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Time Discounting and Time Consistency

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Abstract - In the economic literature the most widely used type of additive time discounting is Exponential Discounting. Recent work however casts doubts on its ability in explaining how individuals effectively choose. In particular a more general form of discounting that gained importance, in both applied and theoretical work, is Hyperbolic Discounting which captures well phenomena such as procrastination and addiction. An important issue related to the additive form assumed for discounting is that time consistent preferences are the case only with Exponential Discounting. This paper shows that forms of Hyperbolic Discounting, in particular close to the so called Quasi-Hyperbolic model, could also be characterized in terms of dynamically consistent choices when individuals discount the welfare of future selves as well as their payoffs.

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1. *Introduction*

The issue of time discounting has recently received increasing attention by a growing body of literature; excellent overviews on the matter are Angeletos et al (2001), Frederick-Lowenstein- O'Donough (2002) and Laibson (2002). A main reason in support of such an effort comes from empirical evidence Ainslie (1992-2002) indicating how intertemporal choices could be better explained by hyperbolic discounting (HD) rather than, the more widely used, exponential discounting (ED). In particular, a version of HD that gained relevance among economists is the Quasi-Hyperbolic Discounting (QHD), Phelps-Pollack (1968), Laibson (1997). Indeed, forms of HD seem to provide a better model than ED to explain phenomena exhibiting time inconsistent behaviour like, for example, procrastination. In fact it is well understood that when an individual's welfare is calculated by an additive functional discounting future (instantaneous utilities) payoffs, time consistent choices can only be the case with ED. Broadly speaking, a preference ordering expressed at time $t=0,1,2..$ on choices available at time $t^*>t$, will never be changed at time t', with $t \le t' \le t^*$ and $t^*=1,2,..$

In this paper we introduce an alternative, though related, view at dynamic consistency and show that such property is enjoyed by generalised versions of ED, that can be close to the most common form of QHD, when an individual is altruistic towards future selves and, as well as her own payoffs, discounts their welfare (Saez-Marti-Weibull, 2002). Then, a main message of the paper is that time consistency is not necessarily a prerogative of standard ED. Such characterisation can also be seen to provide possible foundations for more general models than ED.

The result is elicited from a benchmark environment given by an infinite stream of unitary payoffs, available to an individual over time. The rationale behind it is that a stationary framework should make the essential properties of intertemporal preferences emerge in a more natural way.

The main finding of the work, which confines itself to the analysis of a specific type of altruism towards future selves, establishes that when time consistency holds purely egoistic and the mixed egoistic-altruistic type of behaviour that we investigate imply the same individual welfare, for a given degree of altruism.

2. *The Model and Main Results*

Consider an individual evaluating an infinite sequence x of constant numbers $\{x(t)\}$, with $x(t)=c$ and $t=0,1,2,...$, where the index t is interpreted as time. To keep the exposition simple, and with no loss of generality, let $c=1$. As it is standard, we conceive such an individual as a collection of selves, one for each t.

In the paper we shall be studying the following additive functional form

$$W(s;T,K) = \Sigma_{t=s,s+1,..,s+T-1}\, d(t-s) + \Sigma_{t=s+T,..,s+T+K.}\, W(t)d(t-s) \qquad (1)$$

where $W(s;T,K)$ is a finite number representing self's s welfare, calculated by discounting payoffs up to time $s+T-1$ and then the welfare of future selves from time $s+T$ to $s+T+K$, with $s,K=0,1,2..$and $T=1,2,...$ We interpret K as the degree (extent) of altruism towards future selves, while T determines the first future self to enjoy altruistic behavior on the part of self s. Finally notice that T should have been written as $T(s)$, since we want to contemplate each self to possibly have a different T; this is why our notion of time consistency will be asked to hold for each T. Finally, $d(0)=1$ and $0\leq d(t)\leq 1$, with $t=1,2,..,$ is a converging sequence of numbers. We refer to the sequence $\{d(t)\}$ as the individual's discounting function.

Expression (1) could be given at least two interpretations. The first would simply say that proper welfare calculation is done through recursive relations of some type, rather than by mere payoff discounting, since a rational agent should anticipate her welfare at future times. The second, as we mentioned, simply views it as a form of altruism towards future selves (Saez-Weibull, 2002). In what follows we shall refer to both readings.

It is worth anticipating that in the paper the following specific forms of (1) will have a special part.

$$i)\ W(s;\infty,K) = \Sigma_{t=s,s+1,..}\, d(t-s) \qquad (2)$$

This is the most common (standard) criterion of payoff discounting used to calculate an individual's welfare through additive functional forms. It could be thought of as either a completely egoistic (with respect to future selves) approach or else fully myopic.

$$ii)\ W(s;T,0) = \Sigma_{t=s,s+1,..,s+T-1}\, d(t-s) + W(s+T)d(T) \ \ \text{with } T<\infty \qquad (3)$$

In this simplest mixed type of discounting, the summation considers payoffs up to some time $s+T-1$ and then only the welfare of self $s+T$, where the expression is truncated.

$$iii)\ W(s;T,\infty) = \Sigma_{t=s,s+1,..,s+T-1}\, d(t-s) + \Sigma_{n=s+T,s+T+1,..,}\, W(n)d(n-s) \ \ \text{with } T<\infty \ (4)$$

In (4) altruism (rationality) takes its fully extended form in discounting the welfare of all future selves from T onward.

We now come to the notion of time consistency. To motivate it we observe how in such a stationary framework, represented by the infinite sequence of unitary payoffs (and for given K), there should be no a priori reason for the selves to perceive different welfare, independently of the value of T. Namely, the notion that we have in mind is robust with respect to when self s starts being altruistic, and so sensitive to K only.

This idea is captured by the following definition

Definition *(Time Consistency) We say that an individual's preferences are "Time Consistent" (TC) if, given K, the discounting function d(t) is such that* $W(s;T,K)=W(s';T,K)$, *for all* $s\neq s'=0,1,2...$ *and all* $T=1,2,..$

We are now ready to formulate the main result of the paper.

Theorem *a)* $W(s;T,K)$, *with* $(T<\infty, K<\infty)$ *and* $(T=\infty, K)$, *satisfies TC only if*

$$D[t(K+1)+i] = d(i)D(K)^t \qquad (5)$$

for $i=1,2,....,K+1$; $t=0,1,2,..$ *and* $D(K)<1$, *where* $D(K)=[\Sigma_{i=1,2,..,K+1}\, d(i)]$.

b) $W(s;\, T,\, \infty)=1$ *for all* $T<\infty$.

Proof a) Let $T,K<\infty$. If $W(s;T,K)$ satisfies TC then $W(s;T,K)=W=W(s';T,K)$ for all $s\neq s'=0,1,2...$, $T=1,2,..$ and given K.
Let $s=0$ so that $W(0;T,K) = W= d(0) + [d(1) +..+d(K+1)]W =1+ D(K)W$ from which $W= 1/(1-D(K)) = W(s;T,K)$.

Notice now that

$$W(s;T+1,K) - W(s;T,K) = W - W = Wd(T+K+1) + d(T) - Wd(T)=0$$

from which

$$d(T+K+1)= (1 -1/W)d(T)=D(K)d(T)$$

and it is immediate to verify that the conclusion to the first part follows.

Consider now $T=\infty$; then

$$W(s;\, \infty,K)=1 + \Sigma_{t=0,1,2,..}[\Sigma_{i=1,2,..,K+1}\, d(i)D(K)^t] = 1 + \Sigma_{t=1,2,..}D(K)^t = 1/(1 - D(K))$$

only if $D(K)<1$.

b) Take now $W(s;\, T\text{-}1,\, \infty)= W(s;\, T,\, \infty)$. Hence, $W(s;\, T,\, \infty)-W(s;\, T\text{-}1,\, \infty)=0$ which leads to $d(T)(1-W)=0$, satisfied when $W=1$ or, equivalently, $d(T)=0$ for all $T=1,2,..$

□

In words, the above theorem says that if the stationarity of the environment is captured, and so TC holds, the discounting function must have a form *akin* to the standard ED. In particular, this obtains when $K=0$; indeed, in this case $D(0)=d(1)$ and $d[t(K+1)+i] = d(t+1)= d(1)[d(1)]^t = d(1)^{t+1}$ for $t=0,1,2,..$

However, when $K\geq 1$ expression (5) can specify forms of QHD. In particular, consider the following example.

Example Let $d(i)=[d/(K+1)]$, for $i=1,..,K+1$ and $0<d<1$. Then $D(K)=d$ and $d[t(K+1)+i] = [d/(K+1)](d^t)= [d^{t+1}/(K+1)]$. Putting $[1/(K+1)]=\beta<1$, the discounting function then becomes the sequence

$$1, \beta d,.., \beta d, \beta\, d^2,.., \beta\, d^2,\ldots, \beta\, d^t,..., \beta\, d^t,...$$

which is similar to the standard QHD function, with the difference that now the generic term βd^t characterizes more than one term of the function. The coefficient β would have a simple explanation, being inversely related to the degree of altruism (number of discounted future selves' utilities). The greater K the lower the weight associated to each discounting coefficient; hence, the more concerned about the welfare of future selves an individual is the lower her own welfare. At the limit, when $K\to\infty$, this would tend to *1*.

Notice that with respect to ED of the kind $1, d, d^2, d^3,.., d^t,..$, the above would have an HD type of shape, in the sense of being below ED near the origin.

In words, the above result provides the following, specific, connotation of the generalised forms of QHD identified by (5). In the stationary environment that we investigate, they represent the only type of intertemporal preferences that would make completely "egoistic", as described by (2), and "altruistic" (for a finite degree K of altruism) individuals to perceive the same welfare within (namely independently of T, for T finite) and between the two criteria.

3. *Conclusions*

In this paper we characterized generalized forms of Quasi-Hyperbolic Discounting as the time consistent preferences of an individual evaluating a stationary environment, represented by an infinite stream of unitary payoffs available over time. In particular, the result obtains when an individual discounts the welfare of her future selves as well as her future payoffs; namely, when she has a certain degree of altruism. Such connotation can also be seen as suggesting a way to found alternative, to Exponential Discounting, additive time preferences.

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