A methodology for the study of multi-dimensional and longitudinal aspects of poverty and deprivation

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Abstract

(1) This paper goes beyond the conventional study of poverty based simply on the *poor/non-poor* dichotomy defined in relation to some chosen poverty line. Poverty is treated here as a matter of degree determined in terms of the individual's position in the income distribution. Comparing and contrasting the conventional and this alternative poverty measures illuminates differentials in the level and intensity of poverty among subgroups in the population. (2) By appropriately weighting non-monetary indicators of deprivation to reflect their dispersion and correlation, we construct quantitative indices of deprivation in its various dimensions, thus viewing non-monetary deprivation also as a matter of degree. The states of poverty and deprivation are thus seen as *fuzzy sets*, to which all members of the population belong but to varying degrees. (3) We explain and use the basic theorems of the union and intersection of fuzzy sets to demonstrate the potential of this approach for studying *monetary poverty and non-monetary deprivation in combination*. This allows an estimation of the incidence of what we term 'manifest deprivation' (the joint experience of both poverty and non-monetary deprivation), and 'latent deprivation' (the experience of either) by individuals in the population. (4) Furthermore, we demonstrate the potential of the same approach in studying *dynamically over time* changes in the situation of individuals in terms of income poverty, non-monetary deprivation, and the two forms of deprivations in combination, including the construction of seemingly conventional measures describing the movement of individuals into and out of poverty and the experience of persistent poverty and deprivation.

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1. Introduction

1.1 A multi-dimensional, longitudinal and comparative perspective

For understanding poverty and social exclusion, it is necessary to consider deprivation simultaneously in its *multiple dimensions* – low income as well as diverse non-monetary or life-style aspects of deprivation. Furthermore, these multiple aspects must be considered *longitudinally*, identifying the extent to which households and individuals are subject to persistent deprivation. This paper presents a methodology for *multi-dimensional* and *longitudinal analysis* of poverty and social exclusion in a multi-country *comparative* context. Going beyond the conventional study of poverty based simply on the poor/non-poor dichotomy defined in relation to some chosen poverty line, the approach treats *poverty as a matter of degree determined in terms of the individual's position in the income distribution*: the state of poverty is thus seen as a 'fuzzy set' to which all members of the population belong but to varying degrees. The approach presented here represents a continuation and further development of the work of Cerioli and Zani (1990), Cheli and Lemmi (1995), Cheli (1995), Betti and Verma (1999), Betti *et al.* (2004), Verma and Betti (2002), and others. Aspects of this methodology have been applied in the Eurostat official publication Second European Social Report (Eurostat 2002), to which the present authors contributed as members of a team of researchers (Betti and Verma, 2002).

<u>Multidimensional analysis</u>. Comparing and contrasting the conventional and this alternative poverty measures illuminates differentials in the level and intensity of poverty among subgroups in the population. More importantly, the same methodology permits the inclusion of other dimensions of deprivation: by appropriately weighting non-monetary indicators of deprivation to reflect their dispersion and correlation, we can construct indices of life-style deprivation in its various dimensions. Making use of the basic theorems of the union and intersection of fuzzy sets, the approach allows us to meaningfully combine income and the diverse non-income deprivation indices at the micro-level, to construct what we have termed 'latent' and 'manifest' indicators of deprivation; these indicators illuminate the extent to which purely monetary indicators are insufficient in themselves in capturing the prevalence of deprivation.

<u>Longitudinal and multidimensional analysis</u>. The paper aims to demonstrate the potential of this approach in studying persistence or otherwise of the state of income poverty over time, as well as alternative construction of seemingly conventional indices such as measures describing the movement of individuals into and out of poverty and the experience of persistent poverty. The same applies for non-monetary indicators of deprivation, and also to the combined monetary and non-monetary indicators. We can thus study a whole range of indicators of poverty and deprivation, from cross-sectional monetary poverty rates to multi-dimensional 'latent' and 'manifest' indicators of deprivation dynamically over time.

<u>Multi-country comparative perspective</u>. Patterns and relationships are better understood when they are seen in relation to each other across different countries and other population groupings. Moreover, the measures themselves can be designed and tailored to facilitate such comparisons. We aim to do this on the basis of highly comparable data available for a number of EU countries, namely the European Community Household Panel (ECHP) [2].

1.2 Poverty and deprivation as a matter of degree

Figure 1 illustrates the basic idea of treating poverty and deprivation as a matter of degree, replacing the conventional classification of the population into a simple "poor/non-poor" or "deprived/non-deprived" dichotomy. In principle all individuals in a population are subject poverty or deprivation, but to varying degrees. We say that each individual has a certain *propensity* to poverty or deprivation, the population covering the whole range [0,1]. The conventional approach is a special case of this, with the population dichotomised as {0,1}: in the conventional approach those with income below a certain threshold are deemed

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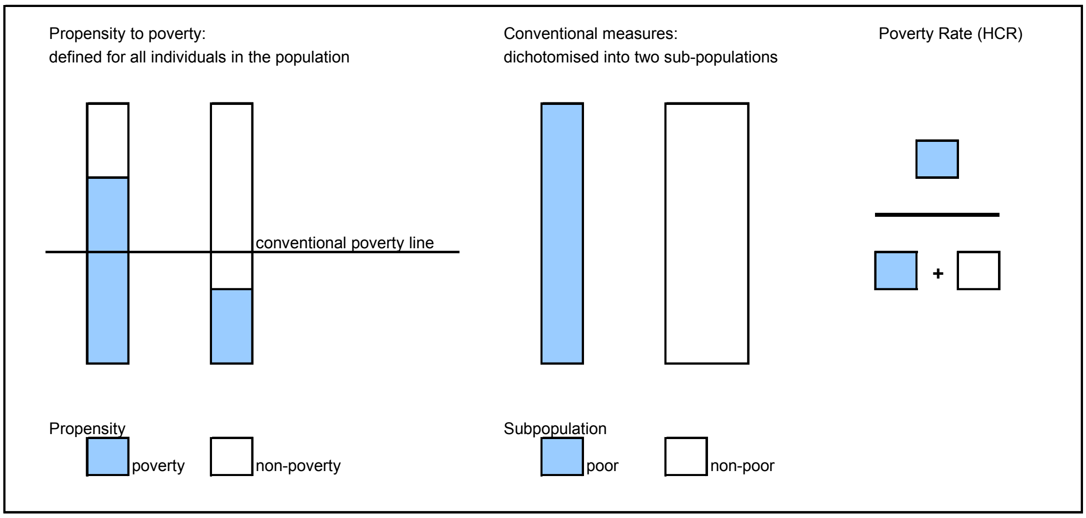
[2] See Annex for information on the data sources used for numerical illustrations, and some further details on the methodology.

to be poor (i.e. are all assigned a constant propensity=1); others with income at or above that threshold are deemed to be non-poor (i.e. are all assigned a constant propensity=0).

Figure 1. The basic idea of poverty or deprivation as a matter of degree:
comparison with the conventional "poor/non-poor" dichotomy



There are several advantages of treating poverty and deprivation as a matter of degree, applicable to all members of the population, rather than as simply a "yes-no" state.

1. Further insight into the relative income situations of individuals and groups can be obtained by incorporating into the poverty rates a measure of the actual levels of incomes received, particularly at the lower end of the income distribution. The same applies in the study of the relative position of demographic and other socio-economic subgroups in the population (children, old persons, minorities, etc.).

2. Life-style or non-monetary deprivation (in this paper we will use these two terms interchangeably) depends on forced non-access to various facilities or possessions determining the basic conditions of life. An individual may have access to some but not to others. Hence life-style deprivation is inherently a matter of degree, and some *quantitative approach* such as the present one is essential.

3. The combined analysis, considering income poverty and life-style deprivation simultaneously, is greatly facilitated by treating each dimension as a matter of degree. The need to divide the population into numerous discrete groups - as would normally be required in the conventional analysis, especially in the longitudinal context - is avoided.

4. Equally important is the potential of this approach in studying poverty in the longitudinal context. To what extent do individuals and households move in and out of poverty from one period to another? The conventional measure traces this as a count of movements across some chosen poverty line. This measures mobility simply in terms of movements across some designated poverty line, and does not reflect the actual magnitude of the changes affecting individuals at all points in the distribution. Consequently, the degree of mobility of persons near to the chosen poverty line tends to be over-emphasised, while that of persons far from that line largely ignored. The same applies in relation to non-monetary and multi-dimensional measures of deprivation.

5. We can expect the resulting measures to be more precise. The sampling error of a distribution is lower than that of a dichotomy with values concentrated at the two end points. We can also expect the measures to be less sensitive to local irregularities in the income distribution curve, and to the particular choice of the poverty threshold.

1.3 Scope of this paper

The concern of this paper is primarily *methodological*, rather than detailed numerical results from specific applications. Nevertheless, some illustrative results based on real, nationally representative and comparable data from EU countries will be presented.

In order to illustrate the richness of this approach, we analyse *five types of measures of poverty and deprivation* in relation to each other: (1) income poverty as conventionally viewed in the form of a poor/non-poor dichotomy; (2) poverty viewed as a propensity to which all individuals are subject to a greater or lesser degree; (3) life-style deprivation in its various dimensions ('domains') determined by the lack of access to non-monetary facilities and opportunities; and two measures of income poverty and life-style deprivation in combination – (4) 'latent deprivation' representing the presence of either dimension, and (5) 'manifest deprivation' representing the situation of individuals subject to both simultaneously.

Then we analyse each of these measures in *five aspects in the time dimensions:* (1) cross-sectional measures (including their averaging over time); (2) the incidence of poverty and deprivation at any time during the interval; (3) the persistence and continuity of the state of poverty/deprivation over time; (4) the dynamic aspects of movements into and out of poverty and deprivation; and (5) the duration of the time spent in that state by individuals in the population.

2. Income poverty

2.1 Conventional income poverty measures

Diverse 'conventional' measures of monetary poverty and inequality are well-known and need not be discussed here. For the most part, we will consider only the most commonly used indicator, namely the proportion of a population classified as 'poor' in purely relative terms on the following lines. To dichotomise the population into the "poor" and the "non-poor" groups, each person j is assigned the equivalised income y_j of the person's household. Persons with equivalised income below a certain threshold or poverty line (say 60% of the median equivalised income) are considered to be poor (assigned a poverty index H_j=1), and the others as non-poor (assigned a poverty index H_j=0)). The conventional income poverty rate (the so-called Head Count Ratio, HCR) is the population average of this poverty index, approximately weighted by sample weights (w_j):

$$\overline{H} = \frac{\Sigma_j w_j . H_j}{\Sigma_j w_j}.$$

The actual computation of even this, seemingly straightforward measure, requires a number of complex decisions and choices, such as: (1) the concept and operational definition of what constitutes 'income'(gross or net disposable, before or after certain transfers, excluding or including certain receipts, etc.); (2) its reference period (past year, month or current), unit (household, person, or some other intra-household grouping), and scale (currency, PPS, etc.) of measurement, and also the data sources and methodology of measurement; (3) unit of analysis (household or person); (4) choice of the reference population (international, national, subnational or other groupings), and threshold (x% of the population mean or median, etc.) for defining the income distribution and poverty line. Furthermore, (5) the income amount as measured needs to be appropriately converted ('equivalised') so as to obtain a meaningful analysis variable which takes into account differences, for instance, in household size and composition.

In the empirical results presented here, we have used the European Community Household Panel data, making analytical choices which are in line with most analyses hitherto performed by Eurostat and other researchers using these data. For some specific details, please see Annex.

Table 1 shows the conventional poverty rates for EU-15 countries (before the recent EU expansion). There are big differences among the countries: from a rate of 10% in Finland, to over 20% in Portugal. Levels of national median incomes are also shown for reference, as, at least for the set of countries considered, there is generally a negative relationship between income level and the relative poverty rate.

With the household's equivalised income ascribed to each of its members, persons with equivalised income below a certain percentage of the national median have been classified as poor. This is done for each ECHP wave separately, and the results shown are averaged over the seven waves so as to gain sampling precision. Furthermore, a weighted average of the rates corresponding to three different poverty thresholds (50%, 60% and 70% of the national median) is taken so as to reduce their sensitivity to a particular choice of the threshold. The weights were determined on the basis of the following considerations.

- Lower thresholds (such as 50% of the median) are more distinguishing among countries, but the HCR values corresponding to them are numerically lower. It is therefore desirable to increase the weight given to them.
- At the EU level the available sample sizes are very large, and therefore averaging across thresholds is hardly necessary. It is desirable to define the measure such that for EU-15 as a whole it *exactly coincides* with the commonly used threshold of 60% of the median.

Hence the averaged head-count ratio H_C for a particular country (C) is taken as

$$H_C = H_{C,50}w_{EU,50} + H_{C,60}w_{EU,60} + H_{C,70}w_{EU,70},$$

with the weights determined by the head-count ratios for EU-15 with the three thresholds:

$$w_{EU,50} = \frac{1}{3}.\left(\frac{H_{EU,60}}{H_{EU,50}}\right), \qquad w_{EU,60} = \frac{1}{3}, \qquad w_{EU,70} = \frac{1}{3}.\left(\frac{H_{EU,60}}{H_{EU,70}}\right).$$

Table 1
Conventional (HCR), Fuzzy Monetary (FM) and Fuzzy Supplementary (FS) rates

	median income*		HCR	FM	FM/HCR	FS	FS/HCR
EU-15	EU	100	16,1	16,1	1,00	16,1	1,00
Finland	FI	94	9,5	9,4	0,98	10,5	1,10
Denmark	DK	125	10,2	9,1	0,89	10,8	1,06
Sweden	SE	(76)	10,3	10,0	0,97	11,9	1,15
Netherlands	NL	106	11,1	11,2	1,01	11,5	1,03
Luxembourg	LU	175	12,0	10,8	0,91	13,0	1,09
Austria	AT	115	12,7	11,9	0,94	13,3	1,05
Germany	DE	114	12,7	12,7	1,00	14,1	1,11
Belgium	BE	117	14,5	14,0	0,96	15,2	1,05
France	FR	106	15,2	14,8	0,98	15,5	1,02
Ireland	IE	90	17,5	16,6	0,95	17,8	1,02
Spain	ES	72	18,8	19,6	1,04	17,7	0,94
UK	UK	106	18,8	18,6	0,99	18,7	0,99
Italy	IT	85	19,5	19,7	1,01	17,3	0,89
Portugal	PT	59	21,5	22,5	1,04	23,4	1,08
Greece	GR	65	21,6	22,2	1,03	21,3	0,99

Results 'consolidated' over three poverty thresholds, 50%, 60% and 70% of the national mediar equivalised income; statistics are averaged over waves 1-7of ECHP. FM and FS have been benchmarked to eaxaclty equal HCR at EU-15 level.
*relative to EU equivalised median income, averaged over ECHP waves 1-7 (11,897 PPS)
( ) due to the use of entirely different type of data source, this figure for SE lack comparability.

2.2 The propensity to income poverty

Apart from the various methodological choices involved in the construction of conventional poverty measures, as noted above, the introduction of fuzzy measures brings in *additional* factors on which choices have to be made. These concern at least two aspects:

- Choice of 'membership functions', such as quantitative specification of the propensity to poverty of each person given the level and distribution of income of the population.
- Choice of 'rules' for manipulation of the resulting fuzzy sets, specifically the rules defining complements, intersections and union of the sets.

Both these choices must meet some basic logical and substantive requirements to be meaningful. It is also desirable that they be useful in the sense of elucidating aspects of the situation not captured (or not captured as well) by the conventional approach.

The issue of choice in relation to the membership function is obvious. However, it may appear surprising that such choices can also exist in relation to the rules defining operations on fuzzy sets. While these rules cannot be discussed fully in this paper, we will need to clarify their application in the study of poverty and deprivation in a later section.

As to the choice of the membership function we have proceeded as follows, on the lines of Cheli (1995) and Betti and Verma (1999). The propensity to income poverty, defined as Fuzzy Monetary (FM) associated with each individual j is related to the person's rank and share in the equivalised income distribution within each country. First we construct an income index:

$$V_j = \sum_{i=j+1}^{n} v_i \text{ , j = 1 to n-1, } V_n = 0 \text{, with } v_i = y_i \Big/ \sum_{i=2}^{n} y_i \text{ ,}$$

v_i being the share of total equivalised income (y_i) received by individual of rank i in the ascending income distribution. V_j varies from $V_1 = 1$ for the poorest, to $V_n = 0$ for the richest individual.[3] It is the share of the total equivalised income received by all individuals less poor than the person concerned. Corresponding to the income index, the propensity to income poverty is defined as:

$$\mathrm{FM_j} = \left(\mathrm{V_j}\right)^{\alpha/\mathrm{H_C}} .$$

where H_C is the conventional poverty rate for the country (C) concerned, as defined in the previous section. Apart from the choice of the functional form, the fuzzy measures need to be bench-marked against some standard. Parameter α should be determined so as to make the mean (FM_C) of the resulting FM_j values the same as (or close to) H_c, in order to guarantee comparability with the conventional approach. Empirically, large values of FM_j would then be concentrated at the lower end of the income distribution, making the propensity to income poverty sensitive to the share of the income received by poorer sections of the population.

We have considered two options in determining parameter α. One is to determine a different α_C value separately for each country (C) such that FM_C exactly equals H_C for the country concerned. In practice, it turns out that - at least among the EU-15 countries considered - the resulting values of α_C are very close (within ± 8%), notwithstanding the large differences in the country poverty rates (see Table 4). This suggests

[3] In order to take into account possible negative incomes, tied rankings (e.g., individuals in the same household sharing the same income) and weighting, the correct formula to be used is the following:

$$\mathrm{V_j} = \frac{\sum_i \mathrm{w_i}\left(\mathrm{y_i} - \mathrm{y_1}\right) | \, \mathrm{i} \in \mathrm{K} : \mathrm{y_K} > \mathrm{y_i}}{\sum_i \mathrm{w_i}\left(\mathrm{y_i} - \mathrm{y_1}\right) | \, \mathrm{i} \in \mathrm{K} : \mathrm{y_K} > \mathrm{y_1}}$$

the alternative of obtaining a single, more robust value of α by pooling together data from all the countries being analysed. This benchmarks the fuzzy poverty rate to be identical to the conventional rate for *the group of countries as a whole*. However, this is only approximately so for any individual country (see Table 1). Nevertheless this pooled approach has the advantage that only a single parameter has to be estimated, which can therefore be done more reliably. This feature can be an important advantage when the same procedure is used for fuzzy supplementary (FS) measures (discussed below), for which very limited data may be available in some countries.

To summarise, we have computed numerical results with two options (the last mentioned above first):

- Option 1: matching the fuzzy with conventional poverty rates only at EU-15 level by estimating a single value of parameter α on the basis of pooled data.[4]

- Option 2: matching the fuzzy with conventional poverty rate at the country level by estimating a separate α_C value for each country.

In either case, comparison of the conventional and fuzzy poverty rate can provide additional insights in the analysis of population subgroups. For instance, for two groups with the same H_C, a higher value of FM_C in one would indicate in it a more acute poverty situation (lower income levels) compared to the other subgroup – a difference which is not captured by the conventional measure. Similarly, the mean value of FM_j among the bottom FM_C% of the population (which has a maximum value of 1.0) would indicate the extent to which the given total 'quantum of poverty' in the population is concentrated among the poorest.

Table 1 has been computed with option (1). As can be seen from the table, the ratio (FM_C/H_C) across countries is quite stable, except that it tends to increase a little with increasing H_C.[5]

3. Life-style deprivation

3.1 Variables and dimensions of life-style deprivation

In addition to the level of monetary income, the standard of living of households and persons can be described by a host of indicators, such as housing conditions, possession of durable goods, the general financial situation, perception of hardship, expectations, norms and values. Quantification and putting together of a large set of non-monetary indicators of living conditions involves a number of steps, models and assumptions.

Firstly, from the large set which may be available, a selection has to be made of indicators which are most meaningful and useful. For our analysis using European Community Household Panel data, a large subset of the available indicators were selected. The most important determining factor in the choice of the set of items for analysis was an assessment – based on a detailed examination of variations in frequency distributions across countries and background knowledge of national situations – of the extent to which an item could be meaningfully included in *comparative analysis*. Generally, the preference has been to include a majority of so-called 'objective' indicators on life-style deprivation, such as the possession of material goods and facilities and physical conditions of life, at the expense of what may be called 'subjective' indicators such as self-assessment of the general health condition, economic hardship and social isolation, or the expressed degree of satisfaction with various aspects of work and life. These latter type of indicators tend to be more culture-specific and hence less comparable across countries and regions.

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[4] Note that when data are so pooled, variables v_j and V_j must still be defined separately with each country, since it is the position of each person within the *national income distribution* which is of interest. Also, the national conventional head count ratio H_C is included in the equation for FM_j as a major determining factor. It is this factor which makes options (1) and (2) rather similar in terms of the results obtained. Incidentally, with our data the common value found was α=2.26, which happens to be close to ln(10).

[5] Tables 1-3 have been computed with option (1). These use full cross-sectional data from first 7 ECHP waves. The remaining tables use option (2) and are based on longitudinally linked data ('balanced panel') for only the first 5 waves. See Annex for further details.

Secondly, it is useful to identify the underlying dimensions and group the indicators accordingly. Taking into account the manner in which different indicators cluster together (possibly differently in different national situations) adds to the richness of the analysis; ignoring such dimensionality can in fact result in misleading conclusions. Using confirmatory factor analysis various dimensions of non-monetary or life-style deprivation were identified (Whelan *et al.*, 2001). The indicators used and their grouping is summarised in Figure 2.

Figure 2. Dimensions and items of deprivation

1 Basic life-style deprivation – these concern the lack of ability to afford most basic requirements:

Keeping the home (household's principal accommodation) adequately warm.
Paying for a week's annual holiday away from home.
Replacing any worn-out furniture.
Buying new, rather than second hand clothes.
Eating meat chicken or fish every second day, if the household wanted to.
Having friends or family for a drink or meal at least once a month.
Inability to meet payment of scheduled mortgage payments, utility bills or hire purchase instalments.

2 Secondary life-style deprivation – these concern enforced lack of widely desired possessions ("enforced" means that the lack of possession is because of lack of resources):

A car or van.
A colour TV.
A video recorder.
A micro wave.
A dishwasher.
A telephone.

3 Housing facilities – these concern the absence of basic housing facilities (so basic that one can presume all households would wish to have them):

A bath or shower.
An indoor flushing toilet.
Hot running water.

4 Housing deterioration – these concern serious problems with accommodation:

Leaky roof.
Damp walls, floors, foundation etc.
Rot in window frames or floors.

5 Environmental problems – these concern problems with the neighbourhood and the environment:

Shortage of space.
Noise from neighbours or outside.
Too dark/not enough light.
Pollution, grime or other environmental problems caused by traffic or industry.
Vandalism or crime in the area.

3.2 Constructing indicators of life-style deprivation

Putting together of categorical indicators of deprivation for individual items to construct composite indices requires decisions about assigning numerical values to the ordered categories and the weighting and scaling of the measures.

Assigning numerical values to deprivation scores

Most of the items under consideration are in fact simple 'yes/no' dichotomies. An obvious choice is to assign a value of (say) 1 to the possession and 0 to the absence of a particular item or facility. In principle, some such items may involve more than two ordered categories. In the same way as for dichotomies, equally spaced values in the range [0,1] can be assigned to an ordered polytomy. Let us consider a set of K non-monetary indicators - housing conditions, possession of durable goods, the general financial situation, etc.

For each k of the K indicators, we define the following indicator for each of the M_k ordered categories of the variable, with m=1 the most deprived to m=M_k the least deprived, m being the category of variable k to which unit j belongs:

$$s_{j,k(m)} = \frac{m-1}{M_k - 1}.$$

Note that s_j has been defined such that, just like v_j for income, a higher value reflects a more privileged situation. As noted, in most of the cases our variables are simple dichotomies, and therefore the indicator $s_{j,k}$ is equal to zero (for the most deprived) or equal to one (for the least deprived).

Composite indicator of overall non-monetary or life-style deprivation

It is simpler to describe first the construction of a single indicator reflecting the individual's overall non-monetary or life-style deprivation, before considering the construction of dimension-specific indicators.

For this purpose, individual indicators are combined to form an index describing an overall degree of deprivation. The individual's score averaged over items (k) is written as the weighted mean:

$$S_j = \Sigma_k \left(w_k . s_{j,k}\right) / \Sigma_k w_k$$

Note that the set of weights w_k are taken to be item-specific only; for a given item they are common to all individuals (j) in the country. For each item and each country, the weights are in fact determined wave by wave and then averaged over waves. The weighting procedure is based on the following statistical considerations taking into account how the items are distributed in the population[6].

Firstly, the weight is determined by the variable's power to 'discriminate' among individuals in the population, that is, by its dispersion. We take this as proportional to the coefficient of variation of deprivation score p=(1-s_{jk}) for the variable concerned. This means that for small proportions, the weight varies inversely to the square-root of the proportion (p). Thus deprivations which affect only a small proportion of the population, and hence are likely to be considered more critical, get larger weights; while those affecting large proportions, hence likely to be regarded less critical, get smaller weights. Note, however, that the contribution of these p individual values to the average level of deprivation in the population resulting from the item concerned turns out to be directly proportional to the square-root of the p. In other words, deprivations affecting a smaller proportion of the population are treated as more intense at the individual person's level but, of course, their contribution to the average level of deprivation in the population as a whole is correspondingly smaller.

Secondly, from a non-redundant point of view, it is necessary to limit the influence of those characteristics that are highly correlated with the others included in the analysis. Even for the overall index, it is reasonable to consider this correlation separately within each of the five dimensions of deprivation identified earlier, i.e., the weight of variable k in deprivation dimension d is taken as the inverse of an average measure of its correlation with all the other variables in that dimension. Thus the results are not affected by arbitrary inclusion or exclusion of items highly correlated with other items in the set.

To surmise, the weight given to an item is directly proportional to the variability of the item in the population and inversely proportional to its correlation with other items in the deprivation dimension to which it belongs. Full description of the calculation of weights is reported in Betti and Verma (1999).

The quantities S_j need to be converted and bench-marked to obtain the "FS_j" measures of non-monetary deprivation. Here we have studied two options which – in contrast to the two options discussed above in relation to the FM measures – do seem to have rather significant implications for the type of FS measures obtained.

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[6] The scaling of the weights can be arbitrary, though scaling them to sum to 1.0 may be convenient.

- Option 1: In this option, the propensity to non-monetary poverty, i.e., Fuzzy Supplementary (FS) deprivation associated with each individual *j* is constructed in the same manner as the propensity to income poverty (FM). This, as in the case of monetary poverty measures H_C or FM, provides a *purely relative* measure of non-monetary deprivation within each country. The resulting measure is closely tied to the country-specific monetary poverty measures.

- Option 2: In this option, the quantities S_j are used directly (i.e., without the exponential transformation involving α and the link to the national poverty rate H_C) to obtain the FS_j measures of non-monetary deprivation. As will seen below, this results in a partly relative measure – a measure of deprivation reflecting a *mixture* of purely relative aspects in terms of the distribution within any country, and of absolute differences among countries in terms of the prevailing average living standards.

Hence we have two measures of non-monetary deprivation, reflecting somewhat different aspects of the situation and each providing additional useful information. Their construction is described below in turn.

3.3 Non-monetary deprivation: a purely relative measure

Overall deprivation measure

As in the Fuzzy Monetary approach, we define the individual's propensity to non-monetary deprivation as the sum of the shares of (S_j) received by individual of rank *j* in the ascending "supplementary" distribution at the country level:

$$V_{(FS)j} = \frac{\sum_i w_i S_i \mid i \in K : S_K > S_j}{\sum_i w_i S_i \mid i \in K : S_K > S_1}.$$

Corresponding to income poverty, the propensity to non-monetary deprivation is defined as:

$$FS_j = \left(V_{(FS)j}\right)^{\alpha/H_C}.$$

As above in the case of FM_j, by matching the FS rate with the monetary poverty rate only at EU-15 level, we can estimate a single value of parameter α on the basis of pooled data.[7] In other words (whether conventional or FM, which are identical in Option 1 of the previous Section 3.2), we have determined parameters α such that for the *EU-15 population as a whole* the mean (FS_C) of the index FS_j is equal to the proportion poor HCR (H_{EC}). The resulting measures are strongly influenced by the national monetary poverty rate H_C, this making them relative measures of the within-country situation.[8]

The last two columns of Table 1 show the estimated Fuzzy Supplementary (overall non-monetary deprivation) index by country, and its ratios to the conventional HCR. The values of the ratio are close to 1.0, confirming that as constructed the index is a more or less purely *relative deprivation* index, just as HCR. There is only a very slight tendency for the ratio to be a little lower at the bottom of the table, i.e. among poorer, less equal countries – opposite to the slight tendency noted earlier in relation to FM.

[7] As before, when data are so pooled, variables V_j and FS_j must still be defined separately with each country, since it is the position of each person within the *national distribution* which is of interest.

[8] Exactly as before, H_C, has been taken as a weighted average of national poverty rates at the three thresholds (50%, 60% and 70% of the national median equivalised income). Incidentally, with our data the value found was α=0.93 (compared with α=2.26 for FM, noted earlier). This is the value averaged over ECHP waves 1-7. The computation has to be wave by wave, though the value is generally stable across waves.

Composite indicators for underlying dimensions

Next, individual indicators within each major dimension (such as housing, environment, etc.) can be combined to form an index describing the degree of deprivation *specific to the dimension concerned*. The individual's score averaged over items in the dimension (d) is written as the weighted mean:

$$S_{d,j} = \Sigma_{k \in d} \left(w_{k.} . s_{j,k} \right) / \Sigma_{k \in d} w_{k.} \ ,$$

where the weights w_k are the same weights used for the overall score.

As in the case of the overall FS index, we can define the individual's propensity to deprivation in a particular dimension (d) as the sum of the shares of ($S_{d,j}$) received by individual of rank *j* in the ascending "non-monetary dimension (d)" distribution within each country:

$$V_{(FS)d,j} = \frac{\sum_i w_i S_{d,i} \mid i \in K : S_{d,K} > S_{d,j}}{\sum_i w_i S_{d,i} \mid i \in K : S_{d,K} > S_{d,1}} .$$

Corresponding to the overall non monetary propensity, the propensity to dimension (d) deprivation is defined as:

$$FS_{(d)j} = \left(V_{(FS)d,j} \right)^{\alpha / H_C}$$

where α and H_C are taken as the values estimated for FS analysis above.

As seen from Table 2, among the poorer, generally less equal countries, dimension-specific deprivation tends to be more pronounced than is the case when only the overall deprivation index (FS) is considered. Table 3 shows results for some indicators of monetary and non-monetary deprivation in combinations, constructed using Option 1. These will be described and commented upon later where the results using Option 2 are also presented.

Table 2
Dimension-specific deprivation rates

		FS	Dim. 1	Dim. 2	Dim. 3	Dim. 4	Dim. 5
EU-15	EU*	16	20	17	16	20	21
Finland	FI	10	15	11	11	12	15
Denmark	DK	11	12	11	10	13	14
Sweden	SE						
Netherlands	NL	11	13	11	11	15	15
Luxembourg	LU						
Austria	AT	13	15	14	13	16	16
Germany	DE						
Belgium	BE	15	17	15	15	19	20
France	FR	15	19	16	15	20	21
Ireland	IE	18	20	18	17	20	21
Spain	ES	18	21	19	18	24	26
UK	UK	19	21	20	17	21	23
Italy	IT	17	24	20	18	22	26
Portugal	PT	23	24	28	28	35	30
Greece	GR	21	36	24	23	27	26

*simple average over available countries

Dim. 1 Basic life-style deprivation
Dim. 2 Secondary life-style deprivation
Dim. 3 Housing facilities
Dim. 4 Housing deterioration
Dim. 5 Environmental problems

Table 3
Latent and Manifest deprivation rates*

		mean(FM,FS)	Lat	Man	Lat/mean	Man/mean	Man/Lat
EU	EU	16.1	25.9	6.4	1.60	0.40	0.25
Finland	FI	9.9	16.7	3.1	1.68	0.32	0.19
Denmark	DK	10.0	17.3	2.7	1.73	0.27	0.15
Sweden	SE	10.9	18.5	3.3	1.69	0.31	0.18
Netherlands	NL	11.3	18.2	4.5	1.61	0.39	0.24
Luxembourg	LU	11.9	19.7	4.2	1.65	0.35	0.21
Austria	AT	12.6	21.1	4.1	1.67	0.33	0.19
Germany	DE	13.4	22.8	4.0	1.70	0.30	0.18
Belgium	BE	14.6	23.6	5.6	1.62	0.38	0.24
France	FR	15.1	23.8	6.5	1.57	0.43	0.27
Ireland	IE	17.2	26.2	8.2	1.52	0.48	0.31
Spain	ES	18.6	29.4	7.8	1.58	0.42	0.27
UK	UK	18.6	29.2	8.0	1.57	0.43	0.27
Italy	IT	18.5	29.4	7.7	1.59	0.41	0.26
Portugal	PT	22.9	35.2	10.7	1.54	0.46	0.30
Greece	GR	21.8	33.4	10.1	1.53	0.47	0.30

*Using purely relative measures of deprivation ('Option 1')

3.4 Non-monetary deprivation: a partly relative measure

We start with the weighted mean of an individual's score averaged over k items expressing access to various items or facilities.

$$S_j = \Sigma_k \left(w_k . s_{j,k}\right) / \Sigma_k \, w_k$$

where the sum is over all items (for the overall score) or over items in a particular dimension. The overall degree of deprivation is then defined as:

$$V_{(FS)j} = 1 - S_j$$

and the propensity to non-monetary deprivation as:

$$FS_j = U.V_{(FS)j},$$

where the scaling factor U is determined on the basis of the following considerations.

For individuals in the population, the overall deprivation index $V_{(FS),j}$ as defined above varies in the range [0,1]. However, with this procedure an index=1.0 is obtained only if the individual lacks *all* the items/facilities included in the analysis. These conditions are too restrictive in defining what should be considered as the 'most deprived' situation (it would be even more restrictive if the number of items included in the analysis is increased). It is more reasonable to define as 'the most deprived' persons who lack at least a certain proportion U<1 of the items considered.

The exact value of U used in our analysis has been determined on the following basis: it is chosen to scale the overall non-monetary deprivation index such that its simple average over EU-15 countries exactly equals the same average for the income poverty rate. This gives a value of U very close to 0.6. In our view, something like U=0.6 is a reasonable choice, meaning that individuals lacking 60 percent (i.e. 15 of the 24 items in our analysis) or more items are considered to be 'most deprived'. This single scaling factor is then used throughout our analysis, for all countries and waves. Note that for any particular country, the average non-monetary deprivation indicator is not scaled to exactly match the country income poverty rate. It should be emphasized that the particular choice of the value of parameter U is of absolutely no consequence for the resulting patterns of variation across items, dimensions, countries or population subgroups discussed in the following sections. It does, however, affect the numerical results when we contrast and combine monetary and non-monetary indices for analysing overall deprivation in all its aspects, as discussed in the next section.

On the basis of wave-specific results (not shown here for reasons of space), in almost all countries we see a strong downward trend in the levels of non-monetary deprivation. This is expected in view of generally increasing levels of income, though the figures from a fixed panel of individuals are likely to over-estimate this trend because of the life-cycle effect.

The main point brought out in Table 4 is that differentials among countries are generally larger in non-monetary deprivation than in income poverty. We believe this is related to the negative correlation between levels and disparities in income among the EU countries, noted earlier. Across countries, the average non-monetary deprivation increases with increasing levels of income poverty and declines with the increasing level of income in the country. In so far as less well-off countries in the EU also tend to be subject to greater inequality of income, the non-monetary deprivation index shows a greater range of variation among the countries, with particularly large values for Portugal and Greece.

In fact, this deprivation index reflects both the relative and absolute dimensions of the levels of living. It is an 'absolute' measure in the sense that it reflects the actual lack of various possessions and facilities to which individuals and households are subject. However, the significance (in statistical terms, the weight) given to the lack of a particular item is determined in the 'relative' context of the level and distribution of the lack in the national population of which the individual forms a part. This is in contrast to the income poverty rate (or the FS measure constructed using 'Option 1') which reflects only the relative distributional aspects.

On the basis of wave-specific results (not shown here for reasons of space), in almost all countries we see a strong downward trend in the levels of non-monetary deprivation as defined in this section. This is expected in view of generally increasing levels of income, though the figures from a fixed panel of individuals are likely to over-estimate this trend because of the life-cycle effect.

Table 4
Cross-sectional poverty/deprivation rates

		Monetary poverty		Non-monetary	Latent	Manifest	ratios			
		H or FM	parameter α	deprivation (FS)	deprivation	deprivation	FS/FM	Lat/mean	Man/mean	Man/Lat
Denmark	DK	8,4	0,86	8,9	15,0	2,3	1,06	1,79	0,27	0,15
Netherlands	NL	10,2	1,08	9,2	15,9	3,5	0,90	1,56	0,34	0,22
Germany	DE	12,8	1,02							
Belgium	BE	17,5	1,06	12,9	24,1	6,3	0,74	1,38	0,36	0,26
France	FR	16,9	0,93	15,0	24,4	7,6	0,89	1,44	0,45	0,31
Ireland	IE	19,2	0,92	16,0	26,2	9,0	0,83	1,37	0,47	0,34
Spain	ES	19,5	1,10	19,7	30,0	9,2	1,01	1,54	0,47	0,31
UK	UK	19,5	0,96							
Italy	IT	19,6	1,05	15,2	26,9	7,9	0,77	1,37	0,40	0,29
Portugal	PT	23,4	1,08	31,6	40,7	14,4	1,35	1,74	0,61	0,35
Greece	GR	21,5	1,05	25,6	35,7	11,4	1,19	1,66	0,53	0,32
simple average		17,1	1,01	17,1	26,5	7,9	1,00	1,55	0,46	0,30

All results from Table 4 onwards are based on longitudinally matched (balanced panel) for ECHP waves 1-5.
The above applies also to the cross-sectional rates presented here.
FM is defined using option 2 of Section 3.2, and FS using option 2 of Section 3.4.

4. Income poverty and non-monetary deprivation in combination

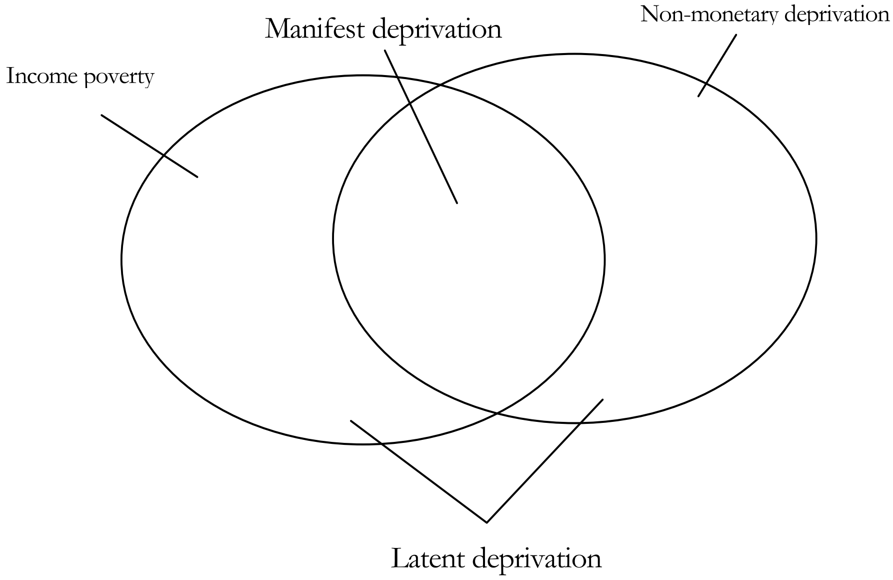
The two measures FM_j propensity to income poverty, and FS_j the overall life–style deprivation propensity, may be combined to construct composite measures which indicate the extent to which the two aspects of income poverty and life-style deprivation overlap for the individual concerned. These measures are as follows.

M_j manifest deprivation, representing the propensity to both income poverty and life-style deprivation simultaneously. It represents the individual being subject to both income poverty and life-style deprivation; one may think of this as the '**m**anifest' or the '**m**ore intense' degree of deprivation.

L_j latent deprivation, representing the individual being subject to at least one of the two, income poverty and/or life-style deprivation; one may think of this as the '**l**atent' or the '**l**ess intense' degree of deprivation.

The two concepts can be seen graphically in Figure 3, where Manifest deprivation can be seen as the intersection of the two fuzzy sets (Income poverty and Non-monetary deprivation), while latent deprivation can be seen as the union of the two.

Figure 3: Latent and manifest deprivation.



4.1 Intersection, union and complement in fuzzy sets theory

The study of the combine effect of income poverty and non-monetary deprivation requires operations on the fuzzy sets describing the individual memberships (FM_j, FS_j), i.e., respectively the propensities to be in monetary poverty and non-monetary deprivation. The basic set operation of interest are complements, intersections and unions of sets. Some explanation and clarification of this operations is required when dealing with fuzzy sets. Fuzzy set operations are a generalisation of the corresponding 'crisp' set operations in the sense that the former reduce to (exactly reproduce) the latter when the fuzzy membership functions, being in the whole range [0,1], are reduced to a {0,1} dichotomy.

Let us use a simplified notation: $a=FM_j$, $b=FS_j$, for the membership functions of the two sets of individual j (subscript j can be dropped when not essential); also $m_1=\min(a,b)$ and $m_2=\max(a,b)$ for the smaller and the larger of the two values. We also denote by c(..), i(..) and u(..) the basic set operations of complementation, intersection and union, respectively. To be meaningful, consistent and useful, these operations must satisfy some essential, and some additional desirable requirements, briefly as follows (Klir and Yuan, 1995)[9]:

Fuzzy complement c(..):
1. reduction to the crisp set operation $c(a)=1-a$ with dichotomous membership {0,1};
2. boundary condition, $c(0)=1$, $c(1)=0$;
3. monotonicity, if $a' \leq a$ then $c(a') \geq c(a)$;
4. $c(a)$ is continuous and involutive, $c(c(a))=a$.

Fuzzy intersection i(..):
1. reduction to the crisp set operation with dichotomous membership {0,1};

[9] We have used this excellent text for substance as well as notation extensively in this section.

2. boundary condition, $i(a,1)=a$, $i(a,0)=0$;
3. monotonicity, if $a'\leq a$ then $i(a',b)\leq i(a,b)$; if $a'<a$ and $b'<b$, then $i(a',b')<i(a,b)$;
4. cumutativity, $i(a,b)=i(b,a)$; associativity, $i(a,i(b,c))=i(i(a,b),c)$; and continuity;
5A. $i(a,a)=a$ (idempotency), <u>or</u> 5B. $i(a,a)<a$ (*sub*idempotency)

Fuzzy union u(..):
1. reduction to the crisp set operation with dichotomous membership {0,1};
2.boundary condition, $u(a,0)=a$, $u(a,1)=1$;
3. monotonicity, if $a'\leq a$ then $u(a',b)\leq u(a,b)$; if $a'<a$ and $b'<b$, then $u(a',b')<u(a,b)$;
4. cumutativity, associativity, and continuity as above;
5A. $u(a,a)=a$ (idempotency), <u>or</u> 5B. $u(a,a)>a$ (*super*idempotency)

<u>Standard fuzzy set operations</u>

The distinction between conditions 5A and 5B is important in our context. Operations satisfying 1-4 and 5A are termed 'standard', because they have certain special properties and are commonly used. These operations are

Standard fuzzy complement c_s(..):

$$c_S(a)=1-a=\bar{a}, \text{ say, } \quad \text{and} \quad c_S(b)=1-b=\bar{b},$$

being the degree to which the individual belongs to the (monetary) "NON-POOR" set, or alternatively, does not belong to the (monetary) "POOR" set.

Standard fuzzy intersection i_s(..):

$$i_S(a,b)=\min(a,b)=m_1, \text{ say,}$$

being the degree to which the individual is subject both to monetary poverty <u>and</u> to non-monetary deprivation.

Standard fuzzy union u_s(..):

$$u_S(a,b)=\max(a,b)=m_2, \text{ say,}$$

being the degree to which the individual is subject to either of the two form of deprivation, monetary or non-monetary (to either one, the other , or both).

The standard operations are commonly used in particular because these are the *only* ones which satisfy the intuitively and substantively desirable condition 5A (idempotency), namely

$$i_S(a,a)=\min(a,a)=a, \quad \text{and} \quad u_S(a,a)=\max(a,a)=a,$$

<u>Other options</u>

It happens that the standard operations defined above are in fact not the only possible and acceptable generalisations of the corresponding crisp set rules. Other options can be more appropriate (convenient, useful, illuminating) depending on the context and objectives of the application. Some important ones are listed in the table below. They all meet the basic requirement of reproducing the corresponding crisp set operation with dichotomous membership {0,1}, satisfy conditions 2-4, but condition 5 only in the form 5B (i.e. are not idempotent – $i(a,a)<a$, and $u(a,a)>a$).

But before proceeding, the reader may well ask: "Why bother with any options other than the standard operation?" The answer is that it is necessary to consider other options so as to clarify and improve our particular application to poverty and deprivation in a multi-dimensional and also a longitudinal context, as will be explained below in relation to the additional requirement of "additivity".

Some important points should be noted in relation to the above panel.

		intersection	union
S	Standard	$i_{max}(a,b) = \min(a,b) = m_1$	$u_{min}(a,b) = \max(a,b) = m_2$
A	Algebraic	$i(a,b) = a * b$	$u(a,b) = a + b - a * b$
B	Bounded	$i(a,b) = \max(0, a + b - 1)$	$u(a,b) = \min(1, a + b)$
D	Drastic	$i_{min}=m_1$ when $m_2=1$; =0 otherwise	$u_{max}=m_2$ when $m_2=0$; =1 otherwise
Example of parametric family covering the whole range			
Y	Yager, w=(0,∞)	$i_w(a,b) = 1 - \min\left(1, \left((1-a)^w + (1-b)^w\right)^{1/w}\right)$	$u_w(a,b) = \min\left(1, \left(a^w + b^w\right)^{1/w}\right)$

There is in fact a whole range of possible options between "D" and "S" above in the choice of the set operations:

$$i_{min} \le i(a,b) \le i_{max}; \qquad u_{max} \ge u(a,b) \ge u_{min} .$$

An example is provided in the last row, the so-called Yager's parametric family, w=(0,∞). We obtain form "D" for w=0, form "B" for w=1, and the standard form "S" for w →∞.

Permissible forms of the two operations, intersection and union, go in pairs: to be consistent, it is necessary to select the two from the same row of the table. This is necessary to satisfy the De Morgan laws of set operations

$$\overline{A \cap B} = \overline{A} \cup \overline{B}; \qquad \overline{A \cup B} = \overline{A} \cap \overline{B}$$

which in the fuzzy case can be written as

<u>Condition 6:</u>

$$c[i(a,b)] = u[c(a), c(b)]; \qquad c[u(a,b)] = i[c(a), c(b)].$$

Any of the above pairs is consistent not only with the standard definition of the complement, $c_S(a)=1-a$, but also with any complement which satisfies conditions 1-4 noted above, such as

$$c_w(a) = \left(1 - a^w\right)^{1/w},$$

termed the Yager's class of fuzzy complements by Klir and Yuan (1995). As noted, the standard operations are the only ones satisfying idempotency, $i_S(a,a)=u_S(a,a)=a$. For the algebraic operations for instance,

$$i(a,a) = a^2 \le a, \qquad u(a,a) = 2a - a^2 \ge a.$$

For our application, the most important implication of the above comments is that with the Standard fuzzy operations, i_S provides the *weakest* (the most loose or the largest) intersection among all the permitted forms (it is for this reason that it has been labelled as i_{max} in the table); all other forms give a smaller, or at least no larger, value for the intersection. By contrast, u_S provides the *strongest* (the most tight or the smallest) union among all the permitted forms (it is for this reason that it has been labelled as u_{min} in the table); all other forms give a larger, or at least no smaller, value for the union.

It is this factor which makes it inappropriate to use the Standard set operations uniformly throughout in our application under discussion. Figure 4.A below lists the various fuzzy sets of interest in the joint study of monetary poverty and non-monetary deprivation. With the simplified notation introduced earlier, $a=FM_j$, $b=FS_j$, and $m_1=\min(a,b)$ and $m_2=\max(a,b)$, the table lists the membership functions for the four possible subsets in terms of the joint incidence of monetary and non-monetary deprivation, using the Standard fuzzy set operations. It can be seen that because of 'expansive' nature of the standard intersection, the sum of the resulting membership functions for the four subsets exceeds 1.0.

Figure 4.A
Application of the Standard operations to all subsets representing combinations of monetary poverty and non-monetary deprivation.

monetary poverty, non-monetary deprivation	membership function	$\delta = 1-(a+b) = (\bar{a}-b) = (\bar{b}-a) \geq 0$				$\delta = 1-(a+b) = (\bar{a}-b) = (\bar{b}-a) < 0$			
		$a \leq b \leq 0.5$	$b \leq a \leq 0.5$	$a \leq 0.5$, $b > 0.5$	$b \leq 0.5$, $a > 0.5$	$a \leq 0.5$, $b > 0.5$	$b \leq 0.5$, $a > 0.5$	$a > 0.5$, $b \geq a$	$b > 0.5$, $a > b$
BOTH ('manifest' deprivation)	$\min(a,b)$	$a=m_1$	$b=m_1$	$a=m_1$	$b=m_1$	$a=m_1$	$b=m_1$	$a=m_1$	$b=m_1$
only monetary	$\min(a,\bar{b})$	$a=m_1$	$a=m_2$	$a=m_1$	$a=m_2$	$\bar{b}=1-m_2$	$\bar{b}=1-m_1$	$\bar{b}=1-m_2$	$\bar{b}=1-m_1$
only non-monetary	$\min(\bar{a},b)$	$b=m_2$	$b=m_1$	$b=m_2$	$b=m_1$	$\bar{a}=1-m_1$	$\bar{a}=1-m_2$	$\bar{a}=1-m_1$	$\bar{a}=1-m_2$
neither of the two	$\min(\bar{a},\bar{b})$	$\bar{b}=1-m_2$	$\bar{a}=1-m_2$	$\bar{b}=1-m_2$	$\bar{a}=1-m_2$	$\bar{b}=1-m_2$	$\bar{a}=1-m_2$	$\bar{b}=1-m_2$	$\bar{a}=1-m_2$
SUM	Σ	$1+2.m_1$	$1+2.m_1$	$1+2.m_1$	$1+2.m_1$	$1+2.(1-m_2)$	$1+2.(1-m_2)$	$1+2.(1-m_2)$	$1+2.(1-m_2)$
EITHER of monetary or non-monetary - one, the other , or both ('latent' deprivation) $u(a,b) = \max(a,b) = c(\min(\bar{a},\bar{b})) = 1-\min(\bar{a},\bar{b}) = m_2$									

However, this is in conflict with substantive requirements of our situation in the following sense. In the conventional analysis, the population is similarly divided into four crisp sets (exhaustive and non-overlapping classes) according to joint incidence of monetary/non-monetary deprivation (yes-yes, yes-no, no-yes, no-no); and by definition, the proportions in the four groups must sum to 1. This should be true with fuzzy sets as well, since it is precisely these proportions that we wish to estimate and compare with the conventional analysis.[10] This substantive requirement may be specified as "condition 7", as follows.

Condition 7:

If a set of fuzzy membership functions are to reflect exhaustive and non-overlapping categories of the conventional (crisp) formulation, then at the individual level (or at least averaged over individuals), the fuzzy membership functions should add up to 1.

In our application, we are primarily interested in two of the sets in the table, which under the Standard operations are

Manifest Deprivation = $i_S(a,b) = \min(a,b) = m_1$

Latent Deprivation = $u_S(a,b) = \max(a,b) = m_2$
or alternatively seen as $c_S\left(i_S\left(\bar{a},\bar{b}\right)\right) = 1 - i_S\left(\bar{a},\bar{b}\right) = 1 - \min\left(\bar{a},\bar{b}\right)$.

Since the Standard operations provide maximal estimates for both the intersections in the above equations, we have a *maximal* estimate for Manifest Deprivation, and a *minimal* for Latent Deprivation. One can (and we do) argue that on substantive grounds, the above is a reasonable (indeed desirable) choice for intersections of 'similar' states. However, then the form of the other intersections in the table $\left\langle\left(\bar{a},b\right),\left(a,\bar{b}\right)\right\rangle$ need to be modified in view of "condition 7" above. Note that these are intersections of 'conflicting' states.

Now it can be seen that the Algebraic form "A", applied to all the four intersections, is the *only one* which meets this condition, as shown in Figure 4.B below.[11] But despite this numerical consistency, we *do not* regards this form to give results which, for our particular application, appear generally acceptable on intuitive or substantive grounds. For instance, for an individual with propensity to monetary poverty FM_j=0.5, and propensity to non-monetary deprivation FS_j=0.4, the resulting Manifest Deprivation would be only 0.5*0.4=0.2 under this rule, while the Latent Deprivation would be as high as 1-(1-0.5)*(1-0.4)=0.7. This indeed is "over-correction" to the apparently much more reasonable result with the Standard operations (0.4 and 0.5 respectively). The same pattern can be seen by considering other values of the membership functions (FM_j,FS_j).[12]

The following is a possible reason for uniform application of the Algebraic rule failing to give reasonable results in our application. If we take the liberty of viewing the fuzzy propensities as probabilities, then the algebraic product rule i(a,b)→joint probability(a,b)=a*b implies zero correlation between the two forms of deprivation, which is clearly at variance with the high positive correlation we expect in the real situation for similar states. The rule therefore seems to provide *unrealistically low* estimates for the resulting membership function; the Standard rules (S), giving higher overlaps (intersections) are more realistic.

---

[10] Of course, when the membership functions are dichotomous {0,1}, the fuzzy sets do produce the corresponding conventional results. The issue here is when the membership functions are fuzzy [0,1].

[11] The SUM will exceed 1 as we move towards the Standard operations, and fall below 1 as we move the other way, towards the Bonded operations.

[12] Such numerical results appear even more striking when we apply the procedure described here to the study of persistence or otherwise of poverty and deprivation, as considered in a later section.

Though in this paper we do not directly deal with the other two sets $\langle(\bar{a},b),(a,\bar{b})\rangle$, the lack of overlap between the two dimensions of deprivation, it appears that here the Algebraic rule (and hence also the Standard rules) tend to give *unrealistically high* estimates for the resulting membership functions. The reasoning similar to the above applies: in the real situation, we expect large negative correlations (hence reduced intersections) between *conflicting* states in the two dimensions of deprivation.

Figure 4.B Application of the Algebraic and Composite set operations.

monetary poverty, non-monetary deprivation	Algebraic set operations	Composite set operations		$a \le b$	$a > b$
both	ab	Standard	$\min(a,b)$	m_1	m_1
only monetary	$a.\bar{b} = a.(1-b)$	Bounded	$\max(0, a+\bar{b}-1) = \max(0, a-b)$	0	m_2-m_1
only non-monetary	$\bar{a}.b = b.(1-a)$	Bounded	$\max(0, \bar{a}+b-1) = \max(0, b-a)$	m_2-m_1	0
neither of the two	$\bar{a}.\bar{b} = (1-a).(1-b)$	Standard	$\min(\bar{a},\bar{b})$	1-m_2	1-m_2
SUM, Σ	1			1	1
Either of monetary or non-monetary (one, the other , or both) – standard set operations: $u(a,b) = \max(a,b) = c(\min(\bar{a},\bar{b})) = 1-\min(\bar{a},\bar{b}) = m_2$					

The Standard complement, $c_S(a)$=1-a, is used throughout.

In view of the above, we are studying the following options.

<u>Algebraic set operations</u>

These meet "condition 7" on consistency, but their limitations in the context of our present application have already been noted.

<u>Composite set operations</u>

This consists of applying two different types of operations depending on the nature of the fuzzy sets under consideration, as detailed in Figure 4.B.

(1) For sets representing 'similar states', such as the presence (or absence) of both types of deprivation, the Standard operations (which provide less restrictive intersections than Algebraic operations) are used.

(2) For sets representing 'conflicting states', such as the presence one type but the absence of the other type of deprivation, the Bounded operations (which provide more restrictive intersections than Algebraic operations) are used.

As seen in Figure 4.B, these two together perfectly meet "condition 7" on consistency.

We consider this to be a more reasonable choice than uniform application of the Algebraic operations. In our application, we have considered measures (Manifest and Latent deprivation) which fall under (1), and in the numerical illustrations presented here, the Standard operations have been applied. This implies that, for consistency, type (2) must be applied in the construction membership functions of "disjoint deprivation" (present in one but not the other dimension). See below for a graphical representation of the such an operational form.

A possible refinement

If the intersections provide by (1) are considered a little too weak and by (2) a little too strong, then these can be adjusted by introducing an element of the Algebraic form into each. A weighted combination, for instance in the form

(1) For sets representing 'similar states', **w.(Standard) + (1-w).(Algebraic)**

(2) For sets representing 'conflicting states', **w.(Bounded) + (1-w).(Algebraic)**

would make the type of adjustment sought, still meeting "condition 7" on consistency, since each of the two weighted components in the combination (1+2) meets this condition. The choice of weight w<1 is a matter of substantive judgement and empirical reality in the particular context. Here w can be expected to be close to 1.0, we think. Parameter w can be thought of a measure of degree to which different states can be distinguished as 'similar' or 'conflicting'. When the distinction is ambiguous (as between the presence and absence of deprivation in different dimensions in our application), w=1. When the categories are neutral with no such distinction, w=0, i.i. a uniform application of the Algebraic rules throughout would be appreciated. This proposal is speculative at this stage and needs testing, of course.

A graphical representation of the fuzzy set operational forms

To elucidate these fuzzy set operational forms, which are central to our methodology, we have developed the following graphical representation. In Figure 5 the "universal set" X (i.e., membership≡1 for any element of the population of interest) is represented by a rectangle of unit length, and within it the membership functions ($0 \le a \le 1$, $0 \le b \le 1$) on two subsets of any element are placed. Different placements correspond to different types of fuzzy set operations.

In the Standard operations, the two similar subsets (a,b) are placed on the same base, so that the smaller (say b) lies completely within the larger (say a). Consequently, their intersection is maximised, to equal the smaller of the subsets. By the same token, their union is minimised, to equal the larger of the subsets. The union is represented in the second figure; it shows separately the amount (a-b, in this case) by which it exceeds the intersection. By placing one set higher than the other within X, the overlap (intersection) is generally reduced, and the underlay (union) generally increased.

In the Algebraic form, set (b) is placed symmetrically over sets (a) and (non-a), i.e. each of the two receiving (b) in proportion to its size, respectively a*b and (1-a)*b. Hence a*b is the overlap (intersection), while the underlay (union) is [a+(1-a)*b]=[a+b-a*b].

In the Bounded form, appropriate for dissimilar sets, set (b) is placed at the opposite end of X, thus further reducing the intersection to (a+b-1) (which is non-zero only if a+b>1); and increasing the union to (a+b), or to 1 if a+b>1.

Representation beyond this form (up to the limits i_{min} and u_{max} introduced earlier) appears possible, but less elegant, involving sliding and stretching the (a,b) to outside X boundaries.

Figure 6 shows graphically the Composite set operations we have suggested and used in part. Similar states (poor/deprived, and non-poor/non-deprived) are treated using the Standard operations. That is, they are placed on the same base, thus maximizing their intersection and minimizing their union. This is reflective of the positive correlation between similar states in reality. Note that it is only the part which is not covered by the sum of the two intersections of the 'similar states' (namely, a-b in this example) which is available as the total of intersections for the two dissimilar states (poor/non-deprived, and non-poor/deprived): this is required by "condition 7" concerning consistency. Sets in a pair representing dissimilar states are placed at opposite ends, resulting in the Bounded form, with appropriately reduced intersection and increased union so as to meet the consistency condition. Note that membership function of one of those sets in pair is always equal to zero. Finally, it may be noted that all these different forms reduce to exactly the same form for corresponding crisp sets with dichotomous {0,1} membership. For non-empty subsets, at least one of membership functions equals 1, i.e., cover entire X, so that it makes no difference as to where the other membership function is placed within X.

Figure 5. Graphical representation of the fuzzy set operations

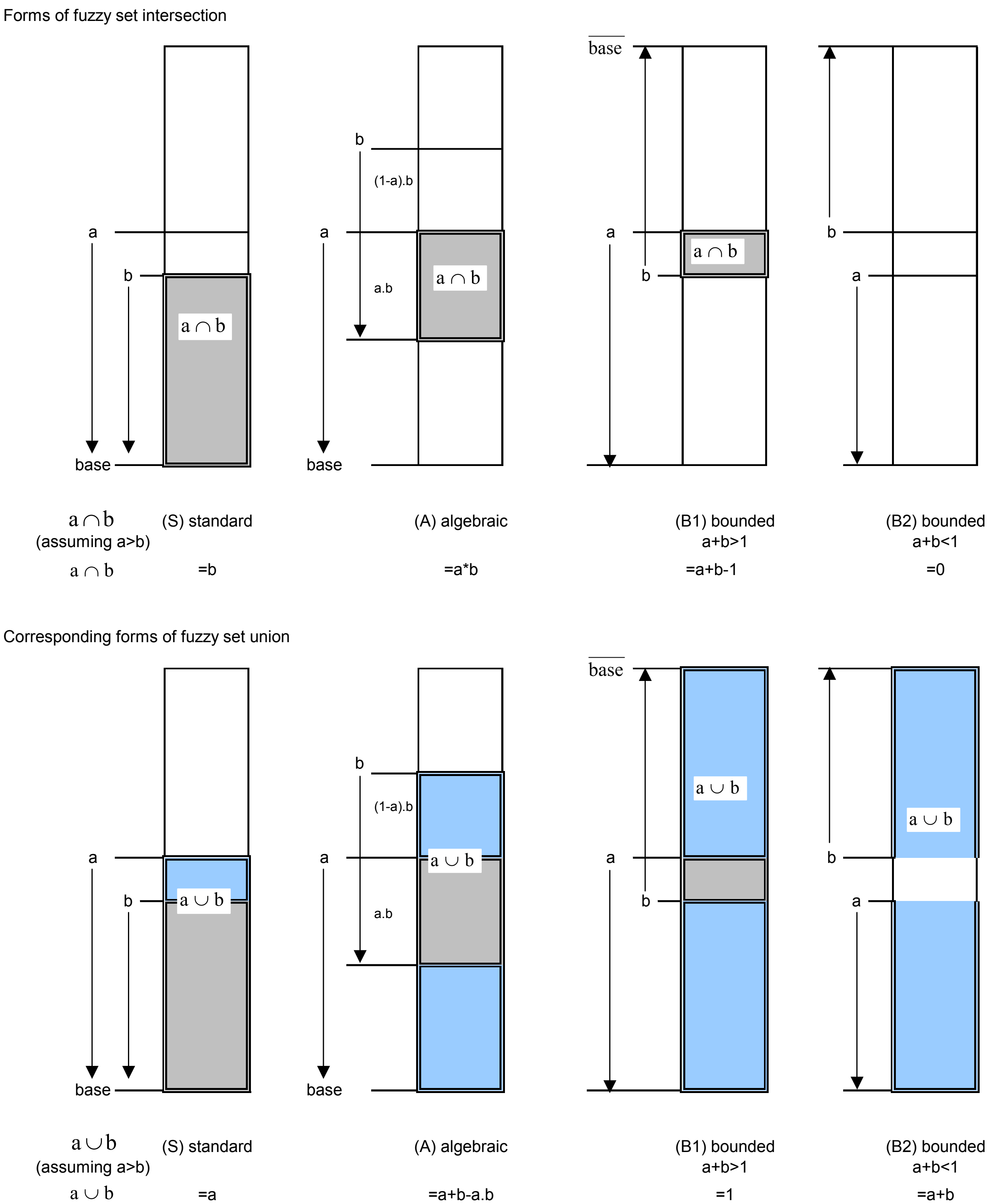
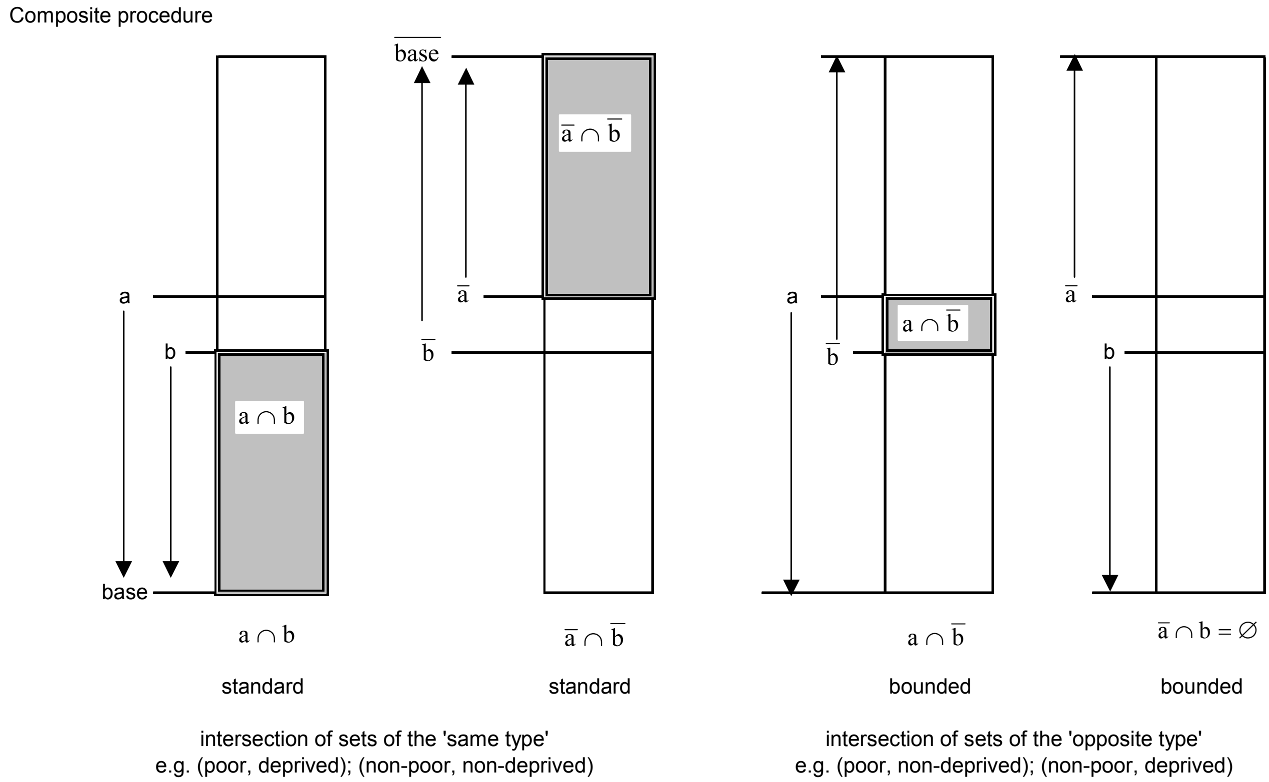


Figure 6. Graphical representation of the proposed system



4.2 Application of the Composite set operations

Once the propensities to income poverty (FM_j) and life-style deprivation (FS_j) have been defined at the individual level (j), the corresponding combined measures are obtained in a straightforward way, using the Composite set operations proposed in the previous section, which can then be aggregated to produce the relevant averages and rates for the population. The 'manifest' deprivation propensity of individual *j* is the intersection (the smaller) of the two measures FM_j and FS_j:

$$M_j = \min\left(FM_j, \ FS_j \right).$$

Similarly, the 'latent' deprivation propensity of individual *j* is the union (the larger) of the two measures FM_j and FS_j:

$$L_j = \max\left(FM_j, \ FS_j \right).$$

Figure 7 shows graphically the latent and Manifest deprivation measures compared with to the traditional view.

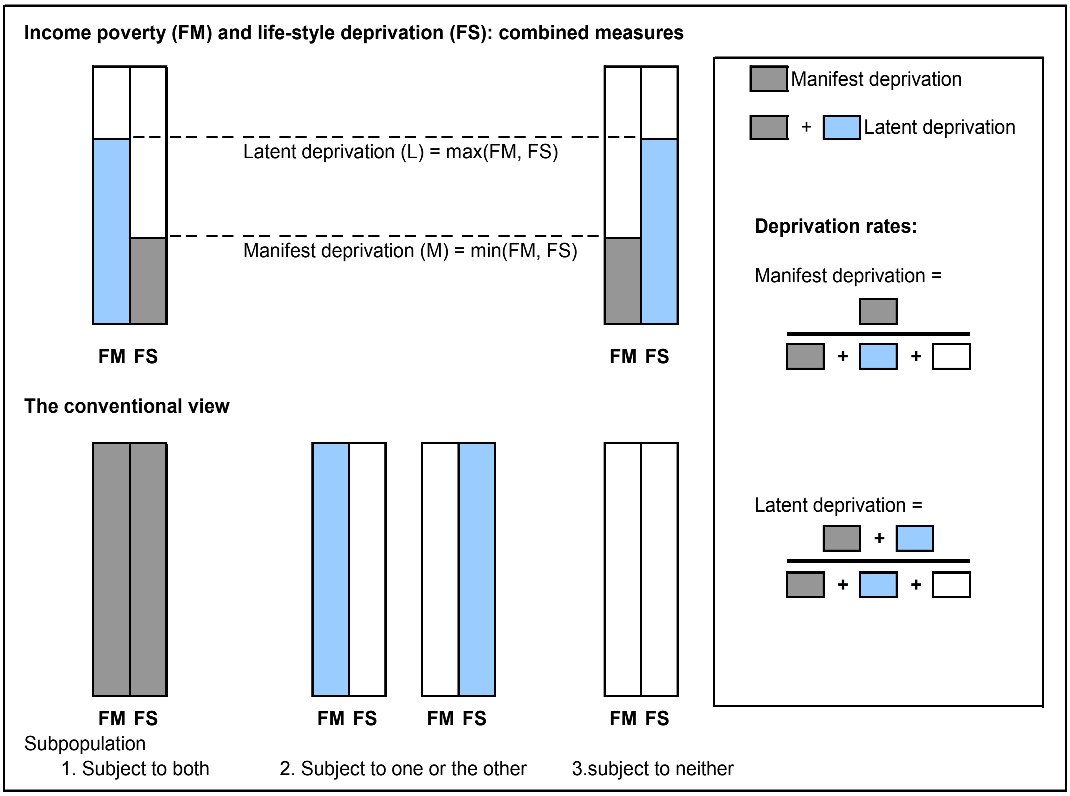
These measures are shown in Tables 3 and 4 (given in Sections 3.3 and 3.4 above) in the same form as those for income poverty and non-monetary deprivation separately in the previous tables.[13]

National differentials in the *latent* deprivation (presence of either form of deprivation) are smaller than those in income poverty or in life-style deprivation taken alone. This is because in countries with lower levels of disparities overall, different dimension of deprivation tend to be less overlapping over

[13] As noted earlier, these two tables use somewhat different data sets and procedures.

the same individuals, than in countries with higher levels of disparities among their populations. For the same reason, national differentials in *manifest* deprivation (joint presence of both forms) tend to be larger than those in either form taken alone. *The high level of overlap between different dimension of deprivation over the same individuals in the already more unequal situation in countries makes the impact of these disparities even more extreme.*

Figure 7. Latent and manifest deprivation:
the present conceptualisation compared with the traditional view.



Averaged over countries the picture is as follows. Around a third of the population subject to either form (i.e., to 'latent') deprivation are in fact subject to both forms of (i.e., to 'manifest') deprivation. Somewhat under 50% of those in monetary poverty are also subject to non-monetary deprivation, and the same applies in reverse.

5. Longitudinal aspects: persistence of poverty and deprivation

5.1 Introduction

Diverse measures

Above we have described five main measures which have been developed and analysed in this paper. These are:

H_j the conventional income poverty index (0,1)

FM_j the propensity to income poverty (continuous in the range 0-1)

FS_j the propensity to life-style deprivation

M_j manifest deprivation, representing the propensity to both income poverty and life-style deprivation simultaneously.

L_j latent deprivation, representing the individual's propensity to being subject to at least one of the two, income poverty and/or life-style deprivation.

In addition, the propensity to life-style deprivation can be analysed separately in its various dimensions, such as the five dimensions (Sup1-Sup5) identified as earlier. Then in principle there are also measures corresponding to FS, M and L in the conventional dichotomous {0,1} form.

Any of these measures can be studied in the time dimension: both in the cross-sectional and the longitudinal contexts. The cross-sectional may refer to levels over single years or to averages over a number (T) of years. In the longitudinal dimension, indicators may be designed to capture the experience of poverty and deprivation at any time during a period, or persistently or continuously over the period. We can also construct individual propensities and average rates of exit and of re-entry into the state of poverty and deprivation, and the distribution of the time spent in such state. And so on.

Longitudinal analysis over two time periods

The methodology for longitudinal analysis of a deprivation measure over two consecutive time periods is, formally and in statistical term, the same as that for cross-sectional analysis of two different measures over a single time period. Here instead of considering the fuzzy sets of monetary poverty and non-monetary deprivation, we consider any kind of fuzzy poverty measured at two consecutive periods, say period (1) and period (2). So if for instance we consider the Fuzzy Monetary measure, we can define $a=FM^{(1)}$ and $b=FM^{(2)}$ as its membership function of being POOR at the two times, and the corresponding NON-POOR complements, thus giving four membership functions.

The first attempt to define joint membership functions over two periods was in Cheli (1995) for what has been called the Totally Fuzzy and Relative (TFR) poverty measures. The four membership functions were taken in the form $g_{k,l} = \min(a, b)$, which corresponds to the Standard fuzzy set operations, but applied so as to meet the meet the marginal constraints (which amount to "Condition 7" described above in the previous section). The actual procedure involved applying the standard procedure independently to any of the four sets, and then determining the remaining three from the marginal constraints. In order to choose one of the four alternative solution (determined by which of the four sets is chosen as the starting point) the author followed Manton *et al.* (1992) in choosing the solution that produced the joint membership function matrix with *minimum entropy*. Later Betti *et al.* (2004) showed that in fact there are only two possible outcomes: starting from either of the two similar sets, ($poor_{(1)}$--$poor_{(2)}$) or (non-$poor_{(1)}$--non-$poor_{(2)}$), produces the same result; a different result is obtained by starting from either of the two dissimilar sets, ($poor_{(1)}$--non-$poor_{(2)}$) or (non-$poor_{(1)}$--$poor_{(2)}$). In any case, an approach like the above can be criticised on two counts. Firstly, it makes no distinction between similar and dissimilar states, disregarding the large positive correlation which can be expected between the former and the large negative correlation between the latter. Secondly, using the minimum entropy actually results in a discontinuity, with a sudden shift in the outcome when the poverty membership functions for the two periods pass from being concordant (both greater than or both less than 0.5) to being discordant (one <0.5 and the other >0.5). Such a discontinuity is not meaningful in any real situation.

The "Composite" solution proposed in the previous section of course explains, and provides a solution, to this problem. In this the similar sets, ($poor_{(1)}$--$poor_{(2)}$) and (non-$poor_{(1)}$--non-$poor_{(2)}$), are treated with the Standard set operations, and dissimilar sets, ($poor_{(1)}$--non-$poor_{(2)}$) and (non-$poor_{(1)}$--$poor_{(2)}$), with the Bounded set operations. The justification for this choice has already been given in Section 4. The procedures described in the previous section for the joint analysis of *cross-sectional* measures of monetary and non-monetary forms of deprivation in fact also apply to *longitudinal analysis over two time periods*.

Longitudinal analysis in general, possible involving more than two periods

Based on our earlier exploration of longitudinal aspects of fuzzy poverty measures (Verma and Betti, 2002), here we try to generalise the concept of persistent, any-time, transient poverty and deprivation by means of the concepts of intersection and union of fuzzy sets. It must be emphasised that *the procedures described below are not definitive at this stage*. They may be further developed or revised; however, they have been applied to obtain results which appear plausible and informative.

We believe that the procedures described in the previous section for the joint analysis of cross-sectional measures can be adapted for application apply to longitudinal analysis over *more than two time periods*. This can be done for either form deprivation or for both forms in combination. In the following we develop diverse measures in the time dimension. The concepts apply to any of the above measures (H, FM, FS, M, L, etc.), and the symbol P is used to represents any of these. Thus:

$P_{t,j}$ = the propensity to poverty (deprivation) at time t of individual j, over some interval t=1 to T

It is also useful to introduce the notation

$P_{(t),j}$ = the ordered set corresponding to the above, such that
$P_{(1),j} \geq P_{(2),j} \geq .... \geq P_{(T),j}$.

Cross-sectional rates

$$\overline{P}_t = \frac{\Sigma_j w_j . P_{t,j}}{\Sigma_j w_j}, \text{ t = 1 to T}$$

where w_j = the sample weight of individual j.

5.2 Longitudinal rates over period t = 1 to T

Consider a panel of individuals (j) over a period (t=1 to T) years, with $P_{t,j}$ the propensity to poverty or deprivation at time t of individual j. In the conventional analysis, $P_{t,j}$ takes the dichotomous values 1 (=poor) and 0 (=non-poor). Here the measure varies in the range [0,1] determined by the level and position of the individual in the income distribution, as defined above.

Any-time poverty

The individual's propensity to 'any-time poverty' (i.e., poverty during at least one year over the interval) is given by the largest of the cross-sectional indices:

$$P_{(1),j} = \max\left(P_{t,j}\right), \text{ t = 1 to T,}$$

the corresponding rate for the population being

$$\overline{P}^{(A)} = \frac{\Sigma_j w_j . P_{(1),j}}{\Sigma_j w_j} .$$

Persistent poverty

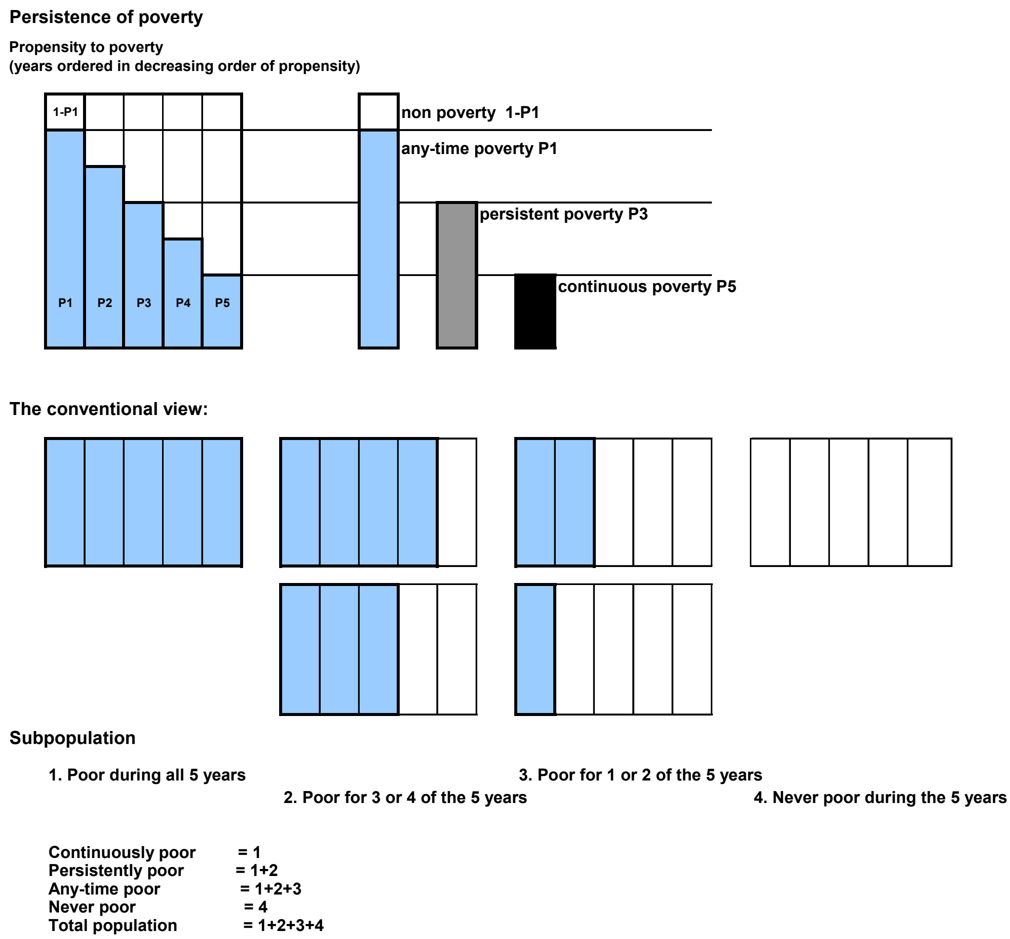
We adopt the following definition of persistent poverty for the numerical results presented here. It refers to poverty during *at least a majority* of the T years, i.e. for at least T' years, where

T' = int(T/2) + 1 (i.e. the smallest integer strictly larger than T/2).

For instance, for a 2 or 3 year interval, persistent refers to poverty for at least 2 years; for T=4 or 5 years, it refers to poverty for at least 3 years, etc.[14] At the individual level, this is the Tth largest value of the annual propensities to poverty, i.e. $P_{(T'),j}$. The corresponding persistent poverty rate is

$$\overline{P}^{(P)} = \frac{\Sigma_j w_j . P_{(T'),j}}{\Sigma_j w_j} .$$

Figure 8. Any-time, persistent and continuous poverty/deprivation: Fuzzy conceptualisation compared with the conventional view



Continuous poverty

The individual's propensity to continuous poverty (i.e., for all the years over the interval) is the smallest of the cross-sectional indices:

$$P_{(T),j} = \min\left(P_{t,j}\right), \text{ t = 1 to T,}$$

the corresponding rate for the population being

$$\overline{P}^{(C)} = \frac{\Sigma_j w_j . P_{(T),j}}{\Sigma_j w_j} .$$

[14] Eurostat recommends that for a period of T=4 years, persistent poverty be defined as poverty for at least 3 of the 4 years, *including the last year*. Except for the additional condition in italics (which is not consistent with the idea of an index covering a whole interval), the above definition is in line with the Eurostat recommendation.

Tables 5 shows longitudinal monetary poverty rates, comparing the conventional and fuzzy approaches. By definition, cross-sectional conventional and fuzzy rates are scaled to be the same at EU-15 level, but differences at the national level are also very small. But note that conventional rates for any-time poverty are on the average around 5% higher than the corresponding fuzzy rates, while rates of continuous poverty (over 5 years) are around 20% lower. This may be because of the conventional approach being excessively sensitive to fluctuations close to the selected poverty line; in part it may also result from methodological differences between the two approaches, which requires further examination. The two types of measures are close for persistent poverty.

Table 5
Longitudinal income poverty rates

		Conventional monetary poverty rates				Fuzzy monetary poverty rates				ratio Conventional/Fuzzy			
		(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
Denmark	DK	8,2	22,6	5,1	1,2	8,4	21,1	5,8	1,7	0,98	1,07	0,87	0,70
Netherlands	NL	9,9	22,1	8,2	2,3	10,2	21,5	8,4	2,9	0,98	1,03	0,97	0,77
Germany	DE	11,9	26,0	9,9	2,9	12,8	25,6	10,7	4,4	0,93	1,02	0,92	0,65
Belgium	BE	18,2	36,3	16,9	4,0	17,5	32,9	16,4	5,2	1,04	1,11	1,03	0,77
France	FR	17,6	31,6	15,7	8,0	16,9	29,5	15,3	7,8	1,04	1,07	1,03	1,02
Ireland	IE	19,1	36,1	17,6	5,4	19,2	33,6	17,7	7,4	1,00	1,07	0,99	0,73
Spain	ES	19,6	40,0	17,0	4,8	19,5	37,0	17,4	6,2	1,00	1,08	0,98	0,77
UK	UK	19,3	37,9	17,3	5,6	19,5	36,0	17,8	7,0	0,99	1,05	0,97	0,81
Italy	IT	19,4	38,0	16,5	5,6	19,6	36,2	17,6	7,3	0,99	1,05	0,94	0,77
Portugal	PT	23,4	41,3	21,8	10,1	23,4	38,8	22,0	10,8	1,00	1,06	0,99	0,94
Greece	GR	21,7	41,5	19,7	6,6	21,5	39,0	19,5	8,2	1,01	1,06	1,01	0,81
simple average		17,1	33,9	15,0	5,1	17,1	31,9	15,3	6,3	1,00	1,06	0,98	0,82

(1) cross-sectional (averaged over balanced panel for ECHP waves 1-5)
(2) any-time (one or more years during the 5 years)
(3) persistent (for 3 or more years during the 5 years)
(4) continuous (over all the 5 years)

Table 6 shows longitudinal measures for non-monetary indicators, and for latent and manifest poverty indicators which combine the monetary and non-monetary dimensions.

Table 6
Longitudinal non-monetary, latent and manifest deprivation rates

		Non-monetary deprivation rates				Latent deprivation rates				Manifest deprivation rates			
		(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
Denmark	DK	8.9	17.8	7.7	2.7	15.0	31.3	12.6	4.7	2.3	6.1	1.3	0.3
Netherlands	NL	9.2	16.9	8.3	3.2	15.9	29.9	14.0	6.1	3.5	7.1	3.0	1.0
Germany	DE												
Belgium	BE	12.9	23.4	11.7	4.9	24.1	41.3	22.6	10.2	6.3	13.0	5.4	1.6
France	FR	15.0	25.6	14.3	6.5	24.4	39.5	22.9	12.3	7.6	13.3	7.0	3.2
Ireland	IE	16.0	28.0	15.0	6.4	26.2	43.1	24.8	12.3	9.0	16.1	8.5	3.1
Spain	ES	19.7	32.9	18.9	8.2	30.0	49.1	28.3	13.8	9.2	17.5	8.2	2.7
UK	UK												
Italy	IT	15.2	26.3	14.1	6.1	26.9	44.9	24.9	12.8	7.9	15.1	7.1	2.7
Portugal	PT	31.6	46.4	31.6	16.6	40.7	58.2	40.6	23.1	14.4	24.8	13.4	5.7
Greece	GR	25.6	41.1	24.9	12.2	35.7	55.4	34.6	18.5	11.4	21.6	10.4	3.7
simple average		17.1	28.7	16.3	7.4	26.5	43.6	25.0	12.6	7.9	15.0	7.1	2.7

The following subsections describes how the various dynamic measures of individuals' movements into and out of poverty, of spells and durations in the state of poverty, etc., can be constructed, even though poverty is treated as a [0,1] propensity rather than simply a "yes-no" state. The more familiar conventional measures are seen only as a special case of the more general formulation below.

5.3 Exits from and re-entries to the state of poverty

Given the state of poverty at time t=1, the objective is to estimate the rates of exit from and re-entry into that state in the following years t=2, 3, 4, …. With

$P_{t,j}$ = propensity to poverty at time t of individual j
$\mathbf{p_{t,j}}$ = min($P_{t,j}$, $P_{1,j}$), being the propensity to poverty at both times 1 and t.

Given poverty at time 1, the individual's propensity at time t to exit from poverty at time (t-1) is

$$E_{t,j} = \max\left(0,\ \left(p_{t-1,j} - p_{t,j}\right)\right).$$

The corresponding "population at risk" is $p_{t-1,j}$, giving the exits rate at time t as:

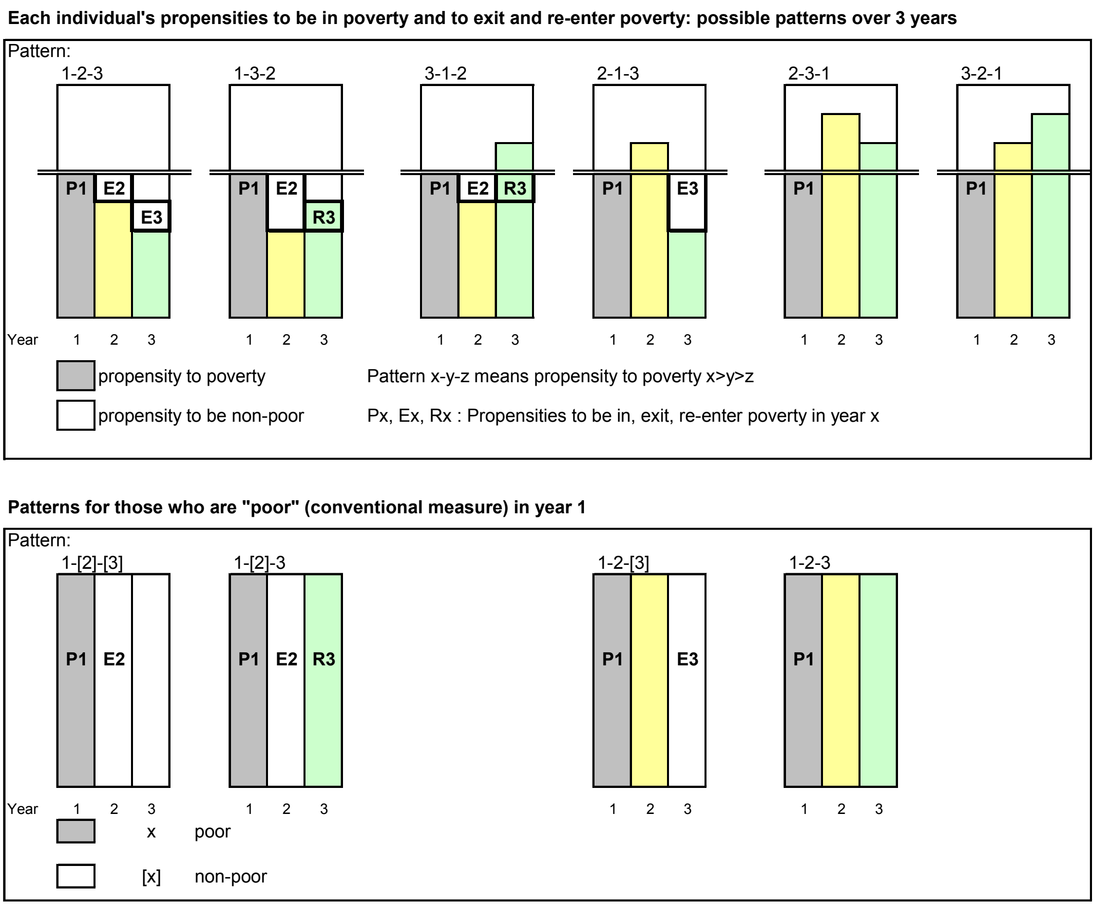
$$e_{t,j} = \frac{\Sigma_j w_j . E_{t,j}}{\Sigma_j w_j . p_{t-1,j}},$$

where w_j is the sample weight of individual j.

Similarly, given poverty at time 1 but having exited from it by time (t-1), the individual's propensity to re-enter poverty at time t is

$$R_{t,j} = \max\left(0,\ \left(p_{t,j} - p_{t-1,j}\right)\right).$$

Figure 9. Graphical representation of individual propensities to exit and re-enter the state of poverty and deprivation



The corresponding "population at risk" is that which has escaped by time (t-1) from poverty at time 1, i.e.,

$p_{1,j}$-$p_{t-1,j}$, giving the re-entry rate at time t as:

$$r_{t,j} = \frac{\Sigma_j w_j . R_{t,j}}{\Sigma_j w_j . \left(p_{1,j} - p_{t-1,j}\right)}.$$

From propensity to poverty at time 1, the gross exit rate over the time t=2 to T is

$$e_T^{(G)} = \frac{\Sigma_j w_j.\left(\Sigma_t E_{t,j}\right)}{\Sigma_j w_j.p_{1,j}}.$$

This measures the gross volume of exits experienced, even if some are followed by subsequent re-entry into poverty. Similarly, the gross re-entry rate over the time t=3 to T is

$$r_T^{(G)} = \frac{\Sigma_j w_j.\left(\Sigma_t R_{t,j}\right)}{\Sigma_j w_j.p_{1,j}}.$$

The difference between the above two is the net exit rate over the interval:

$$e_T^{(N)} = \left(e_T^{(G)} - r_T^{(G)}\right) = 1 - \frac{\Sigma_j w_j.p_{T,j}}{\Sigma_j w_j.p_{1,j}}.$$

Tables 7-9 show exit and re-entry rates in relation to the state of conventional poverty, fuzzy monetary poverty, and non-monetary deprivation. Similar measures can be constructed for combined monetary and non-monetary measures, and compared using the conventional and fuzzy approaches.

Table 7
Of poor in year 1, exits and re-entries into poverty over 5 years (conventional monetary poverty measure)

Of poor in year 1:		(2) Exit in yr 2	Of (2) re-enter in yr 3	(3) Exit in yr 3	Of (3) re-enter in yr 4	(4) Exit in yr 4	Of (4) re-enter in yr 5	(5) Exit in yr 5	Gross rate over 5 year Exit	Gross rate over 5 year Re-entry	net Exit rate
Denmark	DK	0.42	0.27	0.45	0.25	0.40	0.24	0.29	0.97	0.40	0.57
Netherlands	NL	0.41	0.36	0.25	0.19	0.33	0.23	0.35	0.92	0.35	0.57
Germany	DE	0.44	0.20	0.32	0.14	0.34	0.15	0.29	0.89	0.25	0.64
Belgium	BE	0.41	0.39	0.33	0.18	0.35	0.29	0.39	0.97	0.40	0.57
France	FR	0.29	0.18	0.17	0.25	0.22	0.22	0.12	0.62	0.23	0.39
Ireland	IE	0.24	0.28	0.25	0.34	0.28	0.26	0.39	0.84	0.30	0.54
Spain	ES	0.41	0.40	0.37	0.26	0.34	0.27	0.30	0.95	0.42	0.53
UK	UK	0.35	0.25	0.27	0.25	0.20	0.21	0.26	0.78	0.29	0.49
Italy	IT	0.45	0.30	0.26	0.21	0.27	0.29	0.26	0.87	0.38	0.49
Portugal	PT	0.24	0.28	0.23	0.28	0.14	0.20	0.28	0.69	0.23	0.46
Greece	GR	0.40	0.25	0.28	0.34	0.18	0.21	0.26	0.82	0.34	0.48
simple average		0.37	0.29	0.29	0.25	0.28	0.23	0.29	0.85	0.33	0.52

Table 8
Of poor in year 1, exits and re-entries into poverty over 5 years (fuzzy monetary poverty measure)

Of poor in year 1*:		(2) Exit in yr 2	Of (2) re-enter in yr 3	(3) Exit in yr 3	Of (3) re-enter in yr 4	(4) Exit in yr 4	Of (4) re-enter in yr 5	(5) Exit in yr 5	Gross rate over 5 year Exit	Gross rate over 5 year Re-entry	net Exit rate
Denmark	DK	0.38	0.24	0.40	0.24	0.37	0.27	0.34	0.94	0.38	0.56
Netherlands	NL	0.39	0.34	0.21	0.19	0.27	0.23	0.23	0.81	0.32	0.49
Germany	DE	0.38	0.19	0.25	0.15	0.25	0.19	0.20	0.76	0.24	0.52
Belgium	BE	0.37	0.35	0.25	0.19	0.28	0.35	0.35	0.88	0.38	0.50
France	FR	0.28	0.21	0.16	0.24	0.16	0.22	0.14	0.59	0.22	0.37
Ireland	IE	0.21	0.30	0.19	0.26	0.18	0.17	0.27	0.66	0.20	0.46
Spain	ES	0.37	0.37	0.31	0.29	0.26	0.25	0.27	0.86	0.37	0.49
UK	UK	0.31	0.27	0.23	0.28	0.17	0.23	0.21	0.71	0.28	0.43
Italy	IT	0.40	0.33	0.21	0.23	0.22	0.27	0.21	0.78	0.34	0.44
Portugal	PT	0.22	0.23	0.19	0.30	0.15	0.17	0.25	0.64	0.20	0.44
Greece	GR	0.35	0.28	0.22	0.34	0.16	0.24	0.23	0.74	0.32	0.42
simple average		0.33	0.28	0.24	0.25	0.22	0.23	0.25	0.76	0.30	0.47

* Strictly, this of course means all individuals, but weighted according to their fuzzy propensity to monetary poverty in year 1.

Table 9
Of those subject to non-monetary deprivation in year 1, exits and re-entries into poverty over 5 years

Of the deprived in y(		(2) Exit in yr 2	Of (2) re-enter in yr 3	(3) Exit in yr 3	Of (3) re-enter in yr 4	(4) Exit in yr 4	Of (4) re-enter in yr 5	(5) Exit in yr 5	Gross rate over 5 year Exit	Re-entry	net Exit rate
Denmark	DK	0.40	0.35	0.23	0.18	0.41	0.12	0.39	0.95	0.28	0.67
Netherlands	NL	0.35	0.31	0.21	0.29	0.23	0.11	0.41	0.87	0.26	0.61
Germany	DE										
Belgium	BE	0.28	0.31	0.23	0.31	0.23	0.14	0.35	0.80	0.26	0.55
France	FR	0.33	0.26	0.19	0.33	0.19	0.13	0.32	0.78	0.26	0.53
Ireland	IE	0.34	0.36	0.17	0.23	0.20	0.07	0.42	0.85	0.23	0.62
Spain	ES	0.29	0.39	0.19	0.33	0.20	0.11	0.40	0.82	0.26	0.57
UK	UK										
Italy	IT	0.31	0.30	0.22	0.30	0.17	0.11	0.36	0.80	0.24	0.55
Portugal	PT	0.19	0.34	0.12	0.29	0.10	0.07	0.35	0.64	0.15	0.49
Greece	GR	0.27	0.28	0.15	0.27	0.12	0.11	0.34	0.70	0.19	0.51
simple average		0.31	0.32	0.19	0.28	0.21	0.11	0.37	0.80	0.23	0.57

* Strictly, this of course means all individuals, but weighted according to their propensity to non-monetary deprivation in year 1.

5.4 Time spent in the state of poverty

With the conventional "poor/non-poor" dichotomy, any individual spends specific number of year between 0 and T in the state of poverty during an interval T. However, with poverty treated as a matter of degree, a single individual is seen as contributing to the *whole distribution*, from 0 to T, of the number of years spent in poverty. The individual's contribution to exactly t out of T years spent in poverty is

$$\Pi_{t,j} = P_{(t),j} - P_{(t+1),j} \text{ for t = 1 to T,}$$

where, as defined earlier, the propensities to poverty (P) refer to the ordered sequence

$$P_{(1),j} \geq P_{(2),j} \geq .... \geq P_{(T),j}, \text{ with } P_{(T+1),j} = 0 \text{ defined for convenience.}$$

The individual's contribution to zero years (never) in poverty is the remainder:

$$\Pi_{0,j} = 1 - \Sigma_t \Pi_{t,j} = 1 - P_{(1),j}, \text{ the sum being over t = 1 to T.}$$

Obviously, the total time spent by the individual in poverty during the T years is

$$T_j = \Sigma_t t.\Pi_{t,j} = \Sigma_t P_{t,j}.$$

From the above, various measures averaged over the population can be computed, such as the following.

<u>Distribution of the *population* according to the number of years in poverty</u>

$$\overline{\Pi}_t = \frac{\Sigma_j w_j.\Pi_{t,j}}{\Sigma_j w_j}, \quad \overline{\Pi}_0 = 1 - \Sigma_t \overline{\Pi}_t = 1 - \overline{P}^{(A)}, \text{ t = 1 to T.}$$

<u>Mean proportion of the time during T spent in poverty by the population:</u>

$$\overline{T} = \frac{1}{T}.\frac{\Sigma_j w_j.T_j}{\Sigma_j w_j}.$$

<u>Distribution of that *time* according to the number of years in poverty:</u>

$$\overline{T}_t = \frac{\Sigma_j w_j.(t.\Pi_{t,j})}{\Sigma_j w_j.T_j} = t.\left(\frac{\Sigma_j w_j.\Pi_{t,j}}{\Sigma_j w_j.T_j}\right).$$

It is instructive to note how the conventional "poor/non-poor" dichotomous approach is a special case of the above. In that approach, a person is poor during a specific number of years, say T_j in the range 0 to T. Only one of the $\Pi_{t,j}$ values equals 1, the rest being 0.

$$\Pi_{t,j} = 1 \ |T_j = t; \quad \Pi_{t,j} = 0 \ |T_j \neq t; \text{ for t = 0 to T.}$$

The weighted proportion of the population who spend exactly t years in poverty is

$$\overline{\Pi}_t = \frac{\Sigma_j w_j \ |T_j = t}{\Sigma_j w_j}.$$

The mean proportion of the time spent in poverty is as before:

$$\overline{T} = \frac{1}{T}.\frac{\Sigma_j w_j.T_j}{\Sigma_j w_j},$$

the distribution of that time according to the number of years in poverty being:

$$\overline{T}_t = \frac{\Sigma_j w_j.T_j \ |T_j = t}{\Sigma_j w_j.T_j}.$$

Tables 10-12 show some results on the number of years spent in the state of poverty or deprivation during the past 5 years, again in relation to the state of conventional poverty, fuzzy monetary poverty, and non-monetary deprivation. Similar measures can be constructed for combined monetary and non-monetary measures, and compared using the conventional and fuzzy approaches.

Table 10
Distribution of the population according to the number of years in poverty during past 5 years
Conventional monetary poverty measure

		zero	1 yr	2 yrs	3 yrs	4 yrs	5 yrs	mean years during the 5 yr period
Denmark	DK	77.4	13.1	4.4	2.2	1.6	1.2	0.41
Netherlands	NL	77.9	9.5	4.4	3.5	2.4	2.3	0.50
Germany	DE	74.0	11.1	5.1	4.0	3.1	2.9	0.60
Belgium	BE	63.7	13.3	6.1	6.0	6.9	4.0	0.91
France	FR	68.4	10.3	5.6	4.1	3.6	8.0	0.88
Ireland	IE	63.9	10.9	7.6	6.2	6.0	5.4	0.96
Spain	ES	60.0	13.4	9.5	7.2	5.0	4.8	0.98
UK	UK	62.1	12.8	7.9	7.0	4.7	5.6	0.96
Italy	IT	62.0	12.5	9.0	4.6	6.3	5.6	0.97
Portugal	PT	58.7	13.7	5.8	5.5	6.2	10.1	1.17
Greece	GR	58.5	13.9	7.9	6.3	6.8	6.6	1.09
simple average		66.1	12.2	6.7	5.1	4.8	5.1	0.86

Table 11
Distribution of the population according to the number of years in poverty during past 5 years
Fuzzy monetary poverty measure (FM)

		zero	1 yr	2 yrs	3 yrs	4 yrs	5 yrs	mean years during the 5 yr period
Denmark	DK	78.9	11.2	4.0	2.3	1.8	1.7	0.42
Netherlands	NL	78.5	8.9	4.2	2.9	2.5	2.9	0.51
Germany	DE	74.4	9.9	4.9	3.3	3.1	4.4	0.64
Belgium	BE	67.1	11.2	5.3	5.0	6.1	5.2	0.88
France	FR	70.5	9.4	4.9	3.5	3.9	7.8	0.84
Ireland	IE	66.4	9.5	6.4	4.7	5.6	7.4	0.96
Spain	ES	63.0	11.4	8.3	6.0	5.2	6.2	0.98
UK	UK	64.0	11.3	6.9	6.0	4.9	7.0	0.97
Italy	IT	63.8	11.8	6.9	5.0	5.4	7.3	0.98
Portugal	PT	61.2	10.1	6.6	5.3	6.0	10.8	1.17
Greece	GR	61.0	12.2	7.3	5.6	5.8	8.2	1.07
simple average		68.1	10.6	6.0	4.5	4.6	6.3	0.86

Table 12
Distribution of the population according to the number of years spent in non-monetary deprivation during past 5 yea
Fuzzy non-monetary deprivation measure (FS)

								mean years during the
Denmark	DK	82.2	6.4	3.7	2.8	2.2	2.7	0.44
Netherlands	NL	83.1	5.3	3.3	2.6	2.5	3.2	0.46
Germany	DE							
Belgium	BE	76.6	7.3	4.4	3.4	3.4	4.9	0.64
France	FR	74.4	7.2	4.1	4.0	3.8	6.5	0.75
Ireland	IE	72.0	8.2	4.8	4.3	4.3	6.4	0.80
Spain	ES	67.1	8.5	5.6	5.0	5.7	8.2	0.98
UK	UK							
Italy	IT	73.7	7.4	4.9	3.8	4.2	6.1	0.76
Portugal	PT	53.6	8.5	6.3	6.1	8.9	16.6	1.58
Greece	GR	58.9	10.1	6.1	5.9	6.8	12.2	1.28
simple average		71.3	7.7	4.8	4.2	4.7	7.4	0.86

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ANNEX
Data sources for cross-sectional and longitudinal analyses

The first tables (Tables 1-3) use more complete and up-to-date version of the data covering 7 ECHP waves, for survey years 1994-2000. Since these have been used here mostly for constructing cross-sectional measures, the full cross-sectional data sets have been used (i.e. not confined to a balanced panel only). As waves 1-3 do not include all EU-15 countries (missing countries being Austria wave 1, Finland waves 1-2, and Sweden waves 1-3; wave 1 could also not be used for Luxembourg), a simple procedure has been used to adjust the EU-15 figures for these waves. For Germany, the UK and Luxembourg, the results are based on the national panels which replaced the original ECHP samples in the countries as of 1998. Most of the cross-sectional longitudinal analyses have, however, been carried out on 'balanced panels' covering the five survey years 1994-98, comprising of individuals enumerated in the first five rounds of European Household Panel Survey, that is waves 1 to 5 (these analyses were conducted prior to the full set of 8 ECHP waves becoming available to us). For income distribution statistics, this has been possible in all the countries with the following exceptions in the above mentioned 5-years longitudinal set:

- Sweden and Finland: no data are presented for Sweden (where only one reconstructed survey, for 1997, was available), and Finland where only three years (ECHP surveys for 1996-98) were available.
- Austria: analysis could be carried out covering only four years 1995-98, since the ECHP began a year later than other countries.
- Luxembourg: analysis could be carried out covering only three years 1994-96, since the data at the time of longitudinal analysis were only available through 1996, based on the converted PSEL-1 which stopped in 1996, but not for PSEL-2 which replaced it.
- Germany and the UK: Income distribution statistics for the whole duration 1994-98 are based on the national panels which replaced the original ECHP samples in the countries as of 1998. The original ECHP samples for 1994-96 have not been used in these countries for income distribution analysis because they do not cover five years longitudinally. In any case the national panel replacing the ECHP do not collect sufficient information for constructing the life-style deprivation index.

The data are as provided by Eurostat for the purpose of research in ECHP Users' Data Base (UDB). These longitudinal data have been reweighted so as to agree with the distribution of the last wave included in the analysis on some basic characteristics, the latter being the sample weights provided in the data set.

Income

Each person is assigned the equivalised income of the person's household. The equivalised income of a household is obtained by dividing its total disposable income (as recorded in ECHP database, being the income in Purchasing Power Standard (PPS) for the calendar year preceding each wave of the survey) by the household's equivalised size. The equivalised size is obtained by using an equivalent scale which takes into account the actual size and composition of the household. In our empirical analysis here, we have used the Eurostat or so-called 'modified OECD' scale, which assigns a weight of 1 to the first adult in the household, 0.5 to each subsequent person aged 14 and over, and 0.3 to each child aged under 14 in the household.

Imputations of non-monetary deprivation items

Non-monetary deprivation items are not available for all households, thus had in part to be imputed. The imputation procedures have been based on the 'sequential regression multivariate imputation' (SRMI) approach adopted by the imputation software IVE-ware. The method builds the imputed values by fitting a sequence of regression models and drawing values from the corresponding predictive distribution, under the hypothesis of Missing at Random (MAR) mechanism, infinite sample size and simple random sampling (Raghunathan et al, 2001). All those households for which household income was available (~98.5 percent) *and* for which the number of missing non-monetary items was below 7 (~99.5 percent) could be imputed. As a result approximately two percent of households which were originally in the ECHP Users Data Base had to be excluded for the present analysis.