=\Sigma_{x\in \underline{X}}\, p(x \mid b)$.

From Assumptions 6-8 we have that

$$V(X \mid b^{*})= \Sigma_{h\in \underline{X}}\, p(h \mid b)\ \Sigma_{\{x\in \Gamma(h)\cap X\}}\, p(x \mid h,b')[\Sigma_{\{h'\in S(1,x)\cap X\}}\, p(h' \mid x,b')V(X \mid b^{*}) +$$

$$\Sigma_{\{h'\in S(1,x)\text{-}X\}}\, p(h' \mid x,b')E(\Gamma(h') \mid b')]/\, p(X \mid b^{*})$$

and

$$V(X \mid b^{*})=\Sigma_{h\in \underline{X}}\, p(h \mid b)\Sigma_{\{h'\in H\text{-}X \mid P(1,h')\in \Gamma(h)\cap X\}}\, p(h' \mid h,b')\ E(\Gamma(h') \mid b')/[1\text{-}\Sigma_{h\in \underline{X}}$$

$$p(h \mid b)\Sigma_{\{x\in \Gamma(h)\cap X\text{-}\{h\}\}}\, p(x \mid h,b')]$$

so that

$$V(X \mid b^{*})=\Sigma_{h\in \underline{X}}\, p(h \mid b)E(\Gamma(h) \mid b^{*})/p(\underline{X} \mid b)$$

As a consequence, from (2) it is $b \in argmax_{b^* \in B}\, V(X\,|\,b^*)$. Indeed, if this was not so, b could not be the ex-ante optimal strategy. Since this holds true for any information set X, reachable with positive probability by b, dynamic consistency follows.

5. PERSPECTIVES AND CONCLUSIONS

Before concluding the paper, it could be interesting a brief discussion on possible alternative ways to model the DM reasoning at information sets. In particular, Assumption 7 stipulates that, from an history in X, the continuation value of the game when passing to an history h outside X is given by the expected value, under the possibly revised behavioural strategy b', of the subgame starting from h.

However, we could also contemplate that at any X the DM would anticipate the kind of reasoning that she will entertain later in the game at all information sets, possibly exhibiting absentmindedness. Though a complete analysis of the point is not of central interest to the paper, we think it worth to exemplify the matter and illustrate how consistency could still follow. Consider the problem in Fig. 2 below, an extension of Fig. 1 problem.

Insert Fig 2 about here

In it the DM can not distinguish between nodes I and II and between nodes III and IV. So there are two information sets, say X and Y, where $X=\{I,II\}$ and $Y=\{III,IV\}$. If in the ex-ante calculations p and q are the probabilities of c, respectively, in X and Y, then the (ex-ante) expected payoff is

$$\Pi(p,q)=(1-p)V_0 + p(1-p)V_1 + p^2(1-q)V_2+p^2q(1-q)V_3+p^2q^2V_4 \qquad (3)$$

and the optimal probabilities p^* and q^* maximise *(3)* in p and q.

Upon reaching X let now p' and q' be the (possibly) revised probabilities, α again the conditional (to X) belief to be at node I and β the conditional (to Y) belief to be at node III[4].

Then, if $\Pi(X;p',q')$ is the game value at X and $\Pi(Y;p',q')$ the game value at Y we have

$$\Pi(X;p',q')= \alpha[p'\Pi(X;p',q')+(1-p')V_0]+(1-\alpha)[p'\Pi(Y;p',q')+(1-p')V_1] \qquad (4)$$

$$\Pi(Y;p',q')= \beta[q'\Pi(Y;p',q')+(1-q')V_2]+(1-\beta)[(1-q')V_3+q'V_4] \qquad (5)$$

The term $p'\,\Pi(Y;p',q')$ on the RHS of (4) formalises the above considerations; at X the DM anticipates that upon reaching Y she will perform

[4] The DM reasoning concerning the possibility of strategy revision upon Y being reached, which in this case would only pertain the possible updating of q', will be analogous to the example of Fig.1 and so omitted.

a similar kind of reasoning. Hence, at node *II* she will evaluate the continuation of the game by $\Pi(Y;p',q')$ rather than by the expected value $(1-q')V_2+q'(1-q')V_3+(q')^2 V_4$.

Belief consistency would entail $\alpha=1(1+p')$ and $\beta=(p')^2/[(p')^2+(p')^2(q')]$. Solving the system *(4)*-*(5)* immediately entails that

$$\Pi(X;p',q')=(1-p')V_0 + p'(1-p')V_1 + (p')^2(1-q')V_2+ (p')^2(q')(1-q')V_3 + (p')^2(q')^2V_4 \quad (6)$$

and so p^{**} and q^{**}, the optimal conditional (to X) probabilities, will be found by maximising the above expression relatively to (respectively) p' and q'. Since expressions *(3)* and *(6)*, as functions of the pairs (p,q) and (p',q'), are the same dynamic consistency would follow as $p^*=p^{**}$ and $q^*=q^{**}$.

In this paper we have generalised to a large class of finite extensive form decision problems, where nature is not present, an approach to individual choice formation originally suggested by Dimitri (1999) and Segal (2000). The main theorem that we obtain is a general result of dynamically consistent choices, an individual might entertain within a decision problem in which imperfect recall, included absentmindedness, could be present.

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Figures

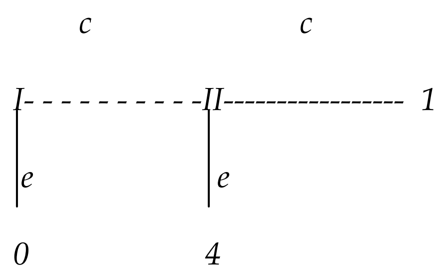

Fig 1

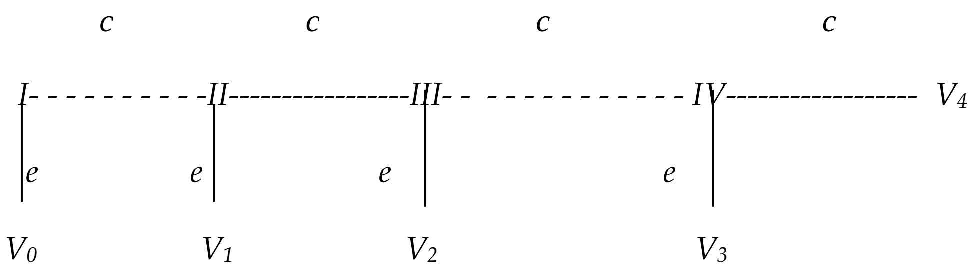

Fig 2