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Francis Ysidro Edgeworth on the regularity of law and the impartiality of chance

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Abstract - This paper proposes a general interpretation of Edgworth's thought based on the recognition of a unitary philosophical project in his contributions to ethics, economics, probability and statistics. This project consists in the search for a common epistemological foundation for the social sciences. The point is illustrated in reference to the coexistence in Edgworthian scientific programme of the 'regularity of law' with the 'impartiality of chance'. The interpretation here proposed challenges the traditional stereotypes according to which Edgeworth was a crass utilitarian, and an ingenuous advocate of a rather primitive neoclassical economics. His plea for the use of mathematics, and his choice of deterministic models for the description of the economic behaviour, appear more innovative when the role of probability is considered.

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‘A man of science may write one great work which adequately expresses his contribution to human knowledge; another may prefer to publish the results of his labours as they are achieved and let the world reap the immediate benefit. There is much to be said for and against each method; but work published in small discrete *quanta* is liable to be become so absorbed in the common stock of knowledge that its sources are forgotten’ (Sanger 1925: 601). In 1925 Charles Percy Sanger did not hesitate in depicting his friend Francis Ysidro Edgeworth as a scientist who doesn’t write treatises, but who could nevertheless influence the work of other scholars. The topic of the scientist unable to write treatises was repeated by John Maynard Keynes (1926) in the most famous –but not equally reliable- paper on Edgeworth. In the mid 1920s Edgeworth’s project appeared completely defeated to Keynes’s eyes; and the promises and great prospects of his *Mathematical Psychics* had not been fulfilled at all.

After eighty years Keynes’s picture appears absolutely unfair. In fact it is even difficult to list all scholars who credit Edgeworth of some path-breaking contributions in economics and statistics. As foreshadowed by Sanger, Edgeworth’s contributions have been absorbed in the common stock of knowledge as devices for modern microeconomics, contributing to the development of neoclassical economics. With the appearance of modern game theory, Edgeworth’s solution acquired a new relevance as for example in Shubik (1959) and Scarf (1962). Ronald Coase (1988), in his ‘Note to the problem of social cost’, attributed to the unconscious reminiscences of Edgeworth’s *Mathematical Psychics* a substantial part in his famous theorem, at the basis of the modern economic analysis of law. Edgeworth’s contributions to the probabilistic foundation of statistics are now considered a substantial part of the ‘statistical revolution’ in the social sciences at the end of Nineteenth Century (Stigler 1978; 1986; 1999); and he is considered the precursor of many modern statistical devices, such as two way analysis of variance, correlation coefficients, tests of significance and Edgeworth expansion.

The fate invariably encountered by his contribution is that to be rediscovered years later; probably because of his idiosyncratic style and pattern of research. His works appear very fragmented and without a first-sight discernible path, as it is possible to infer from his bibliography, counting at least 4 books, 198 journal articles, 204 reviews and 139 entries for *Palgrave* (Baccini

2003). In fact a comprehensive reconstruction of Edgeworth's works is still lacking in the literature; and the interested reader must refer to the partial stories offered by Creedy (1986), Newman (1987) and Mirowski (1994) for economics, Bowley (1928) and Stigler (1978, 1986, 1999) for statistics, Baccini for probability (1997, 2001, 2004).

Nearly all scholars studying Edgeworth papers are puzzled by his pattern of research, especially in the years between 1878 and 1890, when he switched abruptly from ethics, to economics, to probability theory and finally to statistics. Here, there is the problem of the general interpretation of Edgeworth's scientific production, with two possible solution. The first one considers it as schizophrenic, and refers the changing path of his interests to external circumstances –favourable or critic reviews, his personal search for a hero and so on-; the second one suggests that it replies to a general purpose scientific approach.

The thesis of this paper points to the idea that in Edgeworth's works it is possible to isolate a general research design which can explicate his switching of interest in the Eighties. In order to grasp the unitary approach of Edgeworth's analysis, it is necessary to focus our attention to the problems of the foundations of the various disciplines to which Edgeworth contributed, and to the role played by the probability theory. The only scholar to accept this procedure of analysis is Mirowski (1994); his reconstruction, though having some common points, differs radically from the one here presented.

1. Edgeworth was born on 8 February 1845 at Edgeworthstown, County Longford, Ireland. His Irish family was an extraordinary one (Butler-Butler 1927); his grandfather Richard Lovell Edgeworth was an eccentric inventor of mechanical devices and member of circle of savants which included the luminaries of the industrial revolution. Francis Ysidro's father was Francis Beaufort (born 1809) who married, aged 22, a beautiful catalan refugee encountered in London, Rosa Florentina Eroles, aged 16. Edgeworth was the seventh child and sixth son (Barbé 2004: 303). After the deaths of the father (1846) and of the aunt Maria Edgeworth (1849), the Irish novelist, Rosa completed the education of the children by teaching them Spanish and Italian, and providing private tutors in Classics and Mathematics.

He entered Trinity College, Dublin in the academic year 1861-1862, obtaining the first place at the July entrance examination, and gaining two prices of 5 pounds each for the best Greek verse and Greek prose compositions. In the term examinations of 1862 and in the following two years he confirmed his proficiency in the classics; in 1862 he was third in the ranking of the junior freshmen in mathematics. John P. Mahaffy was for several years his college tutor; in 1873 he wrote a testimonial remembering that during the years in the college 'he was considered the very ablest man in his class, and made himself deservedly popular by his genius as well as kindliness'. R.Y. Tyrrel, his professor of Latin, remembered that 'in Trinity College there are two verdicts pronounced on every student of eminence; one that of Lecturers and Examiners, the other that of the elite of the students. The latter verdict was pronounced in favour of Mr. Edgeworth with an enthusiasm to which I remember no parallel'.

The brilliant Francis Ysidro arrived to Oxford in 1867; as student of Balliol College he took a First Class at the Final Examination in *Literae Humaniores* in 1869.

He left Oxford in 1869, and for twelve years, as he wrote in 1881, he was 'engaged in studying and teaching the science which I desire to profess'. Biographical details of this period are lacking. Probably he lived in two small rented rooms at 6, Mount Vernon, Hampstead. He studied commercial law; he was admitted as member of the Inner Temple 31 January 1874, and was called to the BAR on 13 June 1877; but he never practised. In 1875 he was candidate for a professorship of Greek at Bedford College. He lectured on Logic and Mental and Moral Science for two years at Mr. Walter Wren's Institutions to candidates for the India Civil Service.

In 1876 he published his first two-page paper on an argument of ethics (Edgeworth 1876); the paper appeared in the journal *Mind* founded in that same year and devoted to the study of psychology and philosophy. In 1877, his first book was privately published with the title *New and Old Methods of Ethics* (hereinafter NOME). At the end of the Seventies, probably on the indication of William Stanley Jevons, his neighbour in Hampstead, he sent the book to Alfred Marshall. In the years between 1879 and 1881 Edgeworth applied himself to the study of economics, as it will be remarked later on. In 1881 he published privately again his second and best known book *Mathematical Psychics. An Essay on the Application of Mathematics to the Moral Science* (from now on referred to as MP). It was reviewed by Marshall and Jevons, and warmly welcomed by Francis Galton in a private letter.

From 1880 he was for eight years on the staff of King's College, London; lecturing on Logic, and on Political Economy at the Ladies's Class in Kensington. In 1887 he was elected to the office of examiner in Political Economy at the University of London. In 1888 he was candidate for the vacant professorship of the Principles and Practice of Commerce at King's College, London, and then he was appointed to the professorship.

In 1890, Edgeworth became the first editor of the *Economic Journal.* In that occasion, Marshall tried repeatedly to convince Neville Keynes to accept the editorship, and wrote explicitly in his correspondence that Edgeworth was not his 'first choice' (Whitaker 1996, I: 287, n. 260).

In that same year 1890, Edgeworth was appointed to the Tooke Chair of Economics and Statistics again at King's. In 1891 he was appointed to the Drummond Professorship of Political Economy at Oxford, which Marshall welcomed writing to him a 'Hurrah, Hurrah, Hurrah!' (Whitaker 1996, II: 7-8, n. 340). He became a fellow of All Souls College where he lived for the rest of his life in his 'palatial room'. He received several academic honours: twice President of section F of the British Association (1889, 1922), President of the Royal Statiscal Society (1912) fellow of the British Academy. He served as editor of the *Economic Journal* until his death, in Oxford on 13 February 1926 at the age of 81.

2. As we have seen E's first substantial known work was NOME. It is a strange book for Edgeworth's contemporaries and for modern readers. For the latter it is the first part that sounds strange with its discussion of utilitarian ethics with particular reference to the works of Henry Sidgwick and Alfred Barrat. The new elements introduced by Edgeworth in the second part of the book made it also *strange* for his contemporaries: he imported arguments drawn from the frontiers of the psycho-physics research tradition, and applied differential calculus in solving the problem of the maximization of social welfare in utilitarian ethics. The relevance of the context in which Edgworth wrote the book was pointed out by Howey (1960); Stigler (1965: 115) missed the point; Creedy (1986) underlined the new methodological approach, and Newman (1987) contains the complete synthesis of Edgeworth's arguments in modern notation, so it is not useful to repeat it here. It is instead useful to stress some points of interest for our reconstruction.

In NOME, the general objective of utilitarian ethics is the maximization of social welfare: ‘the utilitarian end is the greatest quantity of happiness of sentients, exclusive of number and distribution’ (Edgeworth 1877: 35). The problem to solve is how to distribute ‘among a given set of sentients’ a given amount of ‘stimulus (…), corresponding to a given amount of material means’ in view of the ‘production of the greatest quantity of happiness’ (Edgeworth 1877: 40). Every sentient is represented as a *pleasure machine* whose pleasure function is an increasing concave function of the stimulus. Edgeworth formalized the function as $\pi = k\left|f(y) - f(\beta)\right|$ where y is the quantity of the stimulus, β denotes the sensibility to the stimulus of the sentient, i.e. the ‘threshold’, or the minimum value of the stimulus for which the sentient has any pleasure, and k the capacity for pleasure. Edgeworth then discussed the problem of the distribution of a given quantity of stimulus in the case in which β and k are the same for all the sentients, and in the four cases in which β and k vary. Edgeworth demonstrated that in the first case the solution is an equal distribution of the material means; and that in the other cases an unequal distribution is required ‘such that most means are assigned to those who have most felicific power’ (1877: iii). This rough summary does not do justice to the subtleties of Edgeworth’s reasoning; but our emphasis is not on the results but on the foundations. It is therefore useful to briefly consider the question of the nature of agents.

The problem at hand was on the foundation of ethics, or more precisely the representation of human actions in ethics (Baccini 2007): whether they can be reduced to simple *contractions* (searching for pleasure) and *irritations* (fleeing from pain) of our nervous system, or whether instead they are relative to the sphere of consciousness. Edgeworth was fascinated by Alfred Barrat’s reductionist view that all human actions have their origin in *pleasure,* and was favourably disposed to accept that a physical origin of human actions exists. However individuals experiment a difference between reflex actions and volitive actions, and between hedonistic preferences and non-hedonistic preferences, so he retained that it is difficult to sustain that *all* actions derive directly from pleasure. On the other hand, the idea that human action have their origin in the idea of pleasantness, as in Sidgwick’s perspective, is not tenable.

Edgworth’s proposals was to blend Barrat’s perspective with Sidgwick’s one. Body and mind are interdependent in the sense that every movement in the *material substrate* of the mind is associated with a mental phenomenon, so, at the end, ‘that definite physical phenomena (which Mr. Barratt (…) calls *pleasure*) are the cause of all human action.’(NOME: 10). The search for pleasure and the flight from pain can be explained in relation to the fact that, in the nervous system, certain inter-relations are constituted that associate pleasure and pain to previous experiences of pleasure and pain more or less remote in time. These experiences are of two kinds: some already had by the individual during previous periods of his life; and some handed down to the individual by the species to which he belongs, through the physical connections internal to the structures of the nervous system, which he inherits from the species.

That second kind of experience originates from Herbert Spencer's psychology (1870). According to Spencer human beings are born with a nervous system inside of which connections handed down from the species are defined. These connections, inherited by the specie, determine for the individual a 'preparedness to cognize'. The nervous system is something organic, subject to evolution, which incorporates *a priori* knowledge within its structure. The individual experience not only furnishes the concrete material at the basis of all thoughts, but without it the organised structures of the nervous system would not, on their own, give rise to knowledge. Edgeworth applied this framework to ethics where every action can be attributed either to individual experiences of pleasure/pain or to the experiences handed down to the individual by the species. In this way, it is possible to justify also apparently non-hedonistic actions as referring to an ancestral experience of pleasure and pain.

Edgeworth defined, in this way, an experiential or *physical* basis for ethics. The key concept for the foundation of ethics was extension of the domain of the *experience of pleasure*, including the one carried out by the species and handed down genetically to successive generations. The *ancestral experience* served to legitimately represent, in a utilitarian context, many forms of real behaviour.

All that we have seen above is the nature of the *pleasure machine* studied in NOME. It is useful to note that, as we have seen, not all machines are equal; they differ in respect to their capacity for happiness and to their threshold of sensitivity to pleasure. The solutions of the utilitarian problems depend from the distribution of sentients in a population in relation to their different characteristics. But, 'to sum up the indication of exact utilitarianism. With regard to the theory of distribution, there is no indication that, at any rate between classes so nearly in the same order of evolution as the modern Aryan races, a law of distribution other than equality is to be wished' (Edgeworth 1877: 78).

The passage to this last statement from different pleasure machines and different solutions to the utilitarian problem, is obscure: how is it possible to represent synthetically the plurality of agents as a unique pleasure machine with the same β and k? Or more generally, what is the status of the notion of man as a pleasure machine?

Edgeworth developed this point in a paper published in 1879 and entitled 'The hedonical calculus'. The problem tackled in this paper is a generalization of the utilitarian one posed in NOME, where the maximization of social welfare is developed in relation not only to the distribution of given 'means', but also to labour and the conditions of its reproduction. The egalitarian solution proposed in NOME, was abandoned for an antiegalitarian one, based on the different capacity for pleasure and work of different individuals in society, offering a poisonous brew of utilitarianism and eugenics (Newman 1987: 92-93).

From our perspective, the paper is very important because it contains a relevant reference to probability theory, in reference to the question of the measurability of pleasure. Edgeworth introduced the following axiom: ‘Pleasure is measurable, and all pleasure are commensurable; so much of one sort of pleasure felt by one sentient being equateable to so much of other sorts of pleasure felt by other sentients’ (Edgeworth 1879: 396). The idea behind this is that all pleasure machines can be considered as having the same sensibility to the stimulus β. The idea of a common threshold was drawn from the notion of *Ebenmerklich* or ‘just noticeable (or perceivable) difference’, utilized in psycophysics by Gustav Fechner and William Wundt (Newman 2003: 157-158). If there is an unit of pleasure perceived by every sentient, it is possible to sum up not only all the units of pleasure perceived by the same individual to measure his total pleasure, but it is also possible to sum up the pleasure of different individuals of a group or society, and finally it is possible to compare the pleasures of two or more individuals or groups of individuals. Edgeworth then based the idea of the cardinality of the pleasure function on results drawn from the frontiers of psychology of his time.

But here we are more interested in justifying the cogency of the axiom above. In 1879 this justification was based on the analogy between the methods of measuring pleasure with the method of measuring probability; the axiom at the basis of the utilitarian calculus cannot be rejected precisely as ‘one cannot reject the practical conclusion of Probabilities, though one may object, with Mr. Venn, to speaking of belief being numerically measurable. Indeed these principles of μετρητική (metretikè: science of measuring) are put forward not as proof against metaphysical subtleties, but as practical; self evident *a priori*, or by whatever επαγωγή (epagoghé: argument from induction) or εθισμός (ethismos: habituation) is the method of practical axioms’ (Edgeworth 1879: 395-96). In this citation we can see the reference to the already encountered *a priori* knowledge, but in relation to probability and to John Venn who will have a fundamental importance in the following work of Edgeworth. This passage, in reality, is far from clear. To have a precise statement of the same concept, but not its explication, it is necessary to wait until 1881 when Edgeworth published his *Mathematical Psychics.*

3. MP, in particular the first subdivision of the second part (15-56) and its appendix, contained the origin of many of the basic concepts of modern microeconomic theory: a generalized utility function –which substituted the Fechnerian one of NOME; the indifference curves; the structure of the now called Edgeworth's box; the marginal rate of substitution.

The main novel element of Edgeworth's thought was the subdivision of the 'calculus of pleasure' in two parts: the 'Economical calculus', which 'investigates the equilibrium of a system of hedonic forces each tending to maximum individual utility', flanked the already developed Utilitarian calculus, dedicated to 'the equilibrium of a system in which each and all tend to maximum universal utility' (Edgeworth 1881: 15-16). The switch from utilitarian ethics to economics is universally attributed to the influence of his neighbour in Hampstead Jevons.

The main theme of the economical calculus was the statement of this new theorem on contract and competition: '(α) Contract without competition is indeterminate, (β) Contract with *perfect* competition is perfectly determinate, (γ) Contract with more or less perfect competition is less or more indeterminate' (Edgeworth 1881: 20).

Its logical primitives are *agents*, who may be individuals or combinations of individuals; *articles* of the contract may be economic goods, but also property rights; *actions* are what agents do. All actions can be classified as *war* or *contract* 'according as the agent acts *without,* or *with,* the consent of others affected by his actions'. The action of *recontracting* without the consent of others is war. (Edgeworth 1881: 15-16). 'The *field of competition* with reference to a contract, or contracts, under consideration consists of all the individuals who are willing and able to recontract about the articles under consideration'. A '*perfect* field of competition' is a field with perfect communication between individuals in which: (i) any individual is free to recontract with any out of an indefinite number; (ii) any individual is free to contract with and indefinite number, so each article of contract is perfectly divisible; (iii) any individual is free to recontract with another without the consent of any third party.

Edgeworth proposes two solution concepts for the problem of competition. The first one reads: 'A *settlement* is a contract which cannot be varied with the consent of all the parties to it'. Or in modern jargon: a contract is an optimal Pareto allocation for the parties of the contract. The second one reads: 'A *final settlement* is a settlement which cannot be varied by recontract within the field of competition', i.e. with renegotiation with any or all of the parties outside the contract, but inside the field. This signifies, in modern jargon, that a final settlement lies in the *core*. Therefore, and finally, a contract is indeterminate when there are an infinite number of final settlements.

Edgeworth's central interest was on the indeterminateness of contracts and not, as it to his modern interpreters, but not to Newman, on the conditions of its perfect determination. According to him, contract is indeterminate when the market is not perfect, i.e. when the number of competitors is limited, or the articles of contract are not perfectly divisible, or in case of the existence of *combination* of agents, as trade unions in the labour market. Almost every species of social and political contract is affected by indeterminateness: 'throughout the whole region of in a wide sense *contract*, in the general absence of a mechanism like perfect competition, the same essential indeterminateness prevails; in international, in domestic politics; between nations, classes, sexes. The whole creation groans and yearns, desiderating a principle of arbitration, an end of strifes' (Edgeworth 1881: 51). The principle of arbitration is the utilitarian maximization contained in the 'Hedonical calculus' (Edgeworth 1879) which Edgeworth reprinted with the addition of some notes, as the second subdivision of the second part of MP. (On the logical weakness of this point the interested reader can see Newman 1994).

If this, in a nutshell is the content of MP, it is necessary for us to underline the use of probability in it. MP starts with an analogy between the calculus of probability and the calculus of pleasure. In the first part of the book, dedicated to the discussion of the applicability of mathematical methods to social sciences, Edgeworth introduced the argument with which we have closed the preceding paragraph. The loose quantitative estimate disposable for measurements of pleasure are based on the notion of just perceivable increment of pleasure or of utility, which 'implied equatability of time-intensity unit'. This is 'a first principle incapable of proof' (Edgworth 1881: 7). This principle 'may be compared, perhaps, to the first principles of probabilities, according to which cases about which we are equally undecided, between which we perceive no material difference, count as equal' (Edgeworth 1881: 99); and it 'is doubtless a principle acquired in the course of evolution.' (Edgeworth 1881: 7). As the conception of man as pleasure machine is based on the possibility to roughly measure utility, the calculus of probability too is based on the possibility to roughly measure probability. These two measurement processes depend on a same inherited *a priori* experience which permits to define as axioms two equivalences: the one between atoms of pleasures experienced by different sentients, and the one between probabilities of indistinguishable events. The justification of the second one needed the analysis of the foundation of probability.

In MP, probability appears also in two other contexts. The second one, the less interesting for our narrative, is related to the equiprobability of the points of a final settlement: under imperfect competition the solution is indeterminate, comprehending all the points on a tract of the contract curve. According to Edgeworth every point of this tract has the same *a priori* probability to be selected by 'objectable arts of higgling' (Edgeworth 1881: 30).

The first one instead opens a new perspective. In writing about a market without competition, he affirmed: 'if competition is found wanting, not only the regularity of law, but even the impartiality of chance –the throw of a die loaded with villainy- economics would be indeed a ‘dismal science’, and the reverence for competition would be no more’ (Edgworth 1881: 50).

The explication of this puzzling phrase needs a reference to a two page paper published in *Philosophical Magazine* December 1883, entitled ‘The physical basis of probability’. It illustrates two analogies between the equilibrium of a particle in a system with a plurality of attractive centres, and the expression of a mean of several observations calculated with the method of least squares; and between those and the ‘mathematical theory of exchange’. The process of exchange is described as one in which ‘the forces at work, the taste of buyers and sellers, are of inconceivable complexity. Yet the position of equilibrium is characterized by a feature of geometrical simplicity, uniformity of rate-of-exchange [in modern term: the same marginal rate of substitution]’. The possibility of ‘mathematically representing maximum advantage is due to the same cause in the market as in the observatory: what may be called the law of great numbers. The sum of squares (…) makes its appearance in virtue of the exponential law of error or probability-curve incidental to the method of least squares; and this simple form arises when the observations are independent of each other and indefinitely numerous. Similarly the law of unity of price hold good where the competitors are independent and indefinitely numerous. In both cases uniformity is due to plurality; definite order to infinite numbers.’ (Edgeworth 1883: 434)

In a companion two-page paper, published on the *Journal of the Royal Statistical Society* March 1884 and entitled ‘The rationale of exchange’, Edgeworth presented again this argument and stated clearly ‘the *rationale* why the complex play of competition tends to a simple uniform result is to be sought in (…) law of great numbers’ (1884: 261). It rests, for us, that in the bulk of the construction of perfect market competition and market clearing mechanism, Edgeworth posed his ‘beloved law of error’. The existence of the equilibrium price depends on the great numbers of buyers and sellers, and on the irreducible differentiation of their tastes.

This point is not well covered in the vast mass of literature on Edgeworth’s theory of exchange. It is true that is not an analytical demonstration of the unity of price; and Edgeworth itself wrote, probably with his characteristic ‘chuckle and smile’, that the derivation of this principle is ‘far-fetched’ (Edgeworth 1884:165). He presented a very different rationale for equilibrium in the market in respect to the Smithian invisible hand, or Walrasian auctioneer. This rationale can probably sounds *dated* to modern readers; but it is absolutely original and based, again, on the frontier of knowledge of his time. The application of probability theory and the statistical inference to the social sciences was not yet arrived; and consequently Edgeworth’s proposal can be seen as a step in this direction.

4. At this point, it is simple to understand why Edgeworth turned his attention to probability and statistics. From a very general point of view probability furnished a solution to the two big foundational problems of his MP: the possibility to depict man as a pleasure machine is based on the same knowledge necessary to define a priori probability, and the fact of the price in a market is dependent on the law of error. Probability and the law of error became his new research topic: so from October 1883 to December 1885 he published an impressive series of 13 papers dedicated to the foundation of probability and statistics.

The landscape of those works is characterized by the 'epistemological flog' of the nineteenth century debate on the nature of probability. *The Logic of Chance* by Venn (1888), which canonically systematised the frequentist theory of probability (Chatterjee 2003; Galavotti 2005; Gillies 2000), had a fundamental relevance in this debate, and was Edgeworth's principal reference text. In the frequentist tradition, probability can be defined only in the context of a random experiment or trial, whose outcome is unpredictable beforehand and which is capable of indefinite repetition under identical conditions, at least conceptually. The totality of possible outcomes is the domain, and probabilities are assigned to various sets -the events- in this domain. Primitive probabilities –the simplest kind of probability- are empirically measurable as the relative frequency of events. In the frequentist tradition, probability is not a property attributable to a single event; so all probability statements refer to sets of events, and not to the individual event: (Chatterje 2003: 40; Jonathan Cohen, 1989: 48-49; Galavotti 2005: 83).

Venn named the sets of events, series; the existence of the series and the numerical proportion of their characteristic properties can be ascertained only by resorting to experience which 'is our sole guide.' (Venn 1888: 74-75). Edgeworth's aim was to enlarge Venn's vision. The first step in this direction was to admit that probability must be referred to two different spheres, one *objective*, relative to the frequencies observed in a certain phenomenon, and one *subjective*, relative to the mental condition associated with those frequencies: 'Probability may be described, agreeably to general usage, as importing partial incomplete belief (…) as differing somehow in degree from perfect belief or rather credibility. (…) Thus the object of the calculus is probability as estimated by statistical uniformity.' (Edgeworth 1884a: 223). He considered as primitives the notions of event and series of events. Starting from events and series he defined primitive probabilities 'as measured by the number of times that the event is found by experience to occur, in proportion to the number of times that it might possibly to occur." (Edgeworth 1911: 376; but see also 1884a: 223; 1899: 208).

In Venn's theory the body of evidence utilizable in fixing initial probabilities is very small, based as it is only on experienced statistical evidence. Edgeworth enlarged this body of evidence by a strategy already applied to the foundation of his ethics and economics. 'I only contend that Mr. Venn (…) has not made the foundation wide enough, and that therefore he is unable to carry up the structure to the full height of generality. He is unable to rise an axiom of equal distribution of quantity in general, above the view that, in the absence of any such (specific) information, we are entirely in the dark.'(Edgeworth 1884b: 160-161). So, beside statistical probabilities, Edgeworth introduced primitive *a priori* probability, that is probabilities 'not determined by statistics', (Edgeworth 1884c: 204), or probabilities measured 'when probability, founded upon statistical fact (…) has reached the utmost degree of tenuity' (Edgeworth 1884a: 229). A priori probability emerges when we have probability defined in reference to a sort of experiential knowledge which is handed down through heredity, and which belongs to the species before they belong to the individual. This a priori probability is the experiential basis on which is founded the Laplacean principle of indifference, according to which the same probability value is attributed to the faces of a dice which is known to be unloaded.

The analogy which we have proposed in MP was completely developed in the epistemological foundations of probability. The inherited experiential knowledge at the basis of Edgeworth's theory of probability, has the same nature of the inherited memory of past experience of pleasure at the basis of the *pleasure machine* of NOME and MP (Baccini 2007). As for ethics and economics, also for theory of probability this point is topical. Without this kind of experience, it is impossible to construct a notion of probability utilizable for statistical inference.

In fact, in Edgeworth's theory there are two kinds of primitive probabilities: probabilities derived by statistical knowledge, and a priori probabilities derived from antenatal knowledge. Complex probabilities are the result of the application of the calculus to primitive probabilities. And the existence of a priori probabilities permits to construct the inverse probability that is, crudely speaking, the art of inferring from observed events the probability of their causes. The inverse probability is 'the most perfect type of probability' which presents 'the two aspects: proportion of favourable cases given *a priori* and frequency of occurrence observed *a posteriori*." (Edgeworth 1911: 377). Its principal instrument is Bayes's theorem which requires for its practical application the definition of prior probabilities.

Venn had banned inverse probability from the frequentist approach. He denied that it is possible to define by experience a priori probability and then excluded Bayes's theorem and inverse probability from his theory (Dale 1991: 324-326). In his approach, a priori probability can be defined only as an epistemic or subjective measure, depending on the status of non experiential knowledge of the observer; for this posterior probability –the probability calculated by the Bayes's theorem- also looses its objective characterization. Instead, the enlarged notion of experience allowed Edgeworth to define experientially primitive a priori probability and then to accept Bayes theorem and inverse probability in a frequentist framework. According to Edgeworth, the derivation of the posterior distribution is simply an exercise in inductive inference; the prior distribution being objective, the posterior probability has the same objective interpretation. It is possible, therefore and contrary to Venn's speculation, to develop a complete theory of statistical inference in a frequentist framework.

The relevance of the reasoning about the experiential nature of a priori probability is illustrated by the fact that Edgeworth (1887) dedicated the entire booklet *Metretike* to a deep discussion about probabilities involved in Bayes's theorem. The booklet has at first sight a strange construction, based as it is on variations on the analogy between the measurement of probability in probability theory and of utility in economics. But, in our pattern of reconstruction of his ideas, it does not sound so off-key. Edgeworth was searching for a common foundation of social sciences, the role of probability is central for this scope and *Metretike* represented the non technical synthesis of his works in this direction.

5. With his work on the foundations of probability theory, Edgeworth paved the way for his passage to the theory of statistics. In 1885 he read a series of four papers to the Cambridge Philosophical Society, to the international gathering to celebrate the jubilee of the Royal Statistical Society, and to two meetings of the British Association. The first one entitled 'Observation and Statistics' (Edgeworth 1885) summarized the papers of the previous years on the probabilistic foundation of statistics. The second one on 'Methods of Statistics' (Edgeworth 1885a) was methodological, presenting an exposition an application and interpretation of significance tests for the comparison of means. These two papers became the basic reference - in England and on the continent - for the theory and application of statistical techniques to social and economic data (Stigler 1999: 104) until the appearance of Arthur L. Bowley's *Elements of Statistics* (1901). The third paper 'On the methods of ascertaining variations in the rate of births, deaths and marriages' (Edgeworth 1885b) worked on two-way classifications and contained many ideas that anticipated modern analysis of variance. The fourth one on 'progressive means' (Edgeworth 1886) contained a discussion on the use of linear least squares for detrending time series, and comparing different series.

In this firework exhibition of theoretical ideas, probability theory still occupied the central place. And this is not an unimportant detail. According to Stigler (1978), all those papers contributed to change the intellectual climate at the end of Nineteenth Century by opening the way to the application of probability-based statistical methods in the social sciences.

To illustrate synthetically the point in reference to economics, it is useful to linger on the jubilee meeting of the Royal Statistical Society in 1885 where Alfred Marshall also presented a paper on 'The graphic methods of statistics' The distance between the methodological approach in Marshall's paper and Edgeworth's one is sideral (Baccini 2006). Marshall's paper contained a detailed description of the usefulness of systematically collecting time series data for the social scientist. The implicit model of data systematization was his 'red book', in which he collected lifelong facts and data with the only aim to suggest causal connection between phenomena. Broadly speaking Marshall retained that the usefulness of statistics was primarily heuristic: his red book could suggest new ideas and new connection between facts. Marshall was not interested in statistical inference or estimate; at least in two occasions he wrote of the method of least squares as vitiated by an assumption of symmetry that suggested to him to regard them 'as mathematical toys' (Whitaker 1996, II: 301, n. 634; III: 264-5, n. 977).

Edgeworth's approach was exclusively theoretical, with probability theory and the law of error playing the central role. In a nutshell he conceived two different uses of the law of error: (i) the direct one, according to which, under assigned conditions, we can infer the theoretical probability of a given means and standard deviations; and (ii) the inverse use, according to which we can infer the existence of a 'cause other than chance' by comparing the actual distribution with the counterfactual one represented by the normal law of error. According to Edgeworth, the task of the statistician is to study the techniques which permit a systematic comparison of means, and to evaluate whether differences in *figures* are also differences in *facts*. The statistician does not assume that the law of error is always fulfilled, because it is necessary to prove that 'what is true of games of chance is true' also for social and economic data. He develops tests of significance and of appropriate assumptions, and develops a mathematical methodology for analyzing variance and evaluating deviations in complex situations.

Marshall's major influence in economic disciplines contributed to prevent the diffusion of Edgeworth's methods in the application of statistics to economic data, probably retarding the development of econometrics. Edgeworth's influence was greater in statistics, where he 'oversaw the final break with the past in what has been called the modern revolution in statistics and played (with Karl Pearson and Galton) a vital role in producing that revolution' (Stigler 1999: 127).

6. Edgeworth studies did not stop in 1890 with his academic success, but it is impossible to summarize the hundreds of contributions of the following years on various specific topics in economics, statistics and methodology, flanked by hundreds of reviews of the major books on economics. In collecting his thirty years work in economics alone for the *Papers Relating to*

Political Economy, Edgeworth himself underlined the difficulty of 'palliating' the 'deficiency' of 'unity of design' in 'a collection of papers written at different times and for various destinations' (Edgeworth 1925: viii). To this end he classified his papers in seven groups: value and distribution, monopoly, money, international trade, taxation, mathematical economics and reviews. As we have already said, a complete reconstruction is still lacking in the literature, but it is not our scope to fill the gap of this dimension. Our aim is to suggest a general interpretation of Edgeworth's research path, and at this point, this is possible.

The interpretation proposed in this paper is that the shifting of attention from ethics to economics, to probability and finally to statistics, is the result of a unitary philosophical project: the search for a common foundation for the social sciences. This common foundation consisted of the identification of a unitary epistemological basis, which involved the possibility of utilising cognitive material of the same species, although of a different *genus*. The epistemological foundation of the social sciences permits the coexistence in Edgworthian scientific programme of the 'regularity of law' with the 'impartiality of chance'.

The impartiality of chance is the device which allowed Edgeworth to reduce complexity of human motives for action to the simplicity of the pleasure machine hypothesis. This hypothesis does not give any indication regarding the real behaviour of concrete persons in the individual cases (like frequentist probability for single case), but make it possible to grasp an average trend that is characteristic of the human species (like frequentist probability for the series). The representation of humans as pleasure machines permits to reduce the complexity of human nature and to handle with the mathematics the problems of the economical and the utilitarian calculus, i.e. to treat the human behaviour as if it was governed by the regularity of the law of mathematics.

The impartiality of chance incorporated in the law of error determines the emergence of the price equilibrium in a 'field of competition' in which there are many different agents with different real behaviours. And again, the impartiality of chance is the device which permits the construction of the hypotheses on which are based the practical and theoretical instruments of the statistical inference. Under these hypoteses, statistical inference is governed by the regularity of the law of the calculus.

The interpretation here proposed challenges the traditional stereotypes according to which Edgeworth was a crass utilitarian, and an ingenuous advocate of a rather primitive neoclassical economics. His plea for the use of mathematics, and his choice of deterministic models for the description of the economic behaviour, appear more innovative when the role of chance is considered.

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