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Hyperbolic Discounting can represent Consistent Preferences

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Abstract - Among non Exponential Discounting (ED) models, introduced to capture time inconsistent choices, Hyperbolic Discounting (HD) recently gained particular relevance. This paper points out that, for some particular payoff structures, HD can also represent consistent preferences.

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1 Introduction

Since Strotz (1955) and Pollak (1968) a commonly shared view has prevailed (see, among others, Frederick-Loewenstein-O'Donoghue, 2002; Benhabib-Bisin-Schotter, 2004) suggesting that a decision maker exhibits time consistent choices if and only his discounting function is exponential. While, according to the accepted definition of time consistency (TC), it's certainly true that exponential discounting (ED) entails TC, the issue as to whether TC implies ED does not seem to have been fully investigated. In a simple three period model, this note challenges the above shared view in showing that though the class of discounting functions that represent time consistent choices contains ED, it also includes non-ED models too. Among these we can find hyperbolic discounting (HD) Ainslie (1975-2002), one of the most successful non-ED models, originally proposed to describe dynamically inconsistent behaviour such as addiction and procrastination.

Therefore, the main conclusion of the paper is that TC does not imply ED, namely non-ED models (such as HD) do not always imply time inconsistent choices.

2 The Model

Consider a (discrete) three periods time horizon, $t=0,1,2$, and an individual having to choose between payoff $x(1)>0$, available at $t=1$, and $x(2)>0$ available at $t=2$.

His discounting function is $d(0)=1$, $0\leq d(t)\leq 1$ and he's asked to indicate, first at $t=0$ and subsequently at $t=1$, whether he prefers $x(1)$ at $t=1$ or $x(2)$ at $t=2$.

According to the accepted definition (Frederick-Loewenstein-O'Donoghue, 2002), this amounts to testing the decision maker's TC with respect to the two following payoff profiles,

$$a=(x'(0)=0,\ x(1)>0,\ x'(2)=0);\ b=(x'(0)=0,\ x'(1)=0,\ x(2)>0)$$

At $t=0$, he would prefer $x(1)$ ($x(2)$), namely profile a (b), if

$$x(1) \geq (<) [d(2)/d(1)]x(2) \qquad (1)$$

and would still prefer $x(1)$ ($x(2)$) at $t=1$ if

$$x(1) \geq (<) d(1)x(2) \qquad (1a)$$

With no meaningful loss of generality, we adopted the convention that in case (1) and (1a) hold as equalities then $x(1)$would be preferred.

When both (1) and (1a) are satisfied the individual is dynamically consistent, since his decision would be the same independently of the time at which it's taken. To discuss TC it's convenient to reformulate the two inequalities as follows, where $x=[x(1)/x(2)]$

$$d(2) \leq (>)\ xd(1) \qquad (2)$$

$$d(1) \leq (>)\ x \qquad (2a)$$

If $x \geq 1$ then TC is the case for all pairs $[0 \leq d(1) \leq 1, d(2) \leq Min(1, d(1)x)]$; consistency, however, can only concern $x(1)$ but not $x(2)$. Inconsistent choices realize when $d(1)$ is low and $d(2)$ is big enough, namely when they are sufficiently different, with $d(2)>d(1)$. As x gets large, namely $x(1)$ is much higher than $x(2)$, choices tend to become dynamically consistent for almost all discounting functions. Notice however that if the condition $d(1) \geq d(2)$ is imposed, then any discounting function can represent time consistent preferences. In other terms, if payoffs are decreasing with time then no inconsistency can arise. Hence, study of TC is meaningful when $x(1)<x(2)$, $x<1$, namely when the higher payoff is available later in time.

If $x<1$ then TC realizes for all $[d(1) \leq x, d(2) \leq d(1)x]$ or $[d(1)>x, d(2)>d(1)x]$, namely as long as $d(1)$ and $d(2)$ are not too different. Moreover, quite intuitively, with $x(1)<x(2)$ "high values" of the discount function entail consistent preferences for $x(2)$ while "low values" for $x(1)$. The following observation relates TC to ED.

Fact 1 *Exponential discounting satisfies inequalities (2) and (2a) and represents time consistent choices.*

Proof (i) Suppose $x \geq 1$; then it's immediate to see that the function $d(0)=1$, $0 \leq d(1) \leq 1$ and $d(2)=d(1)^2$ satisfies both (2) and (2a), again only in so far as $x(1)$ is concerned. (ii) If $x<1$ then notice that within the domain $[d(1) \leq x, d(2) \leq d(1)x]$ the function $d(1)^2$ is bounded above by $d(1)x$ while, over the domain $[d(1)>x, d(2)>d(1)x]$ the function $d(1)^2$ is bounded below by $d(1)x$, and the conclusion follows.

Therefore, with both $x<1$ and $x \geq 1$ we have that TC is represented by ED, namely $d(2)=d(1)^2$; however, (2) and (2a) can be satisfied also by other discounting functions. The following section shows that HD is included in such, non-exponential, discounting functions.

3 Time Consistency with Hyperbolic Discounting

We now discuss conditions under which HD could represent dynamically consistent choices.

Fact 2 *Let $x<(1/2)$ or $x\geq(2/3)$ and suppose $d(t)=1/(1+t)$, with $t=0,1,2$. Then $d(t)$ can satisfy inequalities (2) and (2a) and represent time consistent choices.*

Proof (i) Assume $x\geq1$; then since $d(0)=1$, $d(1)=(1/2)$ and $d(2)=(1/3)$, namely decreasing with time, it's easy to see that (2) and (2a) are satisfied, again only in so far as $x(1)$ is concerned. (ii) Suppose now $x<1$; then, for all $x\geq(2/3)$ inequalities (2) and (2a) are satisfied for consistent choices of $x(1)$. Analogously, for all $x<(1/2)$ inequalities (2) and (2a) are satisfied for consistent choices concerning $x(2)$, and the result is proved.

Summarising, the above observation says that if $x(1)<x(2)$ then HD can represent time consistent choices when the two payoffs are either close to each other or else quite different. This suggests that the most common interpretation of HD, as a model formalizing inconsistent choices, is correct but only for some payoff structures. As a matter of fact, in the simple context we worked with, the measure of the relevant parameter space ($x<1$) entailing TC ($5/6$) is significantly larger than the one implying inconsistency ($1/6$).

Fact 2 could be further generalized by showing that, indeed, any non-ED model could represent TC preferences.

Proposition 1 *For any $d(0)=1$, $0<d(1)\leq d(2)\leq1$ there exist $x<1$ such that the discounting function represents TC preferences.*

Proof It's easy to see that any $x\geq Max[d(1), d(2)/d(1)]$ will induce consistent choices of $x(1)$ while any $x<Min[d(1), d(2)/d(1)]$ of $x(2)$, and the conclusion follows.

For a better appreciation and understanding of Fact 2 it's convenient to consider now extending the model to $T>2$ periods. Let now $t=0,1,2,..,T$ and suppose the individual has to choose between payoffs $x(T-1)>0$ and $x(T)>0$, available respectively at $t=T-1$ and $t=T$.
The individual is first asked to choose at $t=0$, then at $t=1$ etc. until $t=T-1$. The following result, the easy proof of which we are going to skip, establishes more general conditions for discounting functions representing TC.

Proposition 2 *Define $x=x(T)/x(T-1)$ and suppose $x<Min[d(t)/d(t-1)]$ or $x\geq Max[d(t)/d(t-1)]$; then $d(t)$ represents time consistent preferences. If $d(t)/d(t-1)$ is increasing with t, as in the case of $d(t)=1/(1+t)$, then the bounds become $x<d(1)=(1/2)$ and $x\geq d(T)/d(T-1)=1-(1/T)$.*

The proposition generalizes Fact 2, establishing that TC occurs when the two payoff values are either similar, in which case the earlier one will be consistently preferred, or else the later payoff is sufficiently larger than the earlier one and chosen at each time. This of curse clarifies that the only model guaranteeing TC for all x<1 is ED.

The result appears to suggest TC to depend upon:

(i) similarity-diversity of payoff available in time

(ii) length of time horizon.

In particular, in so far as HD is concerned the longer the time horizon, the more similar payoffs have to be for $x(T-1)$ to be consistently chosen, and as T gets large the two payoffs will basically have to be the same. Preferences for $x(T)$ however, do not depend upon T, and always concern payoffs such that $x(2)$ is at least twice as large as $x(1)$.

4 Conclusions

The main conclusion of the paper is that non exponential discounting functions do not necessarily represent dynamically inconsistent choices. In a simple three period horizon model we have shown that hyperbolic discounting, as well as exponential discounting, can formalize consistent preferences. In the more interesting case, where the two relevant payoffs increase with time, this can occur when they either do not differ much or else they are quite different. Therefore, hyperbolic discounting can be conveniently used to formalize dynamically inconsistent choices only for specific payoffs streams.

References

J. Behabib, A. Bisin, A. Schotter, 2004, Hyperbolic Discounting; An Experimental Analysis, *Working Paper NYU*.

G. Ainslie, 1992, *Picoeconomics*, (Cambridge University Press, Cambridge UK)

G. Ainslie, 2002, *Breakdown of will* (Cambridge University Press, Cambridge UK)

Frederick S., Loewenstein G. and T. O'Donoghue, 2002, Time discounting and time preferences: A Critical Review, *Journal of Economic Literature*, XL,351-401.

R. Pollak, 1968, Consistent Planning, *Review of Economic Studies*, 35, 201-208.

Strotz R., 1955, Myopia and Inconsistency in Dynamic Utility Maximization, *Review of Economic Studies*, 23, 165-180.